See the Bold-Shaddow *of* Vrania's Glory,
Immortall *in* His Race, *no lesse in* Story:
An Artist *without* Error, *from whose* Lyne,
Both Earth *and* Heav'ns, *in sweet* Proportions twine:
Behold Great EUCLID! *But, behold Him well!*
For 'tis in Him Divinity doth dwell.

G. Wharton.

EUCLID'S ELEMENTS OF Geometry.

In XV. Books:
With a supplement of divers *PROPOSITIONS* and *COROLLARIES.*
To which is added, a *Treatise* of REGULAR SOLIDS,
By *CAMPANE* and *FLUSSAS.*

LIKEWISE

Euclid's DATA:

And MARINUS his Preface thereunto annexed.

Also a Treatise *of the* Divisions *of* Superficies, *ascribed to* Machomet Bagdedine, *but published by* Commandine, *at the request of* John Dee *of* London; *whose* Preface *to the said* Treatise *declares it to be the* Worke *of* EUCLIDE, *the* Author *of these* ELEMENTS.

Published by the Care and Industry of
JOHN LEEKE and GEORGE SERLE, Students in the *MATHEMATICKS.*

LONDON:
Printed, by R. & W. LEYBOURN, for GEORGE SAWBRIDGE at the *Bible* upon *Ludgate-hill,*
M DC LXI.

TO THE

Duke of YORK.

May it please your HIGHNESSE:

BEfore a PRINCE of ſo approved Conduct, I ſhall not enumerate the many wayes that the MATHEMATICKS are Auxiliary in WAR; nor trouble you with ſuch Trite Stories to Court your belief thereof, as that of ARCHIMEDES; who ſitting in his Chair, with his *Nugæ Mathematicæ* (as he called them) threw multiplicity of Deaths amongſt the all-daring Legions of MARCELLUS: This Sir, is a taske needleſſe, and moſt injurious to your Highneſſe Merits; whoſe perfection in theſe

Sciences, could be only made questionable by such exaggerations.

No, great Sir, The sole reason for laying this Dedication: with so much presumption, at your Highnesse Feet, was our consciousness how particular an interest you have in the Prince of Mathematicians, whilst all the world admires you, as the most absolute Proficient both in *Mars*, and the *Muses* Schools.

'Tis EUCLIDE, Sir, that implores your Princely Patrociny, the vouchsafe whereof, as it will procure him the favour of all; so it will be a Glorious Shield, to dazle the eyes of such who would over-nicely censure the Lapses of his Interpretors, and

SIR,

Your Highnesses most humble and most obliged Servants,

JOHN LEEKE,
GEORGE SERLE.

To the unfained lovers of Truth, and constant *Students* of Noble *Sciences*, JOHN DEE of *London*, heartly wisheth grace from Heaven, and most prosperous success in all their honest attempts and *exercises*.

DIvine PLATO, the great Master of many
worthy Philosophers, and the constant avoucher,
and pithy perswader of *Unum*, *Bonum*, and *Ens*:
in his School and Academie, sundry times (be-
sides his ordinary Scholers) was visited of a certain
kind of men allured by the noble fame of *Plato*,
and the great commendation of his profound
and profitable doctrine. But when such Hea-
rers, after long harkening to him, perceived,
that the drift of his discourses issued out, to con-
clude, this *Unum*, *Bonum*, and *Ens*, to the Spiri-
tual, Infinite, Eternal, Omnipotent, &c. Nothing
being alledged or expressed, How; worldly goods:
how, worldly dignity: how, health, strength, or
lustinesse of body: nor yet the means, how a mar-
vellous, sensible and bodily blisse and felicity here-
after, might be atteined: Straightway, the fantasies of those hearers were dampt, their opinion
of *Plato* was clean changed; yea, his doctrine was by them despised; and his School, no
more of them visited. Which thing, his Scholer, *Aristotle*, narrowly considering, found
the cause thereof, to be, For that they had no fore-warning and information, in general,
" whereto his Doctrine tended. For, so, might they have had occasion, either to have
forborn his School haunting: (if they, then, had misliked his Scope and purpose) or con-
stantly to have continued therein: to their full satisfaction: if such a final Scope and
intent, had been to their desire. Wherefore, *Aristotle*, ever, after that, used in brief, to
" fore-warn his own Scholers and hearers, both of what matter, and also to what end, he
" took in hand to speak, or teach. While I consider the diverse trades of these two excellent
Philosophers (and am most sure, both, that *Plato* right-well, otherwise could teach: and
that *Aristotle* might boldly, with his hearers, have dealt in like sort as *Plato* did) I am in
no little pang of perplexity: Because, that which I mislike, is most easie for me to perform
(and to have *Plato* for my example.) And that which I know to be most commendable:
and (in this first bringing, into common handling, the *Arts Mathematical*) to be most
necessary: is full of great difficulty and sundry dangers. Yet, neither do I think it meet,
for so strange matter (as now is meant to be published) and to so strange an audience, to
be bluntly at first put forth, without a peculiar Preface: Nor (imitating *Aristotle*) well
can I hope, that according to the amplenesse and dignity of the *State Mathematical*,
I am able, either plainly to prescribe the material bounds: or precisely to expresse the chief
purposes, and most wonderful applications thereof. And though I am sure, that such as did
shrink from *Plato* his School, after they had perceived his final conclusion, would in these
things have been his most diligent hearers (so infinitely might their desires, in fine and at
length, by our *Art Mathematical*, be satisfied) yet, by this my Preface and fore-warning,
Aswell all such, may (to their great behoof) the sooner, hither be allured: as also
the

the *Pythagorical*, and *Platonical* perfect Scholer, and the constant profound Philosopher, with more ease and speed, may (like the Bee) gather, hereby, both wax and honey.

“ Wherefore, seeing I find great occasion (for the causes alleged, and farther, in respect “ of my *Art Mathematick general*) to use a certain fore-warning and Preface, whose “ content shall be, that mighty, most pleasant, and fruitful *Mathematical Tree*, with his “ chief arms and second (grafted) branches: “ Both, what every one is and also, what com- “ modity, in general, is to be looked for, aswell of grift as stock: And forasmuch as this “ enterprize is so great, that, to this our time, it never was (to my knowledge) by any “ atchieved: And also it is most hard, in these our drery dayes, to such rare and strange Arts, to win due and common credit: Neverthelesse, if for my sincere endeavour, to satisfie your honest expectation, you will but lend me your thankful mind a while: and to such matter as, for this time, my pen (with speed) is able to deliver, apply your eye or ear attentively: perchance, at once, and for the first saluting, this Preface you will find a lesson long enough. And either you will, for a second (by this) be made much the apter: or shortly become, well able your selves, of the Lions claw, to conjecture his royal Symmetrie, and farther property. Now then, gentle, my friends and countrey-men, Turn your eyes, and bend your minds to that doctrine, which for our present purpose, my simple talent is able to yield you.

The intent of this Preface.

All things which are, and have being, are found under a triple diversity general. For, either, they are deemed Supernatural, Natural, or of a third being. Things Supernatural, are immaterial, simple, indivisible, incorruptible, and unchangable. Things Natural, are material, compounded, divisible, corruptible, and changable: Things Supernatural, are of the mind onely comprehended: Things Natural, of the sense exterior, are able to be perceived. In things Natural, probability and conjecture hath place: But in things Supernatural, chief demonstration and most sure Science is to be had. By which properties and comparisons of these two, more easily may be described, the state, condition, nature and property of those things, which, we before termed of a third being: which, by a peculiar name also, are called *Things Mathematical*. For, these, being (in a manner) middle, between things supernatural and natural: are not so absolute and excellent, as things supernatural; Nor yet so base and grosse, as things natural: But are things immaterial, and neverthelesse, by material things able somewhat to be signified. And though their particular Images by Art, are aggregable and divisible: yet the general *Forms* notwithstanding, are constant, unchangeable, untransformable and incorruptible. Neither of the sense, can they, at any time, be perceived or judged. Nor yet, for all that in the royall mind of man, first conceived. But surmounting the imperfection of conjecture, weening and opinion; and comming short of high intellectual conception, are the Mercurial fruit of *Dianœtical* discourse, in perfect imagination subsisting. A marveilous newtrality have these things *Mathematical*, and also a strange participation between things supernatural, immortal, intellectual, simple and indivisible: and things natural, mortal, sensible, compounded and divisible. Probability and sensible proof, may well serve in things natural, and is commendable: In Mathematical reasonings, a probable Argument, is nothing regarded: nor yet the testimony of sense, any whit credited: But onely a perfect demonstration, of truths certain, necessary, and invincible: universally and necessarily concluded: is allowed as sufficient for an Argument exactly and purely Mathematical.

Number.

Note the word *Unite*, to express the Greek *Monas*, and not *Unity*, as we have all, commonly, till now, used.

Of *Mathematical* things, are two principal kinds, namely, *Number* and *Magnitude*. *Number*, we define to be a certain Mathematical Summe, of *Units*. And an *Unit* is that thing Mathematical, Indivisible, by participation of some likenesse of whose property, any thing, which is indeed, or is counted One, may reasonably be called One. We account an *Unit*, a thing *Mathematical*, though it be no *number*, and also indivisible because of it materially, Number doth consist: which principally, is a thing *Mathematical*. *Magnitude* is a thing *Mathematical*, by participation of some likeness of whose nature, any thing is judged long, broad, or thick. A thick *Magnitude* we call a *Solid*, or a *Body*. What *Magnitude* so ever is Solid or Thick, is also broad and long. A broad Magnitude we call a *Superficies* or a Plain. Every plain Magnitude, hath also length. A long Magnitude, we term a *Line*. A *Line* is neither thick nor broad, but onely long: Every certain Line, hath two ends: The ends of a Line, are *Points* called. A *Point* is a thing *Mathematical*, indivisible, which may have a certain determined situation. If a Point move from a determined situation, the way wherein it moved, is also a *Line*, Mathematically produced;

A Point.

produced: whereupon, of the ancient Mathematicians, a *Line* is called the race or course of a *Point*. A Point we define, by the name of a thing Mathematical: though it be no Magnitude, and indivisible: because it is the proper end, and bound of a *Line*: which is a true *Magnitude*. And *Magnitude* we may define to be that thing *Mathematical*, which is divisible for ever, in parts divisible, long, broad, or thick. Therefore though a Point be no *Magnitude*, yet *Terminatively* we reckon it a thing *Mathematical*, (as I said) by reason it is properly the end and bound of a line.

A Line.

Magnitude.

Neither *Number* nor *Magnitude* have any Materiality. First, we will consider of *Number*, and of the Science *Mathematical*, to it appropriate, called *Arithmetick*, and afterward of *Magnitude*, and his Science called *Geometrie*. But that name contenteth me not: whereof a word or two hereafter shall be said. How Immaterial, and free from all matter, *Number* is, who doth not perceive? yea, who doth not wonderfully wonder at it? For, neither pure *Element*, nor *Aristotles Quinta Essentia*, is able to serve for Number, as his proper matter. Nor yet the purity and simplenesse of Substance Spiritual or Angelical, will be found proper enough thereto. And therefore the great and godly Philosopher *Anitius Boëtius*, said: *Omnia quæcunque à primæva rerum natura constructa sunt, Numerorum videntur ratione formata. Hoc enim fuit principale in animo Conditoris Exemplar.* That is: **All things (which from the very first original being of things, have been framed and made) do appear to be Formed by the reason of Numbers. For this was the principal Example or pattern in the mind of the Creator.** O comfortable allurement, O ravishing perswasion, to deal with a Science, whose Subject is so ancient, so pure, so excellent, so surmounting all creatures, so used of the Almighty and incomprehensible wisedome of the Creator, in the distinct creation of all creatures: in all their distinct parts, properties, natures, and vertues, by order, and most absolute number, brought from *Nothing*, to the *Formality* of their being and state. By *Numbers* property therefore, of us, by all possible means, (to the perfection of the Science) learned, we may both wind and draw our selves into the inward and deep search and view, of all creatures distinct vertues, natures, properties, and *Forms*: And also farther, arise, clime, ascend and mount up (with speculative wings) in spirit, to behold in the Glasse of Creation, the *Form* of *Forms*, the *Exemplar Number* of all things *Numerable*: both visible and invisible: mortal and immortal, Corporal and Spiritual. Part of this profound and divine Science, had *Joachim* the Prophesier atteined unto: by *Numbers Formal*, *Natural*, and *Rational*, foreseeing, concluding, and foreshewing great particular events, long before their comming. His books yet remaining, hereof, are good proof. And the Noble Earl of *Mirandula*, (besides that) a sufficient witnesse: that Joachim *in his prophesies, proceeded by no other way, then by Numbers Formal.* And this Earl himself, in *Rome*, * set up 900 Conclusions, in all kind of Sciences, openly to be disputed of: and among the rest, in his Conclusions *Mathematical* (in the eleventh Conclusion) hath in Latine this English sentence. *By numbers, a way is had, to the searching out, and understanding of every thing, able to be known. For the verifying of which Conclusion, I promise to answer to the 74 Quæstions, under written by the way of Numbers.* Which Conclusions, I omit here to rehearse: aswell avoiding superfluous prolixitie: as, because *Joannes Picus, Works*, are commonly had. But, in any case, I would wish that those Conclusions were read diligently, and perceived of such, as are earnest Observers and Considerers of the constant law of Numbers: which is planted in things Natural and Supernatural: and is prescribed to all Creatures, inviolably to be kept. For, so, besides many other things, in those Conclusions to be marked it would appear, how sincerely, and within my bounds, I disclose the wonderful mysteries by numbers, to be atteined unto.

Anno 1488.

Of my former words, easie it is to be gathered, that *Number* hath a treble state: One, in the Creator: another is every Creature (in respect of his compleat constitution:) and the third, in Spiritual, and Angelical Minds, and in the Soul of man. In the first and third state, *Number*, is termed *Number Numbring*. But in all Creatures otherwise, *Number* is termed *Number Numbred*. And in our Soul, Number beareth such a sway, and had such an affinity therewith; that some of the old Philosophers taught, *Mans soul, to be a number moving it self.* And indeed, in us, though it be a very Accident, yet such an Accident it is, that before all Creatures it had perfect being, in the Creator, Sempiternally. *Number*

numbring therefore, is the discretion, discerning, and distincting, of things. But in God
the Creator, This discretion, in the beginning, produced orderly and distinctly all things.
For, his *numbring*, then, was his Creating of all things. And his continual *numbring* of
all things, is the Conservation of them in being. And, where and when he will lack an
Unit: there and then, that particular thing shall be *Discreated*. Here I stay. But our
Severalling, distincting, and *numbring*, createth nothing: but of Multitude considered,
maketh certain and distinct determination. And albeit these things be waighty, and
truths of great importance, yet (by the infinite goodness of the Almighty *Ternarie*,)
Artificial Methods and easie wayes are made, by which the zealous Philosopher may
win neer this Riverish *Ida*, this Mountain of Contemplation: and more then Con-
templation. And also, though *number* be a thing so immaterial, so divine, and eternal:
yet by degrees, by little, and little, stretching forth, and applying some likenesse of it, as
first, to things Spiritual: and then bringing it lower, to things sensibly perceived, as of a
momentany sound iterated: then to the least things that may be seen, numerable: And
at length (most grosly) to a multitude of any corporal things seen, or felt: and so of these
grosse and sensible things, we are trained to learn a certain Image, or likenesse of numbers,
and to use Art in them to our pleasure and profit. So grosse is our conversation, and dull
is our apprehension, while mortal Sense, in us, ruleth the Common-wealth of our little
world. Hereby we say, Three Lyons are three or a *Ternarie*. Which * *Ternaries*, are
☞ each, the *Union, knot* and *Uniformity* of three discreet and distinct *Units*. That is, we
may in each *Ternary*, thrice severally, point, and shew a part, *One, One*, and *One*. Where,
in Numbring, we say One, Two, Three. But how far these visible Ones, do differ from
our Indivisible Units (in pure *Arithmetick* principally considered) no man is ignorant.
Yet from these grosse and material things, may we be led upward, by degrees, so informing
our rude Imagination, toward the conceiving of *numbers*, absolutely: (Not supposing,
nor admixting any thing created, Corporal or Spiritual, to support, contein, or represent
those *numbers* imagined:) that at length, we may be able, to find the number of our
own name, gloriously exemplified and registred in the book of the *Trinity*, most blessed and
eternal.

But farther understand, that vulgar Practisers, have Numbers, otherwise, in sundry
Considerations: and extend their name farther, then to Numbers, whose least part is an
Unit. For the common Logist, Reckenmaster, or Arithmetician, in his using of Num-
bers: of an *Unit*, imagineth lesse parts: and calleth them *Fractions*. As of an *Unit*, he
maketh an half, and thus noteth it, $\frac{1}{2}$, and so of other (infinitely diverse) parts of an *Unit*.
Yea, and farther, hath *Fractions of Fractions, &c.* And forasmuch as *Addition, Sub-
straction, Multiplication, Division*, and *Extraction of Roots*, are the chief, and sufficient
parts of *Arithmetick:* which is, the *Science that demonstrateth the properties of numbers,*
Arithmetick *and all operations, in numbers to be performed.* " How often therefore, these five sundry
Note. " sorts of Operations, do, for the most part of their execution, differ from the five opera-
" tions of like general property and name, in our Whole numbers practisable. So, often,
" (for a more distinct doctrine) we, vulgarly account and name it, another kind of *Arith-
metick*. And by this reason: the consideration, doctrine, & working in whole numbers onely:
1. where, of an *Unit*, is no lesse part to be allowed: is named (as it were) an *Arithmetick*
by it self. And so of the *Arithmetick of Fractions*. In like sort the necessary, wonder-
2. ful, and Secret doctrine of Proportion, and proportionality hath purchased unto it self a
peculiar manner of handing and working: and so may seem another form of *Arithmetick*.
Moreover, the *Astronomers*, for speed, and more commodious calculation, have devised
3. a peculiar manner of ordering numbers, about their circular motions, by Sexagenes and
Sexagesmes. By Signes, Degrees, Minutes, &c. which commonly is called the *Arithme-
tick* of *Astronomical* or *Physical Fractions*. That have I briefly noted by the name of
Arithmetick Circular. Because it is also used in Circles, not *Astronomical, &c.* Practise
4. hath led *Numbers* farther, and hath framed them, to take upon them, the shew of *Magni-
tudes* property: Which is *Incommensurability* and *Irrationality*. (For in pure *Arith-
metick*, an *Unit*, is the common Measure of all Numbers.) And here, Numbers are
become, as Lines, Plains, and Solids, sometimes *Rational*, sometimes *Irrational*. And
have proper and peculiar characters, (as √ȝ. √ce. and so of other. Which is to signifie
Root Square, Root Cubick: and so forth:) and proper and peculiar fashions in the five
principal parts: Wherefore the Practiser esteemed this a diverse *Arithmetick* from the
other.

other. Practise bringeth in, here, diverse compounding of Numbers: as sometime, two,
three, four (or more) *Radical* Numbers diversly knit by signs, of More and Lesse: as
thus √ȝ 12 + √ce 15. Or thus √ȝȝ 19 + √ce 12 − √ȝ 2, &c. And some-
time with whole numbers, or fractions of whole Number, among them: as 20 + √ȝ 24.
√ce 16 + 33 − √ȝ 10. √ȝȝ 44 + $12\frac{1}{4}$ + √ce 9. And so infinitely, may hap
the varietie. After this; Both the one and the other hath fractions incident: and so is this
Arithmetick greatly enlarged, by diverse exhibiting, and use of Compositions and
mixtings. Consider how, I (being desirous to deliver the Student from errour and Cavil-
lation) do give to this *Practise*, the name of the *Arithmetick of Radical numbers:*
Not of *Irrational* or *Surd numbers:* which other while, are Rational: though they have
the Signe of a Root before them, which *Arithmetick* of whole Numbers most usual, would
say they had no such Root, and so account them *Surd numbers*, which, generally spoken,
is untrue: as *Euclid's* tenth Book may teach you. Therefore, to call them, generally
Radical numbers, (by reason of the sign √ prefixed) is a sure way, and a sufficient general
distinction from all other ordering and using of Numbers: And yet (besides all this)
Consider; the infinite desire of knowledge, and incredible power of mans Search and
Capacity: how, they, joyntly have waded farther (by mixing speculation and practice)
and have found out, and atteined, to the very chief perfection (almost) of *Numbers* practi-
cal use. Which thing is well to be perceived in that great Arithmetical Art of *Equation:*
commonly called the *Rule of Coss*, or *Algebra*. The Latines termed it, *Regulam Rei &
Census*, that is, the **Rule of the thing, and his value.** With an apt name, com-
prehending the first and last points of the Work. And the vulgar names, both in Italian,
French, and Spanish depends (in naming it,) upon the signification of the Latin word
Res: **A thing:** unleast they use the name of *Algebra*. And therein (commonly) is a
double errour. The one of them which think it to be of *Geber*, his inventing: the other
of such as call it *Algebra*. For, first, though *Geber* for his great skill in Numbers, Geo-
metry, Astronomy, and other marvellous Arts, might have seemed able to have first
devised the said Rule: and also the name carrieth with it a very neer likenesse of *Geber*
his name: yet true it is, that a *Greek* Philosopher and Mathematician, named *Diophantis*,
before *Geber* his time, wrote 13 books thereof (of which six are yet extant: and I had
them to * use, of the famous Mathematician, and my great frind, *Petrus Montaureus:*) *Anno* 1550.
And secondly, the very name, is *Algiebar*, and not *Algebra:* as by the Arabian *Avicen*,
may be proved: who hath these precise words in Latine, by *Andreas Alpagus* (most
perfect in the Arabick tongue) so translated. *Scientia faciendi Algiebar & Almachabel.*
i. *Scientia inveniendi numerum ignotum, per additum numeri, & divisionem, & equa-
tionem.* Which is to say: **The Science of working Algiebar, and Alma-
chabel: that is: the Science of finding an unknown Number, by
Adding of a Number, and Division, and Equation.** Here have you the
name: and also the principal parts of the Rule, touched. To name it, *The rule, or art of
Æquation*, doth signifie the middle part and the State of the Rule. This Rule hath his
peculiar Characters, and the principal parts of *Arithmetick*, to it apperteining, do differ
from the other *Arithmetical operations*. This *Arithmetick* hath Numbers Simple, Com- 5.
pound, Mixt, and Fractions accordingly. This Rule and *Arihmetick* of *Algiebar*, is
so profound, so general, and so (in manner) conteineth the whole power of Numbers,
Application practical: that mans wit, can deal with nothing more profitable about
numbers: nor match, with a thing, more meet for the divine force of the Soul, (in hu-
mane studies, affairs or exercises) to be tried in. Perchance you looked for, (long ere now)
to have had some particular proof, or evident testimony of the use, profit, and Commodity
of Arithmetick vulgar, in the common life and trade of men. Thereto, then I will now
frame my selfe: But herein great care I have, least length of sundry proofs, might make
you deem, that either I did disdoubt your zealous mind to vertues school, or else mistrust
your able wits, by some, to guesse much more. A proof, then four, five, or six, such, will
I bring, as any reasonable man, therewith may be perswaded, to love and honour, yea learn
and exercise the excellent Science of *Arithmetick*.

And first: Who, neerer at hand, can be a better witnesse of the fruit received by *Arith-
metick*, then all kind of Merchants? Though not alike, either need it, or use it. How
could they forbear the use and help of the Rule, called the Golden Rule? Simple and

 Compound:

Compound: both forward and backward? How might they misse *Arithmetical* help in the Rules of Fellowship: either without time, or with time? and between the Merchant and his Factor? The Rules of Bartering in wares onely: or part in wares, and part in money, would they gladly want? Our Merchant Venturers, and Travellers over Sea, how could they order their doings justly and without losse, unlesse certain and general Rules for Exchange of money, and Rechange, were, for their use, devised? The Rule of Alligation, in how sundry cases, doth it conclude for them, such precise verities, as neither by natural wit, nor other experience they were able else to know? And (with the Merchant then to make an end) how ample and wonderful is the Rule of False positions? especially as it is now, by two excellent Mathematicians (of my familiar acquaintance in their life time) enlarged? I mean *Gemma Frisius* and *Simon Jacob*, Who can either in brief conclude, the general and Capital Rules? or who can Imagine the Myriades of sundry Cases, and particular examples in Act and earnest, continually wrought, tried and concluded by by the forenamed Rules only? How sundry other *Arithmetical practises*, are commonly in Merchants hands, and knowledge: They themselves can at large testifie.

The Mintmaster, and Goldsmith, in their mixture of Metals, either of divers kinds, or divers values; how are they, or may they, exactly be directed, and marvelously pleasured, if *Arithmetick* be their guide? And the honourable Physicians will gladly confesse themselves much beholding to the Science of *Arithmetick*, and that sundry wayes: But chiefly in their Art of Graduation, and compound Medicines. and though *Gallenus*, *Averrois*, *Arnoldus*, *Lullus*, and others have published their positions, aswel in the quantities of the Degrees above Temperament, as in the Rules, concluding the new *Form* resulting: yet a more precise, commodious, and easie *Method*, is extant: by a Countrey-
R. B. man of ours (above 200 years ago) invented. And forasmuch as I am uncertain, who hath the same: or when that little Latin treatise (as the Author writ it,) shall come to be printed: (Both to declare the desire I have to pleasure my Countrey, where in I may: and also, for very good proof of Numbers use, in this most subtile, and fruitful Philosophical Conclusion,) I intend in the mean while, most briefly, and with my farther help, to communicate the pith thereof unto you.

First, describe a circle: whose Diameter let be an inch. Divide the circumference into four equal parts. From the Center, by those 4 sections, extend 4 right lines: each of 4 inches and a half long: or of as many as you list, about 4 without the circumference of the circle: So that they shall be of 4 inches long (at the least) without the Circle. Make good evident marks at every inches end. If you list, you may subdivide the inches again into 10 or 12 smaller parts, equal. At the ends of the lines, write the names of the 4 principal Elemental qualities. *Hot*, and *Cold*, one against the other. And likewise *Moist* and *Dry*, one against the other. And in the circle write *Temperate*. Which *Temperature* hath a good Latitude: as appeareth by the Complexion of man. And therefore we have allowed unto it, the foresaid Circle: and not a point Mathematical or Physical.

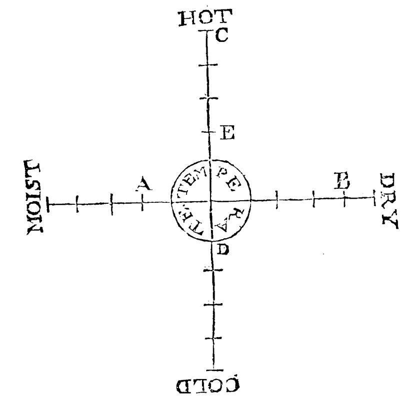
* Take some part of *Lullus* counsel in his Book *De Q. Essentia.*

Now, when you have two things Miscible, whose degrees are * truly known; Of necessity, either they are of one quantity and waight, or of diverse. If they be of one quantity and waight: whether their forms, be contrary qualities, or of one kind (but of divers intentions and degrees) or a *Temperate*, and a contrary, *The form resulting of their Mixture, is in the Middle between the degrees of the forms mixt.* As for example, let *A*, be *Moist* in the first degree: and *B*, *Dry* in the third degree. Adde 1 and 3, that maketh 4: the half or middle of 4 is 2. This 2 is the middle, equally distant from *A* and *B* (for the * *Temperament* is counted none. And for it, you must put a Cypher, if at any time, it be in mixture.) Counting them from *B*, 2 degrees, toward *A*: you find it to be *Dry* in the first degree: So is the *Form resulting* of the mixture of *A*, and *B*, in our Example. I will give you another example. Suppose, you have two things, as *C*, and *D*: and of *C*, the Heat to be in the 4th. degree: and of *D*, the Cold, to be remisse, even unto the *Temperament*. Now, for *C*, you take 4: and for *D*, you take a Cypher: which, added unto 4, yeildeth onely 4. The middle, or half, whereof, is 2. Wherefore the *Form resulting* of *C*, and *D*, is *Hot* in the second degree: for, 2 degrees, accounted from *C*, toward *D*, end just in the second degree of heat. Of the third manner, I will give also an example: which let be this: I have a liquid Medicine whose Qualitie of heat is in the fourth degree exalted: as was *C*, in the example foregoing: and an other liquid Medicine I have: whose Qualitie, is heat, in the first degree. Of each of these, I mixt alike quantitie: Subtract here,

* Note.

Note.

here, the lesse from the more: and the residue divide into two equal parts: whereof the one part, either added to the lesse, or subtracted from the higher degree, doth produce the



degree of the Form resulting, by this mixture of *C*, and *E*. As, if from 4, you abate 1. there resteth 3, the half of 3 is $1\frac{1}{2}$: Add to $1\frac{1}{2}$: you have $2\frac{1}{2}$. Or subtract from 4 this $1\frac{1}{2}$: you have likewise $2\frac{1}{2}$ remaining. Which declareth, the *Form resulting*, to be *Heat*, in the middle of the third degree.

The second Rule.

" But if the quantities of two things Commixt, be divers, and the intensions, (of their " Forms miscible) be in divers degrees, and heigths. (Whether those Forms be of one " kind, or of Contrary kinds, or of a Temperate and a Contrary, *What proportion is of* " *the lesse quantity to the greater, the same shall be of the difference, which is between* " *the degree of the Form resulting, and the degree of the greater quantity of the thing* " *miscible to the difference, which is between the same degree of the Form resulting, and* " *the degree of the less quantity.* As for example. Let two pound of Liquour be given, hot in the fourth degree: and one pound of Liquour be given, hot in the third degree. I would gladly know the Form resulting, in the mixture of these two Liquours. Set down your numbers in order thus.

℔. 2.	Hot. 4.
℔. 1.	Hot. 3.

Now by the Rule of Algiebar, have I devised a very easie, brief, and general manner of working in this case. Let us first suppose that *Middle Form resulting* to be 1 ℭ: as that Rule teacheth. And because (by our Rule, here given) as the weight of 1, is to 2: So is the difference between 4, (the degree of the greater quantity) and 1 ℭ: to the difference between 1 ℭ and 3: (the degree of the thing, in lesse quantity. And withal, 1 ℭ, being always in a certain middle, between the two heigths or degrees. For the first difference, I set 4—1 ℭ: and for the second, I set 1 ℭ—3. And now again, I say, as 1 is to 2. so is 4—1 ℭ to 1 ℭ—3. Wherefore, of these four proportional numbers, the first and the fourth multiplyed, one by the other, do make as much, as the second and the third multiplied the one by the other. Let these multiplications be made accordingly. And of the first and the fourth, we have 1 ℭ— 3, and of the second and third, 8—2 ℭ. Wherefore, your Æquation is between 1 ℭ — 3, and 8—2 ℭ. Which may be reduced, according to the Art of Algiebar: as, here, adding 3, to each part, giveth the Æquation, thus, 1 ℭ=11—2 ℭ. And yet again, contracting or reducing it: Adde to each part, 2 ℭ: Then have you 3 ℭ equal to 11, thus

thus represented 3 ℭ = 11. Wherefore, dividing 11, by 3: the quotient is $3\frac{2}{3}$: the *Value* of our 1 ℭ, *Coss*, or *Thing*, first supposed: and that is the heigth, or Intention of the *Form resulting*: which is *Heat*, in two thirds of the fourth degree: and here I set the shew of the work in conclusion thus.

℔. 2.	*Hot.* 4.
℔. 1.	*Hot.* 3.

$\frac{1}{3}$ $\frac{2}{3}$ $\boxed{3\frac{2}{3}}$ *The Form resulting.*

The proof hereof is easie, by substracting 3, from $3\frac{2}{3}$, resteth $\frac{2}{3}$. Substract the same heigth of the Form resulting, (which is $3\frac{2}{3}$) from 4: then resteth $\frac{1}{3}$. You see that $\frac{2}{3}$ is double to $\frac{1}{3}$: as 2 ℔. is double to 1 ℔. So should it be: by the Rule here given. Note. As you added to each part of the Æquation 3: so if you first added to each part 2 ℭ, it would stand, 3 ℭ − 3 = 8: and now adding to each part 3: you have (as afore) 3 ℭ = 11.

And though, I, here, speak onely of two things miscible: and most commonly more then three, four, five or six, &c. are to be mixed: (and in one Compound to be reduced: and the Form resulting of the same, to serve the turn) yet these Rules are sufficient, duly

Note. repeated and iterated. In proceeding first, with any two: and then with the Form resulting and another, and so forth: For, the last work concludeth the Form resulting of them all: I need nothing to speak, of the mixture (here supposed) what it is. Common Philosophy hath defined it, saying, *Mixtio est miscibilium, alteratorum, per minima conjunctorum Unio.* Every word in the definition, is of great importance. I need not also spend any time, to shew, how, the other manner of distributing of degrees, doth agree to these Rules. Neither need I of the farther use belonging to the Crosse of Graduation (before described) in this place declare, unto such as are capable of that, which I have already said. Neither yet with examples, specifie the manifold varieties, by the foresaid two general Rules to be ordered. The witty and Studious, here, have sufficient: And they which are not able to attein to this, without lively teaching, and more in particular: would have larger discoursing, then is meet in this place to be dealt withall: And other (perchance) with a proud snuffe will disdain this little: and would be unthankful for much more. I, Therefore conclude; and with such as have modest and earnest Philosophical minds, to laud God highly for this: and to marvail that the profoundest and subtilest point, concerning *Mixture of Forms, and Qualities Natural*, is so matcht and married with the most simple, easie, and short way of the noble Rule of *Algebar*. Who can remain, therefore, unperswaded, to love, allow, and honour the excellent Science of *Arithmetick*? For, here, you may perceive that the little finger of *Arithmetick* is of more might and contriving, then a hundred thousands mens wits, of the middle sort, are able to perform, or truly to conclude, without help thereof.

Now, will we farther, by the wise and valiant Captain, be certified, what help he hath, by the Rules of *Arithmetick*: in one of the arts to him apperteining: and of the Greeks

Ταχτικὴ. " named Ταχτικὴ. That is, the skill of ordering Souldiers in Battle ray, after the best " manner to all purposes. This art so much dependeth upon Numbers use, and the Mathematicals, that *Ælianus* (the best writer thereof,) in his work to the Emperour *Hadrianus*, by his perfection, in the Mathematicals, (being greater then others before him had,) thinketh his book to passe all other the excellent work, written of that art, unto his dayes. For, of it, had written *Æneas*: *Cyneas of Thessaly*: *Pyrrhus Epirota*, and *Alexander* his sonne: *Clearchus*: *Pausanias*: *Evangelus*: *Polybius*, familiar friends to *Scipio*, *Eupolemus*, *Iphicrates*, *Possidonius*, and very many other worthy Captains, Philosophers and Princes of immortal fame and memory. Whose fairest flower of their garland (in this feat) was *Arithmetick*: and a little perceiverance, in *Geometrical* Figures. But in many other cases doth *Arithmetick* stand the Captain in great stead. As in proportioning of victuals for the army, either remaining at a stay: or suddenly to be encreased with a certain number of Souldiers: and for a certain time. Or by good arts to diminish his Company, to make the victuals, longer to serve the remanent, and for a certain determined time: If need so require. And so in sundry his other accounts, Reckonings, Measurings, and Proportionings, the wise, expert, and circumspect Captain will affirm the Science of *Arithmetick*, to be one of his chief Counsailors, directers and aiders. Which thing (by good means) was evident to the Noble, the Couragious, the Loyal, and Courteous *John*, late Earl

Earl of *Warwick*. Who was a young Gentleman, throughly known to very few. Albeit his lusty valiantnesse, force, and Skill in Chivalrous feats and exercises: his humbleness, and friendliness to all men, were things, openly, of the world perceived. But what roots (otherwise,) vertue had fastened in his brest, what Rules of Godly and honourable life he had framed to himself: what vices, (in some then living) notable, he took great care to eschew, what manly vertues, in other noble men, (flourishing before his eyes,) he Sythingly aspired after: what prowesses he purposed and meant to atchieve: with what feats of Arts, he began to furnish and fraught himself, for the better service of his King and Countrey, both in peace and war. These (I say) his Heroical Meditations, fore-castings, and determinations, no twain, (I think) beside my self, can so perfectly, and truly report. And therefore, in Conscience, I count it my part, for the honour, preferment, and procuring of vertue, (thus briefly) to have put his Name, in the Register of *Fame Immortal*.

This Noble Earle, died Anno 1552. scarce 24 years of age, having no issue by his wife, Daughter to the Duke Somerset.

To our purpose. This *John* by one of his acts (besides many other: both in *England* and *France*, by me, in him noted) did disclose his hearty love, to vertuous Sciences: and his noble intent, to excell in Martial prowesse: when we with humble request, and instant Soliciting: got the best Rules (either in the time past, by Greek or Roman, or in our time used: and new Stratagems therein devised) for ordering of all Companies, summes, and Numbers of men, (many, or few) with one kind of weapon, or more appointed: with Artillery, or without: on horsback or on foot: to give or take on-set: to seem many, being few: to seem few, being many. To march in battail or journey: with many such feats, to foughten field, Skirmish or Ambush appertaining: and of all these lively designments (most curiously) to be in velame parchment described: with Notes and peculiar marks, as the art requireth: and all these Rules and descriptions Arithmetical, inclosed in a rich Case of Gold, he used to wear about his neck, as his Jewel most precious, and Counsailour most trusty. Thus, *Arithmetick*, of him, was shrined in gold: Of *Numbers* fruit, he had good hope. Now Numbers therefore innumerable, in *Numbers* praise, his shrine shall finde.

What need I, (for farther proof to you) of the Schoolmasters of Justice, to require testimony: how needful, how fruitful, how skilful a thing *Arithmetick* is? I mean the Lawyers of all sorts. Undoubtedly, the Civilians can marvellously declare: how, neither the Ancient Romane Laws, without good knowledg of *Numbers Art*, can be perceived: Nor (Justice infinite Cases) without due proportion, (narrowly considered) is able to be executed. How justly, and with great knowledg of Art, did *Papinianus* institute a law of partition, and allowance, between man and wife after a divorce? But how *Accursius*, *Baldus*, *Bartolus*, *Jason*, *Alexander*, and finally, *Alciatus*, (being otherwise, notably well learned) do jumble, guesse and erre from the equity, art and intent of the Law-maker: *Arithmetick* can detect, and convince: and cleerly, make the truth to shine. Good *Bartolus* tired in the examining and proportioning of the matter, and with *Accursius* Glosse much cumbred: burst out, and said: *Nulla est in toto libro, hac glossa difficilior: Cujus computationem nec Scholastici, nec Doctores intelligunt, &c.* That is: **In the whole book, there is no glosse harder then this: Whose account or reckoning, neither the Scholers nor the Doctours understand, &c.** What can they say of *Julianus* law, *Si ita Scriptum, &c.* Of the Testators will, justly performing, between the wife, sonne, and daughter? How can they perceive the equity of *Aphricanus*, *Arithmetical* Reckoning, where he treateth of *Lex Falcidia*? How can they deliver him, from his Reprovers: and their maintainers: as *Joannes*, *Accursius*, *Hypolitus*, and *Alciatus*? How justly and artificially, was *Africanus* reckoning made? Proportionating to the summes bequeathed, the Contributions of each part? Namely, for the hundred presently receiued, $17\frac{1}{7}$. And for the hundred, received after ten moneths $12\frac{6}{7}$: which make the 30: which were to be contributed by the legataries to the heir. For, what proportion 100 hath to 75: the same hath $17\frac{1}{7}$ to $12\frac{6}{7}$: Which is *Sesquitertia*. that is, as 4, to 3, which make 7. Wonderful many places in the Civil law, require an expert *Arithmetician*, to understand the deep Judgment and just determination of the Ancient Roman Law-maker. But much more expert ought he to be, who should be able to decide with equity, the infinite variety of Cases, which doe, or may happen, under every one of those laws and ordinances Civil. Hereby, easily, you may now conjecture: that in the Canon law: and in the laws of the Realm (which with us, bear the chief authority) Justice and equity might be greatly preferred, and skilfully executed through due

due skill of Arithmetick, and proportions apperteining. The worthy Philosophers and prudent law-makers (who have written many books *De Republica*: How the best state of Common-wealths might be procured and mainteined,) have very well determined of Justice: (which, not onely is the Base and foundation of Common weales: but also the total perfection of all our works, words, and thoughts:) "defining it, to be that vertue, "by which, to every one is rendered, that to him appertaineth. God challengeth this at our hands to be honoured as God: to be loved, as a Father, and to be feared, as a Lord and Master. Our neighbours proportion is also prescribed of the Almighty Law-maker: which is, to do to others, even as we would be done unto. These proportions, are in Justice necessary: in duty, commendable: and of Common-wealths, the life, strength, stay, and flourishing. *Aristotle* in his *Æthicks* (to fetch the seed of Justice, and light of direction, to use, and execute the same) was fain to fly to the perfection, and power of Numbers: for proportions Arithmetical and Geometrical. *Plato* in his book called *Epinomis*, (which book is the Treasure of all his doctrine) where his purpose is to seek a Science, which, when a man had it, perfectly: he might seem, and so be, indeed, *Wise*. He briefly of other Sciences discoursing, findeth them, not able to bring it to passe: But of the Science of Numbers, he saith. *Illa, quæ numerum mortalium generi dedit, id profectò efficiet. Deum autem aliquem, magis quàm fortunam, ad salutem nostram, hoc munus nobis arbitror contulisse, &c. Nam ipsum bonorum omnium Authorem, cur non maximi boni, Prudentiæ dico, causam arbitramur?* **That Science, verily, which hath taught mankind number, shall be able to bring it to passe. And, I think, a certain God, rather than Fortune, to have given us this gift, for our blisse. For, why should we not judg him who is the Authour of all good things, to be also the cause of the greatest good thing, namely, Wisedome?** There, at length, he proveth *Wisedome* to be atteined by good skill of *Numbers*. With which great Testimony, and the manifold proofs, and reasons, before expressed, you may be sufficiently and fully perswaded; of the perfect Science of *Arithmetick*, to make this account: That of all Sciences, next to *Theologie*, it is most divine, most pure, most ample, and general, most profound, most subtile, most commodious, and most necessary. Whose next Sister, is the absolute Science of *Magnitudes*: of which (by the direction and aid of him, whose *Magnitude* is infinite, and of us incomprehensible) I now intend so to write, that both with the *Multitude*, and also with the *Magnitude* of marvellous and fruitful verities, you (my friends and Countreymen) may be stir'd up and awaked, to behold what certain Arts and Sciences (to our unspeakable behoof) our heavenly Father, hath for us prepared, and revealed by sundry *Philosophers* and *Mathematicians*.

Justice.

Both, *Number* and *Magnitude*, have a certain Original seed, (as it were) of an incredible property: and of man, never able, fully to be declared. Of *Number*, an Unit, and of *Magnitude*, a Point, do seem to be much like Original causes: But the diversity neverthelesse, is great. We defined an *Unit* to be a thing Mathematical indivisible: A Point, likewise, we said to be a Mathematical thing indivisible. And farther, that a Point may have certain determined Situation: that is, that we may assign, and prescribe a Point, to be here, there, yonder, &c. Herein (behold) our Unit is free, and can abide no bondage, or to be tyed to any place, or seat: divisible or indivisible. Again, by reason, a Point may have a situation limited to him: a certain motion, therefore, (to a place, and from a place) is to a Point incident, and appertaining. But an *Unit* cannot be imagined, to have any motion. A point, by his motion, produceth Mathematically, a line, (as we said before) which is the first kind of Magnitudes, and most simple: An *Unit*, cannot produce any number. A line, though it be produced of a Point moved, yet it doth not consist of points: Number, though it be not produced of an *Unit*, yet doth it consist of Units, as a material cause. But formally, Number, is the Union and Unity of Units. Which uniting and knitting, is the workmanship of our mind: which, of distinct and discreet Units maketh a Number: by uniformity, resulting of a certain multitude of Units. And so, every number may have his least part given: namely, an Unit: But not of a magnitude, (no, not of a Line,) the least part can be given: because, infinitely, division thereof, may be conceived. All Magnitude, is either a Line, a Plain, or a Solid. Which Line, Plain or Solid, of no Sense, can be perceived, nor exactly by hand (any way) represented: nor of Nature

Number.

Nature produced: But, as (by degrees) Number did come to our perceiverance: So by visible forms, we are holpen to imagine, what our Line Mathematical is. What our Point, is. So precise, are our Magnitudes, that one Line is no broader than an other: for they have no breadth: Nor our Plains have any thicknesse: Nor yet our Bodies, any weight: be they never so large of dimension. Our Bodies, we can have smaller, than either Art, or Nature can produce any: and greater also, then all the World can comprehend. Our least Magnitudes, can be divided into so many parts, as the greatest. As, a Line of an inch long, (with us) may be divided into as many parts, as may the diameter of the whole World, from East to West: or any way extended: What priviledges, above all manual Art, and Natures might, have our two Sciences Mathematical? to exhibit, and to deal with things of such power, liberty, simplicity, purity, and perfection? And in them, so certainly, so orderly, so precisely, to proceed: as excellent, is that workman Mechanical Judged, who neerest can approach to the representing of works, Mathematically demonstrated? And our two Sciences, remaining pure, and absolute in their proper terms, and in their own matter, to have, and allow onely such demonstrations, as are plain, certain, universal, and of an eternal verity? This Science of *Magnitude*, his properties, conditions, and appurtenances: commonly, now is, and from the beginning, hath of all Philosophers, been called *Geometry*. But verily, with a name too base and scant, for a Science of such dignity and amplenesse. And, perchance, that name, by common and secret consent, of all wise men, hitherto hath been suffered to remain: that it might carry with it a perpetual memory of the first and notablest benefit, by that Science, to common people shewed: Which was, when Bounds and Meres of land and ground, were lost, and confounded, (as in *Egypt*, yearly, with the over-flowing of *Nilus*, the greatest and longest river in the World) or, that ground bequeathed, were to be assigned: or ground sold, were to be laid out: or (when disorder prevailed) that Commons were distributed into Severalties. For, where, upon these and such like occasions, Some by ignorance, some by negligence; Some by fraud, and some by violence, did wrongfully limit, measure, encroach, or challenge (by pretence of just content and measure) those lands and grounds: great losse, disquietnesse, murther, and war did (full oft) ensue: Till, by Gods mercy, and mans industry, The perfect Science of Lines, Plains, and Solids (like a divine Justicier,) gave unto every man his own. The people then, by this art pleasured, and greatly relieved, in their lands just measuring: and other Philosophers, writing rules for land measuring: between them both, thus, confirmed the name of *Geometria*, that is, (according to the very Etymologie of the word) Land-measuring. Wherein the people knew no farther of Magnitudes use, but in Plains: and the Philosophers of them had no fit hearers, or Scholers: farther to disclose unto, then of flat, plain *Geometry*. And though, these Philosophers, knew of farther use, and best understood the Etymologie of the word, yet this name *Geometria*, was of them applyed generally to all sorts of Magnitudes: unlesse otherwhile, of *Plato* and *Pythagoras*. When they would precisely declare their own doctrine. Then was * *Geometria*, with them, *Studium quod circa planum versatur*. But, well you may perceive by *Euclid's Elements* that more ample is our Science, then to measure Plains: and nothing lesse therein is taught (of purpose) then how to measure Land. Another name, therefore, must needs be had, for our Mathematical Science of Magnitudes: which regardeth neither clod nor turff: neither hill, nor dale: neither earth nor heaven: but is absolute *Megethelogia*, not creeping on ground, and dasling the eye with pole, perch, rod, " or line: but lifting the heart above the Heavens, by invisible lines, and immortal beams: " meeteth with the reflexions of the light incomprehensible, and so procureth joy, and per" fection unspeakable. Of which true use of our *Megethica*, or *Megethologia*, Divine *Plato* seemed to have good tast and judgment: and (by the name of *Geometry*) so noted it: and warned his Scholers thereof: as, in his seventh *Dialogue*, of the Commonwealth may evidently be seen. Where (in Latin) thus it is; right well translated: *Profectò, nobis hoc non negabunt, Quicunque vel paululum quid Geometriæ gustarunt, quin hæc Scientia, contrà, omnino se habeat, quàm de ea loquuntur, qui in ipsa versantur.* In Enlish, thus. **Verily** (saith *Plato*) **whosoever hath (but even very little) tasted, of Geometry, will not deny unto us, this: but that this Science, is of another condition, quite contrary to that, which they that are exercised in it, do speak of it.** And there it followeth of our *Geometry*. *Quòd quæritur cognoscendi*

Geometry.

Plato 7. de Rep.

cognoscendi illius gratia, quod semper est, non & ejus quod oritur quandóque & interit. Geometria, ejus quod est semper, Cognitio est. At tollet igitur (ô Generose Vir) ad Veritatem, animum: atque ita, ad Philosophandum preparabit cogitationem, ut ad supera convertamus: quæ, nunc, contra quàm decet, ad inferiora dejicimus, &c. Quàm maxime igitur præcipiendum est, ut qui præclarissimam hanc habitant Civitatem, nullo modo, Geometriam spernant. Nam & quæ præter ipsius propositum, quodam modo esse videntur, haud exigua sunt, &c. It must needs be confessed (saith *Plato.*) That [Geometry] is learned, for the knowing of that which is ever: and not of that, which in time, both is bred and is brought to an end, &c. Geometry is the knowledge of that which is everlasting. It will lift up therefore (O Gentle Sir) our mind to the verity: and by that means, it will prepare the Thought, to the Philosophical love of wisedome, that we may turn or convert, toward heavenly things, [both mind and thought] which now, otherwise than becommeth us, we cast down on base or inferiour things, &c. chiefly, therefore Commandement must be given, that such as doe inhabite this most honourable City, by no means, despise Geometry. For even those things [done by it] which, in manner seem to be, beside the purpose of Geometry: are of no smal importance, &c. And besides the manifold uses of *Geometry*, in matters appertaining to war, he addeth more, of second unpurposed fruit, and commodity, arising by *Geometry*, saying: *Scimus quin etiam ad Desciplinas omnes facilius per discendas interesse omnino, attigerit ne Geometriam aliquis, an non, &c. Hanc ergo Doctrinam, secundo loco discendam juvenibus statuamus.* That is. But also, we know, that for the more easie learning of all Arts, it importeth much, whether one have any knowledg in Geometry, or no, &c. Let us therefore make an ordinance or decree, that this Science of young men shall be learned in the second place. This was Divine *Plato* his judgment, both of the purposed, chief, and perfect use of *Geometry:* and of his second, depending, derivative commodities. And for us Christian men, a thousand thousand more occasions are to have need of the help of * *Megethological* Contemplations: whereby to train our Imaginations and Minds by little and little to forsake and abandon, the grosse and corruptible Objects, of our outward senses: and to apprehend by sure doctrine demonstrative, Things Mathematical. And by them readily to be holpen and conducted, to conceive, discourse, and conclude of things Intellectual, Spiritual, Eternal, and such as concern our blisse everlasting: which, otherwise (without special priviledg of Illumination, or Revelation from Heaven.) No mortal mans wit (naturally) is able to reach unto, or to compasse. And, verily, by my small Talent (from above) I am able to prove and testifie that the literal Text, and order of our divine Law, Oracles and Mysteries, require more skill in Numbers and Magnitudes: then (commonly) the Expositors have uttered: but rather onely (at the most) so warned: and shewed their own want therein. (To name any, is needlesse: and to note the places, is, here, no place; But if I be duly asked, my answer is ready.) And without the literal, Grammatical, Mathematical, or Natural verities of such places, by good and certain Art, perceived no Spirituall sense, proper to those places, by Absolute *Theologie*) will thereon depend. "No man, therefore, can doubt, but toward the atteining of knowledg incomparable, and Heavenly Wisdome: Mathematical speculations both of Numbers and Magnitudes: are means, aids, and guides: ready, certain, and necessary. From henceforth, in this my Preface, will I frame my talk, to *Plato* his fugitive Scholers: or, rather, to such, who well can, (and also will,) use their outward senses, to the glory of God, the benefit of their Countrey, and their own secret contentation, or honest preferment on this earthly Scaffold. To them, I will orderly recite, describe, and declare a great Number of Arts, from our two Mathematical fountains, derived into the fields of *Nature*. Whereby such seeds, and roots, as lie deep hid in the ground of *Nature*, are refreshed, quickned, and provoked to grow, shoot up, flour, and give fruit, infinite and incredible. And these Arts shall be such, as upon magnitudes properties do depend more than upon Number. And by good reason we may call them Arts, and Arts Mathematical Derivative, for (at this time) I define An Art, to be a Methodical compleat Doctrine, having abun-

J. D. Herein I would gladly shake off, the earthly name of Geometry.

☞

An Art.

abundance of sufficient and peculiar matter to deal with; by the allowance of the Metaphysical Philosopher: the knowledge whereof, to humane state is necessary. And that I account an Art Mathematical derivative, which by Mathematical demonstrative Method, in Numbers or Magnitudes, ordereth and confirmeth his doctrine, as much and as perfectly as the matter subject will admit. And for that I intend, to use the name and propertie of a *Mechanician*, otherwise, then (hitherto it hath been used, I think it good (for distinction sake) to give you also a brief description, what I mean thereby. A Mechanician or a Mechanical workman, is he whose skill is without knowledg of Mathematical demonstration, perfectly to work and finish any sensible work, by the Mathematician principal or derivative, demonstrated or demonstrable. Full well I know, that he which inventeth or maketh these demonstrations, is generally called *A speculative Mechanician*, which differeth nothing from a *Mechanical Mathematician.* So, in respect of divers actions, one man may have the name of sundry Arts, as sometime of a Logician, sometimes, in the same manner otherwise handled) of Rethorician. Of these trifles, I make (as now, in respect of my Preface), small account: to file them for the fine handling of subtile curious disputers. In other places, they may command me, to give good reason: and here, I will not be unreasonable.

Art Mathematical Derivative.

A Mechanician.

First, then, from the purity, absolutenesse, and Immateriality of principal *Geometrie*, is that kind of *Geometrie* derived, which vulgarly is counted *Geometrie*, and is the Art of measuring sensible magnitudes, their just quantities and contents. I.
This, teacheth to measure either at hand: and the Practiser to be by the thing Measured: and so, by due applying of Compasse, Rule, Square, Yard, Ell, Perch, Pole, Line, Gauging rod, (or such like Instrument) to the Length, Plain; or Solid measured, * to be certified, 1.
either of the length, perimetrie, or distance lineal: and this is called: *Mecometrie.* Or * to 2.
be certified of the content of any plain Superficies: whether it be in ground Surveyed, Board, or Glasse measured, or such like thing: which measuring, is named, *Embadometrie.*
* Or else to understand the Solidity and content of any bodily thing: as of Timber and 3.
Stone, or the content of Pits, Ponds, Wels, Vessels, small and great, of all fashions. Where of Wine, Oil, Beer, or Ale Vessels, &c. the Measuring commonly hath a peculiar name and is called *Gauging.* And the general name of these Solid measures is *Stereometry.*
Or else, this *Vulgar Geometry*, hath consideration to teach the Practiser, how to measure 2.
things, with good distance between him and the thing measured: and to understand thereby, either * how farre, a thing seen (on land or water) is from the measurer: and this 1.
may be called *Apomecometrie:* Or, how high, or deep, above or under the level of the measurers standing, any thing is, which is seen on land or water, called *Hypsometry.* 2.
* Or, it informeth the measurer, how broad any thing is, which is in the measurers view: 3.
so it be on land and water situated: and may be called *Platometry.* Though I use here to condition the thing measured, to be on land or water situated: yet, know for certain, that the sundry height of Clouds, blazing Stars, and of the Moon, may (by these means) have *Note.*
their distances from the earth: and, of the blazing Stars and Moon, the solidity (as well as distances) to be measured: but because, neither these things are vulgarly taught, nor of a common Practiser so ready to be executed. I, rather let such measures be reckoned incident to some of our other Arts, dealing with things on high, more purposely, than this vulgar Land measuring Geometry doth: as in *Perspective* and *Astronomy*, &c.

Geometry Vulgar.

OF these Feats (farther applyed) is sprung the Feat of *Geodesie*, or Land measuring: more cunningly to measure and surveigh Land, Woods, and Waters, afar off. More cunningly, I say: But God knoweth (hitherto) in these Realms of *England* and *Ireland*, (whether through ignorance or fraud, I cannot tell in every particular) how great wrong and injury hath (in my time) been committed by untrue measuring and surveying of Land or wood, any way. And, this I am sure: that the Value of the difference, between the truth and such Surveyors, would have been able to have found (for ever) in each of our two Universities, and excellent Mathematical Reader: to each allowing (yearly) a hundred Marks of lawful money of this Realm: which indeed, would seem requisite, here to be had (though by other wayes provided for) as well as the famous University of *Paris* hath two

Note.

Mathematical Readers, and each two hundred French Crowns yearly, of the French Kings magnificent liberality onely. Now, again, to our purpose, returning :: Moreover, of the former knowledg Geometrical, are grown the skils of *Geography, Chorography, Hydrography*, and *Stratarithmetry.*

" GEOGRAPHIE teacheth ways, by which, in sundry forms, (as *Sphærike, Plain*, " or other) the Situation of Cities, Towns, Villages, Forts, Castles, Mountains, Woods, " Havens, Rivers, Creeks, and such other things, upon the out-face of the earthly Globle " (either in the whole, or in some principal member and portion thereof conteined) may " be described and designed, in commensurations Analogical to Nature and verity: and " most aptly to our view, may be represented. Of this Art how great pleasure, and how manifold commodities do come unto us daily and hourly: of most men is perceived. While some, to beautifie their Hals, Parlers, Chambers, Galeries, Studies, or Libraries with: other some, for things past, as battels fought, earth-quakes, heavenly firings, and such occurents, in Histories mentioned: thereby lively, as it were, to view the place, the Region adjoyning, the distance from us: and such other circumstances. Some other presently to view the large dominion of the Turk: the wide Empire of the Moscovite: and the little morsel of ground, where Christendome (by profession) is certainly known. Little, I say, in respect of the rest, &c. Some either, for their own journeys directing into far lands: or to understand of other mens travails. To conclude, some for one purpose, and some for another, liketh, loveth, getteth, and useth Maps, Charts, and Geographical Globes. Of whose use, to speak sufficiently, would require a book peculiar.

CHOROGRAPHIE, seemeth to be an underling, and a twig of *Geographie*: " and yet neverthelesse, is in practice manifold, and in use very ample. This teacheth " Analogically to describe a small portion or circuit of ground, with the contents: not " regarding what commensuration it hath to the whole, or any parcel, without it, con- " teined. But in the territory or parcel of ground, which it taketh in hand to make de- " scription of, it leaveth out (or undescribed) no notable, or odde thing, above the " ground visible. Yea, and sometimes, of things under ground, giveth some peculiar mark, " or warning: as of Metal-mines, Cole-pits, Stone quaries, &c. Thus a Dukedome, a Shire, " a Lordship, or lesse, may be described distinctly. But marvellous pleasant and profitable is it, in the exhibiting to our eye and commensuration, the plat of a City, Town, Fort, or Palace, in true Symmetry: not approaching to any of them: and out of Gun-shot, &c. Hereby the *Architect* may furnish himself, with store of what paterns he liketh: to his great instruction; even in those things which outwardly are proportioned: either simply in themselves, or respectively to Hils, Rivers, Havens, and Woods adjoyning. So also, term this particular description of places, *Topographie.*

" HYDROGRAPHIE, delivereth to our knowledg, on Globe or in Plain, the per- " fect Analogical description, of the Ocean, Sea-coasts, through the whole World, or " in the chief and principal parts thereof, with the Isles and chief particular places of " danger, conteined within the bounds and Sea-coasts described; as of Quick-sands, " Banks, Pits, Rocks, Races, Countertides, Whirle-pools, &c. This dealeth with the Element of the water chiefly: as *Geographie* did principally take the Element of the Earths description (with his appurtenances) to task. And besides this, *Hydrographie*, requireth a particular Register of certain Land-marks (where marks may be had) from the sea, well able to be skried, in what point of the Sea compasse they appear, and what apparent form, situation, and bignes they have, in respect of any dangerous place in the sea, or neer unto it, assigned: And in all Coasts, what Moon maketh full Sea, and what way, the Tides and Ebbs, come and go, the *Hydrographer* ought to record. The sounding likewise: and the Chanels wayes: their number, and depths ordinarily, at ebbe and floud, ought the *Hydrographer*, by observation and diligence of *Measuring*, to have certainly known, And many other points, are belonging to perfect *Hydrographie*, and for to make a *Rutter*, by: of which, I need not here speak: as of the describing, in any place, upon Globe or Plain, the 32 points of the Compasse, truly; (whereof, scarcely four, in *England*, have right knowledg, because, the lines thereof, are no strait lines, nor Circles.) Of making due projection of a Sphære in plain. Of the Variation of the Compasse, from true North. And such like matters (of great importance, all) I leave to speak of in this place, because, I may seem (already) to have enlarged the bounds and duty of an *Hydrographer*, much more than any man (to this day) hath noted or prescribed: yet am I well able to prove, all these

these things, to appertain, and also to be proper to the Hydrographer. The chief use and end of this Art, is the Art of Navigation, but it hath other divers uses, even by them to be enjoyed, that never lack sight of land.

STRATARITHMETRIE, is the skill (apperteining to the war,) by which a man can set in figure Analogical to any *Geometrical* figure appointed, any certain number or summe of men: of such a figure capable, (by reason of the usual spaces between Souldiers allowed: and for that, of men, can be made no Fractions: yet neverthelesse, he can order the given summe of men, for the greatest such figure, that of them can be ordered) and certifie of the overplus (if any be) and of the next certain summe, which, with the overplus, will admit a figure exactly proportional to the figure assigned. By which skill also, of any army or company of men; (the figures and sides of whose orderly standing, or array is known,) he is able to expresse the just number of men, within that figure conteined: or (orderly) able to be conteined. * And this figure, and sides thereof, he is able to know: either by, and at hand, or a farre off. Thus farre stretcheth the description " and property of *Stratarithmetrie*: sufficient for this time and place. It differeth from " the feat *Tactical, De aciebus instruendis*, because, there is necessary the wisedome and " foresight, to what purpose he so ordereth the men, and skilful ability, also, for any occa- " sion, or purpose, to devise and use the aptest and most necessary order, array and " figure of his Company and sum of men. By figure, I mean, as either of a *Perfect Square*, " *Triangle, Circle, Ovale, long square*, (of the Greeks it is called *Eteromekes*) *Rhombe, Rhomboid, Lunular, Ring, Serpentine*, and such other Geometrical figures: Which in wars, have been, and are to be used for commodiousnesse, necessity and advantage, &c. And no small skill ought he to have, that should make true report, or neer the truth of the numbers and summes, of Footmen or Horsemen, in the Enemies ordering. A farre off, to make an estimate, between neer terms of More and Lesse, is not a thing very rise, among those that gladly would doe it. Great policy may be used of the Captains, (at times fit, and in places convenient) as to use figures, which make greatest shew, of so many as he hath: and using the advantage of the three kinds of usual spaces: (between Footmen and Horsmen) to take the largest: or when he would seem to have few, (being many:) contrariwise in figure and space. The Herald, Pursuivant, Serjeant Royal, Captain, or whosoever is careful to come neer the truth herein, besides the judgment of his expert eye, his skill of ordering *Tactical*, the help of his Geometrical Instrument: Ring or Staffe Astronomical: (commodiously framed for carriage and use.) He may wonderfully help himself by perspective Glasses. In which, (I trust) our posterity will prove more skilful and expert, and to greater purposes, than in these dayes, can (almost) be credited to be possible.

** Note.* The difference between Stratarithmetry, and Tracticie.

J. D. Friend, You will find it hard; to perform my description of this feat. But by Chorography, you may help your self somewhat: where the figures known (in Sides and Angles) are not regular: And where, Resolution, into Triangles can serve, &c. & yet you will finde it strange to deal thus generally with Arithmetical figures: and, that for Battail ray. Their Contents differ so much from like Geometrical figures.

Thus have I lightly passed over the Artificial Feats, chiefly depending upon vulgar *Geometry*: and commonly, and generally reckoned under the name of *Geometry*. But there are other (very many) *Methodical Arts*, which declining from the purity, simplicity, and Immateriality, of our Principal Science of *Magnitudes*: do yet neverthelesse, use the great ayd, direction, and method of the said principal Science, and have proper names, and distinct: both from the Science of *Geometry*, (from which they are derived) and one from the other. As PERSPECTIVE, ASTRONOMY, MUSICK, COSMOGRAPHIE, ASTROLOGY, STATIKE, ANTHROPOGRAPHIE, TROCHILIKE, HELICOSOPHIE, PNEUMATITHMIE, MENADRIE, HYPOGEIODIE, HYDRAGOGIE, HOROMETRIE, ZOGRAPHIE, ARCHITECTURE, NAVIGATION, THAUMATURGIKE, and ARCHIMASTRIE. I think it necessary, orderly, of these to give some peculiar descriptions: and withall, to touch some of their commodious uses, and so to make this Preface, to be a little sweet, pleasant Nosegay for you, to comfort your Spirits, being almost out of courage, and in despair; (through brutish brute.) Weening that *Geometrie*, had but served for building of an house, or a curious bridg, or the roof of Westminster hall, or some witty pretty divise, or engin, appropriate to a Carpenter, or Joyner, &c. That the thing is far otherwise, than the world, (commonly) to this day, hath deemed, by word and work, good proof will be made.

Among these Arts, by good reason, PERSPECTIVE ought to be had, e're of *Astronomical Apparences*, perfect knowledge can be atteined. And because of the prerogative of *Light*, being the first of *Gods Creatures*: and the eye, the light of our body, and

and his sense most mighty, and his organ most artificial, and *Geometrical.* At *Perspective*, we will begin therefore. **Perspective is an Art Mathematical, which demonstrateth the manner and properties of all Radiations, Direct, Broken, and Reflected.** This Description, or Notation, is brief: but it reacheth so far, as the World is wide. It concerneth all Creatures, all Actions, and passions, by Emanation of beams performed. Beams or natural lines, (here) I mean, not of light onely, or of colour (though they, to eye, give shew, witnesse, and proof, whereby to ground the art upon) but also of other *Forms*, both *Substantial* and *Accidental*, the certain and determined active Radial Emanations. By this Art (omitting to speak of the highest points) we may use our eyes, and the light with greater pleasure, and perfecter judgment: both, of things, in light seen, and of other: which by like order of lights Radiations, work and produce their effects. We may be ashamed to be ignorant of the cause, why so sundry ways our eye is deceived, & abused: as, while the eye weeneth a round Globe or Sphere (being far off) to be a flat and plain circle, and so likewise judgeth a plain Square to be a round: supposeth walls parallels, to approach, afar off: roof and floure parallels, the one to bend downward, the other to arise upward, at a little distance from you. Again, of things being in like swiftnesse of moving to think the neerer to move faster, and the farther much slower. Nay, of two things, whereof the one (incomparably) doth move swifter than the other, to deem the slower to move very swift, and the other to stand: What an errour is this, of our eye? Of the Rainbow, both of his Colours, of the order of the colours, of the bigness of it, the place and height of it, (&c.) to know the causes demonstrative, is it not pleasant, is it not necessary? of two or three Suns appearing: of blazing Stars: and such like things: by natural causes, brought to passe, (and yet, neverthelesse, of farther matter significative) it is not commodious for man to know the very true cause and occasion Natural? yea, rather is it not, greatly, against the Soveraignty of mans nature, to be so over-shot and abused with things (at hand) before his eyes? as with a Peacocks tail, and a Doves neck: or a whole ore, in water holden, to seem broken. Things far off to seem neer, and neer, to seem far off. Small things to seem great, and great to seem small. One man to seem an army. Or a man to be curstly afraid of his own shadow. Yea, so much, to fear, that if you being (alone,) neer a certain Glasse, proffer with dagger or sword, to foyne at the glasse, you shall suddenly be moved to give back (in manner) by reason of an Image appearing in the air, between you and the glasse, with like hand, sword or dagger, & with like quicknesse foyning at your very eye, likewise as you doe at the glasse. Strange, this is, to hear off, but more mervailous to behold, than these my words can signifie. And neverthelesse, by demonstration Optical, the order and cause thereof, is certified: even so as the effect is consequent. Yea, thus much more, dare I take upon me, toward the satisfying of the noble courage, that longeth ardently for the wisedome of causes Natural: as to let him understand, that, in *London*, he may with his own eye, have proof of that, which I have said herein. A Gentleman, (which, for his good service, done to his Countrey, is famous and honourable: and for skill in the Mathematical Sciences, and Languages, is the odde man of this land, &c.) even he is able: and (I am sure) will very willingly, let the glasse, and the proof be seen: and so I (here) request him: for the encrease of wisedome, in the honourable, and for the stopping of the mouths malicious; and repressing the arrogancy of the ignorant; ye may easily guesse, what I mean. This Art of *Perspective* is of that excellency, and may be led to the certifying, and executing of such things, as no man would easily believe: without Actual proof perceived. I speak nothing of Natural *Philosophy*, which, without *Perspective*, cannot be fully understood, nor perfectly atteined unto. Nor of *Astronomy*: which without *Perspective*, cannot well be grounded. Nor *Astrology*, naturally verified, and avouched. That part hereof, which dealeth with Glasses (which name Glasse, is a general name, in this Art, for any thing, from which a Beam reboundeth) is called *Catoptrike*, and hath so many uses, both marvellous, and profitable: that, both, it would hold me too long, to note therein the principal conclusions, already known: And also (perchance) some things might lack due credit with you: And I, thereby to lose my labour: and you, to slip into light judgment. * Before you have learned sufficiently the power of Nature and Art.

A marvailous glasse. ☞

S. W. P.

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Now

Now to proceed: ASTRONOMIE, is an Art Mathematical, which demonstrateth the distance, Magnitudes, and all natural Motions, apparences, and passions proper to the Planets and fixed Stars, for any time past, present, and to come: in respect of a certain Horizon, or without respect of any Horizon. By this Art we are certified of the distance of the starry Skie, and of each Planet, from the Center of the Earth, and of the greatnesse of any fixed Star seen, or *Planet*, in respect of the earths greatnesse. As, we are sure (by this Art) that the Solidity, Massinesse, and Body of the *Sun*, conteineth the quantity of the whole Earth, and Sea, a hundred threescore and two times, lesse by $\frac{1}{8}$ one eight part of the earth, but the Body of the whole earthly Globe and Sea, is bigger than the body of the Moon, three and fourty times lesse by $\frac{1}{8}$ of the *Moon*. Wherefore the *Sun* is bigger than the *Moon* 7000 times, lesse by $59\frac{39}{64}$ that is, precisely $6940\frac{25}{64}$ bigger than the *Moon*. And yet the unskilful man, would judg them a like big. Wherefore, of necessity, the one is much farther from us than the other. The *Sun* when he is farthest from the earth (which, now, in our age, is, when he is in the 8 degree, of *Cancer*) is 1179 Semidiameters of the Earth, distant. And the *Moon* when she is farthest from the earth, is 68 Semidiameters of the earth and $\frac{1}{3}$. The neerest, that the *Moon* commeth to the Earth, is Semidiameters $52\frac{1}{4}$. The distance of the starry skie is from us, in Semidiameters of the Earth $20081\frac{1}{2}$. Twenty thousand fourscore, one, and almost a half. Subtract from this, the *Moons* neerest distance, from the Earth: and thereof remaineth Semidiameters of the earth $20029\frac{1}{4}$. Twenty thousand nine and twenty and a quarter. So thick is the heavenly Palace, that the *Planets* have all their exercise in, and most mervailously perform the Commandement and Charge to them given by the Omnipotent Majesty of the King of Kings. This is that, which in *Genesis* is called *Ha Rakia*. Consider it well. The Semidiameter of the Earth, conteineth of our common miles $3436\frac{4}{11}$ three thousand, four hundred thirty six and four eleventh parts of one mile. Such as the whole Earth and Sea, round about, is 21600, one and twenty thousand six hundred of our miles. Allowing for every degree of the greatest circle, threescore miles. Now if you weigh well with your self but this little parcel of fruit *Astronomical*, as concerning the bignesse, distances of *Sun*, *Moon*, *Starry skie*, and the huge massinesse of *Ha Rakia* will you not find your consciences moved, with the Kingly Prophet, to sing the confession of Gods Glory, and say, **The Heavens declare the glory of God, and the Firmament [Ha Rakia] sheweth forth the works of his hands.** And so forth, for those five first staves of that kingly Psalm. Well, well, It is time for some, to lay hold on wisedom, and to judg truly of things: and not so to expound the Holy Word, all by Allegories: as to neglect the Wisdom, Power, and Goodnesse of God, in, and by his Creatures, and Creation to be seen and learned. By Parables and Analogies of whose natures and properties, the course of the Holy Scripture, also, declareth to us very many Mysteries. The whole Frame of Gods Creatures, (which is the whole world) is to us, a bright glasse: from which, by reflexion, reboundeth to our knowledg and perceiverance, Beams and Radiations: representing the Image of his infinite Goodnesse, Omnipotency, and Wisdom. And we thereby are taught and perswaded to glorifie our Creator, as God: and be thankful therefore. Could the Heathenists find these uses, of these most pure, beautiful and mighty Corporal Creatures: and shall we, after that the true *Sonne* of right wisenesse is risen above the *Horizon* of our temporal *Hemisphere*, and hath so abundantly streamed into our hearts the direct beams of his goodnesse, mercy, and grace: Whose heat all Creatures feel: Spiritual and Corporal: Visible and Invisible: Shall we (I say) look upon the *Heaven, Stars*, and *Planets*, as an Oxe, and an Asse doth: no further, careful or inquisitive, what they are: why were they created: how doe they execute that they were created for? Seeing, all Creatures were for our sake created: and both we, and they, created chiefly to glorifie the Almighty Creator: & that by all means to us possible. *Nolite ignorare* (saith *Plato in Epinomis*,) *Astronomiam sapientissimum quiddam esse.* **Be ye not ignorant, Astronomie to be a thing of excellent Wisedome.** *Astronomy* was to us from the beginning commended, and in manner commanded by God himself. In asmuch as he made the *Sun Moon* and *Stars* to be to us for *Signes*, and knowledge of Seasons, and for distinctions of days and years. Many words need not. But I wish, every man should weigh this word *Signes*. And besides that, conferre it also with the tenth Chapter of *Jeremie*. And though Some

Note.

some think, that there they have found a rod: Yet modest reason, will be indifferent judg,
who ought to be beaten therewith, in respect of our purpose. Leaving that: I pray you
understand this: that without great diligence of Observation, Examination and Calcula-
tion, their periods and courses (whereby Distinction of Seasons, Years, and new Moons
might precisely be known) could not exactly be certified. Which thing to perform, is that
Art, which we here have defined to be *Astronomie.* Whereby we may have the distinct
course of Times, Days, Years & Ages: aswel for consideration of Sacred Prophecies, accom-
plished in due time, foretold: as for high Mystical Solemnities holding: And for all
other humane affairs, Conditions, and Covenants upon certain time, between man and
man; with many other great uses: Wherein, (verily,) would be great incertainty, con-
fusion, untruth, and brutish Barbarousnesse: without the wonderful diligence and skill of
this Art: continually learning and determining Times, and periods of Time, by the Re-
cord of the heavenly book, wherein all times are written, and to be read with an *Astrono-
mical Staffe*, instead of a fescue.

MUSICK, of Motion, hath his Original cause. Therefore, after the Motions most
swift, and most slow, which are in the Firmament, of Nature performed: and under the
Astronomers Consideration: now I will speak of another kind of *Motion*, producing
sound, audible, and of man numerable. *Musick* I call here that *Science*, which of the
Greeks is called *Harmonice.* Not medling with the controversie between the ancient
Harmonists and *Canonists.* Musick is a Mathematical Science, which
teacheth by sense and reason, perfectly to judg, and order the di-
versities of sounds high and low. *Astronomy* and *Musick* are sisters, saith
Plato. As for *Astronomy*, the eyes: so for *Harmonious Motion*, the ears were made.
But as *Astronomy* hath a more divine contemplation and commodity, than mortal eye
can perceive: so is *Musick* to be considered, that the * Mind may be preferred before
1. the ear. And from audible sound, we ought to ascend, to the examination: which num-
bers are *Harmonious*, and which not. And why, either, the one are: or the other are not.
I could at large, in the heavenly * motions and distances, describe a marvellous Harmony
2. of *Pythagoras* Harp, with eight strings. Also somewhat might be said of * *Mercurius* two
3. 4. Harps, each of four strings Elemental. And very strange matter, might be alledged of the
Harmony, to our * spiritual part appropriate. As in *Ptolemaus* third book, in the fourth
5. and sixth Chapters may appear. * And what is the cause of the apt bond, and friendly fellow-
6. ship, of the Intellectual and Mental part of us, with our grosse and corruptible body, but a
certain Mean, and *Harmonious Spirituality, with both participating, and of both (in a
manner) resulting? In the * Tune of Mans voice, and also * the sound of Instrument,*
7. 8. what might be said of *Harmonie:* No common Musician would lightly believe. But of
7. D. the sundry mixture (as I may term it) and concourse, diverse collation and application of
these *Harmonies:* as of three, four, five or more: marvellous have the effects been: and
yet may be found and produced the like: with some proportional consideration for our
time and being: in respect of the state, of the things then: in which, and by which, the
wondrous effects were wrought. *Democritus* and *Theophrastus* affirmed that by *Musick*
griefs and diseases of the mind and body might be cured or inferred. And we find in Re-
cord, that *Terpander, Arion, Ismenias, Orpheus, Amphion, David, Pythagoras, Empe-
docles, Asclepiades*, and *Timotheus* by *Harmonical* Consonancy, have done and brought
to passe, things more than marvellous to hear of. Of them then, making no farther dis-
course in this place. Sure I am, that common *Musick*, commonly used, is found to the
Musicians and Hearers, to be so commodious and pleasant, That if I would say and dispute
but thus much: That it were to be otherwise used, then it is, I should find more reprievers,
then I could find privy or skilful of my meaning. In things therefore evident, and better
known, then I can express: & so allowed & liked of, (as I would wish some other things had
the like hap) I will spare to enlarge my lines any farther, but consequently follow my purpose.

Read in *Aristot.* his 8 book of Politicks the 5, 6, & 7 Chap. Where you shall have some occasion farther to think of Musick, then commonly is thought.

Of COSMOGRAPHIE, I appointed briefly in this place, to give you some in-
telligence. Cosmographie is the whole and perfect description of the
Heavenly, and also Elemental part of the World, and their Ho-
mologal application, and mutual collation necessary. This Art re-
quireth *Astronomy, Geographie, Hydrographie*, and *Musick.* Therefore, it is no small
Art, nor so simple, as in common practice, it is (slightly) considered. This matcheth Heaven,
and

and the Earth in one frame, and aptly applyeth parts correspondent: So, as, the Heavenly
Globe, may (in practise) be duly described upon the Geographical and Hydrographical
Globe. And there, for us to consider an *Æquinoctial Circle*, an *Ecliptique line, Colours,
Poles, Stars*, in their true Longitudes, Latitudes, Declinations, and Verticality: also
Climes and Parallels: and by an *Horizon* annexed, and revolution of the Earthly Globe
(as the Heaven, is by the *Primovant*, carried about in 24 equal hours) to learn the Risings
and Settings of Stars (of *Virgil* in his *Georgicks*, of *Hesiod:* of *Hippocrates* in his *Medi-
cinal Sphere, to Perdicca* King of the *Macedonians:* of *Diocles*, to King *Antigonus*,
and of other famous *Philosophers* prescribed) a thing necessary, for due manuring of the
earth, for *Navigation*, for the Alteration of mans body: being whole, sick, wounded, or
bruised, By the Revolution, also, or moving of the Globe Cosmographical, the Rising and
Setting of the Sun: the lengths of days and nights: the Hours and times (both night and
day) are known: with very many other pleasant and necessary uses: Whereof, some are
known: but better remain, for such to know and use: who of a spark of true fire, can
make a wonderful bonfire, by applying of due matter, duly.

Of ASTROLOGIE, here I make an Art, several from *Astronomy:* not by new
devise, but by good reason and authoritie: for, Astrologie is an Art Mathema-
tical, which reasonably demonstrateth the operations and effects,
of the natural beams, of light, and secret influence: of the Stars
and Planets: in every Element and elemental body: at all times
in any Horizon assigned. This Art is furnished with many other great Arts and
experiences: As with perfect *Perspective, Astronomy, Cosmographie, Natural Philoso-
phie*, of the four Elements, the Art of Graduation, and some good understanding in *Musick:*
and yet moreover, with another great Art, hereafter following, though I, here, set this before,
for some considerations me moving. Sufficient (you see) is the stuffe, to make this rare and
secret Art, of: & hard enough to frame to the conclusion Syllogistical: yet both the manifold
and continual travails of the most ancient and wise Philosophers, for the atteining of this
art: and by examples of effects, to confirm the same; hath left unto us sufficient proof and
witness, and we also daily may perceive, That mans body, and all other Elemental bodies
are altered, disposed, ordered, pleasured and displeasured, by the influential working of the
Sun, Moon, and the other *Stars* and *Planets.* And therefore, saith *Aristotle*, in the first of
his *Meteorological* books, in the second Chapter; *Est autem necessariò Mundus iste
supernis lationibus ferè continuus. Ut, inde, vis ejus Universa regatur. Ea siquidem
causa prima putanda omnibus est, unde motus principium existit.* That is: This
[Elemental] World is of necessity, almost, next adjoyning, to the
heavenly motions: That from thence all his vertue or force may
be governed. For that is to be thought the first Cause unto all:
from which, the beginning of motion, is. And again, in the tenth Chapter.
*Oportet igitur, & horum principia sumamus, & causas omnium similiter. Principium
igitur ut movens præcipuumque & omnium primum, Circulus ille est, in quo manifeste
Solis latio, &c.* And so forth. His *Meteorological* books, are full of arguments, and effe-
ctual demonstrations, of the vertue, operation, and power of the heavenly bodies, in and
upon the four Elements, and other bodies of them (either perfectly or unperfectly) com-
posed. And in his second book, *De generatione & corruptione:* in the tenth Chapter.
*Quo circa & prima latio, Ortus & Interitus causa non est: Sed obliqui Circuli latio:
ea namque & continua est, & duobus notibus fit.* In English thus. Wherefore the
uppermost motion is not the cause of Generation and Corruption,
but the motion of the Zodiack, for that both is continual, and is
caused of two movings. And in his second book, and second Chapter of his
Physicks. Homo namque generat hominem, atque Sol. For man (saith he) and
the Sun, are cause of mans generation. Authorities may be brought very
many; both of 1000, 2000, yea and 3600 years Antiquity; of great *Philosophers.*
Expert, Wise, and godly men, for that Conclusion: which, daily and hourly, we men
may discern and perceive by sense and reason. All beasts doe feel, and simply shew by
their actions and passions, outward and inward. All Plants, Herbs, Trees, Flowers and
Fruits, All finally, the Elements, and all things of the Elements composed, do give
Testi-

Testimony (as *Aristotle* saith) that their **Whole dispositions, vertues, and natural motions, depend of the Activity of the heavenly Motions and Influences. Whereby, besides the special order and form, due to every seed: and beside the Nature, proper to the individual Matrix, of the thing produced: What shall be the heavenly impression, the perfect and circumspect Astrologian hath to conclude?** Not onely (by *Apotelesmes*) τὸ ὅτι, but by Natural and Mathematical demonstration τὸ διότι. Whereunto, what Sciences are requisite (without exception) I partly have here warned: and in my *Propædeumes* (besides other matter there disclosed) I have Mathematically furnished up the whole Method. To this our age, not so carefully handled by any, that ever I saw, or heard of, I was (for * 21 years ago) by certain earnest disputations, of the Learned *Gerardus Mercator*, and *Antonius Gogava*, (and other) thereto so provoked: and (by my constant and invincible zeal to verity) in observations of heavenly Influences (to the minute of time,) than so diligent: and chiefly by the Supernatural influence, from the Star of *Jacob*, so directed: That any modest and sober Student, carefully and diligently seeking for the Truth, will both find and confesse, therein, to be the Verity, of these my words: and also become a reasonable Reformer, of three sorts of people: about these influential operations, greatly erring from the truth. Whereof the one is **Light Believers**, the other **Light Despisers**, and the third **Light Practisers**. The first, and most common sort, think the Heavens and Stars, to be answerable to any their doubts or desires: which is not so: and, indeed, they, too much over-reach, The second sort think no influential vertue (from the heavenly Bodies) to bear any sway in Generation and Corruption, in this Elemental World. And to the *Sun*, *Moon* and *Stars* (being so many, so pure, so bright, so wonderful big, so far in distance, so manifold in their motions, so constant in their periods, &c.) they assign a slight, simple office or two, and so allow unto them (according to their capacities) as much vertue, and power influential, as to the Signe of the *Sun*, *Moon*, and seven *Stars*, hanged up (for Signs) in *London*, for distinction of houses, and such grosse helps, in our wordly affairs, and they understand not (or will not understand) of the other workings, and vertues of the Heavenly *Sun*, *Moon* and *Stars*: not so much, as the Mariner or Husbandman: no, not so much, as the *Elephant* doth, as the *Cynocephalus*, as the *Porpentine* doth: nor will allow these perfect and incorruptible mighty bodies, so much virtual Radiation and Force, as they see in a little piece of a *Magnes stone*: which, at great distance, sheweth his operation. And perchance they think, the Sea and Rivers (as the Thames) to be some quick thing, and so to ebbe and flow, run in and out, of themselves, at their own fantasies. God help, God help. Surely these men come too short: and either are to dull: or wilfully blind: or, perhaps too malicious. The third man is the common and vulgar *Astrologian*, or Practiser, who being not duly, artificially and perfectly furnished: yet, either for vain glory, or gain: or like a simple Dolt, and blind Bayard, both in matter and manner erreth: to the discredit of the *Wary* and modest *Astrologian*: and to the robbing of those most noble corporal Creatures, of their Natural Vertue: being most mighty, most beneficial to all elemental Generation, Corruption and the appurtenances: and most Harmonious in their Monarchie: For which things being known, and modestly used: we might highly and continually glorifie God, with the princely Prophet, saying. **The Heavens declare the Glory of God: who made the Heavens in his wisedome: who made the Sun for to have dominion of the Day: the Moon and Stars to have dominion of the night: whereby day to day uttereth talk, and night to night declareth knowledge. Praise him, all ye Stars and Light.** Amen.

An. 1548 and 1549 in Lovain.

Note.

1.

2.

3.

In order, now followeth, of STATIKE, somewhat to say what we mean by that name: and what commodity, doth, on such Art, depend. **Statike is an Art Mathematical, which demonstrateth the causes of heavinesse of all things: and of motions and properties, to heavinesse and lightnesse belonging.** And forasmuch as, by the Bilanx, or Balance (as the chief sensible Instrument) Experience of these demonstrations may be had: we call this Art *Statike*: that is, *the Experiments of the Balance.* Oh, that men wist, what profit (all manner of ways) by this Art might grow, to the able examiner, and diligent practiser. "Thou onely, knowest all "things precisely (ô God) who hast made Weight and Balance, thy Judgment; who "hast created all things in *Number*, *Weight* and *Measure*: and hast weighed the "Mountains and Hils in a Balance: who hast peysed in thy hand, both Heaven and "Earth. We therefore warned by the Sacred Word, to consider thy Creatures: and by "that consideration, to win a glimps (as it were,) or shadow of perceiverance, that thy "wisedome, might, and goodnesse is infinite, and unspeakable, in thy Creatures declared: "and being farther advertised, by thy merciful goodnesse, that, three principal ways, "were of thee, used in Creation of all thy Creatures, namely, *Number*, *Weight*, and "*Measure*. And for as much as, of *Number* and *Measure*, the two Arts (ancient, fa-"mous, and to humane uses most necessary) are, already, sufficiently known and ex-"tant: This third key, we beseech thee (through thy accustomed goodnesse) that it may "come to the needful and sufficient knowledge, of such thy Servants, as in the work-"manship, would gladly find, thy true occasions (purposely of thee used) whereby we "glorifie thy name, and shew forth (to the weaklings in faith) thy wondrous Wisedome "and Goodness. *Amen.*

Marvel nothing at this pang (godly friend, you gentle and zealous Student.) Another day, perchance you will perceive, what occasion moved me. Here, as now, I will give you some ground, and withall some shew, of certain commodities, by this Art arising. And because this Art is rare, my words and practises might be too dark: unlesse you had some light holden before the matter: and that best will be, in giving you, out of *Archimedes* demonstrations, a few principal conclusions, as followeth.

I. The superficies of every Liquor, by it self consisting, and in quiet, is Spherical: the center whereof, is the same, which is the center of the Earth.

II. If Solid Magnitudes, being of the same bigness, or quantity, that any Liquor is, and having also the same Weight: be let down into the same Liquor, they will settle downward, so, that no part of them, shall be above the superficies of the Liquor: and yet, neverthelesse, they will not sink utterly down, or drown.

III. If any Solid Magnitude being Lighter than a Liquor, be let down into the same Liquor, it will settle down, so far into the same Liquor, that so great a quantity of that Liquor, as is the part of the Solid Magnitude, setled down into the same Liquor: is in Weight, equal, to the Weight of the whole Solid Magnitude.

IV. Any Solid Magnitude, Lighter then a Liquor, forced down into the same Liquor, will move upward, with so great a power, by how much, the Liquor having equal quantity to the whole Magnitude, is heavier than the same Magnitude.

V. Any Solid Magnitude, heavier than a Liquor, being let down into the same Liquor, will sinke down utterly: and will be in that Liquor, Lighter by so much, as is the weight or heaviness of the Liquor, having bignesse or quantity equal to the Solid Magnitude.

VI. If any Solid Magnitude, Lighter than a Liquor, be let down into the same Liquor, the weight of the same Magnitude, will be, to the Weight of the Liquor. (Which is equal in quantity to the whole Magnitude,) in that proportion, that the part, of the Magnitude setled down is to the whole Magnitude.

J. D. The cutting of a Sphere according to any proportion assigned, may by this propositiō be done Mechanically by tépering liquor to a certain weight in respect of the weight of the Sphere therein swimming.

By these verities, great errours may be reformed in opinion of the Natural Motion of things Light, and Heavy. Which errours are in Natural Philosophy (almost) of all men allowed: to much trusting to authority, and false Suppositions. As, **Of any two Bodies, the Heavier to move downward faster than the Lighter.** This errour is not first by me, Noted: but by one *John Baptist de Benedictis*. The chief of his Propositions, is this: which seemeth a Paradox.

A Paradox.

If there be two Bodies of one form, and of one kind, equal in quantity or unequal, they will move by equal space, in equal time: So that both their movings be in air, or both in water: or in any one middle.

A common errour noted.

Hereupon, in the feat of GUNNING, certain good discourses (otherwise) may receive great amendment and furtherance, In the intended purpose, also, allowing somewhat to the imperfection of Nature: not answerable to the precisenesse of demonstration. Moreover, by the foresaid propositions (wisely used.) The Air, the Water, the Earth, the Fire, may be neerly known, how light, or heavy they are (Naturally) in their assigned parts: or in the whole. And then to things Elemental, turning your practise: you may deal

N. T. The wonderful use of these Propositions.

deal for the proportion of the Elements, in the things Compounded. Then to the proportions of the Humours in man: their weights, and weight of his Bones, and Fleſh, &c. Then, by weight, to have conſideration of the force of man, any manner of way: in whole, or in part. Then may you of Ships water drawing, diverſly in the Sea and in freſh water, have pleaſant conſideration: and of weighing up of any thing, ſunken in Sea, or in freſh water, &c. And (to lift up your head aloft:) by weight, you may as preciſely, as by any Inſtrument elſe, meaſure the Diameters of *Sun* and *Moon*, &c. Friend, I pray you weigh theſe things with the juſt Balance of Reaſon. And you will find marvels upon marvels: and eſteem one Drop of Truth (yea, in natural Philoſophy) more worth than whole Libraries of Opinions undemonſtrated: or not anſwering to Natures Law, and your experience. Leaving theſe things, thus: I will give you two or three light practiſes to great purpoſe: and ſo finiſh my Annotation *Statical*. In Mathematical matters, by the Mechanicians aid, we will behold here the Commodity of weight. Make a Cube of any one Uniform: and through like heavy ſtuffe: of the ſame ſtuffe make a Sphere or Globe preciſely, of a Diameter equal to the Radical ſide of the Cube. Your ſtuffe, may be Wood, Copper, Tinne, Lead, Silver, &c. (being as I ſaid of like nature, condition, and like weight throughout.) And you may by Say Ballance have prepared a great number of the ſmalleſt weights: which by thoſe Balance can be diſcerned or tried: and ſo have proceeded to make you a perfect Pyle, company and number of weights: to the weight of ſix, eight, or twelve pound weight, moſt diligently tried, all and of every one, the Content known, in your leaſt weight that is weighable. [They that cannot have their weights of preciſeneſſe: may by Sand, uniform, and well duſted, make them a number of weights, ſomewhat neer preciſeneſſe: by halfing ever the Sand: they ſhall, at length, come to a leaſt common weight. Therein, I leave the farther matter, to their diſcretion, whom need ſhall pinch.] The *Venetians* conſideration of weight may ſeem preciſely enough: by eight deſcents progreſſional * halfing, from a grain: your Cube, Sphere, apt Balance, and convenient weights being ready: fall to work. * Firſt, weigh your Cube. Note the number of the weight. Weigh, after that, your Sphere. Note likewiſe, the Number of the weight. If you now find the weight of your Cube, to be to the weight of the Sphere, as 21 is to 11: Then you ſee, how the Machanician and *Experimenter*, without Geometry and Demonſtration, are (as neerly in effect) taught the proportion of the Cube to the Sphere: as I have demonſtrated in the end of the twelfth book of *Euclid*. Often try with the ſame Cube and Sphere. Then, change your Sphere and Cube to another matter: or to another bigneſs: till you have made a perfect univerſal Experience of it. Poſſible it is, that you ſhall winne to neerer termes, in the proportion.

The practiſe Statical, to know the proportion between the Cube and the Square.

J. D.
* For, ſo, have you 256 parts of a grain.

When you have found this one certain Drop of Natural verity, proceed on, to inferre, and duly to make aſſay, of matter depending. As becauſe it is well demonſtrated, that a Cylinder, whoſe height, and Diameter of his baſe, is equal to the Diameter of the Sphere, is *Seſquialter* to the ſame Sphere, (that is, as 3, to 2.) To the number of the weight of the Sphere, adde half ſo much, as it is: and ſo have you the number of the weight of that Cylinder. Which is alſo comprehended of our former Cube: ſo, that the baſe of that Cylinder, is a Circle deſcribed in the ſquare, which is the baſe of our Cube. But the Cube and Cylinder, being both of one height, have their Baſes in the ſame proportion, in the which, they are one to another in their maſſineſſe or ſolidity. But before, we have two numbers, expreſſing their maſſineſſe, ſolidities, and quantities, by weight: wherefore, we have * the proportion of the ſquare, to the Circle, inſcribed in the ſame ſquare. And ſo are we fallen into the knowledge ſenſible and experimental of *Archimedes* great ſecret, of him, by great travail of mind, ſought and found. Wherefore to any Circle given, you can give a ſquare equal: * as I have taught in my Annotation, upon the firſt propoſition of the twelfth book: and likewiſe to any ſquare given, you may give a Circle equal: * If you deſcribe a Circle, which ſhall be in that proportion to your Circle inſcribed, as the ſquare is to the ſame Circle: This, you may do, by my Annotations, upon the ſecond propoſition of the twelfth book of *Euclide*, in my third Problem there. Your diligence may come to a proportion of the ſquare to the Circle inſcribed neerer the truth, than is the proportion of 14 to 11: and conſider, that you may begin at the Circle and Square, and ſo come to conclude of the Sphere and the Cube, what their proportion is: as now, you came from the Sphere to the Circle. For of Silver or Gold, or Latton Lamyns, or plates, (thorough one hole drawn, as the manner is,) if you make a ſquare figure, and weigh it: and then, deſcribing thereon, the

* The proportion of the Square, to the Circle inſcribed.
* The Squaring of the Circle Mechanically.
* To any Square given, to give a Circle equal.

the Circle inſcribed: and cut off, and file away, preciſely (to the Circle) the overplus of the Square: you ſhall then, weighing your Circle, ſee, whether the weight of the Square, be to your Circle, as 14 to 11, as I have noted, in the beginning of *Euclids* twelfth book, &c. after this reſort to my laſt propoſitions, upon the laſt of the twelfth: and there, help your ſelf, to the end, and, here, Note this, by the way. That we may ſquare the Circle, without having knowledge of the proportion of the Circumference to the Diameter: as you have here perceived. And otherwiſe alſo, I can demonſtrate it. So that many have cumbered themſelves ſuperfluouſly, by travailing in that point firſt, which was not of neceſſity: firſt: and alſo very intricate. And eaſily you may, (and that diverſly) come to the knowledge of the circumference: the Circles quantity, being firſt known. Which thing I leave to your conſideration: making haſte to diſpatch another Magiſtral Problem: and to bring it neerer to your knowledge, and readier dealing with, than the world (before this day,) had it for you, that I can tell off. And that is, *A Mechanical doubling of the Cube, &c.* Which may thus be done: **Make of Copper-plates, or Tin-plates a four ſquare upright Pyramis or a Cone: perfectly faſhioned in the hollow, within. Wherein, let great diligence be uſed, to approach as neer as may be) to the Mathematical perfection of thoſe figures. At their baſes: let them be all open: every where, elſe, moſt cloſe, and juſt to. From the vertex, to the Circumference of the baſe of the Cone: and to the ſides of the baſe of the Pyramis: Let four ſtraight lines be drawn, in the inſide of the Cone and Pyramis: making at their fall, on the perimeters of the baſes, equal angles, on both ſides themſelves, with the ſaid perimeters. Then four lines (in the Pyramis, and as many in the Cone) divide one, in 12 equal parts; and another in 24, another in 60, and another in 100, (reckoning up from the vertex.) Or uſe other numbers of diviſion, as experience ſhall teach you. Then * ſet your Cone or Pyramis with the vertex downward, perpendicularly, in reſpect of the baſe, (though it be otherwiſe, it hindereth nothing.) So let them moſt ſtedily be ſtayed.** Now if there be a Cube, which you would have doubled. Make you a prety Cube of Copper, Silver, Lead, Tinne, Wood, Stone, or Bone. Or elſe make a hollow Cube or Cubick coffen, of Copper, Silver, Tinne, or Wood, &c. Theſe you may ſo proportion in reſpect of your Pyramis or Cone, that the Pyramis or Cone, will be able to contein the weight of them, in water three or four times at the leaſt: what ſtuffe ſo ever they be made of. Let not your ſolid angle at the vertex, be too ſharp: but that the water may come with eaſe, to the very vertex, of your hollow Cone or Pyramis. Put one of your ſolid Cubes in a Balance apt: take the weight thereof exactly in water. Poure that water, (without loſſe) into the hollow Pyramis or Cone quietly. Mark in your lines, what numbers the water cutteth: Take the weight of the ſame Cube again: in the ſame kind of water, which you had before: put that * alſo, into the Pyramis or Cone, where you did put the firſt. Mark now again, in what number or place of the lines, the water cutteth them. Two wayes you may conclude your purpoſe: it is to wit, either by numbers or lines. By numbers: as, if you divide the ſide of your Fundamental Cubes into ſo many equal parts, as it is capable of, conveniently, with your eaſe, and preciſeneſſe of the diviſion. For, as the number of your firſt and leſs line (in your hollow Pyramis or Cone) is to the ſecond or greater (both being counted from the vertex) ſo ſhall the number of the ſide of your Fundamental Cube, be to the number belonging to the Radical ſide of the Cube, double to your Fundamental Cube: Which being multiplyed Cubick wiſe, will ſoon ſhew it ſelf, whether it be double or no, to the Cubick number of your Fundamental Cube. By lines, thus: As your leſſe and firſt line (in your hollow Pyramis or Cone) is to the ſecond or greater, ſo let the Radical ſide of your Fundamental Cube, be to a fourth proportional line, by the 12 propoſition of the ſixth book of *Euclid*. Which fourth line, ſhall be the Root Cubick, or Radical ſide of the Cube, double to your Fundamental Cube: which is the thing we deſired. For this, may I (with joy) ſay, ΕΥΡΗΚΑ, ΕΥΡΗΚΑ, ΕΥΡΗΚΑ: thanking the Holy and glorious Trinity, having greater cauſe thereto, then * *Archimedes* had (for finding the fraud uſed in the Kings Crown of Gold:) as all men may eaſily judg: by

Note. Squaring the Circle, without knowing of the proportion between circumference and Diameter.

To double the Cube readily, by Art Mechanical, depending upon demonſtration Mathematical.

J. D.
The 4 ſides of this Pyramis muſt be 4 Iſoſceles triangles alike & equal.

J. D.
* In all workings with this Pyramis or Cone, let their Situations be in all points & conditions, alike, or all one: while you are about one work. Elſe you will erre.

J. D.
* Conſider well when you muſt put your waters together: & when you muſt empty your firſt water, out of your Pyramis, or Cone. Elſe you will erre.

* *Vitruvius* lib. 9. cap. 3.

God be thanked for this invention and the fruit enſuing.

by the diverſity of the fruit following of the one and of the other. Where I ſpeak before of a hollow Cubick Coffin: the like uſe is of it, and without weight. Thus. Fill it with water preciſely full, and pour that water into your Pyramis or Cone: and here note the lines cutting in your Pyramis or Cone. Again, fill your Coffin like as you did before. Put that water, alſo to the firſt. Mark the ſecond cutting of your lines. Now, as you proceeded before, ſo muſt you here proceed. * And if the Cube, which you double, be never ſo great: you have, thus, the proportion (in ſmall) between your two little Cubes: And then, the ſide of that great Cube (to be doubled) being the third, will have the fourth, found, to it proportional: by the 12 of the 6 of *Euclide.*

Note.

* Note, as concerning the Spherical ſuperficies of the water.

Note, that all this while, I forget not my firſt Propoſition Statical, herehearſed: that, the Superficies of the water, is Spherical. Wherein uſe your diſcretion to the firſt line, adding a ſmall hair breadth, more: and to the ſecond half a hair breadth more, to his length. For you will eaſily perceive, that the difference can be no greater, in any Pyramis or Cone, of you to be handled. Which you ſhall thus try. *For finding the ſwelling of the water above level.* " Square the Semidiameter, from the Center of the Earth, to your " firſt waters ſuperficies. Square then, half the ſubtendent of that watry ſuperficies: " (which ſubtendent muſt have the equal parts of his meaſure, all one, with thoſe of the " Semidiameter of the Earth, to your watry ſuperficies:) ſubtract this ſquare from the " firſt: Of the reſidue, take the Root ſquare. That Root, ſubtract from your firſt Semi- " diameter of the Earth to your watry ſuperficies: that which remaineth, is the height of " the water, in the middle, above the Level. Which you will find, to be a thing inſenſible: and though it were greatly ſenſible, * yet, by help of my ſixth Theoreme upon the laſt Propoſition of *Euclides* twelfth book, noted: you may reduce all to a true Level. But farther diligence of you is to be uſed, againſt accidental cauſes of the waters ſwelling: as by having (ſomewhat) with a moiſt ſponge, before, made moiſt your hollow Pyramis or Cone, will prevent an accidental cauſe of ſwelling, &c. Experience will teach you abundantly: with great eaſe, pleaſure, and commodity.

* *Note.*

Note this Abridgment of doubling the Cube, &c.

Thus, may you double the Cube Mechanically, Treble it, and ſo forth, in any proportion. Now will I abridg your pain, coſt, and care herein. Without all preparing of your Fundamental Cubes: you may (alike) work this concluſion. For that was rather a kind of Experimental Demonſtration, than the ſhorteſt way; and all upon one Mathematical Demonſtration depending. Take water (as much as conveniently will ſerve your turn, as I warned before of your Fundamental Cubes bigneſſe.) Weigh it preciſely. Put that water into your Pyramis or Cone. Of the ſame kind of water, then take again, the ſame weight you had before; put that likewiſe into the Pyramis or Cone; for in each time your marking of the lines how the water doth cut them, ſhall give you the proportion between the Radical ſides, of any two Cubes, whereof the one is double to the other, working as before I have taught you: * ſaving that for your Fundamental Cube his Radical ſide: here, you may take a right line, at pleaſure.

Note *

To give Cubes one to the other in any Proportion Rational or Irrational.

Yet farther proceeding with our Drop of Natural truth: **You may now give Cubes one to the other in any proportion given, Rational or Irrational.** On this manner. Make a hollow Parallelipipedon of Copper or Tinne: with one Baſe wanting, or open, as in our Cubick Coffin. From the bottom of that Parallelipipedon, raiſe up, many perpendiculars, in every of his four ſides. Now if any proportion be aſſigned you, in right lines: " Cut one of your perpendiculars (or a line equal to it, or leſſe " than it) likewiſe: by the 10 of the 6 of *Euclide.* And thoſe two parts, ſet in two ſundry " lines of thoſe perpendiculars (or you may ſet them both, in one line) making their be- " ginnings to be, at the Baſe: and ſo their lengths to extend upward. Now, ſet your " hollow Parellelipipedon, upright, perpendicularly, ſteady. Pour in water handſomely, " to the height of your ſhorter line. Pour that water, into the hollow Pyramis or Cone. " Mark the place of the riſing. Settle your hollow Parallelipipedon again. Pour water " into it: unto the height of the ſecond line, exactly. Pour that water * duly into the " hollow Pyramis or Cone: mark now again, where the water cutteth the ſame line, " which you marked before. For, there, as the firſt marked line, is to the ſecond: So " ſhall the two Radical ſides be, one to the other, of any two Cubes: which in their ſo- " lidity, ſhall have the ſame proportion which was at the firſt aſſigned: were it Rational " or Irrational.

* Emptying the firſt.

Thus

Thus, in ſundry wayes you may furniſh your ſelf with ſuch ſtrange and profitable matter: which long hath been wiſhed for. And though it be Naturally done, and Mechanically: yet hath it a good Demonſtration Mathematical. Which is this: Alwayes, you have two like Pyramids: or two like Cones, in the proportions aſſigned: and like Pyramids or Cones, are in proportion, one to the other, in the proportion of their Homologal ſides (or lines) tripled. Wherefore, if to the firſt, and ſecond lines, found in your hollow Pyramis or Cone, you joyn a third and a fourth in continual proportion, that fourth line ſhall be to the firſt, as the greater Pyramis or Cone is to the leſſe: by the 33 of the eleventh of *Euclid.* If Pyramis to Pyramis, or Cone to Cone, be double, then ſhall * Line to Line, be alſo double, &c. But as our firſt line, is to the ſecond, ſo is the Radical ſide of our Fundamental Cube, to the Radical ſide of the Cube to be made, or to be doubled: and therefore, to thoſe twain alſo, a third and a fourth line, in continual proportion, joyned: will give the fourth line in that proportion to the firſt, as our fourth Pyramidal, or Conick line was to his firſt: but that was double or treble, &c. as the Pyramids or Cones were, one to another (as we have proved) therefore, this fourth, ſhall be alſo double or treble to the firſt, as the Pyramids or Cones were one to another: But our made Cube, is deſcribed of the ſecond in proportion of the four proportional lines: therefore * as the fourth line is to the firſt, ſo is that Cube to the firſt Cube: and we have proved the fourth line to be the firſt, as the Pyramis or Cone is to the Pyramis or Cone: Wherefore the Cube is to the Cube, as Pyramis to Pyramis, or Cone is to Cone. But we * ſuppoſe Pyramis to Pyramis, or Cone to Cone, to be double or treble, &c. Therefore Cube is to Cube, double or treble, &c. Which was to be demonſtrated. And of the Parallelipipedon, it is evident, that the water ſolid Parallelipipedons, are one to another, as their heights are, ſeeing they have one baſe. Wherefore the Pyramids or Cones made of thoſe water Parallelipipedons, are one to the other, as the lines are (one to the other) between which, our proportion was aſſigned. But the Cubes made of lines, after the proportion of the Pyramidal or Conick *Homologal* lines are one to the other, as the Pyramids or Cones are, one to the other (as we before did prove) therefore, the Cubes made, ſhall be one to the other, as the lines aſſigned, are one to the other: which was to be demonſtrated. Note. * This my demonſtration is more general, than onely in Square Pyramis or Cone: Conſider well. Thus, have I, both Mathematically, and Mechanically, been very long in words, yet (I truſt) nothing tedious to them, who, to theſe things, are well affected. And verily I am forced (avoiding prolixity) to omit ſundry ſuch things, eaſie to be practiſed: which to the Mathematician, would be a great Treaſure: and to the Mechanician no ſmall gain. * Now may you, **Between two lines given, find two middle proportionals in continual proportion: by the hollow Parallelipipedon, and the hollow Pyramis, or Cone.** Now any Parallelipipedon rectangle being given: three right lines may be found proportional in any proportion aſſigned; of which ſhall be produced a Parallelipipedon, equal to the parallelipipedon given. Hereof, I noted ſomewhat upon the 36 propoſition, of the 11 book of *Euclide.* Now, all thoſe things, which *Vitruvius* in his Architecture, ſpecified, able to be done, by doubling of the Cube. Or, by finding of two middle proportional lines, between two lines given, may eaſily be performed. Now, that Probleme, which I noted unto you, in the end of my Addition, upon the 34 of the 11 book of *Euclide*, is proved poſſible. Now may any regular body be transformed into another, &c. Now, any regular body, any Sphere, yea any mixt Solid: and (that more is) irregular Solids, may be made (in any proportion aſſigned) like unto the body firſt given. Thus, of a *Manneken*, (as the *Dutch* Painters term it) in the ſame *Symmetry*, may a Giant be made: and that, with any geſture, by the *Manneken* uſed: and contrariwiſe. Now, may you, of any Mould or Model of a Ship, make one, of the ſame Mould (in any aſsigned proportion) bigger or leſſer. Now, may you, of any * Gun, or little piece of Ordnance, make another, with the ſame *Symmetrie* (in all points) as great, and as little, as you will. Mark that, and think on it. Infinitely, **may you apply this, ſo long ſought for, and now ſo eaſily concluded: and withall, ſo willingly and frankly communicated to ſuch, as faithfully deal with vertuous ſtudies.** Thus can the Mathematical mind, deal Speculatively in his own Art: and by good means, mount above the clouds and ſtars: And thirdly, he can by order, deſcend, to frame Natural things, to wonderful uſes: and when he liſt, retire home

The Demonſtrations of this doubling of the Cube, and of the reſt.

J. D.

* Hereby help your ſelf to become a preciſe practiſer. And ſo conſider how nothing at all, you are hindred (ſenſible) by the convexity of the water.

* By the 33 of the eleventh Book of *Eucli.*

J. D.

* And your diligence in practice can ſo (in weight of water) perform it: Therefore, now, you are able to give good reaſon of your whole doing.

* Note this Corallary.

* The great Commodities following of theſe new Inventions.

*

Such is the fruit of the Mathematical Sciences and Arts.

home into his own Center: and there, prepare more means, to Ascend or Descend by: and all to the glory of God, and our honest delectation in Earth.

Although, the Printer, hath looked for this Preface, a day or two, yet could I not bring my pen from the paper, before I had given you comfortable warning and brief instructions, of some of the commodities by *Staticke*, able to be reaped: In the rest, I will therefore, be as brief, as it is possible: and withall describing them somewhat accordingly. And that, you shall perceive, by this, which in order commeth next. For whereas it is so ample and wonderful, that an whole year long, one might find fruitful matter therein, to speak of and also in practice is a Treasure endlesse: yet will I glanse over it, with words very few.

This do I call ANTHROPOGRAPHIE. Which is an Art restored, and of my preferment to your service. I pray you think of it, as of one of the chief points of Humane knowledg. Although it be but now first confirmed, with this new name: yet the matter, hath from the beginning, been in consideration of all perfect Philosophers. **Anthropographie is the description of the Number, Measure, Weight, Figure, Situation and Colour of every diverse thing, conteined in the perfect body of MAN with certain knowledge of the Symmetrie, Figure, Weight, Characterization, and due local motion of any parcel of the said Body, assigned: and of numbers, to the said parcel appertaining.** This is one part of the Definition, meet for this place: Sufficient to notifie, the particularity, and excellency of the Art: and why it is, here, ascribed to the Mathematicals. If the description of the heavenly part of the World had a peculiar Art, called *Astronomy*. If the description of the earthly Globe, hath his peculiar Art, called *Geographie*. If the matching of both hath his peculiar Art, called *Cosmographie*. Which is the description of the whole and universal frame of the World: Why should not the description of him, who is the lesse World and from the beginning called *Microcosmus*, (that is, **The lesse World**. And for whose sake and service, all bodily creatures else, were created: Who also participateth with Spirits and Angels, and is made to the Image and similitude of God, have his peculiar Art? and be called the *Art of Arts*: rather than either to want a name, or to have too base and improper a name? You must of sundry professions, borrow or challenge home, peculiar parts hereof: and farther proceed: as God, Nature, Reason and Experience shall inform you. The *Anatomists* will restore to you, some part: *Physiognomists* some: The *Chyromantists* some. The *Metaposcopists* some. The Excellent *Albert Durer*, a good part: the Art of Perspective, will somewhat, for the eye help forward: *Pythagoras*, *Hypocrates*, *Plato*, *Galenus*, *Meletius*, and many other (in certain things) will be Contributaries. And farther, the Heaven, the Earth, and all other Creatures, will each shew, and offer their Harmonious service, to fill up, that, which wanteth hereof: and with your own Experience, concluding: you may Methodically register the Whole, for the posterity: Whereby, good proof will be had, of our Harmonious, and Microcosmical constitution. The outward Image and view hereof, to the Art of *Zographie*, and painting, to Sculpture, and Architecture, (for Church, House, Fort, or Ship) is most necessary and profitable: for that, it is the chief base and foundation of them, Look in * *Vitruvius*, whether I deal sincerely, for your behoof, or no. Look in *Albertus Durerus de Symmetria humani Corporis*. Look in the 27 and 28 chapters, of the 2 Book, *De occulta Philosophia*. Consider the *Ark* of *Noe*. And by that, wade farther, Remember the *Delphical Oracle*, NOSCE TEIPSUM (**know thy self**) so long ago pronounced: of so many a Philosopher repeated: and of the *Wisest* attempted: And then you will perceive how long ago, you have been called to the School, where this Art might be learned. Well. I am nothing affraid, of the disdain of some such, as think Sciences and Arts, to be but seven, perhaps, those such, may, with ignorance, and shame enough, come short of them seven also: and yet neverthelesse, they cannot prescribe a certain number of Arts: and in each certain unpassible bounds to God, Nature, and mans Industry. New Arts daily rise up: and there was no such order taken, that, all Arts, should in one age, or in one land, or of one man, be made known to the world. Let us embrace the gifts of God, and ways to wisedome, in this time of grace, from above, continually bestowed on them, who thankfully will receive them: *Et bonis omnia Co-operabuntur in bonum.*

MAN is the lesse World.

☞ * Microcosmus.

* *Lib. 3. cap. 1.*

☞

TRO-

TROCHILIKE, is **that Art Mathematical, which demonstrateth the properties of all Circular Motions, Simple and Compound.** And because the fruit hereof vulgarly received, is the Wheels, it hath the name of *Trochilike*: as a man would say *Wheel Art*. By this Art, a Wheel may be given, which shall move once about, in any time assigned. Two Wheels may be given, whose turnings about in one and the same time (or equal times) shall have one to the other, any proportion appointed. By Wheels, may a streight line be described: Likewise, a Spiral line in plain, Conical Section lines, and other Irregular lines at pleasure may be drawn. These and such like are principal Conclusions of this Art: and help forward many pleasant and profitable Mechanical works: As Mils, to saw great and very long Deal-boards, no man being by. Such have I seen in *Germany*, and in the City of *Prague*, in the kingdome of *Bohemia*: Coyning Mils, Hand Mils for Corn grinding: And all manner of Mils and Wheel work: By Wind, Smoak, Water, Weight, Spring, Man or Beast moved. Take in your hand *Agricola de re Metallica*: and then shall you (in all Mines) perceive how great need is, of Wheel work. By Wheels, strange works and incredible are done: as will, in other Arts hereafter appear. A wonderful example of farther possibility, and present Commodity was seen in my time, in a certain Instrument: which by the Inventer and Artificer (before) was sold for twenty Talents of Gold: and then had (by misfortune) received some injury and hurt. And one *Janellus* of *Cremona* did mend the same, and presented it unto the Emperour *Charls* the Fifth. *Hieronymus Cardanus*, can be my witnesse, that therein, was one Wheel, which moved, and that in such rate, that, in 7000 years onely, his own period should be finished. A thing almost incredible: But how farre, I keep me within my bounds: very many men (yet alive) can tell.

Saw Mils.

HELICOSOPHIE, is neer Sister to *Trochilike*, and is **An Art Mathematical which demonstrateth the designing of all Spiral lines in Plain, on Cylinder, Cone, Sphere, Conoid, and Sphearoid, and their properties appertaining.** The use hereof in *Architecture*, and diverse Instruments and Engines, is most necessary. For, in many things, the Scrue worketh the feat, which else, could not be performed. By help hereof, it is * recorded, that, where all the power of the City of *Syracusa*, was not able to move a certain Ship (being on ground) mighty *Archimedes*, setting to his Scruish Engine, caused *Hiero* the King, by himself, at ease, to remove her, as he would. Whereat the King wondring: Ἀπὸ ταύτης τῆς ἡμέρας, περὶ παντὸς, Ἀρχιμήδει λέγοντι πιστευτέον. *From this day, forward*, (said the King) *Credit ought to be given to* Archimedes, *whatsoever he saith.*

* *Athenæus lib. 5. cap. 8.*

Proclus pag. 18

PNEUMATITHMIE, **demonstrateth by close hollow Geometrical Figures (regular and irregular) the strange properties (in motion or stay) of the Water, Air, Smoak, and Fire, in their continuity, and as they are joyned to the Elements next them.** This Art, to the Natural Philosopher, is very profitable, to prove that *Vacuum*, or *Emptinesse*, is not in the world. And that, all Nature, abhorreth it so much, that, contrary to ordinary law, the Elements will move, or stand. As, water to ascend rather: than between him and Air, Space or place should be left, more than (naturally) that quantity of Air requireth, or can fill. Again, water to hang, and to descend: rather than by descending, to leave emptinesse at his back. The like is of Fire and Air, they will descend: when, either their Continuity should be dissolved, or their next Element forced from them. And as they will not be extended to discontinuity: So will they not, nor yet of mans force, can be prest or pent in space, not sufficient and answerable to their bodily substance. Great force and violence will they use, to enjoy their natural right and liberty. Hereupon, two or three men together, by keeping Air under a great Cauldron, and forcing the same down, orderly, may without harm descend to the Sea bottome: and continue there a time, &c. Where, Note, how the thicker Element (as the water) giveth place to the thinner (as is the Air:) and receiveth violence of the thinner, in manner, &c. Pumps and all manner of Bellows, have their ground of this art: and many other strange devises: as *Hydraulica*, Organes going by water, &c. Of this Feat, called commonly *Pneumatica*) goodly works are extant, both in Greek and Latine. With old and learned Schoolmen, it is called *Scientia de pleno & vacuo.*

To go to the bottom of the Sea without danger.

MENA-

MENADRIE, **is an Art Mathematical, which demonstrateth, how, above Natures vertue, and power simple: vertue, and force may be multiplyed: and so, to direct, to lift, to pull to, and to put or cast fro, any multiplied or simple, determined vertue, Weight or Force: naturally, not, so, directible or moveable.** Very much is this Art furthered, by other Arts, as in some points by *Perspective*, in some, by *Statick*, in some, by *Trochilike*, and in other by *Helicosophie*, and *Pneumatithmie*. By this Art, all Cranes, Gibbets, and Engines to lift up, or to force any thing any manner of way, are ordered, and the certain cause of their force is known. As, the force, which one man hath with the *Dutch* Waggen Rock: therewith, to set up again, a mighty waggen laden, being overthrown. The force of the Crossebow Rack, is certainly here demonstrated. The reason, why one man doth with a leaver, lift that, which six men, with their hands onely, could not so easily do. By this Art, in our common Cranes in *London*, where power is to crane up the weight of 2000 pound: by two wheeles more (by good order added) Art concludeth, that there may be craned up 200000 pound weight, &c. So well knew *Archimedes* this Art: that he alone with his devices and engines (twice or thrice) spoyled and discomfited, the whole Army and Host of the *Romans*, besieging *Syracusa*, *Marcus Marcellus the Consul*, being their General Captain. Such huge stones, so many with such force, and so far, did he with his engines hail among them out of the City. And by Sea likewise: though there Ships might come to the wals of *Syracusa*, yet he utterly confounded the *Roman* Navy: what with his mighty Stones hurling: what with pikes * of 18 foot long, made like shafts: which he forced almost a quarter of a mile, what, with his catching hold of their ships, and hoysing them up above the water, and suddenly letting them fall into the Sea again: what with his * Burning Glasses: by which he fired their other ships a far off: what with his other policies, devices, and engines, he so manfully acquitted himself: that all the force, courage, and policy of the *Romans* (for a great season) could nothing prevail, for the winning of *Syracusa*: whereupon, the *Romans* named *Archimedes*, *Briareus*, and *Centimanus*. *Zonaras* maketh mention of one *Proclus* who so well had perceived *Archimedes* Art of *Menadrie*, and had so well invented of his own, that with his Burning Glasses, being placed upon the walls of *Bysance*, he multiplyed so the heat of the Sun, and directed the beams of the same against his enemies Navy with such force, and so suddenly (like lightening) that he burned and destroyed both man and ship. And *Dion* specifieth of *Priscus* a *Geometrician* in *Bysance*, who invented and used sundry Engines, of force multiplyed: which was cause, that the Emperour *Severus* pardoned him his life after he had wonne *Bysance*. Because he honoured the Art, wit, and rare industry of *Priscus*. But nothing inferiour to the invention of these Engines of Force, was the invention of Guns. Which, from an *English-man* had the occasion and order of first inventing: though in another land, and by other men, it was first executed. " And they that should see the record, where the occasion and order general of Gunning, " is first discoursed of, would think: that small things slight and common: comming to " wise mens consideration, and industrious mens handling, may grow to be of force incredible.

Plutarchus in Marco Marcello. Synesius in Epistola. Polybius. Plinius. Quintilianus. T. Livius. * *Athenaeus.* * *Galenus. Athenaus.*

Burning Glass.

Guns.

HYPOGEIODIE, **is an Art Mathematical, demonstrating, how under the Spherical Superficies of the Earth, at any depth, to any perpendicular line assigned (whose distance from the perpendicular of the entrance: and the Azimuth likewise, in respect of the said entrance is known) certain way may be prescribed and gone: And how any way above the Superficies of the Earth designed, may under Earth, at any depth limited, be kept: going alwayes perpendicularly, under the way, on Earth designed: and contrariwise, any way, (straight or crooked,) under the Earth, be given: upon the outface, or superficies of the Earth, to Line out the same: so, as, from the Center of the Earth, perpendiculars drawn to the Spherical superficies of the Earth, shall precisely fall in the correspondent points of those two wayes. This, with all other cases and circumstances herein, and appurtenances, this Art demonstrateth.** This Art is very ample in variety of Conclusions, and very profitable sundry wayes to the Common-wealth. The occasion of my inventing this Art, was at the request of two Gentlemen, who had a certain work (of gain) underground and their grounds did joyn over the work: and by the reason of the crookednesse, divers depths, and heights of the way under ground, they were in doubt, and at controversie, under whose ground, as then, the work was. The name onely (before this) was of me published, *De Itinere Subterraneo*. The rest be at Gods will. For Pioners, Miners, Diggers for Metals, Stone, Cole, and for secret passages under ground, between place and place (as this land hath diverse) and for other purposes, any man may easily perceive, both the great fruit of this Art, and also in this Art, the great aid of Geometry.

HYDRAGOGIE, **demonstrateth the possible leading of Water by natures law, and by artificial help, from any head (being a Spring standing or running Water) to any other place assigned.** Long hath this Art been in use: and much thereof written: and very marvellous works therein performed: as may yet appear in *Italy*, by the Ruines remaining, of the *Aqueducts*. In other places of Rivers, leading through the Main land Navigable many a Mile: and in other places, of the marvellous forcings of water to ascend: which all declare the great skill to be required of him, who should in this Art be perfect, for all occasions of waters possible leading. To speak of the allowance of the Fall, for every hundred foot: or of the Ventils (if the waters labour be far and great) I need not: seeing, at hand (about us) many expert men can sufficiently testifie, in effect, the order: though the Demonstration of the Necessity thereof, they know not: Nor yet, if they should be led, up and down, and about Mountains, from the head of the Spring: and then a place being assigned: and of them to be demanded, how low or high, that last place is, in respect of the head, from which (so crookedly, and up and down, they be come: Perhaps, they would not, or could not) very readily or neerly assoil that Question. *Geometry* therefore, is necessary to *Hydragogie*. Of the sundry ways to force water to ascend, either by *Tympane*, *Kettelmils*, *Scrue*, *Ctesibike*, or such like, in *Vitruvius*, *Agricola*, (and other,) fully, the manner may appear. And so, thereby, also be most evident, how the Arts of *Pneumatithmie*, *Helicosophie*, *Statike*, *Trochelike*, and *Menadrie*, come to the furniture of this in speculation, and to the Commodity of the Common-wealth in practice.

HOROMETRIE, **is an Art Mathematical, which demonstrateth how at all times appointed, the precise usual denomination of time, may be known, for any place assigned.** These words are smooth and plain easie *English*, but the reach of their meaning is farther than you would lighly imagine. Some part of this Art, was called in old time *Gnomonice*: and of late, *Horologiographia*: and in *English*, may be termed *Dialling*. Ancient is the use, and more ancient is the Invention. The use, doth well appear to have been (at the least) above two thousand and three hundred years ago: in * King *Achaz* Dial, then, by the Sun, shewing the distinction of time. By Sun, Moon, and Stars, this Dialling may be performed, and the precise time of day, or night known. But the demonstrative delineation of these Dials, of all sorts, requireth good skill both of *Astronomie* and *Geometry* Elemental, Spherical, Phenomenal, and Conical. Then to use the grounds of the Art, for any regular Superficies, in any place offered: and (in any possible apt position thereof) thereon to describe (all manner of ways) how, usual hours, may be (by the *Suns* shadow) truly determined: will be found no sleight Painters work. So to paint and prescribe the Suns Motion, to the breadth of a hair. In this Feat (in my youth) I invented a way, **how in any Horizontal, Mural, or Equinoctial, Dial, &c. at all hours (the Sun shining) the sign and degree ascendent, may be known.** Which is a thing very necessary, for the rising of those fixed stars: whose Operation in the air, is of great might evidently. I speak no further, of the use hereof. But forasmuch as, mans affairs, require knowledge of Times and Moments, when neither Sun, Moon, or Star, can be seen: Therefore, by Industrie Mechanical, was invented first, how by Water, running orderly, the Time and Hours might be known: whereof, the famous *Ctesibius*, was Inventor: a man of *Vitruvius* to the skie (justly) extolled. Then, after that, by sand running, were hours measured: Then

* 2 *Kings* 20.

Then, by *Trochilike* with weight, and of late time, by *Trochilike* with Spring: without weight. All theſe by Sun or Stars direction, (in certain time) require overſight and reformation, according to the heavenly Æquinoctial Motion: beſides the inequality of their own Operation. There remaineth (without parabolical meaning herein) among the Philoſophers, a more excellent, more commodious, and more marvellous way, than all theſe: of having the motion of the Primovant (or firſt Æquinoctial motion) by Nature and Art, imitated: which you ſhall (by further ſearch in weightier ſtudies) hereafter, underſtand more of. And ſo it is time to finiſh this Annotation, of Times diſtinction, uſed in our common, and private affairs: the commodity whereof no man would want, that can tell, how to beſtow his time.

A perpetual Motion.

ZOGRAPHIE, is an Art Mathematical, which teacheth and demonſtrateth, how, the interſection of all viſual Pyramids, made by any plain aſſigned, (the Center, diſtance, and lights, being determined) may be, by lines, and due proper colours repreſented. A notable Art, is this, and would require a whole Volume, to declare the propertie thereof: and the Commodities enſuing. Great skill of *Geometrie*, *Arithmetick*, *Perſpective*, and *Anthropographie*, with many other particular Arts, had the Zographer, need of, for his perfection. For, the moſt excellent Painter, (who is but the proper Mechanician, and Imitator ſenſible, of the Zographer) hath atteined, to ſuch perfection, that ſenſe of man and beaſt, have judged things painted, to be things natural and not artificial: alive and not dead. This Mechanical Zographer (commonly called the Painter) is marvellous in his skill: and ſeemeth to have a certain divine power: as of friends abſent, to make a friendly, preſent comfort, yea, and of friends dead, to give a continual, ſilent preſence: not onely with us, but with our poſterity for many ages. And ſo proceeding, conſider, how in Winter, he can ſhew you, the lively view of Sommers joy, and riches: and in Sommer, exhibite the countenance of Winters doleful ſtate and nakedneſſe. Cities, Towns, Forts, Woods, Armies, yea, whole Kingdomes (be they never ſo far, or great) can he with eaſe, bring with him, home (to any mans judgment) as patterns lively of the things rehearſed. In one little houſe, can he, encloſe (with great pleaſure of the beholders) the portraiture lively, of all viſible Creatures, either on earth or in the earth, living: or in the waters lying, creeping, ſliding or ſwimming: or of any fowl or fly, in the Air flying. Nay, in reſpect of the Stars, the Skie, the Clouds: yea, in the ſhew of the very light it ſelf, that divine creature, can he match our eyes judgment, moſt neerly. What a thing is this? things not yet being, he can repreſent, ſo, as, at their being: the picture ſhall ſeeme (in manner) to have created them. To what Artificer, is not picture, a great pleaſure and commodity? which of them all, will refuſe the direction and aid of Picture? The Architect, the Goldſmith, and the Arras weaver: of Picture, make great account. Our lively Herbals, our portraitures of birds, beaſts, and fiſhes: and our curious Anatomies, which way, are they moſt perfectly made, or with moſt pleaſure, of us beholders? Is it not by Picture onely? and if Picture, by the induſtry of the Painter, be thus commodious and marvellous: what ſhall be thought of *Zographie*, the Schoolmaſter of Picture, and chief Governour? Though I mention not *Sculpture*, in my Table of Arts Mathematical: yet may all men perceive, How, that *Picture* and *Sculpture*, are Siſters Germane: and both, right profitable, in a Commonwealth and of *Sculpture*, aſwel as of *Picture*, excellent artificers have written great books in commendation. Witneſſe I take, of *George Vaſari, Pittore Aretino*: of *Pomponius Gauricus*: and others. To theſe two Arts, (with other,) is a certain odde art, called *Athalmaſat*, much beholding: more, than the common *Sculptor*, *Entayler*, *Carver*, *Cutter*, *Graver*, *Founder*, or *Painter*, (&c.) know their art to be commodious.

An Objection.

ARCHITECTURE, to many may ſeem not worthy, or not meet, to be reckoned among the *Arts Mathematical*. To whom, I think good, to give ſome account of my ſo doing. Not worthy, (will they ſay,) becauſe it is but for building of a Houſe, Palace, Church, Fort, or ſuch like groſſe works: and you alſo, defined the *Arts Mathematical*, to be ſuch as dealed with no Material or corruptible thing: and alſo did demonſtratively proceed in their Faculty, by Number or Magnitude.

The Anſwer.

Firſt, you ſee, that I count here, *Architecture*, among thoſe *Arts Mathematical*, which are derived from the principals: and you know, that ſuch may deal with natural things, and ſenſible matter. Of which ſome draw

John Dee, *his Mathematical Preface.*

draw neerer, to the ſimple and abſolute Mathematical Speculation, then others do. And « though the *Architect* procureth, informeth, and directeth the *Mechanician*, to hand- « work, and the building actual, of Houſe, Caſtle, or Palace, and is chief Judge of the « ſame: yet with himſelf (as chief *Maſter* and *Architect*) remaineth the Demon- « ſtrative reaſon and cauſe of the *Mechanicians* work, in Line, Plain, and Solid: by « *Geometrical*, *Arithmetical*, *Optical*, *Muſical*, *Aſtronomical*, *Coſmographical*, (and « to be brief) by all the former derived *Arts Mathematical*, and other Natural arts « able to be confirmed and ſtabliſhed. If this be ſo, then, may you think, that *Architecture* hath good and due allowance, in this honeſt company of *Arts Mathematical* Derivative. I will herein crave Judgment of two moſt perfect *Architects*, the one being *Vitruvius* the Roman: who did write the Books thereof, to the Emperour *Auguſtus*, (in whoſe days our Heavenly Arch-maſter was born) & the other *Leo Baptiſta Albertus*, a Florentine, who alſo publiſhed ten books thereof. *Architectura* (ſaith *Vitruvius*) *eſt Scientia pluribus diſciplinis, & variis eruditionibus ornata: cujus judicio probantur omnia, quæ ab cæteris Artificibus perficiuntur opera.* That is: Architecture is a Science garniſhed with many doctrines and divers inſtructions: by whoſe Judgement, all works, by other workmen finiſhed, are judged. It followeth. *Ea naſcitur ex Fabrica, & Ratiocinatione, &c. Ratiocinatio autem eſt, quæ, res fabricatas. ſolertia ac ratione proportionis, demonſtrare atque explicare poteſt.* Architecture groweth of Framing, and Reaſoning, &c. Reaſoning is that, which of things framed with fore-caſt, and proportion: can make demonſtration, and manifeſt declaration. Again. *Cùm, in omnibus enim rebus, tùm maximè etiam in Architectura, hæc duo inſunt: quod ſignificatur, & quod ſignificat. Significatur propoſita res, de qua dicitur: hanc autem ſignificat Demonſtratio, rationibus doctrinarum explicata.* Foraſmuch, as in all things: therefore chiefly in Architecture, theſe two things are: the thing ſignified, and that which ſignifieth. The thing propounded, whereof we ſpeak, is the thing ſignified. But Demonſtration, expreſſed with the reaſons of divers doctrines, doth ſignifie the ſame thing. After that. *Ut literatus, ſit peritus Graphidos, eruditus Geometriæ, & Optices non ignarus: inſtructus Arithmetica: hiſtorias complures noverit, Philoſophos diligenter audiverit: Muſicam ſciverit: Medicinæ non ſit ignarus, reſponſa Juriſperitorum noverit: Aſtrologiam, Cœliq́ue rationes cognitas habeat.* An Architect (ſaith he) ought to underſtand Languages, to be skilful of Painting, well inſtructed in Geometry, not ignorant of *Perſpective*, furniſhed with *Arithmetick*, have knowledge of many Hiſtories, and diligently have heard Philoſophers, have skill of Muſick, not ignorant of Phyſick, know the anſwers of Lawyers, and have Aſtronomie, and the courſes Cœleſtial, in good knowledge. He giveth reaſon, orderly, wherefore, all theſe Arts, Doctrines, and Inſtructions, are requiſite in an excellent Architect. And (for brevity) omitting the Latin text, thus he hath. Secondly, it is behoofull for an Architect to have the knowledge of Painting: that he may the more eaſily faſhion out, in patterns painted, the form of what work he liketh: and Geometrie giveth to Architecture many helps: and firſt teacheth the uſe of the Rule, and the Compaſſe: whereby (chiefly and eaſily) the deſcriptions of Buildings, are diſpatched in Ground-plats: and the directions of Squares, Levels, and Lines. Likewiſe, by Perſpective, the Lights of the Heaven, are well led, in the Buildings: from certain quarters of the World. By Arithmetick, the charges of Buildings are ſummed together: the meaſures are expreſſed, and the hard queſtions of Symmetries, are by Geometrical means and methods diſcourſed on, &c. Beſides this, of the Nature of things (which in Greek is called φυσιολογία) Philoſophy doth make declaration. Which it is neceſſary for an Architect, with diligence to have learned: becauſe it hath many and divers Natural queſtions: is ſpecially in Aqueducts. For in their courſes, leadings about, in the level ground, and in the mountings, the natural ſpirits or breaths are ingendred divers wayes: the hindrances, which they cauſe, no man can help, but he, which out of Philoſophy, hath learned the original cauſes of things. Likewiſe, whoſoever ſhall read *Cteſibius*, or *Archimedes* books, (and of others, who have written ſuch Rules) cannot think, as they doe: unleſſe he ſhall have received of Philoſophers, inſtructions in theſe things: and Muſick he muſt needs know: that he may have underſtanding, both of Regular and Mathematical Muſick: that he may temper well his Baliſts, Catapults, and Scorpions, &c. Moreover, the Braſen Veſſels, which in Theatres, are placed by Mathematical order, in ambries, under the ſteps: and the diverſities of the ſounds (which the Grecians call ἠχεῖα) are ordered according to Muſical Symphonies and Harmonies: being diſtributed in

in the Circuits, by *Diatessaron*, *Diapente*, and *Diapason*. That the convenient voice, of the players sound, when it came to these preparations, made in order, there being increased: with that increasing, might come more clear and pleasant, to the ears of the lookers on, &c. And of Astronomy, is known the East, West, South, and North. The fashion of the Heaven, the Æquinox, the Solsticie, and the course of the Stars. Which things, unlesse one know: he cannot perceive, any thing at all, the reason of Horologies. Seeing therefore, this ample Science, is garnished, beautified, and stored, with so many and sundry skils and knowledges: I think, that none can justly account themselves Architects of the sodain. But they onely, who from their childs years ascending by these degrees of knowledges, being fostered up with the atteining of many Languages and Arts, have won to the high Tabernacle of Architecture, &c. And to whom Nature hath given such quick Circumspection, sharpnesse of Wit, and Memory, that they may be very absolutely skilfull in Geometry, Astronomy, Musick, and the rest of the Arts Mathematical: Such surmount and passe the calling, and state of Architects: and are become Mathematicians, &c. And they are found seldom: as in times past was, *Aristarchus Samius*, *Philolaus*, and *Archytas Tarentynes*: *Apollonius Pergeus*, *Eratosthenes Cyreneus*: *Archimedes*, and *Scopas*, *Syracusians*. Who also left to their posterity, many Engines and Gnomonical works: by numbers, and natural means, invented and declared.

A Mathematician.

Thus much, and the same words (in sense) in one onely Chapter of this incomparable *Architect Vitruvius*, shall you find. And if you should, but take his book in your hand, and slightly look through it, you would say straightway: This is *Geometry*, *Arithmetick*, *Astronomy*, *Musick*, *Anthropographie*, *Hydragogie*, *Horometrie*, *&c.* and (to conclude) the store-house of all workmanship. Now, let us listen to our other Judg, our Florentine, *Leo Baptista*, and narrowly consider, how he doth determine of *Architecture*. *Sed antéque ultra progrediar*, *&c.* But before I proceed any further (saith he) I think, that I ought to expresse, what man I would have to be allowed an Architect. For, I will not bring in place a Carpenter: as though you might compare him to the Chief Masters of
" others arts. For the hand of the Carpenter, is the Architects Instrument: But I will appoint
" the Architect to be that man, who hath the skill, (by a certain and marvellous means
" and way) both in mind and Imagination to determine: and also in work to finish:
" what works so ever, by motion of weight and coupling and framing together of bodies,
" may most aptly be commodious for the worthiest uses of Man. And that he may be able to perform these things, he had need of atteining and knowledge of the best and most worthy things, &c. The whole Feat of Architecture in building, consisteth in Lineaments and in framing. And the whole power and skill of Lineaments, tendeth to this: that the right and absolute way may be had, of Coapting and joyning Lines and angles: by which, the face of the building, or frame may be comprehended and concluded. And it is the property of Lineaments, to prescribe unto buildings, and every part of them, an apt place, and certain number: a worthy manner and a seemly order: that so the whole form and figure of the building, may rest in the very Lineaments, &c. And we may prescribe in mind and imagination the whole forms, * all material stuffe being secluded. Which point we shall attein, by noting and fore-pointing the angles, and lines, by a sure and certain direction and connexion. Seeing then, these things are thus: Lineament shall be the certain and constant prescribing, conceived in mind: made in lines and angles: and finished with a learned mind and wit. We thank you Master *Baptist*, that you have so aptly brought your art and phrase, thereof, to have some Mathematical perfection: by certain order, number, form, figure, and *Symmetrie* mental: all natural and sensible stuffe set apart. Now then it is evident, (Gentle Reader) how aptly and worthily I have preferred *Architecture*, to be bred and fostered up in the Dominion of the peerlesse *Princesse*, *Mathematica*, and to be a natural Subject of hers. And the name of *Architecture*, is of the principality, which this Science hath, above all other Arts. And *Plato* affirmeth the *Architect* to be *Master* over all, that make any work. Whereupon, he is neither Smith nor Builder: nor, separately, any Artificer: but the Head, the Provost, the Director, and Judge of all artificial works, and all Artificers. For, the true *Architect*, is able to teach, demonstrate, distribute, describe, and judge all works wrought. And he, onely searcheth out the causes and reasons of all Artificial things. Thus excellent, is *Architecture*: though few (in our days) attein thereto: yet may not the art, be otherwise thought on, than in very deed it is worthy. Nor we may not of ancient Arts, make new and imperfect Definitions in our days: for

Vitruvius.

Who is an Architect.

* The Immaterialitie of perfect Architecture. What Lineament is.

Note.

for scarcity of Artificers: No more than we may pinch in, the Definitions of *Wisedome*, or *Honesty*, or *Friendship*, or *Justice*. No more will I consent, to diminish any whit of the perfection and dignity, (by just cause) allowed to absolute *Architecture*. Under the Direction of this Art, are three principal, necessary *Mechanical Arts*. Namely, *Housing*, *Fortification*, and *Naupegie*. *Housing*, I understand, both for Divine Service, and Mans common usage: publick and private. Of *Fortification* and *Naupegie*, strange matter might be told you: But perchance, some will be tyred, with this Bed-rol, already rehearsed: and other some, will nicely nip my grosse and homely discoursing with you: made in post haste: for fear you should want this true and friendly warning, and tast giving, of the *Power Mathematical*. Life is short, and uncertein: Times are perilous, &c. And still the Printer awayting, for my pen staying: all these things, with farther matter of Ingratefulnesse, give me occasion to passe away, to the other Arts remaining, with all speed possible.

The Art of NAVIGATION, demonstrateth how, by the shortest good way, by the aptest direction, and in the shortest time, a sufficient Ship, between any two places (in passage Navigable) assigned: may be conducted: and in all storms and natural disturbances chancing, how to use the best possible means, whereby to recover the place first assigned. What need the *Master Pilot* had of other Arts, here before recited, it is easie to know: as, of *Hydrographie*, *Astronomy*, *Astrologie*, and *Horometrie*. Presupposing continually, the common Base, and Foundation of all: namely, *Arithmeticke*, and *Geometry*. So that he be able to understand and judg his own necessary Instruments, and furniture necessary. Whether they be perfectly made or no: and also can, (if need be) make them himself. As Quadrants, the Astronomers Ring, the Astronomers Staffe, the Astrolabe Universal. An Hydrographical Globe. Charts Hydrographical, true, (not with parallel Meridians.) The common Sea Compasse: The Compasse of variation: The Proportional and Paradoxal Compasses (of me invented, for our two Muscovy Master Pilots, at the request of the Company) Clocks with spring: hour, half hour, and three hour Sand-glasses: and sundry other Instruments: and also be able on Globe, or Plain to describe the Paradoxal Compasse: and duly to use the same, to all manner of purposes, whereto it was invented. And also, be able to Calculate the Planets places for all times. *An. 1599.*

Moreover, with Sun, Moon or Star (or without) be able to define the Longitude and Latitude of the place, which he is in: So that the Longitude and Latitude of the place, from which he sayled, be given: or by him, be known: whereto appertaineth expert means, to be certified ever, of the Ships way, &c. and by fore-seeing the Rising, Setting, Noonsteding, or Midnighting of certain tempestuous fixed Stars: or their Conjunctions, and Anglings with the Planets, &c. he ought to have expert conjecture of storms, tempests and spouts: and such like Meteorological effects, dangerous on Sea. For (as *Plato* saith,) *Mutationes opportunitatesque temporum presentire, non minus rei militari, quàm Agriculturæ, Navigationique convenit.* **To foresee the alterations and opportunities of times is convenient, no lesse to the Art of War, than to husbandry and Navigation.** And besides such cunning means, more evident tokens in Sun and Moon, ought of him to be known: such (as the Philosophical Poet) *Virgilius* teacheth in his *Georgicks*, where he saith.

Sol quoque & exoriens & quum se condet in undas,
Signa dabit, Solem certissima signa sequuntur, &c.

——— Nam sæpè videmus,
Ipsius in vultu varios errare colores.
Cæruleus, pluviam denunciat, igneos Euros.
Sin maculæ incipient rutilo immiscerier igni,
Omnia tum pariter vento, nimbisque videbis
Fervere: non illa quisquam me nocte per altum
Ire, neque à terra moveat convellere funem, &c.
Sol tibi signa dabit. Solem quis dicere falsum
Audeat? ——————— &c.

Georgic. 1.

And

And so of Moon, Stars, Water, Air, Fire, Wood, Stones, Birds, and Beasts and of many things else, a certain Sympathical forewarning may be had: sometimes to great pleasure and profit, both on Sea and Land. Sufficiently, for my present purpose, it doth appear, by the premisses, how *Mathematical* the *Art* of *Navigation* is, and how it needeth and also useth other *Mathematical Arts*: And now if I would go about to speak of the manifold Commodities, comming to this Land, and others, by ships and *Navigation*, you might think, that I catch at occasions, to use many words, where no need is.

Yet this one thing may I, (justly) say. In *Navigation*, none ought to have greater care, to be skilful, than our *English* Pilots. And perchance, some, would more attempt: and other some, more willingly would be aiding, if they wist certainly, what priviledge, God hath endued this Island with, by reason of Situation, most commodious for *Navigation*, to places most Famous and Rich. And though (of * late) a young Gentleman, a courageous Captain, was in great readinesse, with good hope, and great causes of perswasion, to have ventured for a **Discovery**, (either *Westerly*, by *Cape de Paramantia*: or *Easterly*, above *Nova Zemla*, and the *Cyrimisses*) and was at the very neer time of attempting, called and imployed otherwise (both then, and since,) in great good service to his Country, as the Irish Rebels * have tasted: Yet, I say, (though the same Gentleman, do not hereafter deal therewith) some one or other should listen to the matter: and by good advice, and discreet circumspection, by little and little, winne to the sufficient knowledge of that **Trade and Voyage**; which now I would be sorry, (through carelesness, want of skill and courage) should remain unknown, or unheard of. Seeing, also, we are, herein, half challenged by the learned, by half request published. Thereof, verily might grow commodity, to this Land chiefly, and to the rest of the Christian Common-wealth far passing all riches and worldly Treasure.

** An. 1567 S. H. G.*

** An. 1569.*

THAUMATURGIKE, **is that Art Mathematical, which giveth certain order, to make strange works, of the sense to be perceived, and of men greatly to be wondred at.** By sundry means, this *Wonder-work* is wrought. Some by *Pneumatithmie*: as the works of *Ctesibus* and *Hero*: some by weight, whereof *Timæus* speaketh: some by Strings strained, or Springs, therewith imitating lively Motions: some by other means, as the Images of *Mercury*: and the brazen head, made by *Albertus Magnus* which did seem to speak. *Boethius* was excellent in these feats. To whom *Cassiodorus* writing, saith. **Your purpose is to know profound things, and to shew marvels. By the disposition of your Art, Metals do low: Diomedes of brasse, doth blow a Trumpet loud, a brasen Serpent hisseth: Birds made sing sweetly. Small things we rehearse of you, who can imitate the heaven, &c.** Of the strange self-moving, which at Saint *Denis*, by *Paris*, * I saw once or twice (*Orontius* being then with me, in company) it were to strange to tell. But some have written it: and yet (I hope) it is there, to be seen. And by *Perspective* also strange things, are done: as partly (before) I gave you to understand in *Perspective*: as, to see in the air aloft, the lively image of another man, either waking to and fro: or standing still. Likewise, to come into an house, and there to see the lively shew of Gold, Silver or precious Stones: and comming to take them in our hand, to find nought but Air. Hereby, have some men (in all other matters counted wise) fouly over-shot themselves: misdeeming of the means. Therefore, said *Claudius Cælestinus*, *Hodie magna literatura viros, & magnæ reputationis videmus, opera quædam quasi miranda, supra Naturam putare, de quibus in Perspectiva doctus causam faciliter reddidisset.* That is. **Now a dayes, we see some men, yea at great learning and reputation, to judge certain works as marvellous above the power of nature: of which works, one that were skilful in Perspective, might easily have given the cause.** Of *Archimedes Sphere*, *Cicero* witnesseth. Which is very strange to think on. **For when Archimedes (saith he) did fasten in a Sphere, the movings of the Sun, Moon, and of the five other Planets, he did, as the God, which (in Timæus of Plato) did make the World. That one turning should rule Motions most unlike in slownesse and swiftnesse.** But a greater cause of marvelling we have by *Claudianus* report hereof. Who affirmeth this,

** An. 1551.*

De his quæ Mundo mirabiliter eveniunt, cap. 8.

Archi-

Archimedes work, to have been of Glasse, and discourseth of it more at large: which I omit. The Dove of wood, which the *Mathematician Archytas* did make to flie, is by *Agellius* spoken of. Of *Dædalus* strange Images, *Plato* reporteth. *Homer* of *Vulcans Self-movers*, (by secret wheels,) leaveth in writing. *Aristotle* in his *Politicks* of both, maketh mention. Marvellous was the workmanship of late dayes, performed by good skill of *Trochilike*, &c. For in *Noremberg*, a Flie of Iron, being let out of the Artificers hand, did (as it were) flie about by the guests at the Table, and at length, as though it were wearie, return to his Masters hand again. Moreover, an artificial Eagle, was ordered, to flie out of the same Town, a mightie way, and that aloft in the air, toward the Emperour coming thither: and followed him, being come to the gate of the Town.* Thus you see what Art Mathematical can perform, when skill, will, industrie, and abilitie, are duly applied to proof.

AND for these, and such like marvellous Acts and Feats, Naturally, Mathematically, and Mechannically, wrought and contrived: ought any honest Student, and Modest Christian Philosopher, be counted, and called a **Conjurer**? Shall the folly of Ideots, and the Malice of the Scornful, so much prevail, that He, who seeketh no worldly gain or glory at their hands: But onely, of God, the treasure of heavenly wisdome, and knowledge of pure verity: Shall he (I say) in the mean space, be robbed and spoiled of his honest name and fame? He that seeketh (by S. *Pauls* advertisement) in the Creatures Properties, and wonderful vertues, to finde just cause, to glorifie the Æternal, and Almighty Creator by: Shall that man be (in hugger mugger) condemned, as a Companion of the Hel-hounds, and a Caller, and Conjurer of wicked and damned Spirits? He that bewaileth his great want of time, sufficiently (to his contentation) for learning of Godly wisdome, and Godly Verities in: and onely therein setteth all his delight: Will that man leese and abuse his time, in dealing with the Chief enemy of Christ our Redeemer: the deadly foe of all mankind: the subtile and impudent perverter of Godly Verity: the Hypocritical Crocodile: the Envious Basilisk, continually desirous, in the twinkling of an eie, to destroy all Mankind, both in Body, and Soul, eternally? Surely (for my part, somewhat to say herein) I have not learned to make so brutish, and so wicked a Bargain. Should I, for my xx, or xxv years Study: for two or three thousand Marks spending: seven or eight thousand Miles going and travelling, onely for good learning sake: And that, in all manner of weathers: in all manner of wayes and passages: both early and late: in danger of violence by man: in danger of destruction by wilde beasts: in hunger: and thirst: in perillous heats by day, with toil on foot: in dangerous damps of cold, by night, almost bereaving life: (as God knoweth): with lodgings, oft times, to small ease: and sometime to lesse security. And for much more (than all this) done and suffered, for Learning and atteining of Wisdome: Should I (I pray you) for all this, no otherwise, nor more warily: or (by Gods mercifulnesse) no more luckily, have fished, with so large, and costly, a Net, so long time in drawing and that with the help and advice of Lady Philosophy, and Queen Theology): but at length, to have catched, and drawn up, * a Frog? Nay, a Devil? For, so, doth the Common peevish Pratler Imagine and Jangle: And, so, doth the Malicious scorner, secretly, and bravely and boldly face down, behinde my back. Ah, what a miserable thing, is this kinde of Men? How great is the blindnesse and boldnesse, of the Multitude, in things above their Capacity? What a Land: what a People: what Manners: what Times are these? Are they become Devils, themselves: and by false witnesse bearing against their Neighbour, would they also, become Murderers? Doth God, so long give them respite, to reclaim themselves in, from this horrible slandering of the guiltlesse: contrary to their own Consciences: and yet will they not cease? Doth the Innocent, forbear the calling of them, Juridically to answer him, according to the rigour of the Laws: and will they despise his Charitable patience? As they, against him, by name, do forge, fable, rage, and raise slander, by Word and Print: Will they provoke him by word and Print, likewise, to Note their Names to the World: with their particular devices, fables, beastly Imaginations, and unchristian-like slanders? Well: Well. O (you such) my unkinde Countreymen! O unnatural Countreymen! O unthankful Countreymen! O Brainsick, Rash, Spiteful and Disdainful Countreymen. Why oppresse you me thus violently with your slandering of me: Contrary to Verity: and contrary to your own Consciences? And I, to this hour, neither by word, deed, or thought, have been, any way, hurtful, damageable,

A Digression Apologetical.

** A Proverb, Fair fisht and caught a Frog.*

damageable, or injurious to you or yours? Have I, so long, so dearly, so far, so carefully, so painfully, so dangerously fought and travelled for the learning of Wisdome, and atteining of Vertue: And in the end (in your judgement) am I become, worse, than when I began? Worse than a Mad man? A dangerous Member in the Common-wealth: and no Member of the Church of Christ? Call you this to be Learned? Call you this, to be a Philosopher? and a lover of Wisdome? To forsake the straight heavenly way: and to wallow in broad way of damnation? To forsake the light of heavenly Wisdome: and to lurk in the dungeon of the Prince of darknesse? To forsake the Verity of God, and his Creatures, and to fawn upon the impudent, craftie, obstinate Liar, and continual disgracer of Gods Veritie, to the uttermost of his power? To forsake the Life and Blisse Eternal: and to cleave unto the Author of Death everlasting? that murderous Tyrant, most greedily awaiting the Prey of Mans soul? Well: I thank God, and our Lord Jesus Christ, for the comfort, which I have by the Examples of other men, before my time: To whom, neither in godlinesse of life, nor in perfection of learning, I am worthy to be compared: and yet they susteined the verie like injuries that I do: or rather greater. Patient *Socrates* his *Apologie* will testifie: *Apuleius* his *Apologies*, will declare the brutishnesse of the Multitude: *Joannes Picus*, Earl of *Mirandula*, his *Apologie* will teach you, of the raging slander of the malicious Ignorant against him. *Joannes Trithemius* his *Apologie* will specifie, how he had occasion to make publike Protestation: as wel by reason of the Rude Simple: as also in respect of such, as were counted to be of the wisest sort of men. "Many could I "recite: But I defer the precise and determined handling of this matter: being loth to de-
* ☞ "tect the Folly and Malice of my Native Countreymen. * Who, so hardly, can disgest "or like any extraordinarie course of Philosophical studies: not falling within the Com-"passe of their Capacitie: or where they are not made privie of the true and secret cause, "of such wonderful Philosophical Feats. These men, are of four sorts, chiefly. The first, I may name, *Vain pratling busie-bodies.* The second, *Fond Friends.* The third, *Imperfectly Zealous*: and the fourth, *Malicious Ignorant.* To each of these (briefly, and in cha-
1\. ritie) I will say a word or two, and so return to my Preface. *Vain pratling busie-bodies,* use your idle assemblies, and conferences, otherwise, than in talk of matter, either above your capacities, for hardnesse: or contrarie to your consciences in veritie. *Fond Friends,*
2\. leave off, so to commend your unacquainted friend upon blind affection: As, because he knoweth more, than the common Student: that, therefore he must needs be skilful, and a doer, in such matter and manner, as you term *Conjuring.* Weening thereby, you advance his fame: and that you make other men, great marvels of your hap, to have such a learned friend. Cease to ascribe Impietie, where you pretend Amitie. For, if your tongues were true, then were that your friend *Untrue*, both to God and his Sovereign. Such *Friends* and *Fondlings*, I shake off, and renounce you: Shake you off, your Folly. *Imperfectly*
3\. *Zealous*, to you, do I say: that (perhaps) well, do you mean: but far you misse the Mark: If a Lamb you will kill, to feed the flock with his bloud. Sheep, with Lambs bloud, have no natural sustenance: No more, is Christs flock, with horrible slanders, duly edified. Nor your fair pretense, by such rash ragged Rhetorique, any whit, wel graced. But such, as so use me, will finde a foul crack in their credit. Speak that you know: And know, as you ought: Know not, but Hear-say, when life lieth in danger. Search to the quick, and let charity be your guide. *Malicious Ignorant*, what shall I say to thee? *Prohibe lin-*
4\. *guam tuam à malo. A Detractione parcitur lingua.* **Cause thy tongue to refrain from evil. Refrain your tongue from slander.** Though your tongues be sharpened Serpent-like, and Adders poison lie in your lips: yet take heed, and think, betimes, with your self, *Vir linguosus non stabilietur in terra. Virum violentum venabitur*
Psal. 140. *malum, donec præcipietur.* For, sure I am, *Quia faciet Dominus Judicium afflicti, & vindictam pauperum.*

Thus, I require you my assured friends and Countreymen (you Mathematicians, Mechanicians, and Philosophers, charitable and discreet) to deal in my behalf, with the light and untrue tongued, my envious Adversaries or Fond friends. And farther, I would wish, that at leasure, you would consider, how *Basilius Magnus* layeth *Moses* and *Daniel* before the eyes of those, which count all such studies Philosophical (as mine hath been) to be ungodly or unprofitable. Weigh well Saint *Stephen* his witnesse of *Moses. Eruditus*
Acts 7. 6. *est Moses omni Sapientia Ægyptiorum: & erat potens in verbis & operibus suis.* **Moses was instructed in all manner of wisdome of the Egyptians, and he was of power both in his words and works.** You see this Philosophical Power and Wisdome which *Moses* had, to be nothing misliked of the Holy Ghost. Yet *Plinius* hath recorded *Moses* to be a wicked *Magician*. And that (of force) must be, either for this Philosophical wisdome, learned, before his calling to the leading of the Children of *Israel*: or for those his wonders wrought before King *Pharaoh*, after he had the conducting of the *Israelites*. As concerning the first, you perceive, how Saint *Stephen*, at his Martyrdome (being full of the Holy Ghost) in his Recapitulation of the Old Testament, hath made mention of *Moses* Philosophy: with good liking of it: And *Basilius Magnus*, also avoucheth it, to have been to *Moses* profitable (and therefore I say, to the Church of God necessarie.) But as concerning *Moses* wonders done before King *Pharaoh*: God himself said: *Vide ut omnia ostenta, quæ posui in manu tua, facias coram Pharaone.* **See that thou do all those wonders before Pharaoh, which I have put in thy hand.** Thus, you evidently perceive, how rashly, *Plinius* hath slandered *Moses*, of vain fraudulent *Magick*, saying: *Est & alia Magices Factio, à Mose, Jamne, & Jotape, Judæis pendens: sed multis millibus annorum post Zoroastrem, &c.* Let all such, therefore, who, in Judgement and Skill of Philosophie, are far inferiour to *Pliny*, take good heed, lest they over-shoot themselves rashly, in judging of *Philosophers strange acts*: and the means how they are done. But much more ought they to beware of forging, devising, and imagining monstrous feats, and wonderful works, when and where no such were done: no, not any spark or likelihood, of such, as they, without all shame, do report. And (to conclude) most of all, let them be ashamed of Man, and afraid of the dreadful and just Judge: both foolishly or maliciously to devise: and then divellishly to father their new fond Monsters on me: Innocent in hand and heart: for trespassing either against the law of God or man, in any my Studies, or Exercises, Philosophical, or Mathematical: As in due time, I hope, will be more manifest. Lib. 30. c. 1. 1. ☞ 2. 3.

Now end I, with ARCHEMASTRIE. Which name, is not so new, as this Art is rare. For another Art, under this, a degree (for skill and power) hath been endued with this English name before. And yet this may serve for our purpose, sufficiently, at this present. **This art, teacheth to bring to actual experience, sensible, all worthy conclusions by all the Arts Mathematical purposed, and by true Natural Philosophy concluded: and both addeth to them a farther scope, in the terms of the same Arts, and also by his proper Method, and in peculiar terms, proceedeth, with help of the foresaid Arts, to the performance of compleat Experiences, which of no perticular Art, are able (Formally) to be challenged.** If you remember, how we considered *Architecture*, in respect of all common handworks: some light may you have, thereby, to understand the Sovereigntie and propertie of this Science. *Science* I may call it, rather, than an Art: for the excellencie and Mastership it hath, over so many, and so mightie Arts and Sciences. And because it proceedeth by *Experiences*, and searcheth forth the causes of Conclusions, by *Experiences*: and also putteth the Conclusions themselves, in *Experience*, it is named of some *Scientia Experimentalis.* The **Experimental Science,** *Nicolaus Cusanus* termeth it so, in his *Experiments Statical*, and another *Philosopher* of this Land Native (the flower of whose worthy fame can never die nor wither) did write thereof largely, at the request of *Clement the Sixth.* The Art carrieth with it, a wonderful credit: by reason, it certifieth, sensibly, fully, and compleatly to the utmost power of Nature and Art. This Art certifieth by *Experiences* compleat and absolute: and other Arts, with their arguments and demonstrations, perswade: and in words prove very well their Conclusions. * But words and arguments, are no sensible certifying: nor the full and final fruit of Sciences practicable. And though some Arts have in them *Experiences*, yet they are not compleat, and brought to the uttermost, they may be stretched unto, and applied sensibly. As for example: the natural Philosopher disputeth and maketh goodly shew of reason: And the Astronomer, and the Optical Mechanician put some things in *Experience*: but neither, all, that they may: nor yet sufficiently, and to the utmost, those, which they do. There, then, the *Arch-master*, steppeth in, R. B. ☞ *

in, and leadeth forth on, the *Experiences,* by order of his doctrine *Experimental,* to the chief and final power of Natural and Mathematical Arts. Of two or three men, in whom this Deſcription of *Archimyſtry* was *Experimentally* verified, I have read and heard: and good record, is of their ſuch perfection. So that this Art is no fantaſtical imagination: as ſome Sophiſter, might, *Cum ſuis Inſolubilibus,* make a flouriſh: and dazle your imagination: and daſh your honeſt deſire and courage, from believing theſe things, ſo unheard of, ſo marvellous, and of ſuch importance. Well: as you will. I have forewarned you. I have done the part of a friend: I have diſcharged my Dutie toward God: for my ſmall Talent, at his moſt merciful hands received. To this Science, doth the *Science Alnirangiat,* great Service. Muſe nothing of this name. I change not the name, ſo uſed, and in Print, publiſhed by others: being a name proper to the Science. Under this, commeth *Ars Sintrillia,* by *Artephius,* briefly written. But the chief Science, of the Archimyſtry, (in this world) as yet known, is another (as it were) OPTICAL Science: whereof, the name ſhall be told (God willing) when I ſhall have ſome, (more juſt) occaſion, thereof, to diſcourſe.

Here I muſt end, thus abruptly (Gentle friend, and unfeigned lover of honeſt and neceſſary verities.) For, they, how have (for your ſake, and vertues cauſe) requeſted me, (an old foreworn Mathematician) to take pen in hand: (through the confidence they repoſed in my long experience: and tryed ſincerity) for the declaring and reporting ſomewhat, of the fruit and commodity, by the **Arts Mathematical to be atteined unto:** even they, ſore againſt their wils are forced, for ſundry cauſes, to ſatisfie the workmans requeſt, in ending forthwith: He, ſo feareth this, ſo new an attempt, and ſo coſtly: and in matter ſo ſlenderly (hitherto) among the Common Sort of Students, conſidered or eſteemed.

And where I was willed ſomewhat to alledge, why, in our vulgar ſpeech, this part of the principal Science of *Geometry,* called *Euclids Geometrical Elements,* is publiſhed, to your handling: being unlatined people, and not Univerſitie Scholers: Verily, I think it needleſſe.

1. For, the Honour, and Eſtimation of the **Univerſities and Graduates,** is, hereby nothing diminiſhed. Seeing, from, and by their Nurſe Children, you receive all this Benefit, how great ſoever it be.

2. Neither are their ſtudies any whit hindred. No more than the Italian *Univerſities,* as *Academia Bononienſis, Ferrarienſis, Florentina, Mediolanenſis, Patavina, Papienſis, Perufina, Piſana, Romana, Senenſis* or any one of them find themſelves, any deal diſgraced, or their Studies any thing hindred, by *Frater Lucas de Burgo,* or by *Nicolaus Tartalca,* who in vulgar Italian language, have publiſhed, not onely *Euclids Geometry,* but of *Archimedes* ſomewhat: and in Arithmetick, and Practical Geometry, very large Volumes, all in their vulgar ſpeech. Nor in *Germany,* have the famous *Univerſities,* any thing been diſcontent with *Albertus Durerus,* his Geometrical Inſtitutions in Dutch: or with *Gulielmus Xylander,* his learned tranſlation of the firſt ſix books of *Euclide,* out of the Greek, into the High Dutch. Nor with *Gualterus H. Riffius,* his Geometrical Volume: very diligently tranſlated into the High Dutch tongue, and publiſhed. Nor yet the *Univerſities* of *Spain* or *Portugal,* think their reputation to be decayed: or ſuppoſe any their ſtudies to be hindred by the Excellent *P. Nonnius,* his Mathematical works, in vulgar ſpeech by him put forth. Have you not, likewiſe in the French tongue, the whole Mathematical Quadrivie? and yet neither *Paris, Orleans,* or any of the other Univerſities of *France* at any time, with the Tranſlators, or Publiſhers offended: or any mans ſtudie thereby hindred?

And ſurely, the common and vulgar ſcholer (much more the Grammarian) before his
3. comming to the *Univerſity,* ſhall (or may) be, now (according to *Plato* his Counſel) ſufficiently inſtructed in *Arithmetick* and *Geometry,* for the better and eaſier learning of all manner of *Philoſophy Academical* or *Peripatetical.* And by that means, go more cheerfully, more skilfully, and ſpeedily forward in his ſtudies, there to be learned. And, ſo, in leſſe time, profit more, than (otherwiſe) he ſhould or could do.

4. Alſo many good and pregnant *Engliſh* wits, of young Gentlemen, & of other, who never intend to meddle with the profound ſearch and ſtudie of Philoſophie (in the *Univerſities* to be learned) may neverthe leſſe, now, with more eaſe and liberty, have good occaſion, vertuouſly to occupie the ſharpneſſe of their wits: where elſe (perchance) otherwiſe, they would

would in ſound exerciſes, ſpent (or rather loſe) their time: neither ſerving God: nor furthering the weal, common or private.

And great Comfort, with good hope, may the *Univerſities* have, by reaſon of this *Engliſh* **Geometrie and Mathematical Preface,** that they (hereafter) ſhall be the more regarded, eſteemed, and reſorted unto. For when it ſhall be known and reported, that of the *Mathematical Sciences* onely, ſuch great Commodities are enſuing (as I have ſpecified:) and that in deed, ſome of you unlatined ſtudents, can be good witneſſe, of ſuch rare fruit by you enjoyed (thereby:) as either, before this, was not heard of: or elſe not ſo fully credited: "Well, may all men conjecture, that far greater aid and better furniture, " to win to the perfection of all Philoſophy, may in the Univerſities be had: being the " Store-houſe and Treaſury of all Sciences and all Arts, neceſſary for the beſt and moſt " noble State of Common-wealths. 5.

Univerſities.

Beſides this, how many a common Artificer, is there in theſe Realms of *England* and *Ireland,* that dealeth with Numbers, Rule, and Compaſſe: Who, with their own skill and experience, already had, will be able (by theſe good helps and informations) to find out, and deviſe, new Works, ſtrange Engines, and Inſtruments: for ſundry purpoſes in the Common-wealth? or for private pleaſure? and for the better mainteining of their own eſtate? I will not (therefore) fight againſt mine own ſhadow. For no man (I am ſure) will open his mouth againſt this enterprize. No man (I ſay) who either hath charity toward his brother, (& would be glad of his furtherance in vertuous knowledge:) or that hath any care and zeal for the bettering of the common ſtate of this Realm. Neither any, that make accompt, what the wiſer ſort of men (*Sage* and *Stayed*) do think of them. To none (therefore) will I make any *Apologie,* for a vertuous act doing: and for commending, or ſetting forth, profitable Arts, to *Engliſh* men in their Engliſh tongue. " But, unto God our Creator, let us all be " thankful: for that, **As he, of his goodneſſe, by his power, in his " wiſedome, hath created all things in Number, Weight and Mea- " ſure.** So, to us, of his great mercy, he hath revealed means, whereby to attein the " ſufficient and neceſſary knowledge of the foreſaid his three principal Inſtruments: " Which means, I have abundantly proved unto you, to be the *Sciences* and *Arts Ma- " thematical.* 6.

And though I have been pinched with ſtraightneſſe of time: that, no way, I could ſo pen down the matter (in my mind) as I determined: hoping of convenient leaſure: Yet, if vertuous zeal, and honeſt intent, provoke and bring you to the reading of this Compendious Treatiſe, I doe not doubt, but as the verity thereof (according to our purpoſe) will be evident unto you: So the pith and force thereof, will perſwade you: and the wonderful fruit thereof: highly pleaſure you. And that you may the eaſier perceive and better remember, the principal points, whereof my Preface treateth, I will give you the **Groundplat** of my whole diſcourſe, in a Table annexed: from the firſt to the laſt, ſomewhat Methodically contrived.

The ground Plat of this Preface in a Table.

If haſte, hath cauſed my poor pen, any where to ſtumble: You will (I am ſure) in part of recompence, (for my earneſt and ſincere good will to pleaſure you) conſider the rockiſh huge mountains, and the perilous unbeaten ways, which (both night and day, for the while) it hath toyled and laboured through, to bring you this good News, and Comfortable proof of Vertues fruit.

So I commit you unto Gods merciful direction, for the reſt: heartily beſeeching him to proſper your Studies, and honeſt Intents: to his Glory, and the Commodity of our Country. *Amen.*

Written at my poor Houſe,
At Mortlake

Anno 1570 *Febr.* 9.

Here followeth the GROUND-PLAT of the Mathematical Preface of Master JOHN DEE. By which a brief view of what hath been before at large delivered doth appear.

THIS CHART WILL BE THE
FIRST CHART APPEARING
AT THE END OF THIS FILM.

TO THE READER.

Although it be usual for Authors or Commentators to write somewhat in praise of the Subject of which they treat: Yet we suppose it neither necessary, or (we hope) will it be expected that we speak much or little in praise of the Mathematicks, or of this Book, of the Elements of Geometry, *the ground and foundation of all Mathematical productions; the general use that hath been, and is still made thereof, almost throughout the World, as well in respect of its Original Greek, as the Translations thereof into all Languages, doth sufficiently declare the excellency and worth of the Art, the Book, and the famous Author and Philosopher* EUCLIDE: *An account of whose Original, and the time wherein he flourished, we have thought good to insert; as also that full and learned Preface of the famous Mathematician* John Dee, *then which nothing of that nature can be more ample or satisfactory.*

We shall only advertise the Reader what order and method hath been used and observed by us in this Book (which we now present to his favourable acceptance) which hath not been done by any other that have published these Elements in the English Tongue.

And first, to speak of the Schemes, he is to take notice that throughout the Book, the greater black lines stand for, and denote the Data, *or things given, the lesser black lines the things required, and the pricked lines serve for the Construction and Demonstration; the like method is also observed in the Circular lines given and required.*

Secondly, he is to take notice that the explications of the things given and required, are comprehended in the words of the Proposition, by meanes of the correspondent letters relating to the several parts of the Schemes, in such sort, as the Propositions may be read with them, or without them, and so by this meanes the Demonstrations are much abreviated, yet our endeavours have been to render all things as plain, easie, and intelligible to the Reader as may be, and we hope they will be so found.

Thirdly, Whereas most Authors that have published the Elements of EUCLIDE, *do end at the fifth Proposition of the Fifteenth Book, we have in this Book added the rest of the Propositions belonging to the Fifteenth Book, as also the whole Sixteenth, according to* Flussas, *and also the Treatise of Regular Solids, which Propositions will be found of excellent use in the Geometrie of Solids: Those Propositions also we have reduced to the same method as the precedent, and so consequently they are much abreviated.*

Fourthly, He is to take notice, that at the end of the Second Book, we have added the Ten first Propositions of that Book according to Barlaam *a famous Greek Author, to shew the agreement and correspondency that is between Lines and Numbers, Geometry and Arithmetick, the said Propositions being proved true by both, they having the self same proprieties and passions in Numbers, which* EUCLIDE *in this Second Book demonstrates by magnitudes.*

Fifthly, We have added EUCLIDES Data, *a Book of excellent use in things relating to Algebra, as also a learned Preface thereon, written by the Philosopher* Marinus, *fully unfolding the signification of the Term, and the profit and utility of the Treatise; the same method of bringing the letters of the Scheme into the Propotsition, being observed here as in the other Bookes.*

Lastly, We have added two excellent Treatises of the Divisions

visions of Superficies, one of them written by our EUCLIDE, *though ascribed to* Machomet Bagdedine, *the other by* Frederick Commandine *of* Urbin, *touching the same matter: These two Treatises, together with* EUCLIDES Data, *were never before published in the English Tongue, nor is there any thing extant in English of that nature so full and ample, in all respects, as those are.*

There are some other Treatises ascribed to our Author, viz. *his* Opticks, Catoptricks, Phænomena, *and* Musick, *which we purposely omitted, because they are not Elementary, as also because those Subjects have been more fully treated on by other later Authors. And moreover it is questioned, by many whether they were the Workes of this our* **EUCLIDE,** *or some other.*

Thus as exactly as we can, we have given you a short view of what we thought fit to be premised concerning this Work, which if diligently read, will undoubtedly be attended with as much profit as delight: The greatest Errors (as it is almost impossible but some will passe, in a Work of this Nature) are denoted at the end of the Book: As for the lesse mistakes, we leave them to your goodnesse, either to correct them, or excuse them.

FAREWEL.

An Account of the Author according to Ancient Phylosophers.

THe famous EUCLIDE Institutor of these Mathematical Elements (to premise something concerning the Author, and something also of this his Noble Science) is variously reported of, both for his Person, and the Age and Time in which he flourished. Several Writers are of opinion, as *Campanus* and *Theon*, in their Edition of these Elements, have published that he was borne at *Megara*, a Town adjacent to *Isthmus*, a Hearer of *Socrates*, who denominated a Sect of Philosophers, his Disciples, from that place, which said Sect was otherwise called the *Dialectick*; for that their manner of Writing was by way of Question and Answer, the Property of Logick onely. Of this EUCLIDE *Diogenes Laertius* hath writ much, in his *Lives* of the Philosophers; as also *Cicero* in his Third Book of Academick Questions, where he saith that EUCLIDE was of *Megara*, a Disciple of *Socrates*, whose Scholars affirmed that to be the only good, which was *One*, *Alike*, the *Same*, and *Perpetual*. This relation of EUCLIDE is backt by *Valer. Max.* who in his Eighth Book saith, that *Plato* sent his Scholars to informe themselves of a certain Question, to EUCLIDE the Geometrician: But if credit may be given to *Proclus* a Noble Writer, and other Ancient Writers, This our EUCLIDE was younger then him of *Megara*, and flourished in the Reign of *Ptolomy* the First, who governed *Ægypt* after the death of *Alexander* the Great, in the 115th. Olympiad, and before the Birth of Christ 309 Yeares; which is the most probable opinion, from this Reason: For *Diogenes Laertius*, diligently enumerating the Works of EUCLIDE of *Megara*, makes no mention of this most renowned Volume, of the Geomerical Elements, by which this our EUCLIDE hath decreed to himself a never dying immortal Fame and Glory. Neither is it to be thought, that *Diogenes* a Person so well versed in the Moniments of Philosophers, would have ~~wittingly past over this most~~ eximious piece, or not take notice that it was made by his familiar EUCLIDE. Therefore certainly (though it be uncertaine from whence he had his Birth) this our EUCLIDE was younger then the other: A Philosopher he was, of great praise at first for his Learning in the Academicks, from which he addrest himself wholly to the Study of the Mathematicks, in which he so farre excelled, that by universal consent, and judgment, he is deservedly stiled the Prince of Mathematicians. He writ several Books concerning that Science; in which his accuratenesse, and admirable Doctrine, is very conspicuous; such are his *Opticks*, *Catoptricks*, *Elementary Institutions*, for the Attainment of Musick, *Phœnomena*, and a Book entituled *Data*, *A Treatise of Divisions*, which though ascribed to *Machomet Bagdedine*, is suspected to be his, being a very exact piece, and newly published by the endeavours of *John Dee* of *London*, and *Frederick Commandine* of *Urbin*. He writ also (as *Proclus* saith) the *Conick Elements*, which yet never saw the light, and some such other Tracts. But more especially he wrote this Book of *Geometrical Elements*, never enough (by universal vogue) to be commended, being wrought with such admirable method, and such immense Learning, that no man ever that yet writ of the same, came so much as neer him. In which as he shewed his acute and piercing brain, so did he not publish all things belonging to that Science, but onely the chiefe and most necessary, supported by the most firme and solid Arguments. And what the profit of these his Elements are to the studious, hath by what we have already said, and by what we shall now say, plainly appear. They are called the Elements of Geometry, for this cause, for that no Mathematical Operation can be undertaken, nor no advantage be made, without them. For all Mathematical Writers, as *Archimedes*, *Appollonius*, *Theodosius*, and others, in their Demonstrations use these Elements, as Principles long since known, approved, and demonstrated. Therefore as he, who would learn to read, must first learn his A B C, and daily use them repeated in the forming and expressing of words, so he that would render the Science of the Mathematicks easie to him, must perfectly know these Geometrical Elements: For from these, as from a full Fountaine, all dimension and division of Latitude, Longitude, Altitude, Profundity, as of Fields, Mountains, Islands, take their rise and beginning; from hence the observation (by Instruments) of the Stars in Heaven, the making of all Scioterick Horologies, The force of Engines, Judgment of Weights, all diversity of various apparences (such as are seen in Glasses, in Pictures, in Water, and in the Aire variously radiated) have their Original. By these Principles (further) the Medium and Centre of this whole Fabrick of the World was found out, and the Poles or Hinges on which it is roled; and finally, the certain figure and quantity of it. By the strength of this alone, the rising, setting, depression, return, ascension, descension, and culmination of all the perpetual motions in the Heaven, the diversity of day and night, in all climates, and Latitudes throughout the Year, are infallibly known. The divers Conjunctions also, and Oppositions and Aspects of the Planets, are so easily known, that their places in the Heaven, the Eclipses and defects of the Sun and Moon are so certainly demonstrated, that they can be, and are predicted by the Mathematicks to all after Ages. To summe up all, this vast Work of God and Nature is presented to the eye of our understanding, by the benefit and help of Geometry. Adde hereto, that this Science renders things incredible, and which surpasse all beliefe, most perspicuous and evident to us. Such a Story is that which Historians testifie of *Archimedes* of *Syracusa*, For King *Hieron* having built a Ship (which he purposed to send to *Ptolomy* King of *Ægypt*) of such an immense bulk, that all the Syracusians with their uttermost strength, could not remove, *Archimedes* being a most skilful Geometrician, promised to launch it, which with little labour accordingly in the sight of them all he performed, to such an amazement of

of the King, that he exprest with astonishment, *That what ever Archimedes should from thenceforth say, ought indubitably to be believed.* Not much unlike this, was his other famous Operation before the same King, who had commanded a Crown to be made of Gold, *Archimedes* without melting it down, told the King the respective Weight of Gold and Silver, which the fraud and cunning of the Artificer had mixed and incorporated together. Nor may we passe by in silence that usual Saying of his, *That (trusting to the Force and Efficacy of Geometrical Demonstrations) if there were another World whereon he could set his foot, he would remove this whereon we are, from its place.* And by the same reason, with never so little strength, remove never so ponderous and massie weights. Divers like things have been performed by him, and other industrious Geometricians, which are recorded at large. But such a fame did this Science purchase to *Archimedes*, that *Marcellus* the Roman General, against whom he alone had long defended his Native City of *Syracusa*, by certain Engines invented by Geometrical Demonstrations, having taken the City, and given the plunder and spoil, and sack of it to his Souldiers, forbidding any quarter to be given to the Citizens, did yet by his Proclamation command the saving of the life of *Archimedes*, whom when against that his command, he knew to be killed by the hands of a common Souldier, he did most passionately grieve, and bestowed that Honour upon him being dead, which he could not confer upon him living. And *Cicero* highly pleases himself, and much glories in it, that when he was Quæstor in *Sicily*, he had found out and seen his Monument and Sepulchre. Therefore none may wonder that Geometry was in such esteem among the Grecians. And this also much advances the excellency and utility of Geometry, because its Demonstrations are so clear, that no man can without it be a perfect Methodist; which *Galen* that famous Philosopher and Prince of Physicians, in his Books ingeniously acknowledgeth: Who being every way accomplished in the Dialectick Learning; and having run through all the Schools both of the Stoicks and Peripateticks of his time, and with much intentnesse of mind, considered and studied their Precepts and Rules, declares that they contein nothing in them, which might conduce to the knowledge of Demonstration; but rather most of them were yet in controversie amongst themselves, others quite repugnant to natural reason; so that he was almost brought to the opinion of the *Pyrrhonian* Philosophers, to doubt of every thing, and to determine nothing; but that by the knowledge of Geometry and Arithmetick (in which he was trained up by his Grand-father) he was recovered, and farther reclaimed from that errour. His counsel therefore is, that men apply their minds to study the Arithmetick Characters, and the Demonstration of Lines. So also *Plato* is of this opinion; *Because* (saith he) Geometry *conduceth to the acquiring and right understanding of all other Arts and Sciences.* To conclude, this is the chiefe Glory of Geometry, that it loyters not, or employes it self about these inferiour Machines, from whence it had its Original, but hath soared up into Heaven, and resetled humane minds, (groveling before in the dust) in Cœlestial Seats, and hath capacitated us to the understanding both of this whole Fabrick of the World, and the Administration and Government thereof.

THE

THE FIRST ELEMENT OF EUCLIDE.

THE ARGUMENT.

THis First Book comprehends the properties of the first and chiefest of right lined figures, *viz.* of *Triangles* and *Parallelograms*, as touching the equality, or inequality of their Area, and of their sides and angles, wherein first is declared the original and properties of Triangles compared among themselves: Then intermixing the properties of parallels, the nature of *Parallelograms* and the affections that are in them are demonstrated; after which is shewn the relation of Triangles and Parallelograms, and after what manner a Parallelogram may be made equal to a Triangle; Lastly, it is demonstrated that in a right angled Triangle, the Square that is made of the side subtending the right angle, is equal to the Squares that are made of the sides comprehending the right angle. And that all these things may the better be accomplished, here is also taught the division of a right lined angle, & of a right line into two equal parts, the construction of a perpendicular line, and after what manner a right lined angle may be made equal to a right lined angle, and other things of the like nature.

A DE-

DEFINITIONS.

1 *A point is that which hath no part.* *As* A

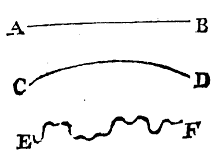
IN Greek it is read σημεῖον, that is to say, a Sign; For seeing that it is void of all Magnitude, that which is externally made, is the signe of that which is conceived in the Minde; and it is (in a manner) the same as a Unite in Number, an Instant in Time, and a Sound in Musick.

2 *A Line is a Length without breadth.* *As* A—B.

THere is no such Line in any material thing, but as the point, so the line which we draw, is the sign of that which we conceive in Mind: For if the point which we conceive be moved, and leaveth an imaginary tract, that shall be a line, long by reason of the motion, but not broad, because the point from whence it proceedeth is void of all extention.

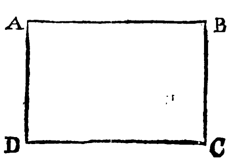
3 *The Extreams or Ends of a Line are points.*

THat is, the beginning and end of a Length, as it is a Length, is a point: because a Mathematician doth not consider Magnitude, but as it is finite: Therefore when *Euclide* speakes of an infinite line, he understandeth an indeterminate line, or a line of any Magnitude.



4 *A Right Line is that which lyeth equally between its points.*

OR whose extreams doth shadow all the middle parts, as saith *Plato*: or the least of all those which have the same extreams, as *Archimedes* will have it: For seeing that a line is conceived to be made by the flux of a point, if it floweth equally between its points, or by the shortest Space, it is called a right line, as A B; If a point be carryed with an uniform motion and distance from a certain point, it is called a circular line as C D; If it moves unevenly, in some place lower, and in others higher, and the extreams do not shadow all the middle parts, it is called a mixt line, as E F: From hence *Aristotle* saith ingeniously *Lib.* 1. *de Caelo*, that according to this threefold distinction of a line, there can be only three motions, two simple, the right and circular, and the third mixt of both.



6 *A Superficies is that which hath only length and breadth.*

AS a line the first Species of continued quantity, is produced by the flux of a point, so a Superficies, the second Species, is conceived to be produced by the flux of a line transversly, or side-wayes, as if the line A D be supposed to move side-wayes to B C, it shalll produce the Superficies A B C D, which may be divided in length as a line, and likewise

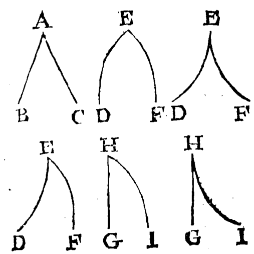
wise in breadth. Conceive a shadow, saith *Proclus*, and thou shalt conceive a Superficies long and broad, yet having no manner of thicknesse.

6 *The Extreams or Ends of a Superficies are lines.*

THis Definition is only to be understood of a plain or a mixt Superficies, and not of a circular, for the extream thereof is only a line, and not lines.

7 *A Plain Superficies is that which lyeth equally between his lines.*

WHat I have said of a right line, the same is to be understood of a Plain Superficies.

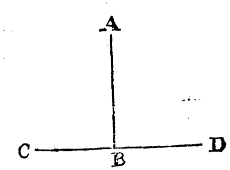


8 *A Plain angle* B A C, *is an inclination or bowing of two lines* A B *and* A C *in a Plain, the one to the other, the one touching the other, and not lying streight forth at length.*

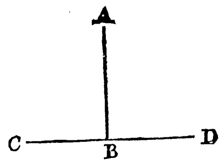
HEre the Causes of an angle are explained, the Material are two lines which touch one another: The Formal is an Inclination of the one to the other. From whence it followeth, First, that those two lines ought not so to touch, that they may lye joyned in a direct line the one to the other, that is, so as they make one right line, but the one ought to incline to the other: Secondly, it followeth that the quantity of an angle consists in a greater or lesser inclination of the lines, and not in the length of the lines: Thirdly, it followeth that it is not necessary that the two lines being produced, should intersect one another after the contact, as *Pelletarius* would have it, for that is only true in right lined angles, but it sufficeth that they touch one another, and be inclined to one another. Lastly, if the angle be in a Plain Superficies; it is called a Plain angle. And in every figure although we call every angle by three letters, yet the middlemost is alwayes to be understood for the Angle.

9 *And when the lines which contein the angle are right lines, it is called a right lined angle.*

ANd if both of them be curved as D E F, it is a curve lined angle, and if one be curved, and the other right as G H I, it is a mixt angle.



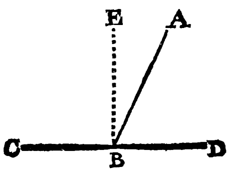
10 *When a right line as* A B, *standing upon a right line as* C D, *maketh the angles on either side,* A B C *and* A B D *equal, then either*



either of those angles is a right angle, and the right line AB, *that standeth erected, is called a perpendicular line to that upon which it standeth* C D.

THen either angle is said to be equal; when the right line A B inclineth not more to C then to D.

That which the Greeks call κάθετος, is rendred in Latine *Perpendicularis*, but the Mathematicians do more frequently use the Greek word then the Latine, chiefly in the Opticks, where there is nothing more used by them then πρὸς κάθετον; alsso in Latine they render it *ad Cathetum*.



11 *An Obtuse angle as* C B A, *is that which is greater than a right angle* E B C.

BEcause the right line A B, doth more recline from the base line C D, then the perpendicular E B.

12 *An Acute angle, as* A B D, *is that which is lesser than a right angle* E B D.

13 *A Term is that which is the extream of any thing.*

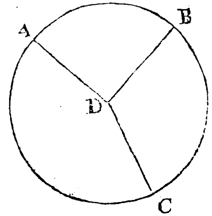
SUch are a point, a line, a superficies; that is to say, a point is the Term of a line, a line of a superficies, and a superficies of a Body.

14 *A Figure is that which is comprehended by one Term, or many.*

IT is said by one Term, because one Term comprehendeth a Circle and an Ellipsis, that is to say, a Circular, or an Elliptical line: but there are alwayes more Terms required to comprehend right lined figures.

Moreover it is to be observed, the Terms ought to encompasse and comprehend the quantity which is called a Figure, and not only terminate it. From whence it followeth, first, that a line properly is no Figure, seeing that points do not encompasse a line, but only terminate it: It followeth, secondly, that there, can be no figure of an infinite Superficies, or an infinite body, if any such may be given; first because every figure ought to encompasse and comprehend the thing figured: Secondly, because it is encompassed with Terms, but a Term is the extream of a thing. And how can that which hath an end and extreams be infinite?

15 A



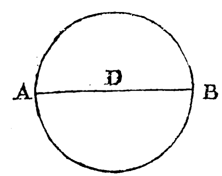
15 *A Circle is a plain figure, comprehended by one line* A B C, *which is called a Circumference: unto which all right lines drawn from one point within the figure, as* D A, D B, D C, *and falling upon the Circumference thereof, are equal the one to the other.*

16 *And that point is called the center of the Circle.*

THeodosius hath the same in his first Book of *Sphericks*, (concerning a Sphere) Definition the first, and second, but in the fifth Definition he thus describeth a Pole.

The Pole of a Circle in a Sphere, is a point in the Superficies of the Sphere, from whence all right lines drawn to the Circumference of the Circle are equal the one to the other.

From which it may be gathered that there is only this difference between the Center and the Pole of a Circle on the Sphere, because the Center is conceived to be within the figure: But the Pole is in the Superficies of the Sphere.



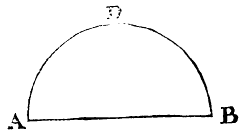
17 *The Diameter of a Circle is a certain right line, as* A B, *drawn by the center* D, *and being terminated by the circumferenee on either side, as* A *and* B, *divides the circle into two equal parts.*

HEre you shall observe three things, first, that all the Diameters of the same Circle are equal to one another, seeing that their halves are equal by the 15th. Definition, the second which followeth from the first is, that although there may be infinite right lines drawn in a Circle not passing by the Center, yet only those right lines which are drawn by the Center and terminated in the Circnmference, are called Diameters; because the Diameter only can measure the Circle: seeing that they only be all equal the one to the other, and of a determinate length; but the others are alwayes unequal and uncertain: For *Ptolomie* saith in his *Analemma* that the measure of any thing ought to be certain, and determinate, not indefinite; Wherefore let not beginners wonder if it be put in the Feminine Gender by the Mathematicians, For Diameter is the same as the dimetient line, or a line dividing into two equal parts.

The

The third is that the Diameter doth divide the Circle into two equal parts, which *Thales* doth thus demonstrate in *Proclus*; Conceive in mind the portion of a semicircle, so to be fitted to another portion, so as the Diameter may be the Base of both of them: If the Circumference of one do altogether agree to the Circumference of the other, it is manifest that those two portions made by the Diameter are equal the one to the other; seeing that neither of them doth exceed the other: But if one Circumference do not agree with the other, but falleth either without or within it, or partly without, and partly within, then the right lines drawn from the Center to the Circumference shall be equal, and not equal, which is absurd.

18 *A Semicircle is a figure which is conteined by the Diameter* A B, *and by that line* A D B. *which is taken away from the circumference of the circle.*



19 Right lined Figures, *are those which are contained under right lines.*

After the Definition of the Circle, *Euclide* intending to give the Description of divers figures, teacheth first of all what figures are called Right lined Figures; for of those he principally treats in the following Books.

Therefore all plain figures enclosed on all sides by right lines, are called Right lined Figures; from whence it is manifest that the plain figures environed by Curve lines, are called Curvilined: but those which are comprehended partly by Right lines, and partly by Curve lines, are called Mixt: Now divers kinds of Figures are here described by *Euclide*.

20 Trilateral, *or three sided figures, are those which are conteined under three right lines.*

When *Euclide* here saith, that Right lined Trilateral figures are such as are environed by three right lines, he shews us clearly how a Triangle ought to be defined: For seeing that in Right lined figures there are as many angles as there are sides, or of right lines of which they are constituted, the Triangle shall be a figure conteined under three right lines whose Species shall be hereafter declared.

21 Quadrilateral, *or four sided figures, are those which are conteined under four right lines.*

In like manner, the Quadrangle, or Quadrilateral figure, is a figure conteined of four right lines, whose divers Species and Kinds, shall be hereafter declared.

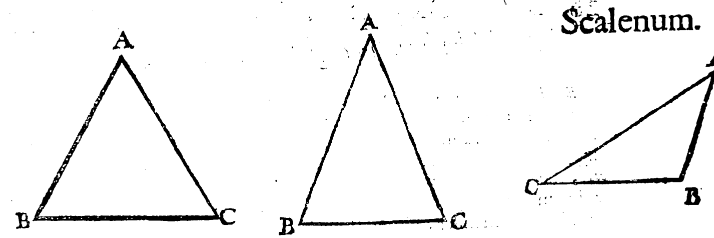
22 Mul-

22 Multilateral, *or many sided figures, are those which are conteined under more than four right lines.*

Forasmuch as the Species or Kinds of Right lined figures are innumerable, by reason of the infinite progression of numbers: For three right lines environing a figure, do constitute the first Kind, under which are conteined all Triangles, four right lines constitute the second Kind, which compriseth all quadrangular or quadrilateral figures; five right lines compound the third Kind, and six the fourth; and so *ad infinitum*: Therefore seeing that right lined figures cannot be infinitely defined, *Euilde* calleth all other right lined figures, comprehended by more than four right lines, by this General Name [*Multilateral*] contenting himself with the Denomination of the Trilateral and Quadrilateral, it may be for this reason, because he treateth principally of those in his Books, and by the knowledg of those the Definition of the rest may be easily understood.

23 *Now of* Trilateral *or three sided figures, that which hath three equal sides is called an* Equilateral *Triangle; as* **ABC**.

Equilateral. Isosceles. Scalenum.



24 *But that which hath only two sides equal, is called an* Isosceles *Triangle; as* **CAB**.

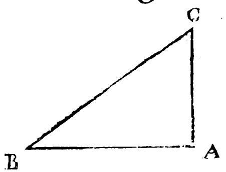
25 *And that which hath the three sides unequal, is called a* Scalenum, *as* **BAC**.

By these three Definitions he exposeth the three Species or Kinds of Triangles, the first and most simple of which is the *Equilateral* Triangle, or Triangle of three equal sides, the second is the *Isosceles* Triangle, which hath onely two sides equal, and the third is the *Scalenum*, which hath all the three sides unequal, the Equilateral Triangles are alwayes uniform, forasmuch as their angles are alwayes equal: But the Isosceles and Scalene Triangles are diversified in infinite manners.

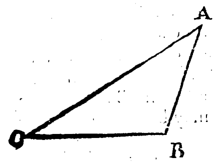
26 More-

26 *Moreover, of three ſided, or* Trilateral *figures, the* Orthogonal, *or Rectangled Triangle, is that which hath one right angle, as* B A C.

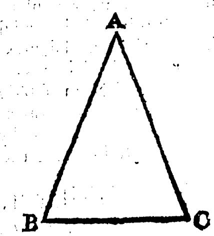
Orthogon.



Ambligon.



Oxigon.



27 *The* Ambligonium, *or obtuſe angled Triangle, is that which hath one angle obtuſe, as* A B C.

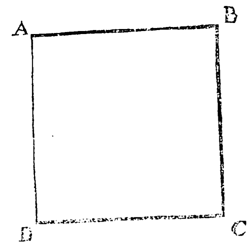
28 *And the* Oxigonium, *or acute angled* Triangle *is that which hath all the angles acute; as* B C A.

EUCLIDE expoſeth Triangles in conſidering them; firſt, according to their ſides; and he doth it now according to the conſideration of their angles, and foraſmuch as there are onely three varieties of Right lined angles, as we have declared, alſo there are three Kinds of Triangles, to wit, the Rectangle, which is that which hath one angle a right angle, which may be either Scalene, or Iſoſceles: The Ambligon Triangle may be alſo in like manner, either Iſoſceles or Scalene, (but not equilateral;) but every Oxigonium, or acute angled Triangle, may be either Equilateral, Iſoſceles, or Scalene.

❡ Here it is to be noted, that in every Triangle, two of the lines being taken for two ſides, the third remaining ſide is uſually called by the Mathematicians the [Baſe] of the Triangle, whether it be the loweſt ſide of the Triangle or not, in ſuch ſort, as each of the lines which encloſeth the Triangle, may be taken for the Baſe.

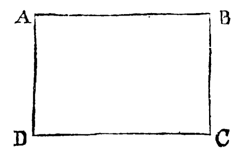
29 *But of* Quadrilateral, *or four ſided figures, the Square is that which is equilateral, and right angled; that is to ſay, which hath the four ſides equal, and the angles right.*

After



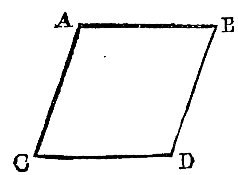
AFter the Trilateral and three ſided figures he expoſeth the Quadrilateral and four ſided figures, ſpeaking only of five ſorts, of which the firſt four are Regular, but the fifth and laſt Irregular: Therefore the firſt figure is called a Square, of which all the ſides are equal to one another, and all the angles right angles, as is the Figure A B C D; and therefore the Quadrangle which is equilateral, and not right angled, or contrarily rectangled, and not equilateral, ſhall not be called a Square.

30 *An* Oblong, *or Long Square is a figure rectangled, but not equilateral.*

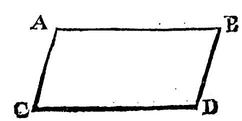


THe ſecond Quadrilateral Figure is called by the Greeks ετερόμηκες, that is to ſay, longer of one part, which is that which hath the four angles right angles, but hath not the four ſides equal to one another, although the oppoſite ſides are equal to one another, as appears by this Figure A B C D, where the ſides A B and D C oppoſite to one another, are equal, and in like manner, the ſides A D and B C.

31 *A* Rhombus, *is a figure equilateral, but not right angled.*



THis Figure which is the third kind of Quadrilateral, or four ſided figures, hath the four ſides equal, but the four angles are not right angles, although that the oppoſite angles are equal to one another, two to two, the angles A and D being equal to one another, but not right angles; and in like ſort, the angles B and C.

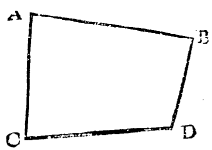


32 *A* Rhomboides *is a Figure which hath the oppoſite ſides, and angles equal, but is neither equilateral, nor right angled.*

This

This Figure called Rhomboides, neither hath the sides equal, nor any angle a right angle, but only hath the opposite sides and opposite angles equal to one another, as the sides A C and B D, are equal to one another, and in like manner, A B and D C, and the opposite angles A and D, and likewise B and C equal to one another; therefore these four Quadrilateral figures may be termed regular, but all other Quadrilaterals are irregular.

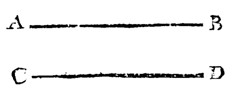
33 *But besides these figures, all other Quadrilateral Figures are called* Trapeziums.



All other Quadrilateral Figures, which differ from those before mentioned; that is to say, which are neither Squares, nor long Squares, Rhombs, nor Rhomboides, are called Trapeziums, and are Irregulars, considering that they may be of divers manners and formes.

34 *Parallels are right lines, which being in one and the same Plain, and prolonged infinitely on both parts, neither meet on the one side, nor on the other, as the lines* AB *and* CD.

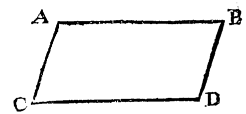
To the end the right lines be parallel, or equi-distant, it sufficeth not, that being prolonged infinitely on both parts, that they meet not at a point, or never intersect, but it is necessary that they be also in one and the same Plain Superficie: For divers right lines being not in one and the same Plain Superficie, prolonged infinitely, will never intersect at one point; yet neverthelesse are not said to be Parallels, such as would be two lines posited transversly, and understood to be in the air, and touch not one another; those I say, can never intersect; Therefore the right lines that are in one and the same Plain Superficie, and which being prolonged infinitely on both sides, (as for Example, the lines A B and C D,) will never intersect, those shall be termed Parallels, or lines equally distant, *Posidonius* thus defineth Parallels.



Parallels, (saith he) are such lines as are neither elevated nor depressed in a Plain, but have all the perpendiculars equal, which are drawn from all the points taken in the one or the other line.

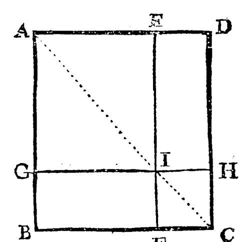
¶ *EUCLIDE* here endeth the Definitions of his First Book, but forasmuch as mention is made therein of Figures, Parallelograms, and of their Complements, we have judged it convenient to explain the two following Definitions, to render the Demonstrations more intelligible.

35 A Parallelogram *is a Quadrilateral, or four sided figure, whose opposite sides are parallel, or equidistant.*



As if in the Figure A B C D, the side A B be parallel to the side opposite C D, and A C parallel to B D, it shall be called Parallelogram; that is to say, a Figure made of lines which are parallels, and of these there are four sorts; to wit, the Square, the long Square, which are also called rectangled Parallelograms; the Rhombe and the Rhomboides, Parallelograms not rectangled, from whence may be collected, that there are only four sorts of Parallelograms.

36 *But when in a Parallelogram there is drawn a Diameter or Diagonal, and two right lines parallel to the sides, cutting the Diameter in one and the same point, in such sort as the Parallelogram be divided by those parallel lines, into four Parallelograms, those two through which the Diameter doth not passe, are called Complements, but the two others, through which the Diameter doth ~~not~~ passe, are said to be about the Diameter.*

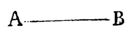


Let A B C D, be a Parallelogram, in which having drawn the Diameter A C, and the line E F, cutting the same in the point I, and in like manner, H G cutting the same Diameter in the same point I, and parallel to A D and BC, it is manifest, that the whole Parallelogram is divided into four Parallelograms, two of which, to wit, D I and I B, through which the Diameter doth not passe, are by Geometricians called Complements, or Supplements of the Parallelograms; the remaining Parallelograms E G and F H, are said to be about the Diameter, forasmuch as the Diameter doth passe through them.

Petitions or Requests.

1 *It is required that it be granted, that a right line may be drawn from any point to any point.*

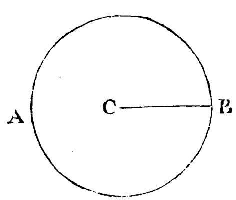
THis first Petition will be manifest, if you consider what hath been heretofore said of a line; For seeing that a line is an imaginary flowing of a point, and by this means a right line is a flowing, proceeding from the direct way, it will happen that if some point be understood to move to another point, a right line will be drawn from one point to another, which *EUCLIDE* requireth by this first Petition; as you may see by the line AB, drawn from the point A, to the point B.



2 *And that a terminated right line may be produced streight forth continually.*

AS if it were understood that this point were moved yet further forth directly, the terminated right line shall be continued; and this production may be continued *ad infinitum*, seeing that we may imagine this point to move infinitely; as if the right line AB were continued to the point C, and then to the point D, &c.

3 *And that upon any Center, and at any distance, a Circle may be described.*



NOw if we conceive in the understanding, some terminated right line, applyed according to one of the extremities at a point fixed, to move about the same, until it return where it began its motion, it will have described a Circle, which is what is required by this Petition, as may be seen by Example in the line CB, which drawn about the Center C, hath described the Circle BA, acording to the quantity, distance, or interval CB.

Others add this fourth Petition.

4 *That any Magnitude being given, it is possible to take another greater or lesser.*

FOr all continued quantity may be augmented by addition, and by division may be infinitely diminished: Therefore there cannot be given a continued quantity so great, but a greater may yet be given; nor one so little, but a lesser may be given; This holds as true in Numbers, by addition, for each number may be augmented infinitely; by the continual addition of Unity, although in the diminishing thereof, you arrive to Unity indivisible.

Common Sentences, or Axioms.

1 *Things that are equal to one and the same, are also equal to one another.*

FRom this first Axiom it is evident, that a thing greater, or lesse than one of the equal things, shall be also greater or lesse than the other; and if one of the equal things be greater or lesse than some Magnitude, the other shall be in like manner greater or lesse than the same Magnitude.

2 *And if to equal things, equal things be added, the whole shall be equal.*

3 *And if from equal things, equal things be taken, the remainders shall be equal.*

4 *And if to unequal things, equal things be added, the whole shall be unequal.*

IN like manner, it is manifest, that if to unequal things, things unequal be added, the greatest to the greatest, and the least to the least, that the whole shall be unequal, the one greater, the other lesse.

5 *And if from unequal things, equal things be taken, the remainders shall be unequal.*

IT follows from hence, that if from unequal things, unequal things be taken; to wit, from the greatest, lesse, and from the least more, the remainders shall be unequal; that is to say, this shall be greater, and that other lesse.

Now in all these Axioms, excepting only the first, by these words, *equal quantities*, you ought also to understand one and the same, common to divers: for if to equal things there be added one common thing, the whole shall be equal; and if from equal things, a common thing be taken, the remainders shall be also equal: and if to unequal things, there be added one and the same common thing, or to one and the same common thing, there be added unequal things, the whole shall be unequal, and if from unequal things there be taken one and the same common thing, or from one and the same common thing, there be taken unequal things, the remainders shall be unequal.

6 *And the things that are double to one and the same thing, are also equal to one another.*

Whence

WHence it followes, that, that which is double to one of the equal things, is alſo double to the other: Likewiſe thoſe things which are triple, quadruple, quintuple, &c. to one and the ſame thing, are equal to one another, as is manifeſt.

7 *And the things which are the one half of one and the ſame thing, are alſo equal to one another.*

COntrariwiſe it follows, that thoſe things which are equal to one another, are the halves of one and the ſame; likewiſe the things which are the third, fourth, or fifth parts, &c. are alſo equal to one another.

In thoſe two Maximes, by one and the ſame quantity, ought alſo to be underſtood, equal quantities, for things double, triple, &c. of things which are equal, are in like manner equal to one another. Alſo the things which are the halves, third, or fourth parts, &c. of equal things, are alſo equal to one another.

8 *And the things which agree to one another, are alſo equal to one another.*

THat is to ſay, that two quantities, the one being poſited on the other, the one exceedeth not the other, but both agree together, thoſe are equal; as two right lines ſhall be ſaid to be equal to one another, when the one being poſited on the other, that which is placed above, totally agrees with the other, in ſuch ſort, as that it neither exceedeth the other, nor is exceeded by it: The ſame alſo of two right lined angles, equal to one another, when the one being poſited on the other, the uppermoſt exceedeth not the other, nor is exceeded by it, but the lines which conſtitute the one, do totally fall upon thoſe which conſtitute the other; for that being ſo, the inclination of the lines are equal; although, neverthe-leſſe, thoſe lines are unequal to one another: but it is to be underſtood, that the Magnitudes which agree to one another, are equal, according to that only in which they agree. Now length agrees only to length, a Superficie to a Superficie, and the inclination of lines, to the inclination of lines, &c.

Clavius willing to convert this Axiom, ſaith thus, [and contrariwiſe the things which are equal to one another, agree with one another, if the one be poſited on the other;] which nevertheleſſe, cannot be, for if a ſquare be deſcribed, and on the Diameter of that Square there be deſcribed another ſquare, the firſt ſquare ſhall be equal to the half of the laſt; that is to ſay, to an Iſoſceles Triangle, and nevertheleſſe, will not agree, being poſited the one on the other: There may be alſo made a Curvilined angle, equal to a right angle, an obtuſe angle, or acute angle, as *Proclus* hath demonſtrated, which nevertheleſſe will not agree, being poſited on one another. Laſtly, a right lined Triangle may be made equal to another right lined Triangle, which may not agree, being poſited on one another, as is manifeſt by the 37 and 41 of the firſt Book: Therefore this Axiom cannot be converted univerſally, but thus it may. Equal right lines agree with one another, being poſited the one on the other: in like manner, equal right lined angles, as alſo plain Superficies, alike and equal, &c.

9 *And*

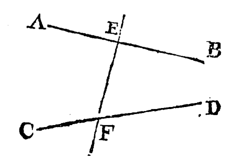
9 *And the whole is greater than its part.*

10 *All right Angles are equal to one another.*

THis Axiom is manifeſt by the 10 Definition, *Pappus* ſhews on this place that this Axiom cannot be converted; for every angle equal to a right angle is not a right angle, as we have ſhewn in the 8th. Common Sentence; but only, that every right lined angle equal to a right angle, is alſo a right angle; and this may be taken for an Axiom.

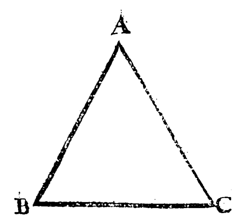
11 *And if on two right lines there fall another right line, making the interior angles on the ſame part leſſe than two right angles; thoſe two right lines being infinitely prolonged, will interſect and cut one another, on that part where the two angles are leſſe than two right angles.*

AS if a right line E F, falling on the two right lines AB and CD, maketh the two interior angles BEF, and DEF on the ſame part BD, leſſe than two right angles; *EUCLIDE* ſaith, that thoſe lines being prolonged on the part BD, where the angles are made leſſe than two right angles, that finally they ſhall interſect at a point, which is manifeſt, for when the two interior angles on the ſame part are equal to two right angles; then the two lines cannot interſect in any part, but are alwayes equally diſtant the one from the other, as ſhall be demonſtrated in the 28th. Prop. of this Book; wherefore if the interior angles on the ſame part be leſſe than two right angles, it is neceſſary that the Space between the lines ſtreighten more and more, and grow leſſe and leſſe, and on the other part that they open and enlarge themſelves more and more: Therefore thoſe lines at laſt will interſect at a point.



12 *Two right lines contein no Space.*

THis Axiom hath no difficulty in it, for if two lines make an angle at a point, they ſpread and dilate themſelves ſtill more and more both wayes; if they be prolonged, as is manifeſt by this Figure, for having drawn a right line from A to B, from the ſame point A, cannot be drawn another right line to the point B, by another way, in ſuch ſort as that between AB and the other line, there ſhould remain a Space incloſed; ſo as that another right line drawn from the point A,



as

as A C, shall make an angle with A B, in such sort as that another line ought yet to be drawn, as B C, to enclose the Space: but a right line and a Curviline, or else two curved lines, may well contein or enclose a Space, as is manifest.

After these Axioms of EUCLIDE, *Clavius* and other Interpreters, have yet added these following Axioms.

1 *Two right lines, have not one and the same Common Section.*

2 *Two right lines meeting at a point, if they be prolonged, they will necessarily intersect.*

There are yet added these six.

1 *If to equal things, unequal things be added, the excesse or difference of the whole shall be equal, or the same with the difference of the things added.*

2 *If to unequal things, equal things be added, the excesse, or difference of the whole shall be equal, or the same with the excesse or difference of those which were at the beginning.*

3 *If from equal things, unequal things be taken, the excesse or difference of the remainders shall be equal, or the same with the excesse or difference of the things taken away.*

4 *If from unequal things, equal things be taken, the excesse or difference of the remainders shall be the same with the excesse or difference of the whole.*

5 *The whole is equal to all its parts taken together.*

6 *If the whole be double to the whole, and the part cut off, double to the part cut off, the rest shall be double to the rest.*

NOw these six pretended Axioms are also hereafter demonstrated by *EUCLIDE*, as shall (God assisting) be made appear. Wherefore seeing that they are proved elswhere, following the Demonstrations of

of *EUCLIDE*, they ought not to be taken for Axioms: We shall demonstrate according to *EUCLIDE* the five first, at the Second Proposition of this Book, and the Sixth, at the fifth Proposition of the Fifth Book.

From what hath been before said, we may gather, with *Proclus* and *Geminus*, this difference between Requests, and Maximes or Axioms; that seeing both the one and the other, are known of themselves, and indemonstrable, that Requests are of the nature of Problemes; forasmuch as by them there is required some thing to be done; but Axioms resemble Theoremes; seeing that by them it is not required that any thing should be done, but only there is proposed some Sentence well known. But the Request or Petition differeth from the Problem, in that the Construction of the Request standeth in need of no Demonstration, but no one will consent to the construction of a Problem, without Demonstration, for that some difficult thing is proposed to be construed. In like manner, there is difference between Axiom and Theorem, for the Axiom ought not to be demonstrated; but the Theorem ought in no wise to be consented to, if not demonstrated: For no one will require the proof or Demonstration of this Proposition, [Things which are equal to one and the same thing, are likewise equal to one another;] but of this there will be required Demonstration, [Of every Triangle, the interior angles are equal to two right angles;] and the same is to be understood of the other Axioms and Theorems, as also of Petitions and Problems.

It is likewise very certain, that of Requests or Petitions, some of them are more proper to Geometry, as those three which *EUCLIDE* hath proposed to us: other some are common to Geometry and Arithmetick, as is this, [A quantity may be infinitely augmented;] for as well the number as the Magnitude, may be augmented by Addition, so as that no end can be found in this augmentation; the same may be said of the Axioms for the Eighth, Tenth, Eleventh, and are proper to Geometry only, but all the others are applyed to the Demonstrations both Geometrical and Arithmetical, for even so as equal Magnitudes taken from equal Magnitudes, do leave equal Magnitudes, be the Magnitudes consisting of Lines, Superficies, or of Solid Bodies; So likewise equal numbers, taken from equal numbers, leave equal numbers; and having said this of these three Kinds of Principles, let us now come to the Demonstrations, by which the nature of these Principles will be more perfectly known; for there are divers principles among Mathematicians, and such as are not easily understood, before their use be known in their Demonstrations, which will be found by experience.

¶ To the end the course of the Demonstrations be not interupted, we have cited the Principles and Propositions of *EUCLIDE* in the Margent, with a Letter of the Alphabet; the same Letter being also put in the Discourse, there where the Proposition ought to be cited, written in the Margent with the same Letter, and may thus be understood;

1 Def.	The first Definition, and so of the rest.
1 Pet.	The First Petition, or Request.
1 c.s. or 1. ax.	The first Common Sentence, or Axiom.
1. 1.	The first Proposition of the first Book.
1. 2.	The first Proposition of the second Book, and so of the rest, in this manner.
Color. 32. 1.	The Corollary of the 32 Proposition of the first Book.

What a Proposition is.

THe principles thus placed and ended, now follow the Propositions, which are Sentences set forth to be proved by reasoning and Demonstrations; and therefore they are again repeated in the end of the Demonstration: For the Proposition is ever the conclusion, and that which ought to be proved.

Propositions are of two sorts, the one is called a Problem, the other a Theorem.

What a Problem is.

A Problem is a Proposition which requireth some action or doing, as the making of some figure, or to divide a figure or line, to apply figure to figure, to adde figures together, or to subtrah one from another, to describe, to inscribe, to circumscribe one figure within or without another, and such like: As of the first Proposition of the first Book is a Problem, which is thus, *upon a given terminated right line, to make an Equilateral triangle.* For in it, besides the Demonstration, and Contemplation of the mind, is required somewhat to be done: namely, to make an Equilateral triangle upon a line given. And in the end of every Problem, after the Demonstration, is concluded after this manner: *Therefore, &c. Which was to be done.*

What a Theorem is.

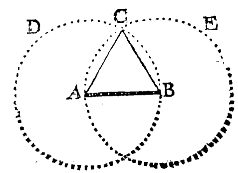
A Theorem is a Proposition which requireth the searching out and demonstration of some property or passion of some figure: Wherein is only Speculation and Contemplation of Mind, without doing or working of any thing, As the seventeenth Proposition of the first Book, which is thus, *Of every triangle two angles are lesse than two right angles, after what manner soever they be taken*; is a Theorem. For in it is required only to be proved and made plain by reason and Demonstration, that two angles are lesse than two right angles, &c. without further working or doing. And in the end of every Theorem, after the Demonstration, is concluded after this manner: *Therefore, &c. Which was to be demonstrated.*

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. PROBLEM 1.



Upon a given terminated right line A B, *to make an Equilateral Triangle* A B C.

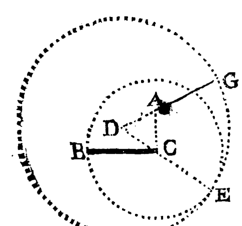
Construction ON the Center A, at the distance A B, [a] describe the Circle B C D; and on the Center B, at the same distance B A, [b] describe the Circle A C E, and from the point C, where the Circles intersect one another, [c] draw the two right lines C A and C B: Then I say, that the Triangle A B C is Equilateral.

a) 3. pet. 1.
b) 3. pet. 1.
c) 1. pet. 1.

Demonstration FOr the right line A C is equal to the right line A B [d], and the right line C B is equal to the same right line A B [e]: Therefore [f] the right lines A C and B C, are also equal the one to the other: And the three right lines C A, A B, and B C, being equal, [g] the Triangle A B C is equilateral: Wherefore upon a given terminated right line A B, is made an Equilateral triangle A B C; Which was required to be done.

d) 15. d. 1.
e) 15. d. 1.
f) 1. ax. 1.
g) 23. d. 1.

PROP. 2. PROBL 2.

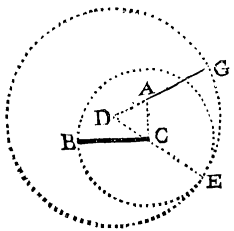


From a point given A, *to draw a right line* A G, *equal to a right line given* B C.

Construction DRaw [a] from A to C, the right line A C, upon which make [b] the equilateral triangle C A D, then on the Center C, at the distance B C, describe [c] the Circle B E,

a) 1. pet. 1.
b) 1. 1.
c) 3. pet. 1.

 and



d) 2. pet. 1.
c) 3. pet. 1.

and [d] produce the side D C to E, and on the Center D, at the distance D E, [e] describe the Circle E G. and produce the side D A to G. Then I say, that the right line A G, (drawn from the given point A) is equal to (the given line) B C.

f) 15. d. 1.
g) 3. ax. 1.

Demonstration FOr the right lines D E and D G are [f] equal: Therefore taking away the equal right lines D A and D C, there shall remaine the equal right lines [g] A G and C E: But C E is equal to C B: Therefore also A G is equal to C B.

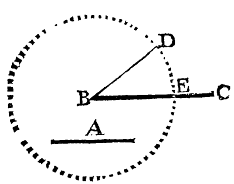
Wherefore from a point given A, is drawn a right line A G, equal to a right line given B C; Which was required to be done.

The position of the point A, in or without the given line B C doth varie the Case, but yet there is alwayes the same Construction and Demonstration.

SCHOLIUM.

The line A G *may be taken equal to* B C *with the Compasses, but in so doing, it answers to no Petition, as* Proclus *well observes.*

PROP. 3. PROBL. 3.



Two unequal right lines A, *and* B C, *being given, to take away from the greater* B C, *a right line* B E, *equal to the lesser* A.

a) 2. 1.
b) 3. pet. 1.

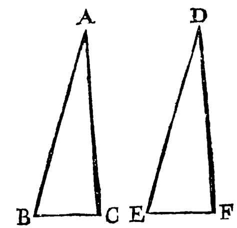
Construction FRom the given point B, [a] draw the right line B D, equal to the given right line A, and on the Center B, [b] at the distance B D describe the Circle D E: Then I say, that the part cut off is equal to A.

c) 15. d. 1.
d) 1. ax. 1.

Demonstration FOr the right line B E is equal to the right line B D [c], and B D is equal to A by Construction; Therefore, the part cut off B E, is equal to the given line A [d]: Which was required to be done.

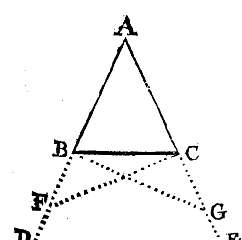
PROP. 4. THEOR. 1.

If two triangles B A C *and* E D F, *have two sides* B A *and* A C *of the one, equal to two sides* E D *and* D F, *of the other, each side to his correspondent side, (that is,* B A *equal to* E D, *and* A C *equal to* D F,) *and the angle* B A C, *conteined by the equal right line of the one, be equal to the angle* E D F, *conteined by the equal right lines of the other: The base also* B C *of the one, shall be equal to the base* E F *of the other; and the one triangle* B A C, *shall be equal to the other triangle* E D F, *and the other angles remaining, shall be equal to the other angles remaining, the one to the other; that is, the angle* A B C *to the angle* D E F, *and the angle* A C B *to the angle* D F E, *by which equal sides are subtended.*



Demonstration FOr the triangle A B C being put upon the triangle D E F, and the point A upon the point D, and the right line A B upon the right line D E the point B shall fall upon the point E, because the right line A B, is equal to the right line D E: Seeing therefore that the right line A B doth agree with the right line D E, and the right line A C with the right line D F; because the angle B A C is equal to the angl E D F: Therefore, also the point C shall agree with the point F, wherefore also the base B C shall agree with the base E F. For if (B agreeing with E, and C with F,) the base B C doth not agree with the base E F: Then two right lines shall include a Superficies, which is absurd. Therefore the base B C, agrees with the base E F, and is equal to it: Wherefore the whole triangle A B C, shall agree with the whole triangle D E F, (for all the angles and sides do agree) and is equal to it, and the remaining angles of the one, shall agree with the remaining angles of the other, and shall be equal the one to the other, *viz.* the angle A B C to the angle D E F, subtended by the equal sides A C and D F, and the angle A C B to the angle D F E, subtended by the equal sides A B and D E.

PROP. 5. THEOR. 2.

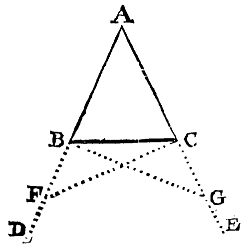


The angles A B C *and* A C B, *which are at the base of* Isosceles *triangles* A B C, *are equal the one to the other, and the equal right lines* A B *and* A C, *being produced, (as to* D *and* E,) *the angles* C B D *and* B C E, *that are under the base, are equal the one to the other.*

Con-

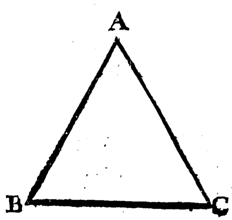
a) 3. 1. b) 1. pet. 1. *Construction* Let any point as F be taken in the line B D, and from the greater line A E, [a] let the lesser A G, be taken equal to the line A F, and [b] draw the right lines F C and G B.

c) by Con. d) by Supp. *Demonstration* Seeing that A F is equal [c] to A G, and A B to A C [d]: Therefore these two F A and A G are equal to these two G A and A B, the one to the other, and they contein a common angle F A G:
e) 4. 1. Therefore [e] the base F C is equal to the base G B, and the triangle A F C is equal to the triangle A G B, and the remaining angles shall be equal to the remaining angles, the one to the other, by which equal sides are subtended, that is, A C F to A B G, and A F C to A G B, *and because* A F *is equal to* A G, *and* A B *to* A C. *Therefore*, B F is equal [f] to C G, and it is proved that F C is
f) 3. ax. 1. equal to G B: Therefore these two B F and F C are equal to these two C G and G B, the one to the other; and the angle B F C is equal to the angle C G B, and they have one common base B C:
g) 4. 1. Therefore [g] the triangle B F C shall be equal to the triangle C G B, and the remaining angles of the one equal to the remaining angles of the other, the one to the other, by which equal sides are subtended: Therefore the angle F B C is equal to the angle G C B, and the angle B C F to the angle C B G: And because the whole angle A B G is equal to the whole angle A C F, of which the angle C B G is equal to B C F; the remaining angle A B C shall be equal to the remaining angle [h] A C B, and they are at
h) 3. pet. 1. the base of the triangle, and it is proved that the angles under the base F B C and G C B are equal: Therefore the angles A B C and A C B, at the base of Isosceles Triangles are equal, &c. Which was to be demonstrated.



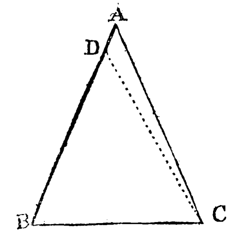
COROLLARIE.

From this Fifth Proposition it also followeth, that every equilateral triangle is equiangled; that is to say, that the three angles of whatsoever equilateral triangle are equal to one another: For let A B C *be an equilateral triangle: Therefore forasmuch as the two sides* A B *and* A C *are equal, the two angles* B *and* C, *upon the base* B C, *shall be also equal. Likewise because the two sides* A B *and* B C *are equal, the two angles* A *and* C *upon the base* A C *shall be equal: Therefore the three angles* A, B, *and* C *shall be equal: Which was to be demonstrated.*



PROP.

PROP. 6. THEOR. 3.



If two angles A B C *and* A C B, *of a triangle* A B C, *be equal to one another, the sides* A B *and* A C *subtending those equal angles shall be also equal to one another.*

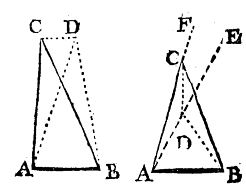
Construction For if they be unequal, one of them shall be greater than the other; Let A B therefore be the greater, (if possible,) and let D B [a] be cut off, equal to A C, and let D C be drawn. a) 3. 1.

Demonstration Forasmuch as D B is equal to A C, and B C common, the two sides D B and B C of the triangle D B C, shall be equal to the two sides A C and C B, of the Triangle A C B, each side to his correspondent side, and the angle A B C is equal to the angle A C B by Supposition; Therefore [b] the base A B shall be equal to the base D C, b) 4. 1. and the whole triangle D B C, equal to the whole triangle A C B; that is, the part equal to its whole; which is absurd; therefore A B shall not be unequal to A C: Therefore if two angles, &c. Which was to be demonstrated.

COROLLARIE.

It follows from this Proposition, that every equiangled triangle (that is to say, all whose angles are equal to one another,) is equilateral; for seeing it hath been demonstrated, that the angles B *and* C *being equal, the subtending sides* A B *and* A C *shall be also equal, it may be concluded in the same manner, that seeing the angles* A *and* B *are equal, the subtending sides* A C *and* C B *shall be also equal, and so of the rest: Therefore the three sides shall be equal, and therefore the triangle is equilateral.* As in the preceding Corollary of the Fifth Proposition.

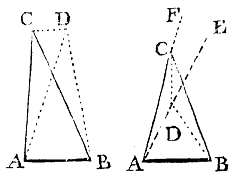
PROP. 7. THEOR. 4.



If two right lines A C *and* B C, *drawn from the extremities* A *and* B, *of one and the same right line* A B, *do meet in a point* C, *there cannot be drawn from the same extremities* A *and* B, *to another point, and on the same side, two other right lines equal to the first, each line to his correspondent line.*

It

IT is most certain, that from the point A, towards C, to another
point as D, a line may be drawn equal to A C, but the line
drawn from B to the same point D, shall not be equal to B C: Like-
wise from B there may be drawn a line equal to B C, on the same
side to some other point: but the line drawn from A to the same point,
shall not be equal to A C: Let there-
a) 2. 1. fore [a] A D be drawn equal to A C, on
the same side, and let B D be joyned:
I say, that B D cannot be equal to B C;
for having drawn C D, the figure
A C D shall be an Isosceles triangle,
A C and A D being equal by constructi-
b) 5. 1. on [b]: Therefore [c] the angle A C D shall
c) 5. 1. be equal to the angle C D A, on the base
d) 9. c. f. C D; But [d] the angle B C D is lesse
than the angle A C D, the part then its



e) 9. c. f. whole; it [e] shall be therefore also lesse than C D A, which is shewn to be
equal to A C D; and therefore yet much lesse than the whole C D B:
But if the line B D were also equal to the line B C, the triangle B C D
f) 5. 1. would be an Isosceles triangle; and therefore [f] the angle B C D should be
equal to the angle C D B, on the base C D. But B C D is shewn to be
much lesse than C D B, it should be then equal, and much lesse, which
is impossible: Therefore B D shall not be equal to B C. Which was
to be demonstrated.

And if from the point B there be drawn a line equal to B C, to some
other point then C, and on the same side, it will be shewn by the same
reason, that the line drawn from the point A, to the same point, shall not
be equal to A C.

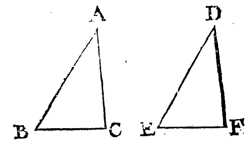
Otherwise, (if it be possible,) from the extremities A and B, let there
be drawn A D and B D, to another point D, towards C, equal to the first
A C and B C, each to his correspondent, to wit A D, equal to A C, ha-
ving the same extremity A, and B D equal to B C, having the same ex-
tremity B, and let D C be joyned; and if the point D be within the tri-
angle, (as in the second figure;) Let A C and A D be prolonged directly
towards E and F, as much as shall be expedient.

Forasmuch as the line A D is put equal to A C, the angle E D C alone,
g) 5. 1. shall be equal to D C B and B C F, taken for one only angle, [g] seeing they
are both of them under the base D C, of the triangle A D C in the first
h) 9. c. f. figure, or as in the second figure: But [h] the whole angle D C F is greater
h) 9. c. f. than the angle D C B alone, [h] the whole, than the part: Therefore the
angle E D C shall be also greater than the angle D C B; again seeing that
the line B C is put equal to the line B D, the angle B C D alone shall be
i) 5. 1. equal to E D C and E D B, taken for one only angle: Seeing [i] that these
angles B C D, E D C, and E D B, are on the base D C of the triangle
B D C: but we have shewn that the angle E D C is greater than the an-
gle B C D: Therefore the same angle E D C, shall be also greater than
the angle E D C and E D B, the part than the whole, which is impos-
sible: Therefore, If two, &c. Which was to be demonstrated.

COROLLARIE.

Hence it follows, that having drawn two lines from the extremities of a right line to one and the same point, as A C and B C, if from the same extremities there be drawn two others, equal to the two first, each to his correspondent, on the same side, they will meet together in one and the same point, being inclined the one to the other, is manifest by the aforegoing figures.

PROP. 8. THEOR. 5.



If two triangles A B C, *and* D E F, *have two sides* A B *and* A C, *equal to two sides* D E *and* D F. *each to his correspondent side, and the base* B C *be equal to the base* E F, *they shall also have the angle* A, *conteined of those sides* A B *and* A C, *equal to the angle* D, *conteined of the sides* D E *and* D F.

Demonstration FOr if it be understood that the triangle A B C be put on
the triangle D E F; to wit, the point B on the point E, and
the base B C on the base E F, the point C will agree with the point F:
[a] Seeing that B C and E F are equal, and the two right lines B A and C A
a) 8. c. f. will agree on the two others E D and F D, [b] being equal to them, each
b) 8. c. f. to his correspondent, and the point A on the point D; and therefore the
angle A shall be equal to the angle D, otherwise they would meet toge-
c) 7. 1. ther in another point, which is impossible; [c] Seeing that if two right
lines drawn from the extremities of another right line, do meet in one
point, &c. Therefore A shall be equal to D: Therefore, If two triangles,
&c. Which was to be demonstrated.

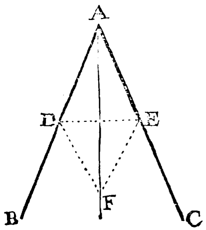
COROLLARIE.

*It may be gathered from this Proposition that the other angles are also equal to
the other angles, each to his correspondent, and the whole triangle, to the whole tri-
angle; for seeing that the angle* A, *is shewn to be equal to the angle* D, *the sides*
A B *and* A C *being equal to* D E *and* D F, *each to his correspondent, it follows*
d) 4. 1. [d] *that the triangle is equal to the triangle, and the other angles equal to the other
angles, each to his correspondent.*

PROP. 9. PROBL 4.

To divide a given right lined angle B A C, *into two equal parts.*

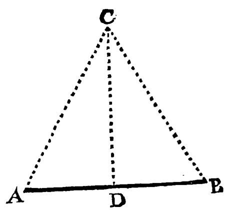
Construction LEt there be taken in the line A B, a point at pleasure D;
and from the greatest line A C prolonged infinitely, let
a) 2. 1. [a] there be cut off A E equal to A D, and let D E be drawn, on which
b) 1. 1. [b] let there be described the equilateral triangle D F E, and let A F be
drawn; I say that the angle B A C is divided into two equal angles.



Demonſtration FOraſmuch as E A hath been taken equal to A D and A F is common, the two ſides D A and A F are equal to the two ſides E A and A F, each to his correſpondent, and the baſe D F equal to the baſe E F; being the ſides of an equilateral triangle; c the angle D A F ſhall be equal to the angle E A F: and therefore the angle B A C ſhall be divided into two equal parts: Therefore the right lined angle, &c. Which was to be done.

c) 8. 1.

PROP. 10. PROBL. 5.



To divide a terminated right line, given A B, *into two equal parts.*

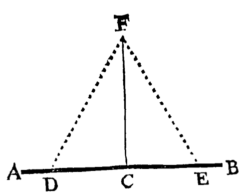
Conſtruction ON A B a let there be deſcribed the equilateral triangle A B C, and then b let the angle A C B be divided in two equal parts by the right line C D: I ſay, that A B is divided into two equal parts in the point D.

a) 1. 1.
b) 9. 1.

Demonſtration FOraſmuch as A C is equal to C B, (they being the ſides of an Equilateral triangle,) and C D is common, the two ſides A C and C D are equal to the two ſides B C and C D, each to his correſpondent, and the angle A C D equal to the angle B C D, by conſtruction; wherefore the baſe A D c ſhall be equal to the baſe B D: Therefore A B ſhall be divided in two equal parts. Which was to be done.

c) 4. 1.

PROP. 11. PROBL. 6.



On a given right line A B, *and from a given point therein* C, *to draw a right line* C F, *at right angles.*

Conſtruction IN the part A C let there be taken any point, as D, then a let C E be taken equal to C D, then on D E let there be deſcribed the equilateral triangle D F E; and let C F be drawn; b I ſay that the ſame C F is at right angles to the line A B.

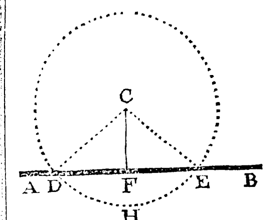
a) 2, 3. 1.
b) 5. 1.

De-

Demonſtration FOraſmuch as C D is equal to C E, and C F common, the triangles D C F and E C F ſhall have the two ſides D C and C F equal to the two ſides E C and C F, each to his correſpondent, and the baſes D F and E F equal, being the ſides of an equilateral triangle, the angles at the point C ſhall be equal; therefore both the one and the other, are right angles c: Therefore C F ſhall be at right angles to A B: Therefore on a right line, &c. Which was to be done.

c) 10. def.

PROP. 12. PROBL. 7.



On an infinite right line given A B, *and from a given point* C, *which is not in it, to draw a perpendicular line* C F.

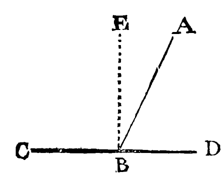
Conſtruction LEt the contingent point H be taken on the other ſide of A B, and from the point C as a Center at the diſtance C H, let the circle D E be deſcribed, cutting A B in the points D and E, then a let D E be divided into two equal parts in the point F, and let C F, C D, and C E be drawn; I ſay that C F is a perpendicular to A B.

a) 10. 1.

Demonſtration FOraſmuch as D F is equal to E F, by Conſtruction, and F C common, the two ſides D F and F C of the triangle D F C, are equal to the two ſides E F and F C of the triangle E F C, each to his correſpondent, and the baſe C D equal to the baſe C E, they being both drawn from the center C to the circumference; b the angles on both ſides at the point F ſhall be equal; and c therefore both the one and the other, are right angles: Therefore C F is a perpendicular drawn to A B: Which was to be done.

b) 8. 1.
c) 10. def.

PROP. 13. THEOR. 6.



When a right line A B. *falling on a right line* C D, *maketh angles* A B D *and* A B C, *either it makes two right angles, or two angles equal to two right angles.*

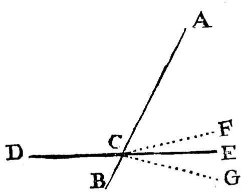
Demonſtration FOr if the angle A B D be equal to A B C; they a ſhall be both right angles; if it be unequal, b let B E be drawn at right angles to C D; therefore E B D and E B C ſhall be right angles; Now foraſmuch as the two angles D B A and A B E c are equal to the right angle E B D, all the parts taken together, to their whole, if d the common angle E B C be added; then, the three angles D B A, A B E,

a) 10. def.
b) 11. 1.
c) 9. c. f.
d) 2. c. f.

 and

and E B C shall be equal to the two right angles D B E and E B C: Again, forasmuch as the angle A B C is equal to the two angles A B E and E B C, the whole, to all his parts taken together, if you add the common D B A, the two angles D B A and A B C shall be equal to the three angles D B A, A B E, and E B C, but those three have been shewn to be equal to two right angles; therefore D B A and A B C shall be also equal to two right angles: Therefore when a right line, &c. Which was to be demonstrated.

PROP. 14. THEOR. 7.



If to a right line A B, *and at a point therein* C, *there be drawn two right lines* D C *and* E C, *not from the same side, making the angles* D C A *and* A C E, *on both sides, equal to two right angles, those right lines* D C *and* C E, *will directly meet with one another.*

Demonstration FOr otherwise D C being prolonged towards E, would fall either above or below C E; Let it fall first of all above C E, (if possible,) as in the point F, in such sort as D C F may be a right line.

a) 13. 1. Forasmuch as the right line A C, falling on the right line D C F, makes the two angles D C A and A C F, [a] the same angles shall be right angles, or equal to two right angles. But the two angles D C A and

b) 10. c. s. A C E are equal to two right angles, by Supposition; and [b] all the right angles are equal to one another; therefore the two angles D C A and

c) 3. c. s. A C F are equal to the two angles D C A and A C E, taking away therefore the common angle D C A, [c] there will remain A C F equal to

d) 9. c. s. A C E, the part to its whole [d], which is impossible. The same absurdity will happen, if it shall be said that D C being prolonged, doth fall below C E, as in the point G; therefore D C and E C shall meet in a direct line. Therefore, &c. Which was to be demonstrated.

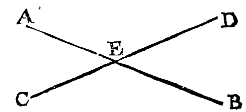
COROLLARIE I.

It follows from this Proposition, that two right lines indirectly meeting with one another, have not one and the same common segment; that is to say, one and the same part common as F C *and* E C, *which meet indirectly in the point* C, *the line* D C *cannot be said to be common to* C E *and* C F; *that is to say, that* D C *and* C E *making a right line* D C *and* C F *will not also make a right line, for otherwise, if as well* D C E, *as* D C F, *were a right line; (as we even now demonstrated,) the angle* H C F *should be equal to the angle* A C E, *the part to the whole, which is impossible.*

COROLLARIE II.

It follows yet that two right lines meeting with one another indirectly in a certain point, if they be both prolonged, they must of necessity cut one another, they being prolonged from the same part where they meet, as if the lines E C *and* F C, *which meet in the point* C, *be prolonged, to wit* E C, *it will fall directly in the point* D, *as hath been shewn; but if* F C *be also prolonged from the part of* C, *it will cut the line* E C, *prolonged at* D; *that is to say, it will fall under* C D: *for otherwise* F C *should fall on* C D; *and therefore* E C *and* F C *should have one and the same common segment, which is shewn to be impossible; and if it should be said that* F C *being prolonged, would fall above* C D, *the same absurdity as before would follow; to wit, that the part should be equal to its whole, which is impossible.*

PROP. 15. THEOR. 8.



If two right lines A B *and* C D, *do cut one another in* E, *they shall make the opposite angles at the head equal to one another,* A E D *to* C E B, *and* A E C *to* B E D.

Demonstration FOrasmuch as D E falls on A B, the two angles A E D and D E B shall be equal to two right angles; Again, forasmuch as B E falls on C D, the two angles C E B and B E D, [a] shall be equal to two right angles, and all the right angles are equal to one another: therefore the two angles A E D and D E B, [b] shall be equal to the two angles C E B and B E D; therefore if the common angle D E B be taken away; [c] there will remaine the angle A E D equal to C E B: in like manner it may be shewn that the angle A E C is equal to the angle D E B: Therefore, If two lines, &c. Which was to be demonstrated.

a) 13. 1.
b) 10. c. s.
c) 3. c. s.

COROLLARIE I.

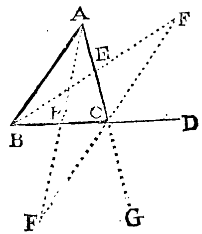
EUCLIDE *gathers from the Demonstration of this Theorem, (according to the opinion of* Proclus) *that two right lines cutting one another, do make four angles equal to four right angles; for it hath been demonstrated, that as well the two angles* A E D *and* D E B, *as the two angles* A E C *and* C E B, *are equal to two right angles, by the* 13th. *Therefore the angles constituted at the point* E, *are equivalent to twice two right angles; and therefore are equal to four right angles.*

COROLLARIE II.

By the same reason we may gather that all the angles constituted at one and the same point, are only equal to four right angles: For if from the point E *in the aforegoing Figure, there be drawn as many right lines as you please, the four angles constituted in the point* E, *shall be divided into divers parts; the which are equal to their whole,* [d] *taken together; therefore those four being equal to four right angles,* (by the first Corollary,) *all their parts taken together, shall be likewise equal to four right angles; from whence it is manifest that every Space, constituted in a Plain, about some point, is equivalent to four right angles, as is manifest: forasmuch as all the angles which may be constituted about that point, are equal to four right angles: In like manner, it is manifest that any number of lines at pleasure, mutually cutting one another, will make the angles at the point of Section, equal to four right angles.* d) Cor. 3. 1.

PROP.

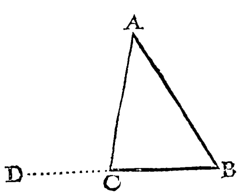
PROP. 16. THEOR. 9.



Of every triangle, as A B C, *one side* B C *being prolonged, as to* D, *the exterior angle* A C D, *is greater than either of the interior and opposite angles* C A B *or* C B A.

Demonstration FOr let the side A C be divided in two equal parts [a]
a) 10. 1. in the point E, and by E let there be
b) 2. 1. drawn B E F; and let [b] E F be made equal to E B, and F C joyned.
Forasmuch as A E is equal to E C, and E F to E B, by Construction, the two sides A E and E B of the triangle A E B, shall be equal to the two sides C E and E F of the triangle F E C, each to his correspondent, and the
c) 15. 1. angles at the head E, conteined by those equal sides, [c] are equal; the base
d) 4. 1. A B shall be equal to the base [d] F C, and the whole triangle to the whole triangle, and the other angles equal to the other angles, each to his correspondent, conteined by the same equal sides; to wit, E C F equal to
e) 9. c. s. E A B: But the whole [e] A C D is greater than E C F, it shall be therefore also greater than its equal E A B: In like manner, having divided B C into two equal parts in the point E, and drawn A E F, in such sort as E F may be equal to E A and C F joyned, and the side A C prolonged to the point G, it will appear that the exteriour angle B C G is greater than the interiour and opposite angle A B C: and
f) 15. pet. therefore A C D which is [f] equal to B C G, shall be also greater than A B C: Therefore, Of every triangle one side, &c. Which was to be demonstrated.

PROP. 17. THEOR. 10.



Of every triangle as A B C, *two angles are lesse than two right angles, after what manner soever they be taken.*

Demonstration FOr first of all the side B C being prolonged to the point D, it will appear that the two angles A B C and A C B, are lesse than two right angles.
a) 16. pet. Forasmuch as the exteriour angle A C D is greater [a] than the opposite
b) 4. c. s. and interiour angle B, if the common angle A C B be added, [b] the two angles A C D and A C B, will be greater than B and A C B; but A C B
c) 13. pet. and A C D are equal [c] to two right angles: Therefore B and A C B are lesse than two right angles: In like manner, the side C A being prolonged to

the point E, it will appear that C A B and A B C are lesse than two right angles, &c. Therefore, Of every triangle, &c. Which was to be demonstrated.

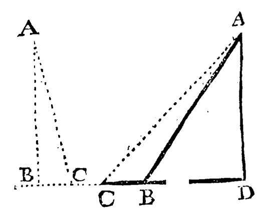
COROLLARIE I.

From these Demonstrations, it follows also that every triangle that hath one angle either right or obtuse, hath the other angles acute: for seeing it is apparent that any two angles of every triangle are lesse than two right angles; it is necessary that if one angle be right or obtuse, that all the other angles be acute, otherwise two angles of a triangle, should be right angles, or greater than two right angles.

COROLLARIE II.

It follows also from this Proposition, that if a right line make two unequal angles with another right line, the one obtuse, the other acute, that the perpendicular line drawn from any point thereof on the other line, will fall on that side where the acute angle is.

Let A B *with* C D *make two unequal angles, to wit,* A B D *acute, and* A B C *obtuse, and from any point, as from* A, *let* A D *be drawn perpendicular to* C B: *I say that* A D *will fall on the side of the acute angle* A B D; *Otherwise, Let it fall (if possible) on the side of the obtuse angle* A B C, *and let it be* A C, *therefore the two angles, to wit, the obtuse* A B C, *and the right angle* A C B *of the triangle* A B C, *shall be greater than two right angles, which is absurd, being they are* [d] *lesse than two right angles,* d) 17. 1. *therefore the perpendicular drawn from the point* A, *on* A B, *shall not fall on the side of the obtuse angle; therefore it shall fall on the side of the acute angle.*



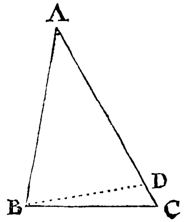
COROLLARIE III.

Likewise it is manifest from this Proposition, that all the angles of an Equilateral triangle, and the two angles on the base of an Isosceles triangle are acute: For seeing that [e] *any two angles of an Equilateral triangle, and the two upper* e) 5. 1.
angles on the base of an Isosceles triangle are equal to one another; and as well the first two [f] *as the last two, are lesse than two right angles; each of them shall be lesse* f) 17. 1.
than a right angle that is to say, it shall be acute, for if one of them were a right or obtuse angle, the two should be equal to two right angles, or greater than two right angles.

PROP. 18. THEOR. 11.

Of every triangle, as A B C, *the greatest side* A C, *subtendeth the greatest angle* A B C.

Demonstration FOr seeing that A C is greater than A B, let [a] A D be cut a) 3. 1.
off equal to A B, and let B D be joyned; forasmuch as A B
and A D are equal, the two angles [b] A B D and A D B shall be equal; but b) 5. 1.
the exterior angle A D B is greater [c] than the interior and opposite c) 16. 1.
angle

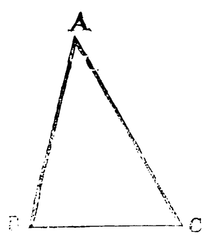


angle C: Therefore its equal ABD, shall be also greater than C: and therefore the whole ABC yet much greater than the angle C. By the same reason it will appear that the angle C is greater than the angle A: Therefore, Of every triangle, &c. Which was to be demonstrated.

COROLLARIE.

It follows from this Proposition, that the three angles of a Scalenum triangle are unequal: For let the triangle ABC *be a Scalenum, whose greatest side let be* AC, *the least* BC, *and the mean* AB; *I say, that all the angles of that triangle are unequal: For seeing that the side* AC *is put greater than the side* AB, *by this Proposition the angle* B *shall be greater than the angle* C, *by the same reason, the angle* C *shall be greater than the angle* A; *forasmuch as the side* AB, *is put greater than the side* BC: *Therefore the three angles are unequal:* B *the greatest,* C *the mean, and* A *the least.*

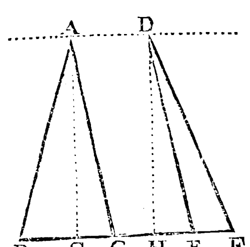
PROP. 19. THEOR. 12.



Of every triangle as ABC, *the greatest angle* B *is subtended by the greatest side* AC, (AC *is greater than* AB.)

Demonstration. For if it be denied, it shall be equal, or lesse: if equal, a) 5. 1. the angle B [a] shall be equal to the angle C, which is absurd; for B is put greater than C, if the side AC be said to be lesse than AB; the angle B subtended of the least side AC, shall be lesse than b) 18. 1. the angle C: subtended of the greatest side AB [b], which is yet more absurd: for the angle B is greater than the angle C, by supposition: Therefore the side AC shall be greater than AB: Likewise it will appear, that the side AC is greater than the side BC, putting the angle B greater than the angle A: Therefore, Of every triangle, &c. Which was to be demonstrated.

COROLLARIE.

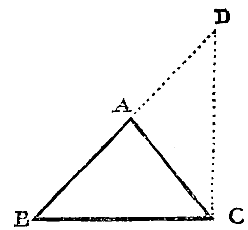


From this Proposition it follows, that of all the right lines drawn from one and the same point, on a right line, that line which is perpendicular, is the least.

For, Let the lines AB, AG, *and* AC, *in the first triangle, be drawn from the point* A, *on the line* BC, *of which let* AG *be perpendicular to* BC: *I say, that* AG *is the least: For seeing that in the triangle* ABG, *the* c) 17. 1. *two angles* AGB *and* ABG *are* [c] *lesse than two right angles, and* AGB *is a right angle:* ABG

ABG *shall be lesse than a right angle; that is to say acute; therefore* [d] AB *shall be greater than* AG, *and in this manner, it might be demonstrated that each of the lines in the other triangle; namely, the triangle* DEF, *is greater than the perpendicular* DH: *Therefore the perpendicular* AG *shall be the least.* d) 19. 1.

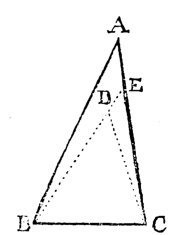
PROP. 20. THEOR. 13.



Of every triangle as ABC, *two sides are greater than the other; after what manner soever they be taken (that is,* BA *and* AC *taken together, are greater than* BC; *likewise* AC *and* CB *are greater than* AB; *and lastly,* AB *and* BC *are greater than* AC.)

Demonstration. Let the side BA be prolonged towards D, [a] in such sort as AD may be put equal to AC; and let CD be drawn: a) 3. 1. Forasmuch as the two sides AD and AC are made equal, the two angles [b] on the base CD; to wit D, and ACD, shall be equal: but the b) 5. 1. angle BCD [c] is greater than the angle ACD, it shall be therefore also c) 9. c. s. greater than D; therefore in the triangle BCD the side BD opposite [d] to the greatest angle BCD, shall be greater than the side BC, oppo- d) 19. 1. site to the least angle D; but the side BD is equal to the two sides AC and AB: (for seeing that AD is equal to AC, if the common side [e] AB e) 2. c. s. be added, the whole BAD shall be equal to AC and AB:) therefore the two sides AB and AC together, shall be greater than the other side CB: In like manner, it may be demonstrated, that two sides (which you please,) of any triangle, are greater than the other side: Therefore, Of every triangle, &c. Which was to be demonstrated.

PROP. 21. THEOR. 14.



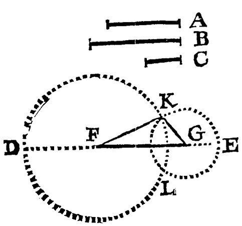
If on one of the sides BC *of a triangle* ABC, *and from the extremities* B *and* C, *are drawn two right lines* BD *and* CD *within, meeting in a point* D, *those lines* BD *and* CD *shall be lesse than the two other sides* BA *and* CA *of the triangle* BAC: *But they shall contein a greater angle, (the angle* BDC *is greater than the angle* BAC.

Demonstration FOr let B D be prolonged to the point E of the side A C: forasmuch as in the triangle B A E, the two sides B A and
a) 20. 1. A E, a are greater than the other B E, if the common side C E be added,
b) 4. c. s. b the sides B A and A C shall be greater than the sides B E and E C.

Again, forasmuch as, in the triangle C E D, the two sides C E and
c) 20. 1. E D are c greater than the other C D, if the common side D B be added, the
d) 4. c. s. two sides C E and E B, d shall be greater than C D and D B; but B A and A C have been shewn to be greater than B E and E C; therefore B A and A C shall be yet greater than B D and C D, which was in the first place proposed.

e) 16. 1. Secondly, forasmuch as the exterior angle B D C is e greater than the interior angle D E C, and the same D E C is greater than B A C, the angle B D C shall be much greater than the angle B A C: which was in the second place proposed: Therefore, If on one of the sides, &c. Which was to be demonstrated.

PROP. 22. PROBL. 8.



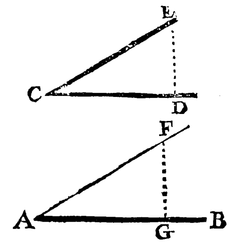
Of three right lines F K, G K, *and* F G, *equal to three given right lines,* A, B, *and* C; *to constitute a triangle* F G K: *But two of them, after what manner soever they are taken, ought to be greater than the other; forasmuch as of every triangle, two sides after what manner soever they be taken, are greater than the other side.*

Construction LEt there be taken some right line, as D E, prolonged infinitely towards E, and of the said line D E. Let there be ta-
a) 3. 1. ken D F equal to a A and F G equal to B; and lastly G E equal to C, and from the point F as a center at the space F D. Let the circle D K be described againe from the center G, and with the space or distance G E, let the circle E K be described; and let F K and G K be drawn: I say, that the triangle F G K is made of three lines equal to the three given lines A, B, and C.

Demonstration FOrasmuch as the point F is the center of the circle D K,
b) 15. def. 1. the line b F K shall be equal to F D, by the construction;
c) 1. c. s. therefore F K and A shall be c equal to one another: Again, forasmuch as G is the center of the circle E K, the line G K shall be equal to G E, by the same reason: But the line C is equal to G E by the construction: therefore G K and C shall be equal, and F G is taken equal to B: Therefore the three right lines F K, F G, and G K, are equal to the three given right lines A, B, and C, &c. Which was to be done.

PROP.

PROP. 23. PROBL. 9.

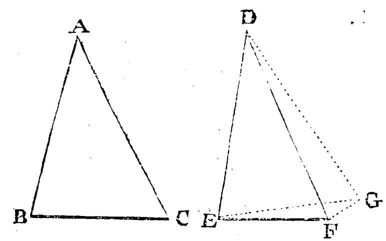


To a given right line A B, *and at a point therein* A, *to constitute a right lined angle* F A G, *equal to a given right lined angle* D C E.

Construction LEt the contingent points C and E be taken in both the one and the other lines D C and D E; and let C E be drawn; then of three lines equal to the right lines D C, C E, and E D, a let there be constituted the triangle A F G; in such sort, as A F be put equal to D C, F G to D E, and A G to C E. a) 22. 1.

Demonstration FOrasmuch then as the two sides A F and A G of the triangle A F G, are equal to the two sides C D and C E, of the triangle C D E, each to his correspondent, and the base F G equal to the base E D; the angle F A G shall be b equal to the angle C D E: b) 8. 1.
Therefore to a given right line, &c. Which was to be done.

PROP. 24. THEOR. 15.



If two triangles A B C, *and* D E F, *have two sides* A B *and* A C *equal to two sides* D E *and* D F, *each to his correspondent, and the angle* A *conteined of those equal sides* A B *and* A C, *greater than the angle* E D F, *they shall have the base* B C *greater than the base* E F.

Demonstration FOr seeing that the angle A is greater than the angle E D F; at the line E D, and at a point therein D, let there be constituted the angle a E D G, equal to the angle A; and let D G be a) 23. 1.
taken, equal to one of the two A C or D F; and let E G and F G be drawn: Forasmuch then as the triangles A B C and E D G have the two sides A B and A C equal to the two sides D E and D G each to his correspondent, and the angle A, equal to the angle E D G, conteined of the same equal sides, b the base B C shall be equal to the base E G: Again, b) 5. 1.

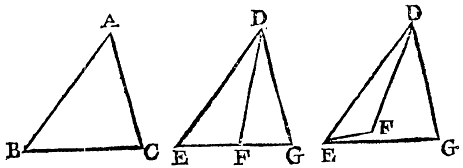
 for-

forasmuch as D G is equal to D F, the triangle D G F is an Isosceles triangle: and therefore c the two angles D G F and D F G, on the base, shall be equal: But the angle D G F is d greater than the angle E G F: therefore D F G equal thereto, shall be also greater than E G F, and therefore the whole E F G, yet greater than E G F: Therefore in the triangle E F G the side E G which doth subtend the greatest angle E F G, shall be greater than E F, which subtendeth the lesser angle E G F: But B C is shewn to be equal to E G: Therefore B C shall be also greater than E F.

c) 9. c. s.
d) 19. 1.

Secondly, Let the base E G fall on E F, it e shall be equal to the base B C, as before; but E G is greater than F E, the whole than part thereof: Therefore B C f shall be also greater than E F.

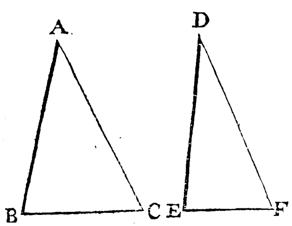
e) 4. 1.
f) 9. c. s.



Lastly, let E G fall under E F; forasmuch as the two sides D G and and E G are g greater than the two sides D F and F E, and D F is made equal to D G, h taking away the equal sides D G and D F, there will remaine E F lesse than E G or B C, which we have shewn to be equal to E G: Therefore, If two triangles, &c. Which was to be demonstrated.

g) 21. 1.
h) 5. c. s.

PROP. 25. THEOR. 16.



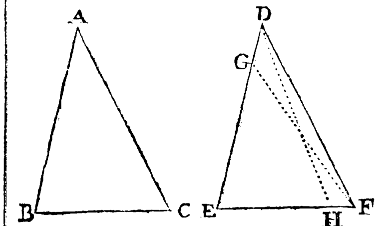
If two triangles A B C *and* D E F, *have two sides* A B *and* A C, *equal to two sides* D E *and* D F, *each to his correspondent, and that they have the base* B C *greater than the base* E F, *they will have the angle* A, *conteined by the same equal right lines* A B *and* A C, *greater than the angle* D, *conteined by the equal right lines* D E *and* D F.

Demonstration. FOr if A were not greater, it should be equal or lesse, if equal, the sides A B and A C being equal to the two sides D E and D F, each to his correspondent; a the base B C should be equal to the base E F, which is absurd, for B C is put greater than E F: If lesse, A B and A C being equal to D E and E F, each to his correspondent; B C should be b lesse than E F, which is more absurd, for it is greater by

a) 4. 1.
b) 24. 1.

supposition: Therefore seeing that it cannot be either equal, or lesse, it shall be greater. Therefore, If two triangles, &c. Which was to be demonstrated.

PROP. 26. THEOR. 17.



If two triangles A B C, *and* D E F, *have two angles* B *and* C, *equal to two angles* E, *and* D F E, *each to his correspondent, and a side equal to a side; that is to say, either* B C *equal to* E F, *that which is adjacent to those equal angles* B *and* C, *or else* A B *and* D E, *that which subtendeth one of those equal angles* C *and* D F E, *they will have the other sides* A C *and* C B, *equal to the other sides* D F *and* F E, *each to his correspondent, and the other angle* A, *equal to the other angle* D.

Demonstration. FOr if A B be not equal to D E, one of them shall be greater, which let be D E, (if possible,) from which let G E be cut off a equal to A B, and let G F be drawn; Forasmuch as, the two sides A B and B C of the triangle A B C, are equal to the two sides G E and E F of the triangle G E F, each to his correspondent, and the angle B equal to the angle E, by supposition, the base b G F shall be equal to the base A C, and all the triangle G E F, equal to all the triangle A B C, and the angle G F E equal to the angle C; but the angle D F E, is equal to the angle C by supposition; therefore the angle G F E shall be in like manner equal to the angle D F E, the part to its whole, which is absurd: therefore the side A B is not equal to D E: Therefore seeing that the two sides A B and B C are equal to the two sides D E and E F, each to his correspondent, and the angles B and E conteined of those sides, also equal, the base c A C shall be equal to the base D F, and the other angles A and also equal, which was proposed.

a) 3. 1.
b) 4. 1.
c) 4. 1.

Secondly, Let the sides A B and D E, subtending the equal angles C and D F E, be equal; I say again, that the other sides A C and C B are equal to the other sides D F and F E, each to his correspondent, B C to E F, and A C to D F, and the other angle A equal to the other angle D; For if B C be not equal to E F: Let one of them, to wit E F, be the greater, from d which let E H be taken, equal to B C, and let D H be drawn.

d) 3. 1.

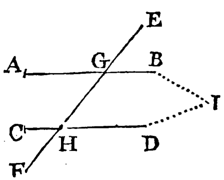
Forasmuch then as the two sides A B and B C, are equal to the two sides D E and E H, and the angles B and E conteined of them, being put equal,

c) 4. 1. equal, the base is equal to the base, and the angle D H E c equal to the angle C: therefore the angle D H E shall be equal to the same angle D F E, the exterior angle to its interior and opposite angle, which is
f) 16. 1. f absurd. Therefore B C is not unequal to E F: Therefore by the fourth Proposition of this Book it may be gathered as before, that the two other sides, are equal to the two other, and the other angle to the other angle: Therefore, If two triangles, &c. Which was to be demonstrated.

COROLLARIE.

It is easie to gather from this Proposition, that the whole triangle is equal to the whole triangle, for by consequence, as the two sides A B *and* B C *are equal to the two* D E *and* E F, *each to his correspondent, conteining those equal angles, the whole triangle is also equal to the whole triangle; which hath been already demonstrated.*

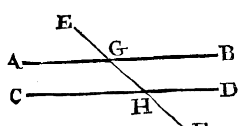
PROP. 27. THEOR. 18.



If a right line E F, *falling on two other right lines* A B *and* C D, *makes the alternate angles* A G H *and* G H D, *equal to one another; those right lines* A B *and* C D, *shall be parallel to one another.*

Demonstration. FOr if they be not parallel, they being infinitely prolonged on both parts, they will meet either on the part of A C or of B D, otherwise they shall be parallels; Let them meet then (if possible,) on the part of B D, as in the point I; and so G I H shall be a triangle,
a) 16. 1. whose exterior angle a A G H shall be greater than the interior and opposite G H D, which is absurd; for A G H is equal to G H D, by Supposition:
b) 35. def. Therefore A B and C D shall not meet, and b therefore they shall be parallels: In like manner, it might be demonstrated also that they can never meet on the part of A C: Therefore, If a right line, &c. Which was to be demonstrated.

PROP. 28. THEOR. 19.

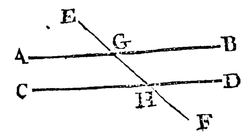


If a right line E F *falling on two right lines* A B *and* C D, *doth make the exterior angle* A G E, *equal to the interior and opposite angle* G H C, *and on the same part, or the interior angles on the same part,* A G H *and* C H G, *equal to two right angles; those right lines* A B *and* C D *shall be parallel to one another.*

Demonstration. FOrasmuch as the angle G H C is equal to the angle F G A by supposition; and that the angle B G H a is equal to the same angle E G A, being both opposites at the head G, the two angles B G H and G H C, which are alternate and opposite angles, b shall be also equal to one another: Therefore c A B and C D shall be parallel to one another. a) 15. 1. b) 1. c. s. c) 27. 1.

Secondly, If E F falling on A B and C D doe make the interior angles, and on the same part; to wit A G H and C H G, equal to two right angles: I say, that A B and C D are also parallel: For seeing that A G E and A G H d are equal to two right angles, and that the interior angles A G H and C H G are put equal to two right angles, the angles A G E and A G H shall be equal to the two angles A G H and C H G, taking away therefore the common angle A G H, there will remaine the exterior angle, equal to the interior and opposite angle on the same part G H C: Therefore A B and C D shall be parallels by the first part of this Proposition: Therefore, If a right line, &c. Which was to be demonstrated. d) 13. 1. 3. c. s.

PROP. 29. THEOR. 20.



If a right line E F *doth fall on two parallel right lines* A B *and* C D, *it will make the alternate angles* A G H *and* D H G *equal to one another, and the exterior angle* E G B, *equal to its interior and opposite angle* G H D, *and on the same part; and the two interior angles* B G H *and* G H D, *and on the same part, equal to two right angles.*

Demonstration. FOr if they be not equal, let the one of them, to wit A G H, be the greater; forasmuch then as the angle A G H is greater than the angle D H G; if the common angle B G H be added, a the two angles A G H and B G H, shall be greater than the two angles B G H and D H G; but the two angles A G H and B G H b are equal to two right angles: Therefore B G H and D H G shall be lesse than two right angles; therefore A B and C D shall not be parallels, c which is absurd: for that they are put parallels: Therefore the angle A G H is not greater than the angle D H G, and therefore is equal thereto; if it were said to be lesse, the same absurdity would happen. a) 4. c. s. b) 13. 1. c) 11. c. s.

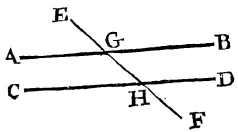
Secondly, I say that the exterior angle E G B is equal to its interior and opposite angle, and on the same part G H D: Forasmuch as the angle G H D is shewn to be equal to the angle A G H, and that the angle E G B is d equal to the same angle A G H, the angles E G B and G H D shall e be equal to one another; and so it may be shewn that E G A is equal to G H C. d) 15. 1. e) 1. c. s.

Thirdly, I say that the two interior angles, and on the same part, B G H and G H D, are equal to two right angles.

For seeing that the exterior angle E G B is shewn to be equal to G H D, if the common angle B G H be added; [f] the two angles B G H and G H D shall be equal to the two angles E G B and B G H; [g] but E G B and B G H are equal to two right angles: Therefore B G H and G H D shall be also equal to two right angles, by the same reason, the two angles A G H aud G H C will appear to be equal to two right angles: Therefore, &c. Which was to be demonstrated.

f) 3. c. s.
g) 13. per.

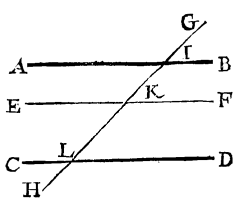
Otherwise, Let A B and C D be parallels: I say that the alternate angles A G H and H G D are equal to one another: If it be not so, the one of them shall be the greater: Let then A G H be the greater (if possible.) Wherefore adding the angle H G B, the two angles H G B and D H G shall be lesse than the two angles H G B and A G H; but H G B and A G H are equivalent to two right angles; therefore H G B and D H G shall be lesse than two right angles; therefore A B and C D shall not be parallel, which is contrary to Supposition: Therefore the angle A G H is equal to the angle G H D.



Secondly, The exterior angle E G B is equal to the interior and opposite angle, and of the same part G H D; for the angle E G B is equal to the angle A G H, and the angle G H D is shewn to be eqnal to the angle A G H: Therefore the angle E G B shall be equal to the angle G H D, the exterior to the interior angle.

Lastly, The two interior angles, and on the same part, B G H and G H D are equal to two right angles: For seeing that the angle E G B is shewn to be equal to the angle G H D, if the angle B G H be added to each of them; the two angles G H D and B G D shall be equal to the two angles E G B and B G H: But the angles E G B and B G H are equal to two right angles; therefore the angles B G H and D H G shall be also equal to two right angles: Which was to be demonstrated.

PROP. 30. THEOR. 21.



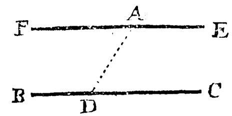
The right lines A B *and* C D, *parallel to one and the same right line* E F, *are also parallel to one another.*

Demonstration Let G H be drawn, cutting the same right lines in the points I, K, and L; forasmuch as the right line G H doth fall on the parallels A B and E F, the angle A I K shall be equal [a] to the alternate angle F K I; Again, forasmuch as G H falls on the parallels C D and E F, the angle D L K shall be equal to the angle I K F, the interior, to the exterior angle, or the exterior to the interior and opposite on the same part But A I K is equal to the same I K F; therefore A I K [b] and D L K shall be equal to one another, which being alternate angles, [c] the lines A B and C D shall be parallel to one another: Therefore, The right lines, &c. Which was to be demonstrated.

a) 29. 1.
b) 1. c. s.
c) 27. 1.

PROP.

PROP. 31. PROBL 10.



From a given point A, *to draw a right line parallel to a given right line* B C.

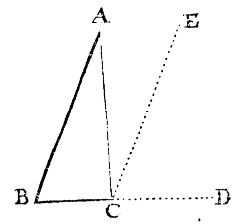
Construction Let the contingent point D be taken in the given line B C; and let A D be drawn, making any angle at pleasure with B C, as the angle A D C, and in the point A of the given right line A D, let there be constituted the [a] right lined angle F A D, equal to the given right lined angle A D C, and let F A be prolonged directly towards E.

a) 23. 1.

Demonstration Forasmuch as the two alternate angles F A D and C D A are equal by construction, the line A F [b] shall be parallel to B C, and drawn from the given point A. Which was to be done.

b) 27. 1.

PROP. 32. THEOR. 22.



Of every triangle, as A B C, *one of the sides* B C *being prolonged, the exterior angle* A C D *is equal to the two interior and opposite angles* A *and* B, *and the three interior angles* A, B, *and* A C B, *of any triangle, are equal to two right angles.*

Construction For from the point C let C E be drawn parallel to A B.

Demonstration Forasmuch as A C doth fall on the parallels A B and C E, the alternate angles [a] A and A C E are equal; Again, forasmuch as B D falls on the Parallels A B and C E, the angles B and E C D shall be [b] equal, the exterior angle to the interior and opposite angle on the same part: Therefore the two angles A C E and E C D; that is to say, the whole A C D shall be equal to the two angles A and B, the exterior angle to the two interior and opposite angles; Which was proposed.

a) 29. 1.
b) 29. 1.

Secondly, I say that the three interior angles of the same triangle A B C; to wit, A, B, and A C B, are equal to two right angles.

Forasmuch as A and B are equal to the angle A C D, as hath been shewn, if the common angle A C B be added, the three angles [c] A, B, and A C B, shall be equal to the two angles A C B and A C D; but the two angles A C B and A C D are equal to two [d] right angles; therefore the

c) 2. c. s.
d) 13. 1.

the

the three angles A, B, and A C B are equal to two right angles: Therefore of every triangle, &c. Which was to be demonstrated.

COROLLARIE I.

From this 32 Proposition may be gathered that the three angles of any triangle taken together, are equal to three angles taken together, of any other triangle: forasmuch as the three angles of the one, as the three angles of the other, are equal c) 32. 1. *c to two right angles. Therefore if two angles of one triangle are equal to two angles of another triangle, the third angle of the one shall be also equal to the third angle of the other. Therefore if two angles of one triangle are equal to two angles of another triangle, each to his correspondent, the other angle of the one shall be equal to the other angle of the other, and those triangles shall be equiangled.*

COROLLARIE II.

It is also evident, that in every Isosceles triangle, whose angle contained of the equal sides is a right angle, that each of the other angles which are on the base is the half of a right angle, for both of them together do constitute a right angle: f) 32. 1. *f Seeing the three angles are equal to two right angles, and that the third angle is put one right angle; therefore seeing that the two remaining angles are equal to one other, each of them shall be the half of a right angle: But if the angle contained of the equal sides be obtuse, each of the other angles shall be lesse than half of a right angle, and the two angles together, lesse than a right angle; If lastly, the above mentioned angle be acute, each of the other angles shall be greater than half a right angle: forasmuch as the two together, are greater than a right angle.*

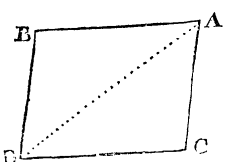
COROLLARIE. III.

It is likewise manifest, that each angle of an equilateral triangle, is the two third parts of a right angle, for the two right angles, g) 32. 1. *g to which the three angles of an equilateral triangle are equal, being divided into three angles, do make each angle the two third parts of a right angle.*

COROLLARIE IV.

It is also evident, that if a perpendicular line be drawn from one of the angles of an equilateral triangle on the opposite side there will be constituted two Scalene triangles, each of which shall have the angle adjacent to the perpendicular a right angle, another shall be the two thirds of a right angle; to wit, that which is also an angle of the equilateral triangle: Lastly, the other angle shall be the third part of a right angle.

PROP. 33. THEOR. 23.



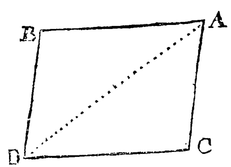
The right lines B D *and* A C, *which joyn together two right lines* B A *and* D C *equal and parallel, and on the same part, the same right lines* B D *and* A C *are in like manner equal and parallel.*

De-

Demonstration. For let A D be drawn, forasmuch as A D falls on the parallels B A and C D, a the alternate angles B A D and A D C shall be equal to one another, and forasmuch as B A is equal to D C by supposition, and A D common, the two sides B A and A D of the triangle A B D shall be equal to the two sides D C and D A of the triangle D C A, each to his correspondent, and the angle B A D equal to the angle C D A conteined of those equal sides, the base b B D shall be equal to the base A C, and the whole triangle A B D shall be equal to the wholle triangle D C A, and the other angles equal to the other angles, each to its correspondent; to wit, C equal to B, and D A C equal to B D A, which being alternate angles, c the lines B D and A C shall be parallel: But they are also shewn to be equal: Therefore they shall be equal and parallel: Therefore, &c. Which was to be demonstrated.

a) 29. 1.
b) 4. 1.
c) 27. 1.

PROP. 34. THEOR. 24.



In Parallelograms the opposite sides A B *and* C D, *and the opposite angles* D B A *and* A C B, *are equal to one another, and their Diameter* A D *doth divide them into two equal parts.*

Demonstration. For seeing that B A and D C are parallels, and that A D falleth on them, a the alternate angles B A D and A D C are equal to one another; Again, seeing that B D and A C are parallels, the alternate angles B D A and C A D are also equal: Therefore seeing that the two angles B A D and B D A of the triangle A B D, are equal to the two angles C A D and C D A of the triangle A C D, each to his correspondent, and the side adjacent to those equal angles, common to both the triangles, b the two sides B D and B A, shall be equal to the two other sides D C and A C, each to his correspondent: to wit, B A equal to its opposite D C, and B D equal to its opposite A C; and the angle B equal to the angle C: And forasmuch as the two angles at the point A are shewn to be equal to the two angles at the point D, the whole angle A shall be equal to its opposite and whole angle D: Therefore the sides and the angles opposite to one another, are equal.

But forasmuch as in the triangle A B D and D C A, the two sides B A and B D are equal to the two sides C D and C A, each to its correspondent, and the angles B and C, conteined of those sides, are equal, as hath been shewn, c the whole triangle B A D shall be equal to the whole triangle A C D: Therefore the Diameter doth divide the Parallelogram into two equal parts: Therefore the sides and the angles, &c. Which was to be demonstrated.

a) 29. 1.
b) 26. 1.
c) 4. 1.

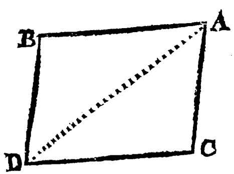
COROL-

COROLLARIE. I.

From this Propoſition may be in the firſt place gathered, that every quadrilateral or four ſided figure, which hath the oppoſite ſides equal, is a Parallelogram; for the two ſides B D *and* B A *being equal to* A C *and* C D, *each to his correſpondent, and the baſe* A D *common, the angles* B *and* C *ſhall be equal, and the two other angles alſo ſhall be equal to the two other angles, each to his correſpondent; ſo all the alternate angles which are at the points* A *and* D, *ſhall be equal to one another: Therefore the figure ſhall be a Parallelogram, all the ſides being Parallels.*

COROLLARIE II.

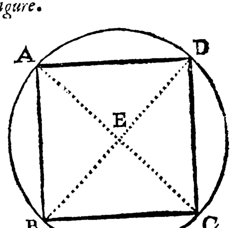
It follows alſo, that every four ſided figure which hath the oppoſite angles equal, is a Parallelogram, as in the former figure, the oppoſite angles being equal, to wit, A *to* D, *and* B *to* C: *And by what hath been ſhewn in the 32 Propoſition, the four angles being equal to four right angles, the two angles* B *and* B A C *ſhall be equal to two right angles: In like manner* B *and* B D C *ſhall be alſo equal to two right angles: therefore as well* B A *and* C D *as* B D *and* A C *ſhall be parallels; and therefore the figure propoſed is a Parallelogram, and the ſame would happen if all the angles were right angles.*



We ſhall alſo ſay, that although the Diagonal line divideth every Parallelogram into two equal triangles, it doth not follow, that every figure of four ſides which divideth it ſelf into two equal triangles by the Diagonal, is a Parallelogram, as is eaſie to be underſtood in drawing from the point B *towards* A, *a line equal to* A C, *and from the point* D *a line equal to* B A, *on the ſame part, which will meet with one another at another point, as is manifeſt by what hath been heretofore demonſtrated; the facility whereof requireth no figure.*

COROLLARIE III.

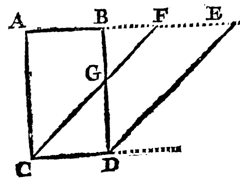
It follows alſo that in the long Square and Rhomboides (for there are but four ſorts of Parallelograms, as we have already ſaid) the Diagonal line doth cut the angles unequally, becauſe of the inequality of the ſides in each triangle. See the former figure.



But in the Square and the Rhombe, the ſaid Diagonal line doth cut the angles into two equal parts; foraſmuch as the two ſides of one of the triangles are alſo equal to the two ſides of the other triangle, each to his correſpondent, and the angle conteined of them equal to the angle, and as appears by this ſquare A B C D, *in which the Diagonal lines* A C *and* B D *are alſo equal, and do cut one the other into two equal parts in the point* E, *becauſe of the equality of all the angles which are in the points* A, B, C, D, *and in the point* E.

But if the figure A B C D *were a Rhomb, the two angles oppoſite ſhould be obtuſe, and the two others acute: Wherefore the Diagonal lines would be unequal; and yet neverthseleſſe would cut each other in two equal parts, as is eaſie to be underſtood by what hath already been demonſtrated, and alſo in the long Square and Rhomboides, and all figures of four ſides, in which the Diagonal lines do cut one another into two equal parts, is a Parallelogram; All which things are ſo eaſily demonſtrated, if the Propoſitions and Demonſtrations before mentioned be underſtood, as there was ſmall need of making any mention of them here.*

PROP. 35. THEOR. 25.

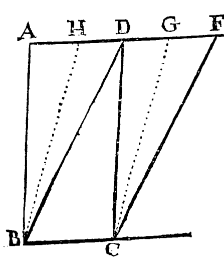


The Parallelograms A C D B *and* F C D E, *conſtituted on one and the ſame baſe* C D, *and between the ſame parallels* A B *and* C D *are equal to one another.*

Demonſtration Foraſmuch as A C D B is a parallelogram, A B is [a] equal to C D, by the ſame reaſon F E is equal to C D: [b] therefore A B and F E are equal to one another; therefore if the common line B F be added, [c] the whole A F ſhall be equal to the whole B E; [d] but A C is alſo equal to B D; therefore the two ſides F A and A C of the triangle F A C ſhall be equal to the two ſides E B and B D of the triangle B E D, each to his correſpondent, and the angles A and B conteined of thoſe ſides, [e] are equal, the exterior angle to the interior angle, and contrarily: therefore the baſe [f] F C ſhall be equal to the baſe E D, and the whole triangle F A C to the whole triangle E B D, from which the common triangle B G F being taken away, [g] the trapezium A B G C will remain equal to the Trapezium E F G D; to which Trapeziums if you again adde the common triangle C D G, the whole Parallelogram A C D B ſhall be equal to the whole Parallelogram E D C F: Therefore the Parallelograms conſtituted, &c. Which was to be demonſtrated.

a) 34. 1.
b) 1. c. ſ.
c) 2. c. ſ.
d) 34. 1.
e) 29. 1.
f) 4. 1.
g) 2. c. ſ.

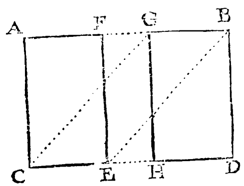
But if the Parallelograms conſtituted on one and the ſame baſe B C, were A B C D and D B C F, as in this ſecond figure; as well the ſide A D as D F, ſhould be equal to B C, as aforeſaid, and A B to D C, and the exterior angle F D C, to the interior and oppoſite angle A: Wherefore the two ſides F D and D C being equal to D A and A B, each to his correſpondent, and the angle F D C equal to the angle A, the whole triangle F D C ſhall be equal to the whole triangle D A B. Therefore if to both, there be added the common triangle B C D, the Parallelogram A B C D ſhall be equal to the Parallelogram D B C F.



And laſtly, if the Parallelograms conſtituted on the baſe B C, were A B C D and H B C G, as well A D as H G would yet be equal to B C, and

and therefore equal to one another: Therefore taking away the common line HD, there would remaine DG, equal to AH, and as before, the triangle GDC ſhall be equal to the triangle HAB: Therefore if to each of them there be added the common Trapezium HBCD, the whole Parallelogram ABCD ſhall be equal to the whole Parallelogram HBCG. Which was to be demonſtrated.

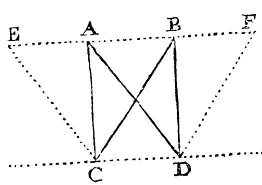
PROP. 36. THEOR. 26.



The Parallelograms ACEF *and* BGHD, *conſtituted on equal baſes* CE *and* HD, *and between the ſame parallels* AB *and* CD *are equal to one another.*

Demonſtration FOr let the right lines CG and EB be drawn; foraſmuch as GB is [a] equal to HD, and CE is put equal to the ſame
a) 34. 1. HD the two ſides, [b] GB and CE ſhall be equal to one another, the
b) 1. c. ſ. which being parallels by ſuppoſition, the lines CG and EB which joyn
them, [c] ſhall be alſo equal and parallel, and BG and CE ſhall be a Pa-
c) 33. 1. rallelogram: Therefore the Parallelogram ACEF ſhall be equal to the
d) 35. 1. Parallelogram GCEB, being [d] on the ſame baſe CE, and between the
ſame Parallels, by the ſame reaſons, the Parallelogram GHDB ſhall be
equal to the ſame Parallelogram GCEB, being on the ſame baſe GB,
and between the ſame Parallels: Therefore the Parallelograms ACEF,
e) 1. c. ſ. and DBGH [e] are equal to one another: Therefore the Parallelograms,
&c. Which was to be demonſtrated.

PROP. 37. THEOR. 27.

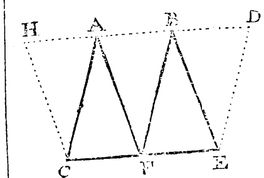


The triangles ACD *and* BCD, *conſtituted on one and the ſame baſe* CD, *and between the ſame parallels* AB *and* CD, *are equal to one another.*

a) 13. 1. *Demonſtration* LEt CE be drawn parallel to AD, [a] and DF parallel to CB, meeting with AB, prolonged on both parts in the
points E and F, the figures ADCE and BCDF, ſhall be Parallelo-
grams, the which being conſtituted on one and the ſame baſe CD, and
b) 35. 1. between the ſame Parallels, [b] ſhall be equal to one another; but the Dia-
c) 34. 1. meters AC and BD doth divide them into [c] two equally: Therefore
d) 7. c. ſ. their halves (which are the triangles propoſed ACD and BCD) [d] ſhall
be alſo equal to one another: Therefore the triangles, &c. Which was
to be demonſtrated.

PROP.

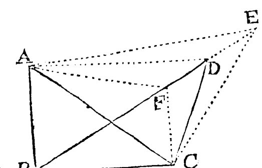
PROP. 38. THEOR. 28.



The triangles ACF *and* BEF, *conſtituted on equal baſes* CF *and* FE, *and between the ſame Parallels* AB *and* CE *are equal to one another.*

Demonſtration FOr let CH and ED be drawn [a] parallel to FA and FB, a) 31. 1.
meeting AB prolonged on both parts in the points H and
D, AFCH and BFED ſhall be Parallelograms, and [b] equal to one an- b) 36. 1.
other, of which the triangles propoſed ACF and BFE are [c] the halves: c) 34. 2.
Therefore thoſe triangles ſhall [d] be alſo equal to one another: Therefore d) 7. c. ſ.
the triangles, &c. Which was to be demonſtrated.

PROP. 39. THEOR. 29.



The equal triangles ABC *and* DBC, *conſtituted on one and the ſame baſe* BC, *and on the ſame part* AD, *are alſo between the ſame parallels; that is to ſay, that the line drawn from* A *to* D, *is parallel to* BC.

Demonſtration FOr if AD were not parallel to BC, there might be drawn
from the point A a parallel to the ſame BC, which would
paſſe above AD, or under it: Let therefore [a] AF be drawn as a parallel a) 31. 1.
to BC, if AD be not a parallel; and let CF be joyned: Foraſ-
much as AF and BC are parallels, the triangle FBC ſhall be equal
to the triangle [b] ABC; but the triangle DCB is equal to the ſame tri- b) 37. 1.
angle ABC by Suppoſition: therefore the triangles [c] FBC and DCB c) 1. c. ſ.
ſhall be equal to one another, the part to the whole, which is abſurd.

If it be ſaid that the line parallel to BC drawn from the point A towards D, ſhall paſſe above AD, the ſame abſurdity will happen; therefore AD is parallel to BC: Therefore the equal triangles, &c. Which was to be demonſtrated.

PROP. 40. THEOR. 30.

The equal triangles ABC *and* DEF, *conſtituted on equal baſes* BC *and* EF, *and on the ſame part* AD, *are alſo between the ſame parallels; that is to ſay, that the line drawn from the point* A *to* D, *is parallel to* BF.

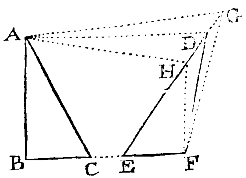
Demon-

a) 31. 1.

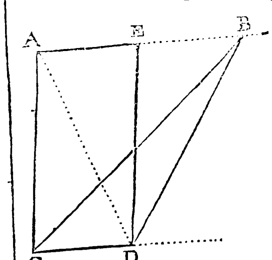
Demonstration For otherwise there might be drawn from the point A a line parallel to [a] B F ; which should fall above or under A D.

Let A G be drawn parallel to B F, (if possible,) and let G F be joyned : Forasmuch as the lines A G and B F are parallels, the triangle G E F shall be equal to the triangle A B C, but the triangle D F E is equal to the same triangle A B C by Supposition : Therefore the triangles D F E and [b] G E F shall be equal to one another, the part to the whole, which is absurd if the parallel be said to passe below A D, the same absurdity will happen : Therefore A D is parallel to B F : Therefore, The equal triangles, &c. Which was to be demonstrated.

b) 1. c. s.



PROP. 41. THEOR. 31.

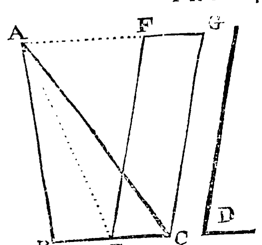


If a Parallelogram A C D E *hath the same base* C D, *as a triangle* B C D, *and is between the same parallels* A B *and* C D, *as is the triangle* B C D, *the Parallelogram* A C D E *shall be double to the triangle* B C D.

a) 37. 1.
b) 34. 1.

Demonstration For having drawn the Diameter A D, the triangle A C D [a] shall be equal to the triangle B C D: But the Parallelogram A C D E is double to the triangle A C D : forasmuch [b] as the triangles A C D and A E D are equal : Therefore the same Parallelogram A C D E is also double to the triangle B C D : Therefore, If a Parallelogram, &c. Which was to be demonstrated.

PROP. 42. PROBL. 11.



To constitute a Parallelogram E F C G *equal to a given triangle* A B C, *in a given right lined angle* D.

That is to say, that hath one angle equal to a given right lined angle.

a) 10. 1.

Construction Let one of the sides of the triangle A B C, (which you please) be divided ; (to wit B C,) into two [a] equal parts in the point E,

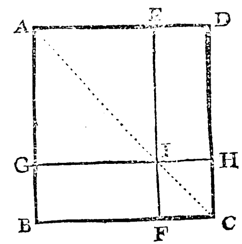
b) 31. 1.

E, and let A E be joyned ; and let the angle C E F be made equal to the given angle D ; then let [b] A F be drawn parallel to B C, meeting E F in the point F ; also let C G be drawn from the point C, parallel to E F, meeting A F prolonged in the point G, so shall the Parallelogram F E C G be constituted in the angle C E F, which is equal to the given angle D, which said Parallelogram I say is equal to the given triangle A B C.

c) 38. 1.
d) 41. 1.
e) 6. c. s.

Demonstration Forasmuch as the triangles A B E and A E C are equal; [c] being constituted on the equal bases B E and E C, and between the same parallels A F and B C, the whole triangle A B C shall be double to the triangle A E C : But the Parallelogram F E C G is [d] also double to the same triangle A E C : Therefore the triangle A B C [e] and the Parallelogram F C shall be equal to one another, and the parallelogram F C shall have the angle C E F equal to the given angle D, by construction : Therefore we have constituted, &c. Which was to be done.

PROP. 43. THEOR. 32.



Of every parallelogram, as A B C D, *the complements* D I *and* I B *of the parallelograms* G E *and* F H, *which are about the Diameter* A C, *are equal to one another.*

a) 34. 1.

Demonstration For seeing that the triangles A B C and A D C [a] are equal to one another (for the Diameter A C divideth the parallelogram into two equal parts, and that the two triangles A G I and A E I are also equal to one another, by the same reason, if those two later be cut off from the two first, [b] the Trapeziums G B F C I and E D H C I, shall remain equal to one another ; But the triangles F I C and H I C are [c] also equal to one another ; therefore if these triangles be taken from the Trapeziums, the complements [d] D I and I B, will remain equal: Therefore, Of every, &c. Which was to be demonstrated.

b) 3. c. s.
c) 34. 1.
d) 3. c. s.

PROP. 44. PROBL. 12.

On a given right line A, *to apply a parallelogram,* L M F H, *equal to a given triangle* B, *in a given right lined angle* C.

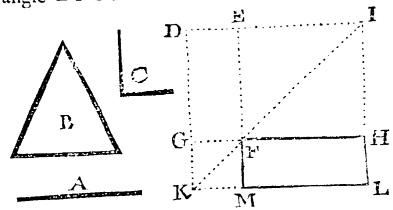
a) 42. 1.
b) 31. 1.

Construction Let the parallelogram D E F G be constituted, [a] equal to the given triangle B, having the angle G F E, equal to C ; then let G F be prolonged to the point H, in such sort as F H may be equal to A, and by H let there be drawn H I, [b] parallel to F E, meeting D E prolonged in the point I ; after that let the Diameter I F be drawn, by the point F, meeting D G prolonged in K, and by K let K L be drawn,

c) 31. 1. parallel c to GH, meeting IH prolonged in L, and let EF be prolonged to the point M: I say, that the Parallelogram LMFH is the Parallelogram required.

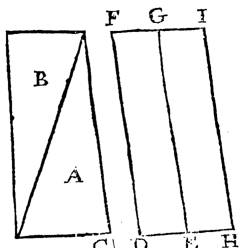
Demonstration FOr it hath the side FH equal to the given line A, by the construction, and the angle HFM is d equal to the angle EFG, the which is made equal to the angle C: Therefore HFM shall be also equal to C, and lastly, the parallelogram FL is equal to the parallelogram GE, the complement equal to the complement, which said parallelogram GE is made equal to the triangle B: Therefore FL shall be also equal to B: Therefore on a given, &c. Which was to be done.

d) 15. 1.



PROP. 45. PROBL. 13.

To constitute a parallelogram DI, *equal to a given right lined figure* AB, *in a given right lined angle* C.



Construction LEt the Parallelogram DG be made a equal to the triangle A, having the angle FDE equal to the given right lined angle C; then to the line GE, let the parallelogram EI be applyed, equal b to the triangle B, having the angle GEH equal to the angle C; and so what was required is done.

a) 42. 1.
b) 44. 1.

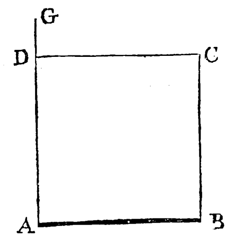
Demonstration FOrasmuch as each of the angles FDE and GEH, is made equal to the angle C, c they shall be equal to one another; therefore if the angle GED be added; d the two angles GED and GEH shall be equal to the two angles GED and FDE; but FDE and GED are equal to two right angles; therefore GED and GEH shall be also equal to two right angles; Therefore e DEH shall be a right line: By the same reasons it may be proved that FG and GI do make a right line. Now seeing that each of these lines DF and HI are equal and parallel f to EG, they shall be equal and parallel to one another, and the right lines FI and DH which joyn them together, g shall be also equal and parallel: Therefore DI shall be a parallelogram, made of the two parallelograms DG and EI, equal to the two triangles A and B: Therefore the parallelogram DI is equal to the right lined figure AB, having the angle D equal to the given right lined angle C: Which was to be done.

c) 1. c. s.
d) 2. c. s.
e) 14. 1.
f) 34. 1.
g) 30. 1.

PROP.

PROP. 46. PROBL. 14.

On a given right line AB, *to describe a Square* ABCD.



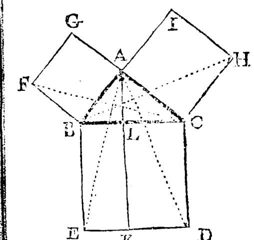
Construction LEt the right line AG be drawn a at right angles to AB, and from the point D let b DC be drawn parallel to AB, likewise let BC be drawn parallel to AD, meeting DC in the point C, the figure ABCD shall be a Parallelogram, and therefore the side DC c is equal to AB, and AD equal to BC: But AD is put equal to AB; therefore BC d shall be also equal to AB; therefore ABCD shall be equilateral: I say it is also a Rectangle.

a) 11. 1.
b) 31. 1.
c) 34. 1.
d) 1. c. s.

Demonstration FOrasmuch as AD and BC are parallel, the e angles A and B are equal to two right angles, but A is made a right angle, therefore B shall be also a right angle, and the two opposite angles D and C f are equal to them, and therefore right angles: Therefore the Parallelogram AC is rectangled, and equilateral, and therefore shall be a Square: Therefore, On a right line, &c. Which was to be done.

e) 28. 1.
f) 34. 1.

PROP. 47. THEOR. 33.



In right angled triangles as ABC, *the square* BD *of the side* BC, *which subtendeth the right angle* BAC, *is equal to the squares* AF *and* CI, *of the sides* AB *and* AC, *which do contein the same right angle* BAC.

Demonstration LEt AK be drawn a parallel to BE or to CD, cutting BC in the point L, and let the right lines AD, AE, CF, and BH be joyned: Forasmuch as the angles BAC and BAG are right angles, b GA and AC shall make one only right line, by the same reason BA and AI shall also make one right line.

a) 31. 1.
b) 14. 1.

Again, Forasmuch as the angles ABF and CBE are equal (as being right angles, if the common angle ABC be added, the whole ABE shall be c equal to the whole CBF; likewise the whole ACD shall be equal to the whole BCH: Forasmuch therefore as the two sides AB and BE of the triangle ABE, are equal to the two sides CB and BF of the triangle CBF, each to his correspondent, (as appears by the Definition of Squares,) and the angles ABE and CBF conteined by those sides are shewn to be equal: d Those triangles ABE and CBF shall be equal; but the Parallelogram AF is e double to the triangle

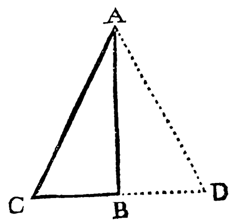
c) 2. c. s.
d) 4. 1.
e) 41. 1.

 CBF,

CBF, being on the same base, and between the same parallels: It shall be therefore also double to the triangle ABC, and the Parallelogram BK is double to the same triangle ABC, it being on the same base, and between the same parallels: Therefore the parallelogram BK shall be equal to the square AF.

By the same discourse it might be shewn that the Parallelogram LD is equal to the square CI: Therefore the square EC shall be equal to the two squares AF and CI: Therefore, In triangles, &c. Which was to be demonstrated.

PROP. 48. THEOR. 34.



If the square described of one of the sides AC *of a triangle* ABC *be equal to the squares of the two other sides* AB *and* BC *of the same triangle* ABC, *the angle* ABC *is a right angle.*

Demonstration LEt BD be drawn equal to BC, and at [a] right angles to AB, and let AD be joyned: Seeing that ABD is a
a) 11. 1. right angle, the square of AD [b] is equal to the two squares of AB and
b) 47. 1. BD: but the square of BD is equal to the square of BC, seeing that the lines BD and BC are made equal, wherefore the square of AD shall be equal to the squares of BA and BC: Therefore seeing that the square of
c) 1. c. s. AC is equal to the same squares of BA and BC, [c] the same squares of AC and AD shall be equal; and therefore the lines AD and AC are equal: Forasmuch then, as in the triangles ABD and ABC, the two sides BD and BA are equal to the two sides BA and BC, each to his correspondent, and the bases AD and AC equal, the angles at the point B
d) 8. 1. conteined of those [d] sides shall be equal to one another: But ABD is made a right angle: Therefore ABC shall be also a right angle: Therefore the square, &c. Which was to be demonstrated.

The End of the First Element of EUCLIDE.

THE

THE
SECOND ELEMENT
OF
EUCLIDE.

THE ARGUMENT.

EUCLIDE treats in this second Book of the power of right lines, seeking the quantity of the squares of the parts of every right line divided, and of rectangled Parallelograms conteined under the parts of the same divided line; compared as well to one another, as with the square of the whole, &c. and for that cause he here unfoldeth in the first place, by these two Definitions two things which are necessary for the well understanding of what is to be demonstrated hereafter.

DEFINITIONS.

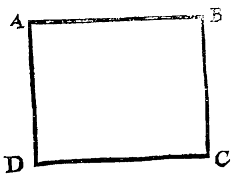
I *Every rectangled Parallelogram is said to be conteined under two right lines, which do contein the right angle.*

IN this first he declares under what lines the rectangled parallelogram is said to be conteined, and what is to be understood by a parallelogram to be conteined under two right lines; which for the more easie understanding, you must first know that every rectangled parallelogram is that which hath all his angles right angles, and of those there are two sorts, to wit, the square, and the oblong or long square, longer on one part then on the other: for in those all the angles are right angles, as we have

have already said in the Definitions : And in every parallelogram if one angle be given a right angle, the three others will of necessity be right angles, For example, in the parallelogram A B C D, Let the angle A be a right angle, I say that the three others B, C, and D, are also right angles; for seeing that A B and D C are parallels, a the two interior angles A and B, are equal to two right angles: But A is a right angle by supposition; therefore B shall be also a right angle: But b forasmuch as each one of them is equal to his opposite, as the angle A to the angle C, and the angle B to the angle D, the angles D and C shall be also right angles: Therefore *EUCLIDE* saith that every parallelogram right angled is said to be conteined under two right lines, which contein a right angle,

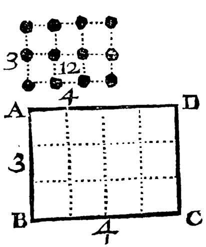
a) 29. 1.

b) 34. 1.



as the Rectangled parallelogram A B C D, is said to be conteined under A B and A D, or under A D and D C, or lastly, under A B and B C: forasmuch as each two of those lines so taken, do expresse the magnitude or space of the whole parallelogram, to wit, one of them, as A B or D C its length, and the other as A D or B C its breadth: Therefore these two lines of the rectangled parallelogram, which contein the right angle being expressed, we conceive immediately the whole quantity thereof, and also its length and breadth is understood: It happens also that by the imaginary motion of one of the lines, according to the other, the whole parallelogram is constituted; for if it be conceived in the understanding, that the right line A B be moved transversly according to A D, in such sort as that it may alwayes constitute a right angle with A D, until the point A arrive to the point D, and the point B to the point C, the whole parallelogram A D C B shall be described: the same will happen if A D be proposed to move transversly according to A B; therefore by good reason, a right angled parallelogram is said to be conteined under two such right lines.

Now this hath a great affinity and neernesse with the multiplication of one number by another, for even so, as by the multiplication of 3 by 4, 12 is produced, which is constituted in form of a parallelogram, from whence it is said to be conteined under 3 and 4: So the parallelogram A B C D, conteined under the two right lines A B and B C, (of which A B conteins 3 Palmes, or other measures, and B C 4) conteins 12 square Palmes, or other measures, which is made by the imaginary passing or motion of the line A B, of 3 Palmes in the line, or according to the line B C of 4 Palmes, as is represented by the figure; and is well known to Arithmeticians and Geometricians,



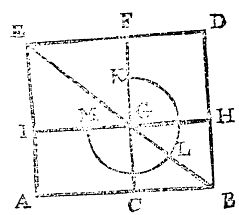
and is demonstrated by *Regiomontanus* in his first Book of triangles, Prop. 16. Whence it comes, that some say, that the rectangled parallelogram is

is constituted, or made of the product of two lines which are about the right angle, the one being multiplyed by the other; as the fore-mentioned Parallelogram is produced by the multiplication of the line A B, in the line B C, or (which is the same thing.) of the line B C in the line A B, for the same parallelogram is produced, whether the lesser line be applyed to the greater, or the greater to the lesser. In like manner, also the same number is produced, if the lesser number be multiplyed by the greater, or the greater by the lesser, as it is demonstrated by *EUCLIDE* in the Seventh Book, Proposition 15. For that 12 is as well produced by 3 multiplyed in 4, as by 4 multiplyed in 3.

¶ The Reader is here advertised that in this second and the other following Books, the rectangled parallelogram is termed by *EUCLIDE* simply Rectangle, which other Geometricians do also observe, so that by Rectangle is to be understood alwayes Rectangled Parallelogram: Again, to the end the same letters be not so often repeated, Geometricians are accustomed to expresse the Parallelogram as well Rectangled, as not Rectangled, by two letters only; to wit, those which are diametrically opposite, as to denote the Parallelogram before mentioned, we say the Parallelogram A C, or else B D.

2 *Of every Parallelogram, one of the Parallelograms (which you please,) described about the Diameter, together with the two complements, is called* Gnomon.

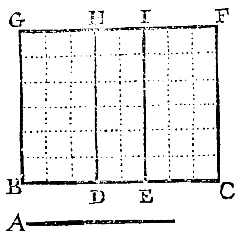
In the Parallelogram A B D E, be it rectangled, or not: Let it be divided into four Parallelograms, as was shewn in the 37th. Definition of the First Book, of which C H and I F, are said to be about the Diameter, and the two others A G and G D are said to be complements; the figure compounded of which you please of the Parallelograms which are about the Diameter, as of C H, with the two complements A G and G D, such as is the figure M L K is called Gnomon: by the same reason, the figure H D E I C G, compounded of the Parallelogram I F, which is about the Diameter, and the two complements A G and G D shall be called Gnomon.



PRO-

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.



If there be two right lines A *and* BC, *and that one of them* BC, *be divided into as many parts as you please* BD, DE, *and* EC, *the Rectangle conteined under those two right lines* A *and* BC, *is equal to the Rectangles conteined under the undivided line* A, *and under each of the parts of the divided line* BC, *to wit, under* A *and* BD, *under* A *and* DE, *and under* A *and* EC.

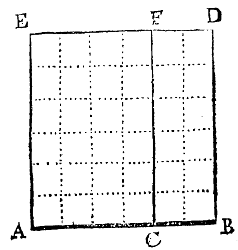
Demonstration. Let the Rectangle be BF, comprised under A and BC;
that is to say, that BG be put equal to A, which may
a) 11.1. thus be done, draw BG and GF [a] at right angles, to the right line BC,
and make each equal to A, and joyn the right line GF, so BG and CF
b) 28.1. shall be [b] parallels, because of the right angles B and C, and also equal
c) 1.c.s. to one another; seeing [c] that each of them is put equal to A: therefore
d) 33.1. the right lines GF and BC which joyn them, [d] shall be also equal and
parallel, therefore BF shall be a Rectangle, and conteined under BC
and A; that is to say, his equal BG, according to the first Definition of
e) 31.1. this Book: Then from the points D and E, draw DH and EI, [e] parallel
f) 30.1. to BG, and therefore parallel [f] to one another: And seeing that by the
same construction BH and BI are Parallelograms, the right lines DH
g) 34.1. and EI shall [g] be equal to BG; that is to say to A: Seeing then that BG
is equal to A, the Rectangle BH shall be conteined under the undivided
line A, and the segment BD; and the Rectangle DI, shall be conteined
under the undivided line A, and the segment DE: In like manner EF shall
be

be conteined under A, and the segment EC: Now seeing that all those parts taken together, are equal to their whole BF, it is evident that the Rectangle comprised of the two lines A and BC, shall be equal to each of the Rectangles comprised of A, and each of the segments of BC: Therefore if there be two lines, &c. Which was to be demonstrated.

SCHOLIUM.

We have divided these figures into equal parts, to render the Demonstrations more easie, and to fit them to numbers, (See the precedent figure:) Let the line BG *equal to* A, *be 6 feet, or other measures, and the line* BC 8, *by the first Definition of this Book, the Rectangle* BF *shall be* 48 *Superficial feet, and by the same reason, the Rectangle* BH *shall be* 18, DI 12, *and* EF *also* 18, *which three Rectangles together do make also* 48, *to wit, all the parts taken together, equal to their whole.*

PROP. 2. THEOR. 2.



If a right line AB *be divided at pleasure in* C, *the Rectangles* CD *and* AF, *conteined under the whole* AB, *and each of the parts* AC *and* CB, *are equal to the square of the whole* AB.

Demonstration. Describe [a] AD the square a) 46.1.
of the whole line AB,
and by C [b] draw CF parallel to AE or b) 31.1.
BD, it shall be [c] equal to AE; that is to say, to AB, to which AE is c) 34.1.
equal, by the Definition of a square: Forasmuch then as AE is equal to AB, the Rectangle AF shall be conteined under the whole AB, and the segment AC: In like manner, forasmuch as CF is equal to AB, the Rectangle CD is conteined under the whole AB, and the other segment CB: Therefore seeing the Rectangles CD and AF are equal to the square AD, it is manifest that the Rectangles conteined under the whole AB, and each of the segments AC and CB are equal to the square of the whole AB: Therefore, If a line, &c. Which was to be demonstrated.

SCHOLIUM.

This may also be seen by numbers, by dividing 6 in 4 and 2, as appeares by the figure, applying it to numbers, AF *shall be* 24, *and* CD 12, *according to the first Definition, which make* 36, *and so much is the square of* AD 6.

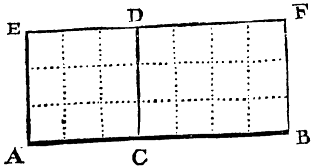
PROP. 3. THEOR. 3.

If a right line AB *be divided at pleasure in* C, *the Rectangle* AF, *conteined under the whole* AB, *and one of the*

parts A C, *is equal to the Rectangle* C F, *conteined under those parts, and* A D *the square of the part* A C *first taken.*

a) 31. 1.

Demonstration. DEscribe A D the square of the part A C, and by the point B draw B F a parallel to A E, meeting E D prolonged at F.

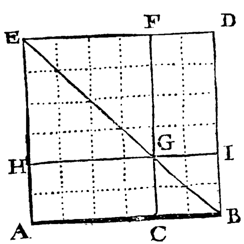


Forasmuch the[...]as the right line A E is equal to the segment A C, by the Definition of a square, the Rectangle A F shall be conteined under the whole A B and the part A C: Again, forasmuch as C D is equal to C A, by the same reason, the Rectangle C F shall be conteined under the parts A C and C B; therefore seeing that the Rectangle A F is equal to the square A D, and to the Rectangle C F: It is manifest that the Rectangle conteined under the whole A B, and the segment A C is equal to the Rectangle conteined under the part A C and C B, and to the square of A C first taken: Therefore, If a right line, &c. Which was to be demonstrated.

SCHOLIUM.

To fit this Theorem to numbers: Let A B 7 *be divided into* 4 *and* 3: *I say the number* 21, *the product of* 3 *by* 7, *is equal to* 12, *the product of the two segments* 4 *and* 3 *multiplyed by one another, and to the number* 9 *the square of the first part* 3: *In like manner* 23 *the product of* 7 *by* 4, *is equal to* 12 *the product of* 4 *in* 3, *and to the number* 16, *the square of* 4 *the number first taken.*

PROP. 4. THEOR. 4.



If a right line A B *be divided at pleasure in* C, *the square* A D *of the whole* A B, *is equal to the squares* H F *and* C I, *of the parts* A C *and* C B, *and to twice the Rectangle* A G *or* G D, *conteined under the same parts* A C *and* C B.

Demonstration. DEscribe the square of the line A B, to wit, A D, and having drawn the Diameter B E, draw through the point C the right line C F, parallel to A E or B D, cutting the Diameter in the point G, and by G draw I G H parallel to A B, or to D E, and so the square of A B shall be divided into four Parallelograms.

a) 5. 1.

Forasmuch as in the triangle A B E, the sides A B and A E are equal, the two angles A B E and A E B a on the base E B are equal: But the angle

angle in the square, and the three angles A, A E B, and A B E b are equal to two right angles, and therefore the two angles A B E and A E B, are each half a right angle, being equal to one another, and forasmuch as H I and A B are parallels, the exterior angle G H E c shall be equal to its interior and opposite angle, and therefore a right angle: But the three angles of the triangle d E G H are equal to two right angles, and G E H is shewn to be half of a right angle, and E H G a right angle: Therefore the other H G E shall be also half a right angle; therefore in the triangle E G H, the two angles G E H and E G A shall be equal, being each of them half a right angle; therefore the two sides H E and e H G shall be equal: But the two opposite sides E F and F G f are equal unto them, each to its correspondent: Therefore the Parallelogram H F shall be a square, having all the sides equal, and the angles right angles: For one of the angles, to wit E H G, being a right angle, all the four shall be right angles, as is shewn by the first Definition of this Book, and by the same reasons it may be shewn that C I is a square: Therefore F H and C I, g are the squares of the parts A C and C B; seeing that H G is equal to A C.

b) 32. 1.
c) 29. 1.
d) 32. 1.
e) 6. 1.
f) 34. 1.
g) 34. 1.

The Rectangles A G and G D shall be in like manner conteined under the segments A C and C B: forasmuch as C G and G I are equal to C B, because of the square C I, and F G is equal to G H, by reason of the square F H; that is to say h to A C: Therefore seeing the square A D is equal to the two squares H E and C I, and to the two Rectangles A G and G D, it is manifest that A D the square of the whole A B, is equal to the squares of the parts A C and C D, and to twice the Rectangle conteined under the same part A C and C B: Therefore, If a right line, &c. Which was to be demonstrated.

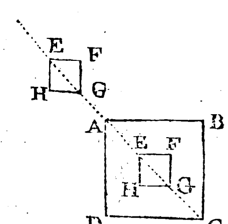
h) 34. 1.

COROLLARIE I.

By this Demonstration it is manifest that the Parallelograms described about the Diameter of a square are square.

Which is apparent by the Demonstration of this Theorem, where it is demonstrated that the Rectangles C I *and* F H, *which are about the Diameter* B E *are squares: for the same Demonstration serves in all other squares.*

Neverthelesse, this Corollary ought to be understood of Parallelograms, which are about the Diameteter of the square, and which have some angle common with the whole square, as are the Parallelograms C I *and* F H, *for the one hath the angle* A B D, *and the other the angle* A E D, *common with the square.*



It is also true of any Parallelograms which are about the Diameter of the square, being prolonged, although it have no angle common with the square, provided their sides be parallel to the sides of the square.

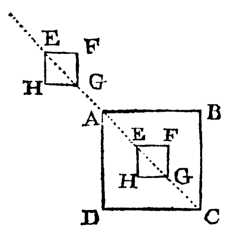
For about the Diameter A C *of the square* B D, *describe the Parallelogram* F H, *be it without or within the square, which notwithstanding must have the sides parallel to the sides of the square: I say that* F H *is a square.*

For seeing that A B *and* E F *are parallels* i *the angles* B A C *and* F E G *shall be equal, the exterior to the interior, and by the same reason the angles* B C A *and* F G E *shall be equal also: But the angles* B A C *and* B C A,

i) 29. 1.

BCA are the halves of right angles, as hath been already demonstrated: Therefore the angles FEG *and* FGE *shall be also the halves of right angles;* [k] *and therefore the sides* EF *and* FG *opposite to them, shall be equal, and* [l] *the angle* F *shall be a right angle: Therefore seeing that the sides* EF *and* FG [m] *are equal to their opposite sides* GH *and* HE, *the Parallelogram* FH, *shall be equilateral; but it is also rectangled, as hath been shewn by the first Definition of this Book: Forasmuch as one of the angles* F *is demonstrated to be a right angle: Therefore* FH *shall be a square: Which was proposed.*

k) 6. 1.
l) 32. 1.
m) 34. 1.



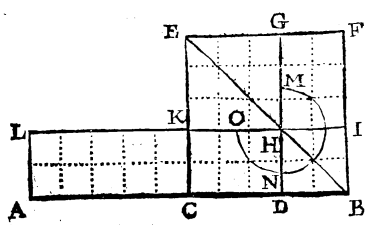
COROLLARIE. II.

It follows also from the Demonstrateon of this Proposition that the Diameter of any square doth divide its angle into two equal parts, for it hath been demonstrated that the angles AEB *and* DEB *in the fore-going Proposition, are the halves of right angles, which is also demonstrated by the Eighth, and Thirty fourth Proposition of the first Book.*

SCHOLIUM.

This fourth Theorem shall be also shewn by numbers; (See the figure belonging to this Proposition) let the line AB *of 6 feet be divided into* AC *and* CB, *4 and 2, the square of* AB *shall be 36 Superficial feet, and the squares of* AC *and* CB, *to wit,* HF *and* CI *shall be 16 and 4, and the two rectangles under* AC *and* CB, *which are* AG *and* GD, *are each 8, all which parts together make 36; to wit, as much as the square of the line* AB, *which is* ABDE, *as is manifest.*

PROP. 5. THEOR. 5.



If a right line AB *be divided into two equal parts* AC *and* CB, *and into two unequal parts* AD *and* DB, *the Rectangle* AH, *conteined of the the unequal parts* AD *and* DB, *of the whole* AB, *with the square* KG, *of the intermediate part* CD, *is equal to the square* CF, *described of* CB, *the half of the whole* AB.

Demonstration Describe CF the square [a] of CB, and draw the Diameter BE, and by the point D draw [b] DG, parallel to BF, or CE, cutting the Diameter in the point H, and by the point H draw KI, parallel to AB, and from the point A draw AL, parallel to CE, meeting

a) 46. 1.
b) 33. 1.

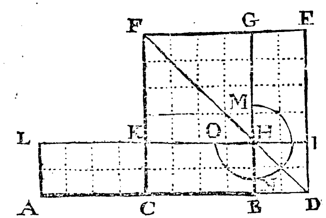
meeting with IK prolonged in the point L; The Parallelograms KG and DI, about the Diameter BE shall be [c] squares, and therefore DH equal to DB and KH to CD: therefore the Rectangle AH shall be conteined under AD and DB, and KG shall be the square of CD, the which AH with KG, is equal to the square CF: For seeing that [d] the complements CH and HF are equal, if the common square DI be added, the Parallelogram DF shall be equal to the Parallelogram CI: But the Parallelogram AK is [e] equal to the Parallelogram CI, being constituted on equal bases AC and CB, and between the same parallels AB and LI: Therefore AK shall be equal to DF, to which if you adde the common CH, the Gnomon MNO shall be equal to the Rectangle AH: Wherefore seeing that the Gnomon MNO and the square KG, are equal to the square CF, the Rectangle AH with the square KG, shall be equal to the same square CF: Therefore, If a line, &c. Which was to be demonstratod.

c) C. 1. 4, 2.
d) 43. 1.
e) 36. 1.

SCHOLIUM.

The same is also manifest in applying it to numbers: Let the line AB *be 10; then as well* AC *as* CB *shall be 5, and let* CD *be 3,* AD *shall be 8, and* DB *2: Therefore the Rectangle* AH *shall be 16, and the square* KG *9, and the Gnomon also 16, which with the square* KG, *is 25, which is the square* CF, *equivalent to the Rectangle* AH *16, and* KG *9, which together make 25, as is manifest.*

PROP. 6. THEOR. 6.



If a right line AB, *be divided into two equal parts in* C, *and there be added to it another right line* BD, *directly, the Rectangle* AI, *conteined under the whole* AB, *with the added line* BD, *to wit* AD, *and the added line* BD, *with the square* KG, *of the half* CB, *is equal to the square* CE, *described of the line* CD, *compounded of the half* CB, *and the added line* BD, *as of one line.*

Demonstration On CD describe the square CE, and having drawn the Diameter DF by B, draw BG parallel to DE or CF, dividing the Diameter in the point H; and by H draw IK, parallel to CD, and by A draw AL parallel to CF, meeting IK prolonged in the point L: Therefore BI and KH shall be squares, and DI shall be equal to BD, [a] and KH is equal to CB: Wherefore the Rectangle AI shall be conteined under AD and DB; and KG shall be the square of CB, which Rectangle AI and square KG, are equal to the square CE: for seeing that

a) 34. 1.

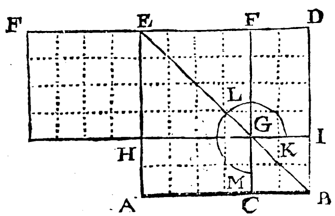
b) 36. 1. that the Parallelogram A K is equal to the Parallelogram C H; ᵇ Seeing that the baſes A C and C B are equal, and the Parallelogram H E is equal to the ſame C H, the complement to the complement, A K and H E ſhall be equal to one another: Therefore if you add to them the common C I, the Gnomon M N O ſhall be equal to the Rectangle A I: Therefore ſeeing the Gnomon M N O, with the ſquare K G, is equal to the ſquare C E, the Rectangle A I with the ſquare K G, ſhall be alſo equal to the ſquare C E: Therefore, If, &c. Which was to be demonſtrated.

This is alſo eaſie to be underſtood by the figure, where the Rectangle A K and H E are each 8 equal parts, to each of which if you add the Rectangle C I 12, the Rectangle A I will be equivalent to the Gnomon, to wit 20: Therefore A I with K G ſhall be equal to the ſquare C E.

SCHOLIUM.

To apply this Theorem to Numbers: Let 8 be divided into two equal parts, to wit into 4 and 4; and let 2 be added, it will be ſeen how the number 20, the product of the whole compound number 10, multiplied by the added 2 with 16, the ſquare of the half 4, is equal to 36, the ſquare of 6 compounded of the half 4, and of the added number 2, as may be alſo underſtood by the figure it ſelf.

PROP. 7. THEOR. 7.



If a right line A B, *be divided at pleaſure in* C, *the two ſquares together, to wit,* A D, *that of the whole* A B, *and* F H *that of one of the ſegments* A C, *are equal to twice the Rectangle* A F, *conteined under the whole* A B, *and the ſaid ſegment* A C, *and to* C I *the ſquare of the other ſegment* C B.

Demonſtration. Deſcribe the ſquare of the line A B, to wit A D, and having drawn the Diameter B E by C, draw C F parallel to A E or B D, cutting the Diameter in the point G, by which point draw H I parallel to A B, by the firſt Corollary of the fourth Propoſition, C I and
a) 34. 1. H F ſhall be ſquares: And ᵃ foraſmuch as G H is equal to A C, and the figure H F ſhall be the ſquare of the ſegment A C: Again, foraſmuch as A E is equal to A B, the Rectangle A F ſhall be conteined under the whole A B, & the ſegment A C, by the ſame reaſon, the Rectangle H D ſhall be conteined under the ſame A B and A C: Foraſmuch as D E and E H are equal to A B and A C, by reaſon of the ſquares A D and H F.

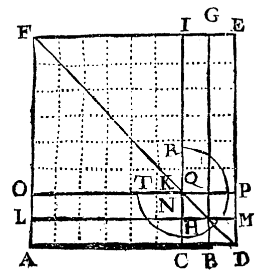
Foraſmuch then as the ſquare A D is equal to the Rectangles A F and F B; that is to ſay, to the Gnomon K L M, and to the ſquare C I, if you add

add the common ſquare H F, the ſquares A D and H F ſhall be equal to the Rectangles A F and D H, (each of which is conteined under the whole A B and the ſegment A C,) and to C I the ſquare of the other part C B: Therefore, If a right line, &c. Which was to be demonſtrated.

SCHOLIUM.

This is manifeſt alſo if it be applyed to numbers, for A B *being divided into 6 parts,* A C *ſhall be 4, and* C B *2, and* A F *ſhall be 24, and* G D *with* H F *alſo 24, and* C I *4, which parts together, make 52 equal parts: And the two ſquares together* A D *and* H F *do alſo make 52 ſuch parts.*

PROP. 8. THEOR. 8.



If a right line A B, *be divided at pleaſure in* C, *four times the Rectangle* A H, *conteined under the whole* A B, *and one of the ſegments* C B, *with the ſquare* O I *of the other ſegment* A C, *is equal to the ſquare* A E, *deſcribed of the whole* A E, *and of the ſaid ſegment* C B, *as of one line* C D.

Demonſtration. Prolong A B towards D, and put B D equal to B C, and on the whole A D deſcribe the ſquare A E, and having drawn the Diameter D F, draw C I and B G parallels to D E, cutting the Diameter in the points H and K, through which points draw L M and O P, parallel to A D, which doth divide the firſt Parallels in the points N and Q.

In the firſt place the Parallelograms O I, N Q, B M, L G, and C P, which are about the Diameter D F ſhall be ſquares, by the firſt Corollary of the fourth Propoſition: And foraſmuch as ᵃ O K is equal to a) 34. 1.
A C; O I ſhall be the ſquare of the ſegment A C: Again, ſeeing that N H is equal ᵇ to C B, N Q ſhall be the ſquare of the ſegment C B: b) 34. 1.
and therefore equal to the ſquare B M; ſeeing that C B and B D are equal, therefore each of the lines B H and H Q is equal to the ſegment C B, and ſo the two Rectangles A H and L Q, ſhall be compriſed under the whole A B, and the ſegment C B: ᶜ Seeing that L H is equal to c) 34. 1.
A B, by the ſame reaſon, N G and H E ſhall be compriſed under A B and C B: Seeing that the right lines N H and H M are equal to C B and B D; and G H and E M equal to F L, that is to ſay to L H, or to his equal A B: And foraſmuch as the ſquares N Q and B M are equal; if you add the common rectangle K G, B M and K G together, ſhall be equal to the Rectangle N G: Wherefore the five rectangles A H, L Q, H E, B M, and K G, which conſtitute the Gnomon R H T, are equal to four times the Rectangle conteined under the whole A B, and the ſegment C B:

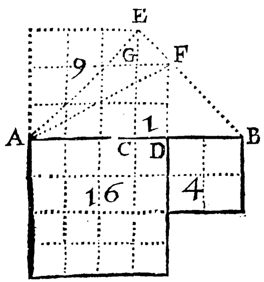
Now

Now ſeeing that the Gnomon R S T, and the ſquare O I, are equal to the ſquare A E; four times the Rectangle conteined under the whole A B, and the ſegment C B, with the ſquare of the other ſegment A C, is equal to the ſquare of the line A D, compounded of A B and of the ſame ſegment C B: Therefore, If a line, &c. Which was to be demonſtrated: And this may be alſo underſtood by the figure divided into equal parts.

SCHOLIUM.

Now divide 10 at pleaſure, into 6 and 4, the number 240, which is made of the whole 10, multiplyed by the part 6, 4 times, with the number 16 the ſquare of the other part 4; that is to ſay, 256 is equal to the ſquare number of 16, which is compounded of the given number 10, and of the ſaid part 6, as is manifeſt: In the ſame manner, the number 160, which is made of the whole 10, 4 times multiplyed by the part 4 with 36, the ſquare of the other part 6; that is to ſay 196, is equal to the ſquare of the number 14, compounded of 10 and 4 as is evident.

PROP. 9. THEOR. 9.



If a right line A B, *be divided into two equal parts in* C, *and into two unequal parts in* D, *the ſquares* 16 *and* 4, *of the unequal ſegments* AD *and* DB, *of the whole* AB, *are double to the ſquare* AE, *of the half* AC, *and the ſquare* 1 *of the intermediate Section* C D.

Demonſtration FRom the point C draw C E at right angles to A B, and equal to A C, and joyn A E and E B together; and from D draw alſo D F at right angles to A B, cutting E B in the point F, by which point draw F G, parallel to A B, dividing C E in the point G; and laſtly joyn A F; Foraſmuch as in the triangle A C E, the two ſides A C and C E are equal, the two angles C A E and C E A ſhall be

a) 32. 1.

equal: but A C E is a right angle; therefore the two [a] others are equivalent to a right angle: and therefore the angle A E C ſhall be the half of a right angle: By the ſame reaſon, the angle B E C ſhall be ſhewn to be half a right angle, and by conſequence the whole A E B ſhall be a angle.

b) 29. 1.

Again, foraſmuch as in the triangle F G E, the angle E G F [b] is equal to the right angle E C B, the exterior angle to the interior, the two

e) 32. 1.

others [c] ſhall be equal to a right angle: But it is demonſtrated that the angle F E G is half a right angle; Therefore F E G ſhall be alſo half a

d) 6. 1.

right angle, and therefore are both equal: Therefore [d] the ſides E G and G F ſhall be equal to one another: In the ſame manner it ſhall be ſhewn that the two lines F D and D B are equal to one another.

Now foraſmuch as in the triangle A C E, the angle C is a right angle, the

c) 47. 1.

[e] the ſquare of the ſide A E ſhall be equal to the two ſquares of A C and C E, the which are equal to one another, being the lines A C and C E are equal: Therefore the ſquare of AE ſhall be double to the ſquare of A C: Again, foraſmuch as in the triangle E G F, the angle G is a right angle,

f) 47. 1.

and the two ſides G E and G F equal [f] the ſquare of E F ſhall be double to the ſquare of G F; that is to ſay, of the ſquare of C D, which is

g) 34. 1.

[g] equal to G F: Therefore the two ſquares of A E and E F, are double to the two ſquares of A C and C D: But theſe ſaid ſquares of A E and

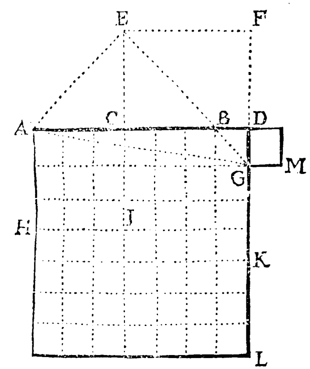
h) 34. 1.

E F [h] are equal to the ſquare of A F: Therefore the ſquare of A F ſhall be double to the two ſquares of A C and C D: and the ſquare of A F is equal to the two ſquares of A D and D F: Therefore the two ſquares of A D and D F are double to the two ſquares of A C and C D: But the ſquare of D F is equal to the ſquare of D B: for thoſe lines are ſhewn to be equal: In like manner, therefore the ſquares of the ſegments A D and D B ſhall be alſo double to the ſquares of A C and C D, the half of the line A B, and of the middle Section: Therefore, If a line, &c. Which was to be demonſtrated,

SCHOLIUM.

Let 10 be divided equally into 5 and 5, and unequally into 7 and 3, the middle Section ſhall be 2, as by the fifth Propoſition, the ſquares of the unequal parts 7 and 3, which are 49 and 9, are double to 25 and 4, the ſquares of the half and of the middle Section, as is manifeſt..

PROP. 10. THEOR. 10.

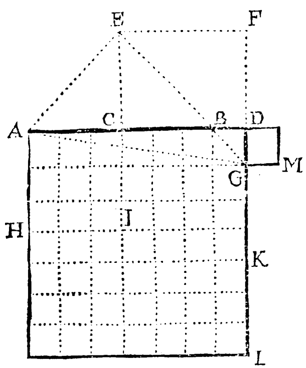


If a right line A B, *be divided into two equal parts in* C, *and there be added to it directly a right line* BD, *the two ſquares* A L *and* B G *together, of the whole* A B, *with the added line* B D, *are double to the ſquare* H C, *deſcribed of the half* AC, *and to the ſquare* C K, *compounded of the half* C B, *and of the added line* B D, *as of one line.*

Demonſtration DRaw C E at right angles to the line A B, and make it equal to the half A C, and joyn the right lines A E and E B, and by D draw F D G, parallel to C E,

CE, meeting with EB prolonged in the point G, and by E draw EF
parallel to CD, meeting DF in the point F, and let the right line AG
be joyned, we shall now shew that the angle AEB is a right angle
as in the foregoing Proposition, and CEB half a right angle;
a) 19. 1. a and therefore its alternate angle EGF in like manner half a right
b) 34. 1. angle: But the angle b F is a right angle being opposite to the right
c) 34. 1. angle C in the Parallelogram CF; c therefore the other angle FEG shall
be also half a right angle, and therefore equal to EGF: Wherefore EF
d) 6. 1. and FG d opposite to the angles FEG and EGF; are in like manner equal;
by the same discourse it shall be shewn that BD and DG are equal;
forasmuch as the angle BDG is a right angle, and BGD half a
right angle, &c.

e) 47. 1. Forasmuch therefore, as the square of AE is e equal to the equal squares of the equal right lines AC and CE, the same square of AE shall be double to the square of AC: Again, forasmuch as the square of EG
f) 47. 1. f is equal to the equal squares of the equal right lines EF and FG, the same square of EG shall be in like manner double to the square of EF, or
g) 34. 1. of CD its equal; for g CD is equal to EF: therefore the two squares of AE and EG are double to the squares of AC and CD; But the square of
h) 47. 1. AG is equal to the squares h of AE and EG, and therefore double to the squares of AC and CD, and the squares of AD and DG are equal to the square of AG: Therefore the squares of AD and DG are double to the squares described of AC and CD: Therefore seeing that the square of DG is equal to the square of its equal BD, the squares of AD and DB shall be in like manner double to the squares described of AC and CD: Therefore, If a right line, &c. Which was to be demonstrated.



SCHOLIUM.

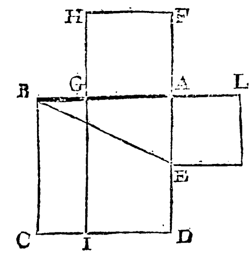
Divide the number 10 *into two equal parts* 5 *and* 5, *to which add some other number as* 3, *in such sort as the whole compound number may make* 13, *whose squares* 169 *and* 9, *of the numbers* 13 *and* 3, *are double to the squares* 25 *and* 64, *which are produced of the numbers* 5 *and* 8, *as is manifest.*

PROP. 11. PROBL. 1.

To divide a given right line AB, *in such sort as that the Rectangle* CG *conteined under the whole* AB, *and one of the segments* BG, *may be equal to the square* AH *of the other segment* AG.

Demon-

Construction Describe the square AC of the right line AB; and having divided the side DA into two equal parts in the point E, draw EB, and prolong DA to the point F, in such sort as EF be put equal to EB, and from AB cut off AG equal to AF, for AB is greater
than AF: Seeing a that EA and AB are greater than EB, or EF its a) 20. 1.
equal: Therefore taking away the common part EA, there will rest AB greater than AF: I say that AB is divided in the point G, in such sort as the Rectangle conteined under AB and BG, is equal to the square of AG, in such sort as that BG is the lesser segment, and AG the greater.



Demonstration For describe AH the square of AG, and prolong the side HG towards I, in such sort as that HI may divide DC
in the point I, which b shall be parallel to FD and BC; therefore CG and DH shall be rectangled Parallelograms, and CG shall be conteined under the whole AB, and the segment BG; for CB is equal to AB; which said Rectangle is equal to the square GF of the other segment AG. b) 34. 1.

For seeing that DA being divided into two equal parts in E, and AF
added thereto, c the Rectangle conteined under DF and FA; that is to c) 6. 2.
say, the Rectangle DH, (for FH is equal to FA,) with the square of the half AE, is equal to the square of EF; that is to say, to the square of
EB, which is equal to EF: But the square of EB is equal d to the squares d) 47. 1.
of EA and AB: Wherefore the Rectangle DH, with the square of AE shall be also equal to the squares of AE and AB; taking away therefore the common square of AE, there will remain the Rectangle DH, equal to the square of AB, which is the square AC: Therefore if the common Rectangle DG be taken away, there will remain the Rectangle GC, equal to the square GF, which was proposed: Therefore we have divided, &c. Which was to be done.

SCHOLIUM.

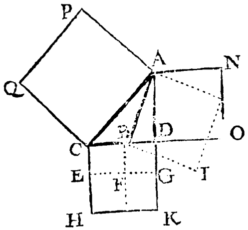
This Problem cannot in any kind be demonstrated by numbers: For no number can be divided in such sort into two parts, as the number produced of the whole multiplyed by one of the parts, may be equal to the square number of the number remaining, as shall be demonstrated in the fourteenth Proposition of the Ninth Book.

¶ The Demonstration of the ten foregoing Theorems in numbers, according to *Barlaam*, shall follow at the End of this Second Book.

PROP. 12. THEOR. 14.

In Ambligonium *triangles* ABC, *the square* AQ, *of the side* AC, *which subtendeth the obtuse angle*

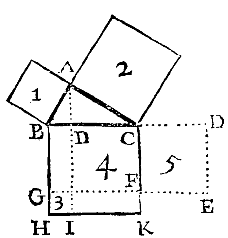
 ABC,



ABC, is greater than the squares AI *and* HF, *of the sides* AB *and* BC, *which do contein the obtuse angle* ABC, *by twice the Rectangle* CF, *conteined under one of the sides which are about the obtuse angle, to wit,* CB, *on which being prolonged, there doth fall the perpendicular* AD, *and the line* BD, *taken without, between the perpendicular* AD, *and the obtuse angle* ABC.

a) 4. 2. *Demonstration* For seeing that CD is divided at adventure in B, [a] the square of CD, to wit, CK, is equal to the two squares of CB and BD, and to twice the Rectangle conteined under CB and BD, adding therefore the common square of AD, to wit, DN, the two squares of CD and DA shall be equal to the three squares of the lines CB, BD, and DA, and to twice the Rectangle conteined under CB and BD: But
b) 47. 1. the square of AC [b] is equal to the two squares of CD and DA, the same square of AC shall be therefore also equal to the three squares of CB, BD, & DA, and to twice the Rectangle conteined under CB and BD;
c) 47. 1. therefore seeing that the square of BA is [c] equal to the squares of BD and DA, the square of AC shall be equal to the squares of CB and BA, and to twice the Rectangle conteined under CB and BD, which is proposed: Therefore, In, &c. Which was to be demonstrated.

PROP. 13. THEOR. 12.



In Oxigonium *triangles* ABC, *the square* I *of the side* AB, *which subtendeth the accute angle* ACB, *is lesse than the squares* 2, *and* BK, *of the sides* AC *and* CB, *which contein the accute angle* ACB, *by twice the Rectangle* BF, *conteined under one of the sides* BC, *which are about the accute angle* ACB, *to wit, that on which the*

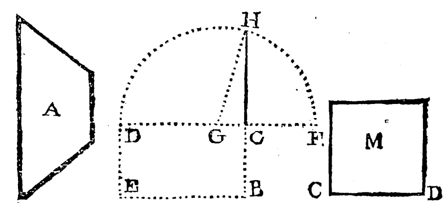
perpendicular AD *doth fall, and the line* DC *taken from within, between the perpendicular* AD, *and the accute angle* ACB.

Demonstration For seeing that the right line BC is divided at adventure in the point D, the squares of [a] BC and of CD, are equal to twice the Rectangle conteined under BC and CD, and to the square BD: Therefore if you add the common square of DA, the three squares of BC, CD, and DA, shall be equal to twice the Rectangle conteined under BC and CD, and to the two squares of BD and DA: But [b] the square of CA is equal to the two squares of CD and DA; therefore the two squares of BC and CA are equal to twice the Rectangle conteined under BC and CD, and to twice the squares of BD and DA; Therefore seeing that the square of AB is equal to the two squares of BD and DA, the two squares of BC and CA shall be equal to twice the Rectangle conteined under BC and CD, and to the square of AB, which was proposed. a) 7. 2. b) 47. 1.

In the same manner, we shall demonstrate that the squares of AB and BC are equal to twice the Rectangle conteined under CB and BD, and to the square of AC, that is to say, that the square of the side AC is lesse than the squares of the sides AB and BC, by twice the Rectangle conteined under CB and BD: Therefore, In Oxigonal triangles, &c. Which was to be demonstrated.

PROP. 14. PROBL. 2.

To constitute a square M, *equal to a given right lined figure* A.



Construction Make [a] the rectangled Parallelogram BCDE, equal to the right lined figure A, and prolong one of its sides, as DC to F, and make CF equal to CB, and divide DF into two equal parts in the point G, which shall be the same as the point C, or not; if it fall on the point C, the right line BC, (seeing it is made equal to CF,) shall be equal to CD; therefore the Rectangle BD shall be a square, and seeing [b] that the sides BE and ED are equal to the opposite sides BC and CD, you have what was sought: But if the point G doth not fall on the point C, from the point G as a center, and at the distance GF or GD, describe the semicircle FHD, and prolong BC a) 45. 1. b) 34. 1.

BC to the Circumference H, and draw also GH: I say that the square of the right line CH is equal to the right lined figure A.

Demonstration. FOrasmuch as DF is divided into two equal parts in G, and in two unequal parts in C, the Rectangle conteined under DC and CF, to wit the Rectangle BD, [c] with the square of GC
c) 5. 2. is equal to the square of GF, or of GH equal thereto: But the square of GH is [d] equal to the two squares GC and CH: Therefore the
d) 34. 1. Rectangle BD, with the square of GC, is equal to the two squares of GC and CH: Therefore if you take away the common square GC, there will remain the Rectangle BD; that is to say, the right lined figure A equal to the square of CH: Therefore, We have constituted, &c. Which was to be done.

The End of the Second Element of EUCLIDE.

The Demonstration of the ten first Theorems of this Second Book of EUCLIDE in Numbers, according to *Barlaam.*

¶ PRINCIPLES.

1 *A number is said to multiply another number: when the number multiplyed is so oftentimes added to it self, as there be unities in the number which multiplyeth: whereby is produced a certain number, which the number multiplyed, measureth by the unities which are in the number which multiplieth.*

2 *And the number produced of that multiplication is called a plain or superficial number.*

3 *A square number is that which is produced of the multiplication of any number into its self.*

4 *Every lesse number compared to a greater, is said to be a part of the greater, whether the lesse measure the greater, or measure it not.*

5 *Numbers, whom one and the self same number measureth equally, that is, by one and the self same number are equal the one to the other.*

6 *Numbers that are equimultiplices to one and the self same number, that is, which contein one and the same number equally and alike, are equal the one to the other.*

PROPOSITION I.

C....4
A..2 D..2 E..2 B
F........................24
G........8 H........8 I........8 K

Two numbers being given, if one of them be divided into any numbers, how many soever: the plain or superficial number which is produced of the multiplication of the two numbers first given, the one into the other, shall be equal to the superficial numbers which are produced of the multiplication of the number not divided into every part of the number divided.

Demonstration. SUppose that there be two numbers AB and C. And divide the number AB into certain other numbers, how many soever, as into AD, DE, and EB. Then I say that the superficial number which is produced of the multiplication of the number C into the number AB, is equal to the superficial numbers which are produced of the multiplication of the number C into the number AD, and of C into DE, and of C into EB. For let F be the superficial number produced of the multiplication of the number C into the number AB, and let GH be the superficial number produced of the multiplication of C into AD: And let HI be produced of the multiplication of C into DE: and finally, of the multiplication of C into FB, let there be produced the number IK: Now forasmuch as AB multiplying the number C produced the number F: Therefore the number C measureth the number F by the unities which are in the number AB. And by the same reason may be proved that the number C doth also measure the number GH, by the unities which are in the number AD, and then it doth measure the number HI, by the unities which are in the number DF; and finally, that it measureth the number IK by the unities which are in the number EB. Wherefore the number C measureth the whole number GK by the unities which are in the number AB. But it before measured the number F by the unities which are in the number AB, wherefore either of these numbers F and GK is equimultiplex to the number C: But numbers which are equimultiplices to one and the self same numbers; are equal the one to the other [a]: Wherefore the number F is equal to the number GK: a) Def. 6. But the number F is the superficial number produced of the multiplication of the number C into the number AB: and the number GK is composed of the superficial numbers prodnced of the multiplication of the number C not divided into every one of the numbers AD, DE, and EB: If therefore there be two numbers given, and the one of them be divided, &c. Which was to be demonstrated.

PROPOSITION II.

If a number given, be divided into two other numbers: the su-

superficial numbers, which are produced of the multiplication of the whole into either part, added together, are equal to the square number of the whole number given.

A....4 C..2 B
D....................................36
B.........................24 F.............12

Demonstration Suppose that the number given be A B: and let it be divided into two other numbers A C and C B. Then I say that the two superficial numbers, which are produced of A B into A C, and of A B into B C, those two superficial numbers (I say) being added together, shall be equal to the square number produced of the multiplication of the number A B into it self. For let the number A B multiplying it self produce the number D. Let the number A C also multiplying the number A B, produce the number E F: Again, let the number C B multiplying the self same number A B, produce the number F G: Now forasmuch as the number A C multiplying the number A B, produced the number E F: therefore the number A B measureth the number E F by the unites which are in A C. Again, forasmuch as the number C B multiplyed the number A B, and produced the number F G: therefore the number A B measureth the number F G, by the unites which are in the number C B: But the same number A B before measured the number E F by the unites which are in the number A C: Wherefore the number A B measureth the whole number E G, by the unites which are in A B. Farther, forasmuch as the number A B multiplying it self, produced the number D: therefore the number A B measureth the number D by the unites which are in himself. Wherefore it measureth either of these numbers; namely, the number D, and the number E G, by the unities which are in himself. Wherefore how Multiplex the number D, is to the number A B, so Multiplex is the number E G to the same number A B. But numbers which are equimuliplices to one and the self same number, are equal the one to the other. Wherefore the number D is equal to the number E G. And the number D is the square number made of the number A B, and the number E G is composed of the two superficial numbers produced of A B into B C, and of B A into A C. Wherefore the square number produced of the number A B, is equal to the superficial numbers produced of the number A B into the number B C, and of A B into A C, added together: If therefore a number be divided into two other numbers, &c. Which was to be demonstrated.

PROPOSITION III.

A....4 C..2 B
D............12
E........8 F....4 G

If a number given, be divided into two numbers: the superficial number which is produced of the multiplication of the whole into one of the parts, is equal to the superficial number which is produced of the parts the one into the other, and to the square number produced of the aforesaid part.

Demon-

Demonstration Suppose that the number given be A B, which let be divided into two numbers A C and C B. Then I say that the superficial number which is produced of the multiplication of the number A B into the number B C, is equal to the superficial number which is produced of the multiplication of the number A C into the number C B, and to the square number produced of the number C B. For let the number A B multiplying the number C B, produce the number D. And let the number A C multiplying the number C B, produce the number E F: and finally, let the number C B multiplying himself produce the number F G. Now forasmuch as the number A B multiplying the number C B, produced the number D. Therefore the number C B measureth the number D, by the unites which are in the number A B. Again, forasmuch as the number A C multiplying the number C B, and produced the number E F; therefore the number C B measureth the number E F, by the unites which are in A C. Again, forasmuch as the number C B multiplied it self and produced the number F G: therefore the number C B measureth the number F G by the unites which are in it self. But as we have before proved, the self same number C B measureth also the number E F by the unites which are in the number A C; wherefore the number C B measureth the whole number E G, by the unites which are in the number A B. And it also measureth the number D by the unites which are in the number A B. Wherefore the number C B equally measureth either number, namely, the number D, and the number E G. But those numbers whom one and the self same number measureth equally, are equal the one to the other. Wherefore the number D is equal to the number E G. But the number D is a superficial number produced of the multiplication of the number A B into the number B C, and the number E G is the superficial number produced of the multiplication of the number A C into the number C B, and of the square of the number C B. Wherefore the superficial number produced of the multitiplication of the number A B into the number C B, is equal to the superficial number produced of the number A C into the number C B, and to the square of the number C B. If therefore a number be divided into two numbers, the superficial number, &c. Which was required to be proved.

A....4 C..2 B
D............12
E........8 F....4 G

PROPOSITION IV.

If a number given be divided into two numbers, the square number of the whole, is equal to the square numbers of the parts, and to the superficial number which is produced of the multiplication of the parts the one into the other twice.

Demonstration Suppose that the number given be A B; which let be divided into two numbers A C and C B. Then I say, that the square number of the whole number A B, is equal to the squares of the parts; that is, to the squares of the numbers A C and C B, and to the superficial number produced of the multiplication of the numbers A C

and

and C B, one into the other twice. Let the ſquare number produced of the multiplication of the whole number A B, into himſelf be D. And let C A multiplyed into himſelf produce the number E F: And C B multiplyed into it ſelf let it produce G H: and finally, of the multiplication of the numbers A C and C B, the one into the other twice, let there be produced either of theſe ſuperficial numbers F G and H K. Now foraſmuch as the number A C multiplying it ſelf produced the number E F: therefore the number A C meaſureth the number E F by the unites which are in it ſelf. And foraſmuch as the number C B multiplyed the number C A,

A......6 C..2 B
D...64
E...............................36 F............12 G....4 H............12 K

and produced the number F G: therefore the number A C meaſureth the number F G by the unites which are in the number C B. But it before alſo meaſured the number E F by the unites which are in it ſelf. Wherefore the number A B multiplying the number A C, produceth the number E G. And therefore the number E G is the ſuperficial number produced of the multiplication of the number B A into the number A C. And by the ſame reaſon may we prove that the number G K is the ſuperficial number produced of the multiplication of the number A B into the number B C. Farther the number D is the ſquare of the number A B. But if a number be divided into two numbers, the ſquare of the whole number is equal to the two ſuperficial numbers which are produced of the multiplication of the whole into either the parts, (by the ſecond Theorem) Wherefore the ſquare number D is equal to the ſuperficial number E K. But the number E K is compoſed of the ſquares of the numbers A C and CB, and of the ſuperficial number which is produced of the multiplication of the number AC & CB the one into the other twice: and the number D is the ſquare of the whole number AB. Wherefore the ſquare number produced of the multiplicatiō of the number AB into its ſelf is equal to the ſquare numbers of the parts, that is, to the ſquare numbers of the numbers A C and C B, and to the ſuperficial number produced of the multiplication of the numbers A C and C B, the one into the other twice: If therefore a number given be divided into two numbers, &c. Which was required to be proved.

PROPOSITION V.

A....4 C..2 D..2 B
E...............16
F...........12 G....4 H
K....4 L....4 M....4 N....4 X

If an even number be divided into two equal parts, and again, alſo into two unequal parts, the ſuperficial number which is produced of the multiplication of the unequal parts the one into the other, together with the ſquare of the number ſet between the parts, is equal to the ſquare of half the number.

Demonſtration. Suppoſe that A B be an even number, which let be divided into two equal numbers A C and C B, and into two unequal numbers A D and D B. Then I ſay, that the ſquare number which is produced of the multiplication of the half number C B into it ſelf, is equal to the ſuperficial number produced of the multiplication of the unequal numbers A D and D B, the one into the other, and to the ſquare number produced of the number C D, which is ſet between the ſaid unequal parts. Let the ſquare number produced of the multiplication of the half number C B into it ſelf be E. And let the ſuperficial number produced of the multiplication of the unequal numbers A D and D B, the one into the other, be the number F G; and let the ſquare of the number D C which is ſet between the parts be G H. Now foraſmuch as the number B C is divided into the numbers B D and D C; therefore the ſquare of the number B C, that is, the number E, is equal to the ſquares of the numbers B D and D C, and to the ſuperficial number which is compoſed of the multiplication of the numbers B D and D C, the one into the other twice, (by the fourth Propoſition of this Book:) Let the ſquare of the number B D be the number K L; and let N X be the ſquare of the number D C: and finally, of the multiplication of the numbers B D and D C, the one into the other twice, let be produced either of theſe numbers L M and M N. Wherefore the whole number K X is equal to the number E. And foraſmuch as the number B D multiplying it ſelf produced the number K L; therefore it meaſureth it by the unites which are in it ſelf. Moreover, foraſmuch as the number C D multiplying the number B D, produced the number L M; therefore alſo D B meaſureth L M by the unites which are in the number C D: but it before meaſured the number K L by the unites which are in it ſelf. Wherefore the number D B meaſureth the whole number K M by the unites which are in C B. But the number C B is equal to the number C A. Wherefore the number D B meaſureth the number K M by the unites which are in C A. Again, foraſmuch as the number C D multiplying the number D B produced the number M N: therefore the number D B meaſureth the number M N, by the unites which are in the number C D: but it before meaſured the number K M by the unites which are in the number A C. Wherefore the number B D meaſureth the whole number K N by the unites which are in the number A D. Wherefore the number F G is equal to the number K N. For numbers which are equimultiplices to one and the ſelf-ſame number, are equal the one to the other. But the number G H is equal to the number N X: for either of them is ſuppoſed to be the ſquare of the number C D. Wherefore the whole number K X is equal to the whole number F H: But the number K X is equal to the number E. Wherefore alſo the number FH is equal to the number E. And the number F H is the ſuperficial number produced of the multiplication of the numbers A D and D B the one into the other, together with the ſquare of the number D C. And the number E is the ſquare of the number C B. Wherefore the ſuperficial number produced of the multiplication of the unequal parts A D and DB, the one into the other, together with the ſquare of the number D C, which is ſet between thoſe unequal parts, is equal to the ſquare of the number C B, which is the half of the whole number A B: If therefore an even number be divided into two equal parts, &c. Which was required to be proved.

PROPOSITION VI.

If an even number be divided into two equal numbers, and unto it be added ſome other numbers, the ſuperficial number which is made of the multiplication of the number compoſed

sed of the whole number and the number added, into the number added, together with the square of the half number, is equal to the square of the number composed of the half and the number added.

A . . . 3 C . . . 3 B . . . . 4 D
E 49
F . . . . 40 G . . . . 9 H
K . . . . 16 L . . . . 12 M . . . . 12 N . . . . 9 X

Demonstration Suppose that A B be an even number, and let it be divided into two equal numbers A C and C B: and unto it let there be added an other number B D. Then I say, that the superficial number produced of the multiplication of the number A D into the number D B, is equal to the square of the number C D. For let the square number of the number C D be the number E, and let the superficial number produced of the multiplication of the number A D into the number D B, be the number F G; and finally, let the square number of C B be the number G H: And forasmuch as the square of the number C D is (by the 4th. Proposition) equal to the squares of the numbers D B and B C together, with the superficial number which is produced of the multiplication of the numbers D B and B C, the one into the other twice. Let the square of the number B D be the number K L: and let the superficial numbers produced of the multiplication of the numbers D B and B C, the one into the other twice be either of these numbers L M and M N: and finally, let the square of the number B C be the number N X. Wherefore the whole number K X shall be equal to the square of the number C D: But the square of the number C D is the number E. Wherefore the number K X is equal to the number E. And forasmuch as the number B D multiplying it self, produced the number K L: therefore the number B D measureth the number K L, by the unites which are in it self, but it also measureth the number L M by the unites which are in the number C B. Wherefore the number D B measureth the whole number K M by the unites which are in the number C D. The number D B also measureth the number M N by the unites which are in the number C B: and the number C B is equal to the number C A by supposition. Wherefore the number D B measureth the whole number K N by the unites which are in the number A D. But the number D B doth also measure the number F G, by the unites which are in the number A D: for by supposition, the number F G is the superficial number produced of the multiplication of the numbers A D and D B, the one into the other. Wherefore the number F G is equal to the number K N. But the number H G is equal to the number N X: for either of them is the square number of the number C B. Wherefore the whole number F H is equal to the number K X, and the number K X is proved to be equal to the number E. Wherefore the number F H shall also be equal to the number E. And the number F H is the superficial number produced of the multiplication of the numbers A D and D B, the one into the other together with the square of the number C B, and the number E is the square of the number C D: Wherefore the superficial number produced of the multiplication of the numbers A D and D B, the one into the other, together with the square of the number C B, is equal to the square of the number C D: If therefore an even number, &c. Which was to be demonstrated.

PRO-

PROPOSITION VII.

If a number be divided into two numbers: the square of the whole number together with the square of one of the parts, is equal to the superficial number produced of the multiplication of the whole number into the foresaid part twice, together with the square of the other part.

A . . . . 5 C . . . 3 B

Demonstration Suppose that the number A B be divided into the numbers A C and C B. Then I say, that the square numbers of the numbers B A and A C are equal to the superficial number produced of the multiplication of the number B A into the number A C twice, together with the square of the number B C. For forasmuch as (by the 4th. Prop. of this Book,) the square of the number A B is equal to the squares of the numbers B C and C A, and to the superficial number produced of the multiplication of the numbers B C and C A, the one into the other twice: add the square of the number A C common to them both. Wherefore the square of the number A B, together with the square of the number A C is equal to two squares of the number A C, and to one square of the number C B, and also to the superficial number produced of the multiplication of the numbers B C and C A, the one into the other twice. And forasmuch as the superficial number produced of the multiplication of the numbers B A and C A, the one into the other once, is equal to the superficial number produced of the multiplication of B C into C A once, and to the square of the number C A (by the third Proposition of this Book): therefore the number produced of the multiplication of B A into A C twice, is equal to the number produced of the multiplication of B C into C A twice, and also to two squares of the number C A. Add the square number of B C common to them both. Wherefore two squares of the number A C, and one square of the number C B, together with the superficial number produced of the multiplication of B C into C A twice, are equal to the superficial number produced of the multiplication of the number B A into the number A C twice together, with the square of the number C B. Wherefore the square of the number A B, together with the square of the number A C is equal to the superficial number produced of the multiplication of the number B A into the number A C twice, together with the square of the number C B. If therefore a number be divided into two numbers, &c. Which was required to be demonstrated.

8		64	the square of the whole A B.
8		25	the square of the part A C.
64	the square of the whole	89	
5		80	the superficial number.
5		9	the square of the other part B C
25	the square of the part	89	

8   8
      5
40   40

80 the ſuperficial number produced of the multiplication of the whole into the part twice.

3
3
9 the ſquare of the other part.

PROPOSITION VIII.

If a number be divided into two numbers, the ſuperficial number produced of the multiplication of the whole into one of the parts four times, together with the ſquare of the other parts, is equal to the ſquare of the number compoſed of the whole number and the foreſaid part.

A......6 C..2 B..2 D

Demonſtration Suppoſe that the number A B be divided into two numbers A C and C B. Then I ſay, that the ſuperficial number produced of the multiplication of the number A B into the number C B four times, together with the ſquare of the number A C, is equal to the ſquare of the number compoſed of the numbers A B and C B. For unto the number B C let the number B D be equal. Now foraſmuch as the ſquare of the number A D is equal to the ſquares of the numbers A B and B D, and to the ſuperficial number produced of the multiplication of the numbers A B and B D, the one into the other twice, (by the 4th. Prop. of this Book.) And the number B D is equal to the number B C: therefore the ſquare of the number A D is equal to the ſquares of the numbers A B and B C, and to the ſuperficial number produced of the multiplication of the numbers A B and B C, the one into the other twice. But the ſquares of the numbers A B and B C are equal unto the ſuperficial number produced of the multiplication of the numbers A B and B C, the one into the other twice, and to the ſquare of A C (by the eighth Propoſition.) Wherefore the ſquare of the number A D is equal to the ſuperficial number produced of the multiplication of the numbers A B and B C, the one into the other four times, and to the ſquare of the number A C. But the ſquare of the number A D is the ſquare of the number compoſed of the numbers A B and B C: for the number B D is equal to the number B C. Wherefore the ſquare of the number compoſed of the numbers A B and B C, is equal to the ſuperficial number produced of the multiplication of the numbers A B and B C, the one into the other four times, and to the ſquare of the number A C. If therefore a number be divided into two numbers, &c. Which was to be demonſtrated.

8   8   8   8
2   2   2   2
16   16   16   16

64 the ſuperficial number produced of the multiplication of the numbers A B and B C, the one into the other four times.

6
6
36 the ſquare of A C.

10
10
100 the ſquare of the number compoſed of A B and B C.

64 the ſuperficial nũber produced of the multipl. made 4 times.
36 the ſquare number of A C.
100

PROPOSITION IX.

If a number be divided into two equal numbers, and again be divided into two unequal parts: the ſquare numbers of the unequal numbers, are double to the ſquare which is made of the multiplication of the half number into it ſelf, together with the ſquare which is made of the number ſet between them.

A.....5 C...3 D..2 B

Demonſtration For let the number A B being an even number be divided into two equal numbers A C and C B: and into two unequal numbers A D and D B. Then I ſay that the ſquare numbers of A D and D B are double to the ſquares which are made of the multiplication of the numbers A C and C D into themſelves. For, foraſmuch as the number A B is an even number, and is divided alſo into two equal numbers A C and C B, and afterward into two unequal numbers A D and D B: therefore the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other, together with the ſquare of the number D C, is equal to the ſquare of the number A C, (by the fifth propoſition.) Wherefore the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other twice, together with two ſquares of the number C D, is double to the ſquare of the number A C. Foraſmuch as alſo the number A B is divided into two equal numbers A C and C B, therefore the ſquare number of A B is quadruple to the ſquare number produced of the multiplication of the number A C into it ſelf (by the 4th. Propoſition) Moreover foraſmuch as the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other twice together, with two ſquares of the number D C, is double to the ſquare number of C A: and foraſmuch

as

as there are two numbers, of which the one is quadruple to one and the ſelf-ſame number, and the other is double to the ſame number: therefore that number which is quadruple, ſhall be double to that number which is double. Wherefore the ſquare of the number A B is double to the number produced of the multiplication of the numbers A D and D B, the one into the other twice together, with the two ſquares of the number D C. Wherefore the number which is produced of the multiplication of the numbers A D and D B, the one into the other twice, is leſſe than half of the ſquare of the number A B, by the two ſquares of the numbers D C. And foraſmuch as the number produced of the multiplication of the numbers A D and D B, the one into the other twice, together with the number compoſed of the ſquares of the numbers A D and D B is (by the fourth Propoſition) equal to the ſquare of the number A B: therefore the number compoſed of the ſquares of the numbers A D and D B is greater than the half of the ſquare number of A B, by the two ſquares of the number D C. And the ſquare of the number A B is quadruple to the ſquare of the number A C. Wherefore the number compoſed of the ſquares of the numbers A D and D B is greater than the double of the ſquare of the number A C by two ſquares of the number D C. Wherefore the ſaid number is double to the ſquares of the numbers A C and C D: If therefore a number be divided, &c. Which was to be demonſtrated.

8
8
64 the ſquare of the unequal part A D.

2
2
4 the ſquare of the unequal part B D.

5
5
25 the ſquare of the half A C.

3
3
9 the ſquare of C D, namely, of the number ſet between.

25
9
34 the ſquares of the half, and of the number ſet between.

64
4
68 the ſquares of the unequal parts.

68 | 2
34 |

PROPOSITION X.

If an even number be divided into two equal numbers, and unto it be added any other number: the ſquare number of the whole number compoſed of the number and of that which is added, and the ſquare number of the number added: theſe two ſquare numbers (I ſay,) added together, are double to theſe ſquare numbers; namely, to the ſquare of the half number, and to the ſquare of the number compoſed of the half number, and of the number added.

A ... 3 C ... 3 B .. 2 D

Demonſtration. Suppoſe that the number A B being an even number, be divided into two equal numbers A C and C B: and unto it let be added an other number B D. Then I ſay, that the ſquare numbers of the numbers A D and D B, are double to the ſquare numbers of A C and C D. For, foraſmuch as the number A D is divided into the numbers A B and B D: therefore the ſquare numbers of the numbers A D and D B are equal to the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other twice, together with the ſquare of the number A B, (by the ſeventh Propoſition.) But the ſquare of the number A B is equal to four ſquares of either of the numbers A C or C B (for A C is equal to the number C B): wherefore alſo the ſquares of the numbers A D and D B are equal to the ſuperficial number produced of the multiplication of the numbers A D and D B, the one into the other twice, and to four ſquares of the number B C or C A. And foraſmuch as the ſuperficial number produced of the multiplication of the numbers A D and D B the one into the other, together with the ſquare of the number C B, is equal to ſquare of the number C D, (by the ſixth Propoſition): therefore the number produced of the multiplication of the numbers A D and D B, the one into the other twice together, with two ſquares of the number C B, is equal to two ſquares of the number C D. Wherefore the ſquares of the numbers A D and D B are equal to two ſquares of the number C D, and to two ſquares of the numbers A C. Wherefore they are double to the ſquares of the numbers A C and C D. And the ſquare of the number A D is the ſquare of the whole and of the number added: And the ſquare of D B is the ſquare of the number added: the ſquare alſo of the number C D is the ſquare of the number compoſed of the half and of the number added: If therefore an even number be divided, &c. Which was required to be proved.

8
8
64 the ſquare of A D.

2
2
4 the ſquare of D B

5
5
25 the ſquare of C D, namely, of the number compoſed of the half and of the number added.

3
3
9 the ſquare of the half A C.

25
9
34

64
4
68

68 | 2.
34 |

The End of the ten firſt Propoſitions according to Barlaam.

THE THIRD ELEMENT OF EUCLIDE.

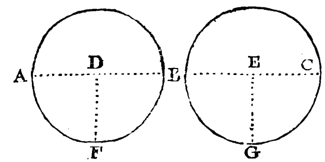
THE ARGUMENT.

This Third Book of *EUCLIDE* treateth of the circle, which of all figures is the most perfect, the properties and passions whereof are herein set forth and demonstrated. Here also is declared the nature and property of lines applyed to circles, either touching or cutting one another, either within or without the circle, as also of right lined angles drawn in the segments of circles, &c. With diverse useful Propositions applicable to diverse necessary and profitable uses.

DEFINITIONS.

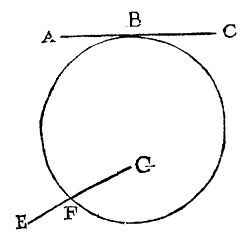
1 *Equal Circles are such whose diameters are equal, or whose right lines drawn from the centers to the circumferences, are equal.*

Forasmuch as *EUCLIDE* demonstrateth in this Third Book divers properties of Circles, he unfoldeth in the first place certain terms whose use is frequent therein: he shewes therefore first of all, that those circles are equal whose diameters or semidiameters are equal; for seeing the circle is described by the revolution of the semidiameter about one



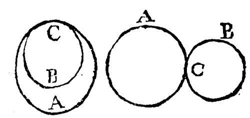
one of its extremities fixed and unmoveable, as is shewn in the First Book: It is manifest that those circles are equal, whose semidiameters or right lines, drawn from the centers to the circumferences are equal: as if the diameters A B and B C are equal, or the right lines D F and E G drawn from the centers D and E, the circles A F B and B G C shall be equal; and contrarily, if the circles be equal, the diameters or right lines drawn from the centers to the circumferences, shall be equal.

Whence it appeares, that the circles (whose diameters or right lines drawn from the centers to the circumferences are unequal,) are unequal; and therefore that circle that hath the greater Diameter, is also the greatest circle; and contrarily of unequal circles, the diameters are unequal.

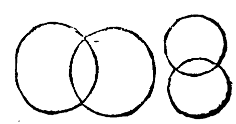


2 *A right line is said to touch a circle, which touching it, if it be prolonged, doth not cut it.*

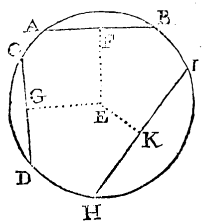
As A B shall be said to touch the circle, if it so touch it at B, as that being prolonged to C, doth not cut it; But forasmuch as E F doth so touch touch the circle in the point F, that being prolonged to the point G, it cutteth it, and falls within, it shall not be said to touch the circle, but to cut it.



3 *Those circles are said to touch one another, which in touching one another, do not intersect or cut one another.*

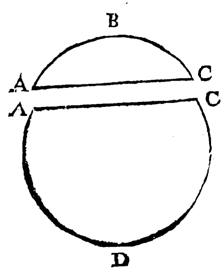


As the two circles A C and B C are said to touch at C, if they touch in such sort as that they cut not one another, be the one within the other or without it: But if two circles touch one another otherwise, they shall be said to cut one another.



4 *In a circle right lines are ſaid to be equally diſtant from the center, when the perpendiculars which are drawn on them from the center, are equal; but that is ſaid to be fartheſt diſtant from the center, on which the greateſt perpendicular doth fall.*

AS the two right lines A B and C D in the circle A B C D ſhall be ſaid to be equally diſtant from the center, if the perpendiculars E F and E G be equal: But the line C D ſhall be ſaid to be more diſtant from the center than the line H I, if the perpendicular E G be greater than the perpendicular E K.



5 *A Segment, or Section of a circle is a figure compriſed under a right line, and the circumference of the circle.*

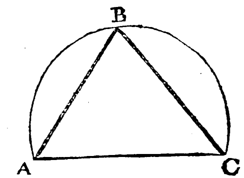
IF in the circle A B C D, let there be drawn a right line by chance as A C, as well the figure A B C conteined under the right line A C, and the circumference A D C ſhall be ſaid to be a Segment, or Section of a circle.

From whence may be gathered, that there are three ſorts of ſegments; to wit, the ſemicircle, when the right line which with the circumference, conſtituteth the ſegment, doth paſſe through the center: The ſegment greater than the ſemicircle, when the center of the circle is within it, as A D C: and the leaſt ſegment is when the figure of the ſaid ſegment is leſſe than the ſemicircle, as A B C. The right line A C ſhall be alſo called a Chord, and the circumference A B C or A D C an Arch.

6 *The angle of the ſegment, or of the ſection, is that which is compriſed under a right line, and the circumference of a circle.*

EUCLIDE now defineth three kinds of angles which are conſidered in circles. In the firſt place the angle of the ſegment; as is the mixt angle B A C or B C A, conteined under the right line A C, and the circumference A B C, and if the ſegment be a ſemicircle, it ſhall be called the angle of the Semicircle, if the ſegment be greater, it ſhall be termed

termed the angle of the greater ſegment: if leſſe than the ſemicircle, it ſhall be termed the angle of the leſſer ſegment.

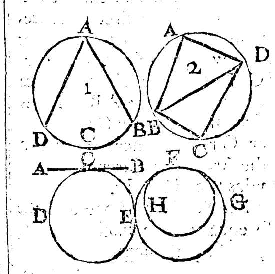


7 *But an angle is in a ſegment or in a ſection, when there is taken ſome point in the circumference of the ſegment, and from it are drawn two right lines, on the extremities of the right line, which is the baſe of the ſegment, and is that (I ſay,) which is conteined under thoſe drawn right lines.*

LEt A B C be a ſegment of a circle, and let the right line A C be the baſe, and from any point as B taken in the circumference, draw the two right lines A B and B C, to the points A and C, which are the extremities of the baſe A C, the right lined angle A B C conteined of them, is ſaid to be an angle in a ſegment.

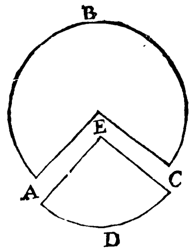
8 *But when the right lines which do contein the angle, do take or receive ſome circumference, the angle is ſaid to inſiſt upon it.*

AS the angle D A B conteined of the two right lines A B and A D, which do receive the circumference B C D, ſhall be ſaid to be, or to inſiſt thereon, and differeth not from the afore-going Definition, but only in the term; For if you draw a right line from B to D for the baſe, it ſhall be ſaid to be in the ſegment, according to the precedent Definition, and by this, to reſt or inſiſt on the circumference B C D: Neverthelesse, it refers to divers things, for the angle in the ſegment refers to the ſegment in which it is; and the angle inſiſting on the circumference refers it ſelf to the circumference which is its baſe: Therefore if you take ſome ſegment of a circle, as B C D, of the circle A B C D, the angle which is in this ſegment, ſhall not be the ſame with that which inſiſteth or reſteth upon it, for the angle which is therein ſhall be B C D: But the angle which inſiſteth or reſteth on the ſame circumference, ſhall be the angle B A D which is different from it.



Beſides, theſe three ſorts of angles aforeſaid, the Geometricians do conſider alſo an angle of contingence, which is conteined of a right line, touching the circle, and of the circumference of a circle

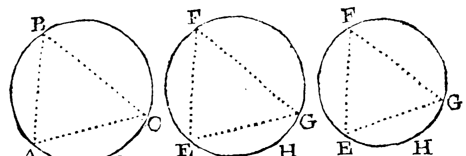
circle, or of two circumferences; touching one another either without or within, as for example, if the right line A B touch the circle C E D in the point C, the mixt angle A C D or B C E shall be termed the angle of contingence: In like manner, the curvilined angle G F H or E F H, made of the circumferences of two circles touching one another, shall be also termed the angle of contingence.



9 *A Sector of a circle, is a figure conteined under two right lines, which constitute an angle at the center, and the circumference comprised between the same lines.*

As the figure A E C D, conteined under the right lines A E and E C, which make the angle at the center E, and under the circumference A D C, comprised between A and C, the extremities of the same lines, is termed a Sector of a circle.

10 *Like Segments or Sections of Circles, are those which receive equal angles, or in which the angles are equal to one another.*



As if in the circles A B C D and E F G H, the right lined angles B and F in the Sections A B C and E F G, are equal, those Sections which do receive those said equal angles, or in which those equal angles are, shall be said to be like Sections, and the circumferences A B C and E F G in like manner shall be alike, that is to say, that the Section A B C shall be such part of the whole circle A B C D, as the Section E F G is of the circle E F G H, and the circumference A B C shall be the same part of the whole circumference A B C D, as the circumference E F G is of the whole circumference E F G H.

By the same reason, the Arches or Circumferences on which equal angles insist, are also alike, as are the Arches or Circumferences A D C and E H G, on which the equal angles B and F insist.

From these things may be gathered that like Sections of one and the same circle, or of equal circles, are also equal to one another, being the same parts of one and the same thing, or of equal things.

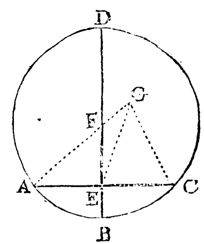
P R O-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. PROBLEM 1.

A circle A B C D *being given, to find the center* F.



Construction Draw any line therein at adventure, as A C, [a] which may be divided in two equal parts in the point E, and by E draw B D at right angles, terminating on both sides of the circumference in the points B and D, and let it be divided into two equal parts in the point F: I say, that F is the center of the proposed Circle. a) 10. 1.

Demonstration For no other point therein can be the center, considering that every other point doth divide it unequally: Therefore if F be not the center of the circle, let G be the center thereof without the line B D, from which draw the right lines G A, G E, and G C.

Forasmuch as, in the triangle A E G, the two sides A E and E G are equal to the two sides C E and E G of the triangle C E G, and the base A G equal to the base C G, (for that they are said to be drawn from the center,) [b] the angles A E G and C E G shall be equal; and therefore right angles. But the angle A E F is a right angle by construction; therefore the right angles A E F and A E G shall be equal, the part to the whole, which is absurd: Therefore G is not the center of the circle; nor by the same reason, is any other point out of the line D B: Therefore F shall be the center: Therefore we have, &c. Which was to be done. b) 8. 1.

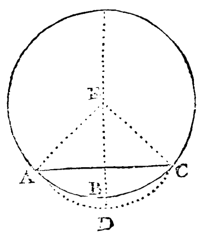
COROLLARIE.

From tis Proposition it is evident that if in a circle a right line be divided into two equal parts, and at right angles, by another right line, the center of the circle shall be in that dividing line, provided it end and terminate on both sides of the circumference, or at least, that it be not lesse than the semidiameter of the circle, or that the divided line doth passe by the center, considering that it is demonstrated

that

that the center of the circle ABCD, *cannot be other-where then in the point* F *of the line* BD, *which divideth* AC *into two equal parts, and at right angles.*

PROP. 2. THEOR. 1.



If in the circumference of a circle ABC, *there be taken two points at pleasure* A *and* C, *the right line* AC *joyning those points* A *and* C, *will fall within the circle* ABC.

Demonstration. For if it fall not within the circle, let it (if possible,) fall without such as is the right line ADC, (as the contradictor will have it,) and having [a] found the center thereof E, draw the right lines EA, EC, and ED, to some point as at D, of the line ADC, in such sort as that it cut the circumference of the circle ABC in the point B.

a) 1. 3.

Forasmuch as the two sides EA and EC of the triangle EAC, to which the line ADC is put for base, are equal, (being drawn from the center,) [b] the angles EAD and ECD shall be equal. But the exterior [c] angle EDA is greater than its interior and opposite angle ECD: seeing that the side CD is prolonged to the point A. Therefore the same angle EDA [d] shall be also greater than the angle EAD, which is equal to ECD: Therefore EA opposite to the greatest angle EDA, shall be greater than ED, opposite to the least EAD: Therefore EB equal to EA should be also greater than ED, the part than the whole, which is absurd: Therefore the right line drawn from A to B, will not fall without the circle, but within it: In like manner, we might shew that it will not fall on the circumference: Therefore, If in a circle, &c. Which was to be demonstrated.

b) 5. 1.
c) 16. 1.
d) 19. 1.

COROLLARIE.

From the Demonstration of this second Proposition it is manifest, that the right line which toucheth the circle, in such sort as that it divide not the said circle, toucheth it only in one point, for if it touch it in two points, [e] *the part of the line between those points would fall within the circle; and therefore would divide it, which is contrary to Supposition.*

e) 2. 3.

PROP. 3. THEOR. 2.

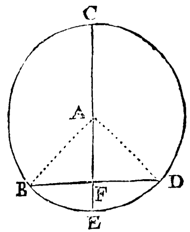
If in a circle BCDE, *some right line* EC *passing by the center* A, *doth cut some other right line* BD, *which doth not passe by the center* A, *into two equal parts in* F, *it will also cut it at right angles, and if it divide it at right angles, it will also divide it into two equal parts.*

Demonstration. For having drawn the two lines AB and AD, the two sides AF and FB of the triangle AFB, are equal to the two sides AF and FD of the triangle AFD, and the bases AB and AD are

are equal [a]: Therefore the angles AFB and AFD shall be equal; that [b] is to say right angles, which was first proposed.

a) 8. 1.
b) 10. def.

Now let CE be at right angles to BD: I say that BD is divided in two equal parts in the point F, by the right line CE; for the two lines AB and AD, being equal, [c] the two angles ABD and ADB shall be equal on the base BD, of the Isosceles triangle ABD: Forasmuch therefore as the two angles AFB and ABF of the triangle ABF, are equal to the two angles AFD and ADF of the triangle ADF, and that the sides AB and AD opposite to the right angles in the point F, are in like manner equal: [d] the sides FD and FB shall be equal, which was in the second place proposed: Therefore, If in a Circle, &c. Which was to be demonstrated.



c) 5. 2.
d) 26. 1.

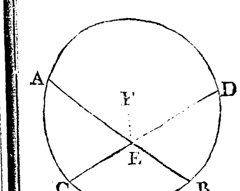
This second part may be thus demonstrated: Let AF be at right angles to BD, the [e] square of AB shall be equal to the two squares of AF and FB; in like manner, the square of AD shall be equal to the two squares of AF and FD: but the squares of AB and AD are equal, seeing the same lines are equal: Therefore the squares of AF and FB shall be equal to the squares of AF and FD: Wherefore taking away the common square AF, the squares of FB and FD will remain equal, and therefore their sides FD and FB shall be equal.

e) 47. 1.

COROLLARIE

Hence it follows that in every Isosceles or Equilateral triangle, the line which shall divide the base into two equal parts, shall be perpendicular to the base: provided it be drawn from the angle opposite to the base on the base, otherwise the conclusion would be nothing: For a line may divide another line into two equal parts, and yet neverthelesse, not be at right angles to the divided line; and contrarily, the line perpendicular to the base drawn from the opposite angle, will divide it into two equal parts: For seeing that, in the triangle BAD, AF *doth divide* BD *into two equal parts, it hath been demonstrated that the angles in the point* F *are right angles, and for that the angles in the point* F *are right angles, it hath been shewn that* AF *doth divide* BD *in two equal parts.*

PROP. 4. THEOR. 3.



If in a circle ABCD, *be drawn two right lines* AB *and* CD, *dividing one another in* E, *being not drawn by the center* F, *they will not divide one another into two equal parts.*

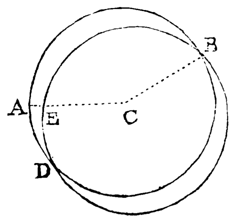
Demonstration. For if one of them passe by the center, it is manifest, that it shall not be divided in two equal parts,

parts, for it can only be divided in two equal parts in the center, b which center the other line is propoſed not to paſſe, but if neither the on nor the other paſſe by the center, although that the one of them ma ſometimes be divided into two equal parts by the other, the other never theleſſe ſhall not be divided into two equal parts.

a) 1. 3. For let them (if poſſible) divide one another in two equal parts, in ſuc ſort as A E may be equal to E B, and C E to E D: and [a] having foun the center F of the ſaid Circle, from the ſaid center draw F E to th point E, where they interſect.

b) 3. 3. Foraſmuch as F E paſſing by the center divides A B, [b] not paſſing by th center into two equal parts in E, it will divide it at right angles: There fore the angle F E B ſhall be a right angle, by the ſame reaſon it ſhall alſ divide C D at right angles: Therefore the angle F E D ſhall be alſo right angle, and therefore equal to the angle F E B, the part to th whole, which is impoſſible: Therefore they ſhall not divide one anothe into equal parts: Therefore, If in the circle, &c. Which was to b demonſtrated.

PROP. 5. THEOR. 4.



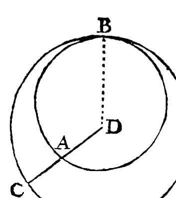
If two circles A B D *and* E B D *do divide one another in* B *and* D, *they ſhall not have the ſame center.*

Demonſtration. OTherwiſe, if they have one and the ſame center: Let C be the center of both (if it may be) and joyn C B to the point of Section, and draw C E A, dividing the circumferences in the points E and A.

Foraſmuch as C is the center of A B D, the line C A ſhall be equal to C B. Again, foraſmuch as C is the cente of the circle B E D, the line C E ſhall be equal to C B: But C A hath

a) 1. c. ſ. been ſhewn equal to C B: therefore [a] C A and C E ſhall be equal, the part and the whole, which is impoſſible: Therefore C is not the center of the two circles: Therefore, If two circles, &c. Which was to be demonſtrated.

PROP. 6. THEOR. 5.



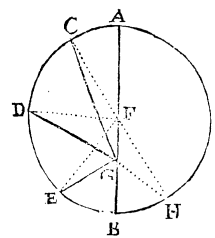
If two circles A B *and* C B, *do touch one another within* B, *they will not have one and the ſame center* D.

Demonſtration. FOr (if it be poſſible, let D be the center of both, and joyn to it the right line D B (at pleaſure) the line D A C dividing the circumferences in the points A and C.

Foraſmuch as D is the center of the circle B C, the line D C ſhall be equal

equal to D B. Again, foraſmuch as D is alſo the center of the circle A B, the line D A ſhall be equal to D B, But D C is demonſtrated alſo to be equal to D B: therefore [a] D A and D C ſhall be equal, the part to the whole, which is impoſſible: Therefore the point D is not the center of both the circles A B and C B: Therefore, If two circles, &c. Which was to be demonſtrated. a) 1. c. ſ.

PROP. 7. THEOR. 6.



If in the diameter A B *of the circle* A C D E B, *ſome point* G *be taken, which is not the center of the circle, and from this point* G *to the circumference* A C D E B, *there fall right lines* G C, G D, G E, *the greateſt line ſhall be* G A, *that line wherein the center* F *is; but the leaſt ſhall be* G B, *that which remaines (to wit, of the diameter) of the others* G C *and* G A, *alwayes the neereſt* G C, *to that* G A *which paſſeth by the center* F, *is greater than* G D, *that which is more remote; and from this ſame point* G *there will only fall two equal right lines* G E *and* G H, *to the circumference on both ſides of the leaſt line* G B.

Demonſtration. DRaw from the center F to the points C, D, and E, the right lines F C, F D, and F E, foraſmuch as the two ſides G F and F C of the triangle G F C, are greater than the other G C: But G F and F C [a] are greater than the other ſide G C, but G F and F C are equal to G F and F A; that is to ſay, to the whole G A, the ſame G A ſhall be alſo greater than G C, and by the ſame reaſon greater than G D, and ſo of the others: Therefore G A ſhall be the greateſt of all thoſe which fall from the point G to the circumference. a) 20. 1.

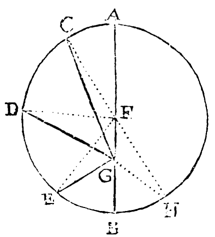
Secondly, Foraſmuch as in the triangle E F G, the ſide E F is [b] leſſe than the two ſides E G and G F: but E F is equal to F B, therefore F B ſhall be leſſe than the two ſides F G and G E: Therefore the common part F G being taken away, there will yet remain G B, leſſe than G E, and by the ſame reaſon G B ſhall be ſhewn to be leſſe than any of the others G D and G C: Therefore G B is the leaſt of all thoſe which are drawn from G to the circumference. b) 20. 1.

Thirdly, Foraſmuch as the two ſides G F and F C of the triangle G F C, are equal to the two ſides G F and F D of the triangle G F D, and the angle G F C is greater than the angle G F D, the whole than the part, [c] the baſe G C ſhall be greater than the baſe G D; by the ſame reaſon c) 24. 1.

reason G C shall be greater than G E; therefore the line which is neerest to that which passeth by the center, is greater than that which is more remote.

Lastly, make the angle B F H on the other side of the line F B, equal to the angle B F E, and draw the right line G H: forasmuch as the two sides E F and F G of the triangle E F G are equal to the two sides H F and F G of the triangle H F G, each to his correspondent side, and that the angles E F G and H F G conteined of those sides are equal by the construction, d the right lines G E and G H, on both sides of the lesser line G B, shall be equal to one another. Now that no other line can be equal to either of them is manifest; for if from the point G you draw another line on that side whereon H is, it will fall below H, and therefore is lesse than G H, as being more remote from that which passeth by the center, as hath been demonstrated: and therefore also is lesse than G E, equal to G H: If it passe above G H, it shall be greater by the same reason: Therefore there cannot be drawn two right lines equal to one another, but only on each side of the least line G B: Therefore, If in the diameter, &c. Which was to be demonstrated.

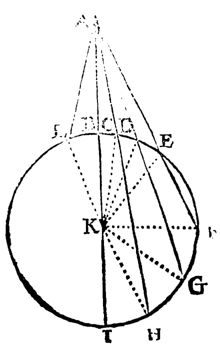
d) 4. 1.



COROLLARIE.

It follows that from every point taken in the Circle, there can be drawn only two right lines to the circumference equal to one another: forasmuch as that from every point taken in a circle, there can be drawn but one diameter.

PROP. 8. THEOR. 7.



If without the circle B C D E *there be taken some point* A, *and from that point there be drawn right lines to the circle, one of which* A I *passeth by the center* K, *and the others* A H, A G, A F *at pleasure; of all the right lines which do fall on the concave circumference, the greatest* A I, *is that which passeth by the center* K, *but of the others alwayes the neerest to that which passeth by the center, shall be greater than that which is more remote: But of those which fall on the convex circumference, the least is that which is put between the point* A *and the diameter* B I, *and of the others, that which is the nearest to the least* A B, *is alwayes lesse than that which is more remote, and from this point* A *can be drawn only two right lines equal to one another, to the circle on both sides of the least line.*

Demonstration 1. FOr from the center K draw the right lines K C, K D, K E, K F, K G, K H: forasmuch as the two sides A K and K H of the triangle A K H a are greater than A H. But A K and K H are equal to A K and K I, that is to say, to the whole A K I, the right line A I shall be also greater than A H, by the same reason, A I shall be also greater than each of the others A G and A F: Therefore A I is the greatest of all those which are drawn from the point A into the circle. a) 20. 1.

Secondly, forasmuch as the sides A K and K H of the triangle A K H, are equal to the two sides A K and K G of the triangle A K G, and that the whole angle A K H is greater than the angle A K G its part, b the base A H shall be greater than the base A G, by the same reason, A H shall be greater than A F: In like manner, A G shall be greater than A F: Therefore the line next to that which passeth by the center is greater than that which is more remote from the center. b) 24. 1.

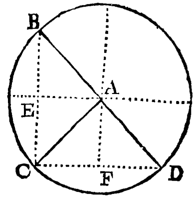
Thirdly, forasmuch as in the triangle A C K the right line A K c is lesse than the two others A C and C K: if the equal lines B K and C K be cut off, there will yet remain A B lesse than A C: by like reason, A B shall be lesse than A D, and lesse also than A E: Wherefore A B is the least of all the lines drawn from the point A without the circle, to the convex circumference. c) 20. 1.

Fourthly, forasmuch as in the triangle A D K there do fall the two right lines A C and C K, comming from the extremities of the side A K, d the same lines A C and C K shall be lesse than A D and D K: Therefore having taken away the equal lines C K and D K, there will yet remaine A C, lesse than A D, and in like manner, A C shall be lesse than A E; and A D also lesse than A E: Therefore the line neerest to the least line A B, is lesse than that which is more remote. d) 21. 1.

In the last place; Make the angle A K L equal to the angle A K D, and draw A L: Forasmuch as the two sides A K and K D of the triangle A K D, are equal to the two sides A K and K L of the triangle A K L: But the angles A K D and A K L conteined of those sides are equal; e the right lines A D and A L on both sides of the least line A B shall be equal to one another, and no other line drawn from the point A to the said circumference, shall be equal to any of them, for it would fall above A L towards B, and therefore lesse than A L; that is to say, than A D, equal to A L; or on the other side of A L, and therefore greater, as hath been demonstrated: Therefore there can be only drawn two equal right lines to the circumference on both sides of the least line A B: Therefore, If you take &c. Which was to be demonstrated. e) 4. 1.

PROP.

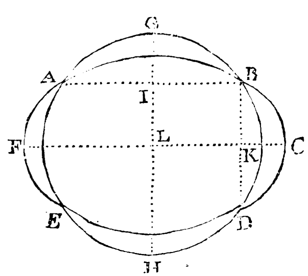
PROP. 9. THEOR. 8.



If within a circle B C D, *there be taken a certain point* A, *and from that point* A *there do fall more than two equal right lines* A B, A C, *and* A D, *to the circumference: the point taken* A, *is the center of the circle.*

Demonstration. JOyn the right light lines B C and C D, which lines shall be divided in two equal parts in the points E and F, and by the point A draw the right lines A E and A F, and prolong them on both sides to the circumference: Forasmuch then as E B is equal to E C, and E A common, the two sides A E and E B of the triangle A E B shall be equal to the two sides A E and E C of the triangle A E C, and the base
a) 8. 1. A B is equal to the base A C, by supposition; a the angle A E B shall be
b) 10.def. equal to the angle AEC; b and therefore are both right angles, by the same reason it shall be shewn that the angles at the point F are right angles: Therefore seeing that the right lines A E and A F do divide the right lines B C and C D in two equal parts, and at right angles; in each of the same dividing lines A E and A F, shall be found the center of the circle, being prolonged to the circumference: Therefore the point A in which
c)Cor.1. 3. they intersect shall be the center of the c circle, it being not to be found any other-where than in the common point where they intersect and divide one another, it being both in the one and the other: Therefore, If within a circle, &c. Which was to be demonstrated.

PROP. 10. THEOR. 9.

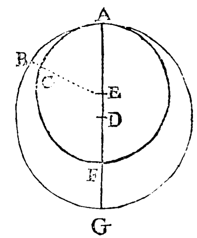


A circle A B C D E F, *doth not divide another circle* A G B D H E *in more than two points.*

Demonstration. LEt the circle A B C D E F divide the circle A G B D H E in more than two points, A, B, and D, (if possible,) and having joyned the right lines A B and B D, divide them in two equal
a) 11. 1. parts in the points I and K, and by those points a draw I L and K L at right angles to the lines A B and B D, and let them be prolonged on both sides to the circumference.

Forasmuch as the right lines I L and K L do divide the right lines A B and B D in two equal parts, and at right angles in the circle A G B D H E,
b) Cor.1.3. both the one and the other shall passe b by the center thereof: Therefore the point L by which point they intersect, shall be the center of the said circle: In the same manner, we might demonstrate that the point L is the center of the circle A B C D E F. Therefore two circles cutting one
c) 5. 3. another shall have one center, c which is impossible, &c. Therefore &c. Which was to be demonstrated.

PROP. 11. THEOR. 10.



If two circles A B G *and* A C F *do touch one another within, (at the point* A,*) and that their centers* E *and* D *be taken, the right line* D E *joyning those centers* E *and* D *being prolonged, will fall on the point of touching* A, *of those circles.*

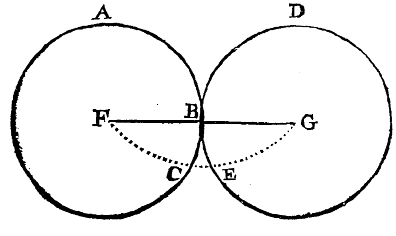
Demonstration. OTherwise it shall fall on one side or the other of the point of touching: Let it then fall (if possible) elsewhere, as at B, cutting the circumferences in the points of C and B, and let B E D be directly prolonged, (which is said to be a right line,) from the other side of the circumference in the point G, and so B E D G shall be the diameter of the circle A B G, and from the point E to the point of touching A, draw E A; forasmuch then as in the diameter G E B of the circle A B G, there is taken the point E, without the center D, the line E B shall be the
a) 7. 3. least a of all those that are drawn from the point E, to the circumference of the circle A B G; and therefore lesse than E A, but E C is equal to E A: Seeing that E C and E A are drawn from the center E of the circle A C F to the circumference A C: Therefore E B shall be also lesse than E C, the whole than its part: Therefore D E being prolonged, can fall no other-where then in the point of touching A, as is the line D E A: Therefore, If two circles, &c. Which was to be demonstrated.

PROP. 12. THEOR. 11.

If two circles A B C *and* D B E *touch one another without in* B, *the right line* F G *drawn from one center* F, *to the other* G, *shall passe by the point of touching* B.

Demonstration. FOr draw the right line F B, and the right line G B, both to the line of touching B, those two lines do make one only right line: Therefore the right line drawn from F to G, doth passe by B: For if it be not so, they will make an angle on one side or the other of B: Let them then make an angle (if possible,) towards G E, and draw from F to G a right line F C E G dividing the circumferences in the points C

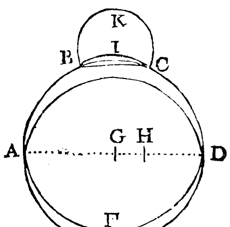
and E: Forasmuch then as the two sides B F and B G of the triangle B F G: (for F B and G B are proposed to make an angle in the point B, and F C E G is a right line, (as the contradictor will have



a) 10. 1. it,) are [a] greater than the other side F C E B:
b) 15. def. 1. But F C is equal to F B, and G E [b] equal to G B, being drawn from the centers; therefore F C & G E, which are no other then a part of the whole F C E G, shall be also greater than the same F C E G, the part than the whole, (seeing that F C E G conteineth yet C E; besides E C and G E,) which is impossible; Therefore if two circles, &c. Which was to be demonstrated.

PROP. 13. THEOR. 12.

A circle A B C D, *doth not touch a circle* A D F, *in more than one point, whether it touch it without or within.*



Demonstration. Let the two circles A B C D and A D F, touch one another within, (if it be possible,) in more than one point, to wit, in the points A and D: and let there be taken the centers of those
a) 6. 3. circles G and H, [a] which shall be different.
b) 11. 3. Therefore the line drawn by the [b] centers G and H prolonged both wayes, shall fall on the points of touching A and D, as A G H D: and forasmuch as G is the center of the circle A B C D, the line A G shall be equal to G D; therefore A G shall be greater than H D, being equal to G D the greatest; therefore A H shall be yet much greater than H D: Again, forasmuch as H is the center of the circle A F D, the line A H shall be equal to H D; but it hath been demonstrated to be much greater (which is impossible:) Therefore a circle toucheth not another circle in more than one point within.

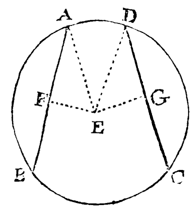
I say also that it toucheth it not without: For (if it be possible,) Let the circle B C K touch the circle A B C D without, in more than one point, to wit, in B and C, and that the right line B C be drawn, it shall fall within one of the circles, and therefore without the other circle, which is contrary to the same second Proposition of this Book: Therefore, a circle toucheth not, &c. Which was to be demonstrated.

PROP. 14. THEOR. 13.

In a circle A B C D, *the equal right lines* A B *and* C D, *are*

are equally distant from the center E, *and those that are equally distant from the center are equal to one another.*

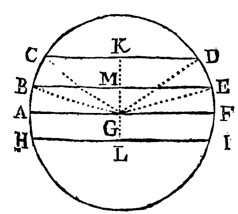
Demonstration. For from the center E draw E F and E G perpendicular to A B and C D, and joyn E A and E D: the lines E F and E G shall divide [a] A B and C D in two equal parts, which being put equal, a) 3. 3.
their halves A F and D G shall be equal.



Demonstration. Forasmuch as the squares of the equal lines E A and E D are equal: and [b] that the square of b) 47. 1.
E A is equal to the squares of A F and F E: In like manner, the square of E D is equal to the squares of D G and G E, the squares of A F and F E shall be likewise equal to the squares of D G and G E; therefore having taken away the equal squares of the equal right lines A F and D G, the square of F E and G E will remain equal: Therefore E F and E G shall be equal: Therefore [c] A B and C D shall be equally distant from the center E. c) 4. def. 3.

Again, Let A B and C B be equally distant from the center E: I say that they are equal to one another; for (the construction made as before,) we shall shew that E F and E G [d] are equal, and that they divide [e] A B d) 4. def. 3.
and C D in two equal parts, and the squares of the equal right lines E A e) 3. 3.
and E D are equal to one another: But the square of A E is [f] equal to the f) 47. 1.
two squares of A F and F E, and that of E D also equal to the two squares of D G and G E: therefore the squares of A F and F E are equal to the squares of D G and G E; having then taken away the equal squares of the equal lines E F and E G: there will remain the squares of A F and D G equal, and therefore their doubles A B and C D are also equal: Therefore, in a circle, &c. Which was to be demonstrated.

PROP. 15. THEOR. 14.



In a circle A B C D, *the greatest line is the diameter* A F, *but of the others* C D, *and* H I, *alwayes that* H I *which is neerest to the center* G, *is greater than* C D, *that which is more remote.*

Demonstration 1. For draw from the center G the perpendiculars G K and G L to C D and H I: and forasmuch as C D is more remote from the center than H I, the perpendicular G K shall be greater [a] than G L: Therefore let G M be cut off from the line G K equal to G L, a) 4. def. 3.
and by M draw B M E, perpendicular to G K, and joyn the right lines G B, G C, G D, and G E.

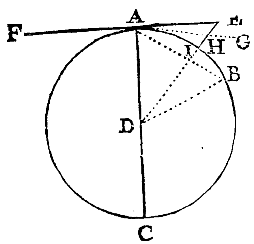
Forasmuch as the perpendiculars G M and G L are equal, B E and H I shall be equally distant from the center, and therefore equal to one another; Again, forasmuch as G B and G E are b) greater than B E, but are equal to the diameter A F, the diameter A F shall be also greater than B E; by the same reason it shall be shewn that A F is the greatest of all the other lines.

b) 20. 1.

Lastly, forasmuch as the sides G B and G E of the triangles B G E are equal to the sides G C and G D of the triangle C G D; and the angle B G E is greater than the angle C G D, c) the base D E shall be greater than the base C D. And therefore H I which hath been shewn to be equal to B E, shall be in like manner greater than C D: Therefore, In a circle, &c. Which was to be demonstrated.

c) 24. 1.

PROP. 16. THEOR. 15.



The right line E F *drawn from the extremitie* A*, of the diameter* A C *of the circle* A B C*, at right angles to the same diameter, shall fall without the circle* A B C*, in the space comprised between the same right line* E F*, and the circumference* A B C*, and there shall not fall another right line* G A*, and the angle of the semicircle* D A B *is greater than any acute right lined angle: But the rest* E A H *is lesse.*

Demonstration IF it be said that E F doth not fall without the circle: Let it (if possible,) fall within, as the line A B, and draw D B; a) the angles D A B and D B A shall be equal; but D A B is a right angle: therefore D B A shall be also a right angle, which is absurd; b) seeing that the two angles of a triangle are lesse than two right angles: In like manner we shall shew that it will not also fall on the circumference; therefore it shall fall without at A E.

a) 5. 1.
b) 17. 1.

I say now, that between A E and the circumference B A, there cannot fall another right line drawn from the point A: For let A G fall there (if possible,) and so then the angle D A G shall be acute, and on A G draw D H perpendicular to A G, cutting the circumference in I, D H shall only fall on the side of the acute angle D A G. Forasmuch c) as in the triangle D A H d) the two angles D H A and D A H are lesse than two right angles, and D H A is a right angle, by the construction, D A H shall be lesse than a right angle; and therefore D A, that is to say, D I equal thereto; shall be e) greater than D H, the part than the whole, which is impossible: Therefore, not any right line shall fall between the circumference A B, and the right line perpendicular A E, but will cut the circle.

c) Co. 17. 1.
d) 17. 1.
e) 19. 1.

I say, Lastly, that the angle of the semicircle conteined of the diameter A C, and the circumfereece B A is greater than any right lined acute angle;

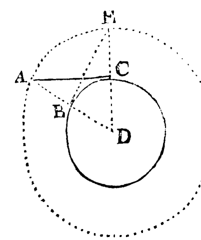
angle: for seeing it hath been shewn that every right line drawn from the point A, below the perdendicular A E, doth fall within the circle, it will of necessity make with the line A C a right lined acute angle, lesse than the angle of the semicircle: But with A E it will constitute a right lined acute angle, greater than the angle of contingence, considering that the angle of contingence is a part of the angle of the semicircle; but this acute angle is some whole, in respect of the angle of contingence, which is manifest, the line A B being drawn at adventure, below A E: for seeing that the same A B doth fall within the circle, as hath been demonstrated; the right lined acute angle C A B is lesse than the angle of the semicircle, conteined under the diameter A C, and the circumference A B C: Seeing that the acute angle is part of the angle of the semicircle: But the angle of contingence conteined under the tangent A E, and the circumference A B C is lesse than the right lined angle B A E, seeing that it is a part thereof: Therefore the angle of the semicircle is greater than any right lined acute angle, and the remainder the angle of contingence lesse: Therefore, the line, &c. Which was to be demonstrated.

COROLLARIE.

It is manifest from hence, that the right line drawn from the extremity of the diameter at right angles, doth touch the circle; for it hath been demonstrated that it falls without the circle: Wherefore it reacheth the circle in this point (the extream of the diameter) only.

As if from the point A *in the circumference of the circle* A B C*, there were to be drawn a line which should touch the circle in the point* A*, we shall draw a line from* A *to the center* D*, and to that draw another,* A E *at right angles, and that line shall touch the circle in the point* A*, as hath been demonstrated.*

PROP. 17. PROBL. 2.



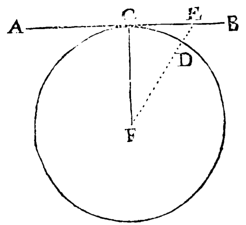
From a given point A*, to draw a right line* A C*, which toucheth a given circle* B C.

Construction FInd the center D, and joyne the line A D, which divideth the circle in the point B, and on the center D, with the distance D A, describe the circle A E, and from the point B draw B E at right angles to A D, dividing the circle A E in the point E; and lastly, draw D E, dividing the circle B C in the point C, and joyn C A: I say, that A C doth touch the given circle in the point C.

Demonstration FOr seeing that the two sides B D and D E of the triangle B D E are equal to the two sides C D and D A of the triangle C D A, each to his correspondent, as appears by the Definition of a circle, and the angle D conteined of those sides is common, the a) bases B E and C A, and the angles D B E and D C A shall be equal. a) 4. 1.
But D B E is a right angle by Construction: Therefore D C A shall be also a right angle: Therefore seeing that C A is drawn from the extremity

b) Co.16.3. C, of the semidiameter D C at right angles, it will b touch the circle: Therefore from the given point A, &c. Which was to be done.

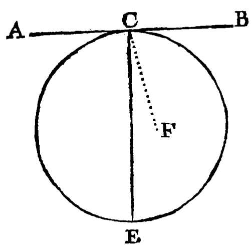
PROP. 18. THEOR. 16.



If some right line A B, *doth touch a circle* C D, *and from the center* F *to the point of touching* C, *there be joyned some right line* F C, *that joyned line* F C *shall be perpendicular to the touch line* A B.

Demonstration FOr if it be denyed, Draw F E perpendicular to A B, cutting the circumference in the point D:
a) 17. 1. Forasmuch as in the triangle C F E, a the two angles F C E and E F C are lesse than two right angles, and E F C is a right angle, by construction,
b) 18. 1. F C E shall be lesse than a right angle: Therefore b C F, that is to say, F D its equal, shall be greater than F E the part than the whole, which is impossible: Therefore F C is perpendicular to A B: Therefore if a right line, &c. Which was to be demonstrated.

PROP. 19. THEOR. 17.



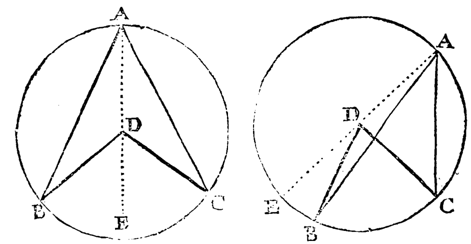
If some right line A B, *doth touch a circle* C E, *and from the point of touching* C, *there be drawn a right line* C E *at right angles to the line touching in that same line* C E, *shall be the center of the circle.*

Demonstration FOr if it be said that the center is not in the line C E, but without; Let it (if possible,) be in the point F, from
a) 18. 1. which point F draw the right line F C, a which shall be perpendicular to A B. Therefore the right angle F C B shall be equal to the right angle E C B, the part to the whole, which is impossible: Therefore the center of the circle shall not be out of the line C E: Therefore, If a right, &c. Which was to be demonstrated.

PROP. 20. THEOR. 18.

In a circle A B C, *the angle* B D C, *which is at the center* D,

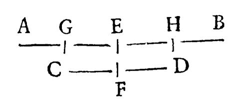
D, *is double to the angle* B A C, *which is at the circumference, when they have for their base one and the same circumference* B C.



Demonstration JOyn A D together, and prolong it to E; forasmuch as the
two lines D A and D B are equal, D A D and D B A shall
a be equal: But the exterior angle B D E is equal b to the two interior an- a) 5. 1.
gles D A B and D B A; and therefore double to each of them, as of b) 32. 1.
D A B: In like manner, we shall shew that C D E is double to the angle
D A C: Wherefore the whole angle B D C shall be double to the whole
angle B A C which was proposed: Again, let the angle C A B not enclose
the angle C D B, and joyn A D, prolonged to E; forasmuch as the two
sides D A and D C are equal, c the angles D A C and D C A shall be c) 5. 1.
equal, but the exterior angle C D E is equal d to them: Therefore it d) 32. 1.
shall be double to D A C, but the angle E D B at the center, is double to
E A B, as is demonstrated; Therefore the remainder B D C shall be
double to the remainder B A C; e for when the whole is double to the e) 10. 1.
whole, and the part cut off to the part cut off, the remainder shall be
double to the remainder, as followeth.

LEMMA.

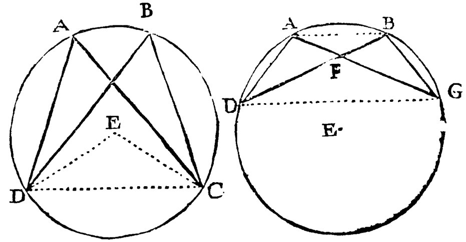
Let the whole A B *be double to the whole* C D, *and* A E *the part cut off, double to* C F *the part cut off, the rest* E B *shall be double to the rest* F D; *for seeing that* A B *is double to* C D, *being divided in two equal parts at* E, *as well* A E *as* E B, *shall be equal to* C D: *Therefore* A E *being also double to* C F, C D *equal to* A E, *shall be also double to* C F, *and therefore double to its other half* F D: *Therefore the rest* E B *equal to* C D, *shall be also double to the rest* F D: *Which was to be demonstrated.*



PROP. 21. THEOR. 19.

In a circle A B C D, *the angles* D A C *and* C B D, *which are in one and the same segment* C B A D, *are equal to one another.*

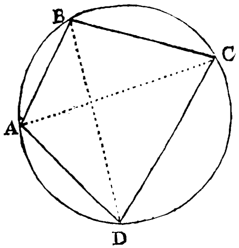
De-



Demonstration FOr let E be the center, to which joyn the lines E D and E C: Forasmuch as the angle D E C is constituted in the center, it [a] shall be double to D A C, the angle at the circumference: for they have the same circumference D C for their base, by the same reason, it shall be double to the angle C B D: Therefore each of the two angles A and B, shall be the half of D E C, and therefore equal to one another: Therefore, In a circle, &c. Which was to be demonstrated.

a) 10. 3.

PROP. 22. THEOR. 20.



The opposite angles B A D *and* B C D *of quadrilateral figures* A C, *inscribed in a circle* A B C D, *are equal to two right angles.*

Demonstration FOr, joyn the two diagonal lines A C and B D; forasmuch as the two angles A C D and A B D, are in one and the same segment A B C D, they shall be equal to one another, by the same reason, the two angles B C A and B D A, in the segment B C D A, shall be also equal to one another; therefore the whole angle B C D shall be equal to the two angles A B D and A D B; therefore having added the common angle B A D, the two angles B C D and B A D shall be equal to the three angles B D A, D B A, and B A D, of the triangle A B D, but the three angles of the triangle A B D [a] are equal to two right angles; therefore the two opposite angles B A D and B C D shall be also equal to two right angles: In like manner, it might be shewn that the two opposite angles at the points D and B, are equal to two right angles: Therefore the angles, &c. Which was to be demonstrated.

a) 32. 1.

PROP. 23. THEOR. 21.

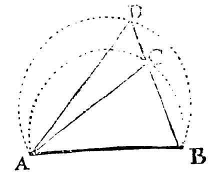
On one and the same right line A B, *there cannot be constituted two segments of circles* A D B *and* A C B, *alike and unequal, and on the same side.*

De-

Demonstration FOr if it be possible, Let two segments of circles be constituted alike and unequal: On the line A B, and on the same part or side A C B and A D B, those segments will only cut one another in the points A and B; Seeing [a] that a circle cuts not another circle in more than two points: Therefore the circumference of one of the segments shall be without the circumference of the other. Draw then the right line B C D, cutting the circumference in the points C and D, and joyn A D and A C: Forasmuch as the segments A C B and A D B are put alike segments by the tenth Definition of this Book, the angle A C B shall be equal to the angle A D B, which is absurd, [b] for the exterior angle A C B is greater than the interior A D B, they are not then like segments: Therefore on one and the same, &c. Which was to be demonstrated.

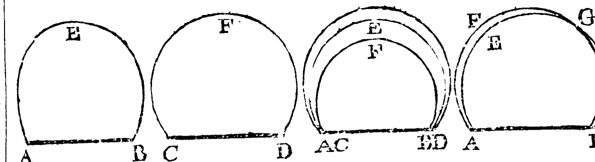
a) 10. 3.

b) 16. 1.



PROP. 24. THEOR. 22.

Like segments of circles A E B *and* C F D, *constituted on equal right lines* A B *and* C D *are equal to one another.*



Demonstration FOr the lines A B and C D being equal, they will agree to one another, if it be understood that they be placed the one on the other, to wit, A B on C D, and the segment A E B will agree with the segment C F D: otherwise the segment A E B shall fall without the segment C F D, or else within it, or a part without, and a part within; and therefore on one and the same right line are constituted like segments of circles, [a] which is absurd: For if part fall without, and part within, they should cut one another in more than two points, as at B and G, which is impossible: For a circle doth not cut another circle in more than two points: Therefore they shall agree, and therefore shall be equal to one another: Therefore like segments, &c. Which was to be demonstrated.

a) 23. 3.

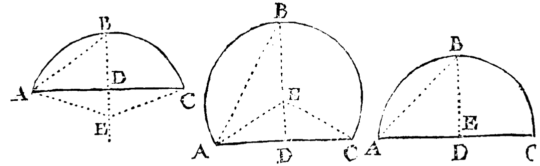
PROP. 25. PROBL. 3.

The segment A B C, *of a circle, being given, to describe the circle of which it is a segment.*

Construction JOyn A C, and divide it into two equal parts in the point D, by which draw D B at right angles to A C, and joyn A B: The angle D B A shall be greater than the angle D A B, or equal or lesse.

Demon-

Demonstration First of all, Let it be greater, (which will happen when the segment shall be lesse than the semicircle,) for then B D prolonged, will passe [a] by the center, which is without the segment; seeing
a) Co. 1. 3. that it is lesse than the semicircle, and [b] D A shall be greater than D B,
b) 7. 3. for D B is the rest of the Diameter: Therefore [c] D B A shall be greater
c) 18. 1. than D A B: Let the angle B A E be made equal to D B A, and let the right line A E cut B D prolonged, at E: I say, that E is the center of the circle, of which A B C is the segment.



For having drawn E C, the two sides A D and D E of the triangle A D E shall be equal to the two sides C D and D E of the triangle C D E, and the angles conteined of them are right angles; [d] therefore A E and
d) 4. 1. E C are equal, but E A is also equal to E B: Seeing [e] that the angles
e) 16. 1. E A B and E B A are equal: Therefore the three sides E A, E B, and
f) 9. 3. E C are equal: [f] Therefore F shall be the center of the circle A B C; seeing that from that point there falls more than two equal right lines, to the circumference.

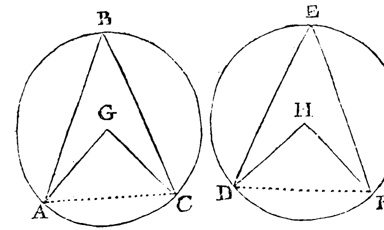
g) 6. 1. Secondly, Let the two angles D B A and D A B be equal: [g] D A shall be equal to D B: But D C is also equal to D B. Seeing that A C is divided into two equal parts in the point D, all the three lines D C, D B, and D A shall be equal; therefore D shall be the center of the circle. and the segment proposed shall be a semicircle.

Lastly, if the angle D B A be lesse than the angle D A B: Let the angle B A E be made equal to the angle A B E; and let E C be joyned: I say that E is the center of the circle: For it will be shewn by the same reasons as above, that the three lines E A, E B, and E C, are equal; and therefore also the segment proposed will exceed the semicircle: Therefore a segment, &c. Which was to be done.

PROP. 26. THEOR. 23.

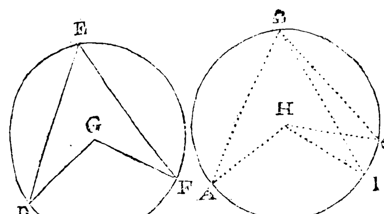
In equal circles A B C *and* D E F, *the equal angles* D H F *and* A G C, *insist on equal circumferences* A C *and* D F, *whether they be constituted at the centers* G *and* H, *or on the circumferences* A C *and* D F.

Demonstration For joyn the right lines A C and D F; forasmuch as the two sides G A and G C of the triangle A G C, are equal to the sides H D and H F of the triangle D H F; because of the equality
a) 4. 1. of the circles, and the angles G and H equal by supposition, [a] the bases A C and D F shall be equal: but the angles B and E constituted at the cir-



circumferences, are equal by supposition, and insist on the same bases: Therefore the segments A B C and D E F shall be alike [b], and there- b) 24. 3.
fore equal; which being subtracted from the equal circles, there will remaine the segments A C and D F equal; and therefore the circumferences A C and D F equal. Which was to be demonstrated.

PROP. 27. THEOR. 24.



In equal circles A B C *and* D E F, *the angles* A H C *and* D G F *that insist on the equal circumferences* A C *and* D F, *are equal to one another, whether they be constituted at the centers* H *and* G, *or else on the circumferences* A C *and* D F.

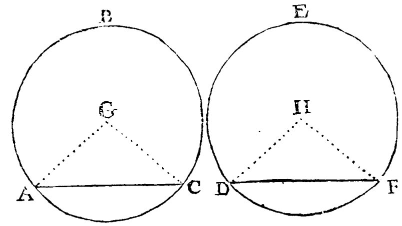
Demonstration For if they be not equal: Let the angle H be the greater, and make the angle A H I equal to the angle D H F [a]: a) 26. 3.
therefore the circumference A I shall be equal to the circumference D F: But the circumference A C is equal to the circumference D F, by supposition; therefore A I and A C shall be equal to one another, the part to the whole, which is impossible; therefore the angles A H C and D G F are equal.

Secondly, Constitute the angles B and E at the circumferences, having equal circumferences, A C and D F for bases; by the same reason it might be shewn that they are equal to one another: Which was to be demonstrated.

PROP. 28. THEOR. 25.

In equal circles A B C *and* D E F, *equal right lines* A C *and* D F *do take away equal circumferences* A B C *and* D E F, *to wit, the greatest from the greatest, and the least from the least.*

Demonstration For let the centers G and H be taken, and the right lines AG, GC, DH, and HF, be drawn; the two sides GA and GC of the triangle AGC are equal to the two sides HD and HF, of the triangle DHF, and the bases AC and DF are equal by supposition, therefore the angles a G and H



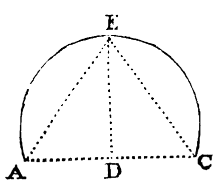
a) 8. 1. are equal: Therefore b the circumfe-
b) 26. 3. rences AC and DF, on which they insist are equal, which being taken from the equal circles, there will remain the circumferences ABC and DEF, also equal: Therefore, In circles, &c. Which was to be demonstrated.

PROP. 29. THEOR. 26.

In equal circles ABC *and* DEF, *equal circumferences* ABC *and* DEF, *subtend equal right lines* AC *and* DF.

Demonstration For, Let the centers G and H be taken, and let the right lines AG, GC, DH, and HF, be drawn: Forasmuch as the two sides AG and GC of the triangle AGC, are equal to the two
a) 27. 1. sides DH and HF of the triangle DHF; a and the angles G and H
b) 4. 1. equal; seeing they instsist on equal circumferences, b the bases which are the right lines AC and DF, are equal: Therefore in circles, &c. Which was to be demonstrated. As by the fore-going figure is evident.

PROP. 30. PROBL 4.



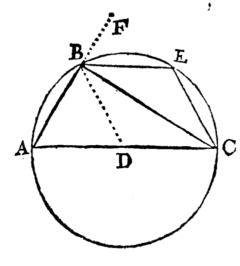
To divide a given circumference AEC, *into two equal parts.*

Construction Draw the right line AC, which divide in two equal
a) 10. 1. parts, a at the point D, by which point
b) 11. 1. draw the right line DE, at b right angles to AC, and joyn the right lines AE and CE, the circumference shall be divided in two equal parts, in the point E.

Demonstration For seeing that AD and CD are equal, and DE common, the two sides AD and DE of the triangle ADE shall be equal to the two sides CD and DE of the triangle CDE; and the angles
c) 4. 1. at the point D are equal, to wit, right angles; c the bases AE and CE
d) 28. 3. shall be equal; d and therefore the circumference AE and CE also equal: Therefore we have divided, &c. Which was to be done.

PROP.

PROP. 31. THEOR. 27.



In a circle ABC, *the angle* ABC, *which is in the semicircle is a right angle: But that which is in the greatest segment* BCA, *is lesse than a right angle, and that which is in the least segment* BEC, *is greater than a right angle; And moreover, the angle* ABC *of the greatest segment is greater than a right angle: But the angle* EBC *of the least segment is lesse than a right angle.*

Demonstration For draw BD to the center D, and prolong AB directly towards F: Forasmuch as DA and DB are equal, the angles a DAB and DBA shall be equal to one another; by the same reason, the two angles DBC and DCB shall be also equal; for DB and DC are also equal to one another: Therefore the two angles DBA and DBC, that is to say, the whole ABC, is equal to the two angles BAC and BCA: But the exterior angle CBF b is equal to the same two interior and opposite angles BAC and BCA: Therefore the two angles ABC and CBF shall be equal to one another; and therefore right angles: Therefore the angle ABC in the semicircle is a right angle, which was in the first place proposed. a) 5. 1. b) 32. 1.

I say, secondly, that the angle BAC in the greatest segment BAC, is lesse than a right angle: Forasmuch as in the triangle ABC, the two angles ABC and BAC, are lesse than two right angles, and ABC is demonstrated to be a right angle, the angle BAC shall be lesse than a right angle; which is in the second place proposed.

Thirdly, I say, that the angle BEC in the least segment BEC, is greater than a right angle: Forasmuch as in the quadrilateral figure ABEC, described in the circle, the two opposite angles c E and A are equal to two right angles, and the angles BAC is shewn to be lesse than a right angle; therefore the other angle BEC shall be greater than a right angle; which was in the third place proposed. c) 22. 3.

Fourthly, I say, that the angle of the greatest segment BAC, which is the angle ABC, conteined of the circumference AB, and of the right line BC, is greater than a right ang'e: for seeing that the right angle ABC is conteined of the right lines AB and BC, and is a part thereof, the whole angle ABC, conteined of the circumference AB, and the right line BC, shall be greater than a right angle; which was in the fourth place proposed.

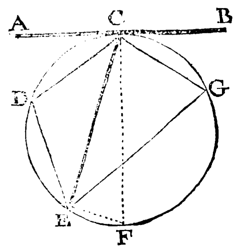
Lastly, I say, that the angle of the least segment, which is the angle CBE, conteined of the right line CB, and of the circumference BEC, is lesse than a right angle; for that it is lesse than the right angle FBC, the

the part than the whole, which was in the last place proposed, &c. Which was to be demonstrated.

COROLLARIE.

Hence it is manifest that the angle of a triangle which is equal to two others, (which is made with- is a right angle; forasmuch as the angle contiguous thereto, is equal to the same; which appeares by out the triangle, the side being prolonged,) it is the half of the three angles of a triangle, which are the first Demonstration; equivalent to two right angles.

PROP. 32. THEOR. 28.



If some right line AB *do touch a circle* CDE, *and from the point of touching* C, *there be drawn some right line* CE, *to the circle dividing it, the angles* ACE *and* BCE, *that it makes with the touch line* ACB, *shall be equal to the angles which are in the alternate segments* CDE, *and* CGFE, (*viz.* ACE *equal to* G, *and* BCE *equal to* D.

Demonstration FOr, Draw the right line CF by the center, and joyn EF, the line [a] CF shall be perpendicular to AB, and the angle CEF shall be a right angle; and therefore the two remaining angles EFC and ECF shall be equal to a right angle, as to the right angle ACF: Therefore if the common angle ECF be taken away, there will remain the angle ACE, equal to the angle EFC, which is equal to the angle G: Forasmuch as they are in one and the same segment: Therefore the angle ACE shall be also equal to the angle G.

a) 18.1.

And forasmuch as in the quadrilateral figure DEGC, inscribed in the circle, the two opposite angles D and G are equal [b] to two right angles, and the two angles ACE and BCE are also equal to two right angles; if the two equal angles ACE and CGE be taken away, there will remain the angle BCE, equal to the angle D, in the alternate segment CDE: Therefore, if a right line, &c. Which was to be demonstrated.

b) 21. 3.

PROP. 33. PROBL 5.

On a right line given AB, *to describe a segment of a circle, which may receive an angle equal to a given right lined angle* C.

First, Let the angle given C, be a right angle.

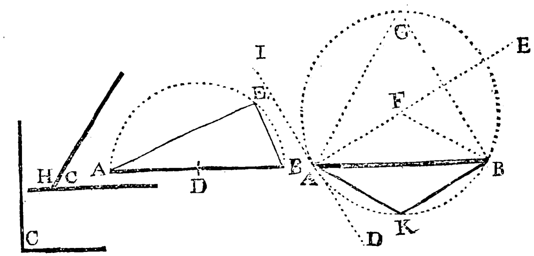
Construction DIvide the right line AB into two equal parts in the point D, and from the point D as a center, and with the distance DA,

DA, describe the semicircle, in which make the angle AEB which is a right angle; and therefore [a] equal to the given right angle C. a) 31. 3.

Secondly, Let the given angle C, be acute.

Construction THen from the point A, of the given right line AB, make the angle DAB equal to the acute angle C, and from the point A draw AE at right angles to AD, which shall fall above AB; then make the angle ABF equal to the angle FAB, and let BF cut the right line AE in the point F, [b] the lines FA and FB shall be equal. Therefore if from the center F, with the distance FA, the circle AG be described, it will passe by the point B: I say then that the angle which shall be made in the segment AGB, described on the line AB, shall be equal to the angle C. Therefore let the angle AGB, be made in the said segment. b) 6. 1.

Demonstration FOrasmuch as AE doth passe by the center, and that from the extremity A, there is drawn AD at right angles thereto, DA will [c] touch the circle in the point A: Wherefore the angle DAB is to say, the given angle C shall be equal to the angle G, in the alternate segment AGB. c) C. 16. 3.



Thirdly, Let the given angle H be obtuse.

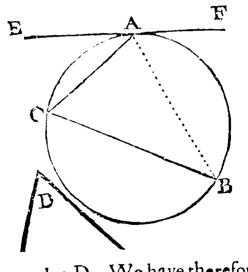
Construction MAke again the angle IAB equal to the angle H, and from the point A, draw AE, at right angles to AI, which will fall above AB: finish the rest as is before taught, and so there shall be described on the line AB the segment AKB, in which the angle K is equal to the obtuse angle given H.

Demonstration FOr the angle IAB, that is to say, the given angle H, is equal to the angle K, in the alternate segment AKB, for that it is the same Demonstration: Therefore, On a given, &c. Which was to be done.

PROP. 34. PROBL. 6.

From a given circle ABC, *to cut off a segment, which may receive an angle, equal to a given right lined angle* D.

Con-



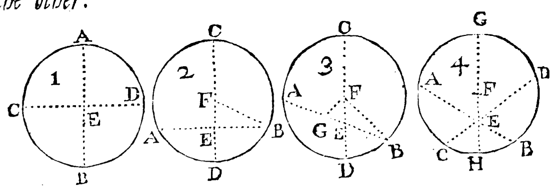
Conſtruction DRaw the right line E F, touching the circle in the point A, then make the angle F A B equal to the given angle D, and draw the right lines A C and B C: I ſay, that the angle A C B in the ſegment A C B is equal to the given angle D.

Demonſtration THe angle F A B, which is made equal to the given angle D, is [a] equal to the angle C, in the alternate ſegment A C B: Therefore the ſame angle C, ſhall be in like manner equal to D: We have therefore cut off from the given circle, the ſegment A C B, &c. Which was to be done.

a) 32. 3.

PROP. 35. THEOR. 29.

If in a circle A B C, *two right lines* A B *and* C D, *do divide one another (in the point* E,*) the Rectangle conteined under the two parts of the one* A E *and* E B, *is equal to the Rectangle conteined under the two parts* C E *and* E D, *of the other.*



Demonſtration LEt the point of interſection E, in the firſt place be the center of the circle, as in the firſt figure, the four ſegments ſhall be equal, each of them being the ſemidiameter of the circle: Therefore it is manifeſt that the Rectangle conteined under the two parts of the one, is equal to the Rectangle conteined under the two parts of the other.

Secondly, Let C D only paſſe by the center, dividing A B into two equal parts in the point E, [a] and therefore at right angles, and joyn the right line B F: Foraſmuch as C D is divided into two equal parts in the center F, and unequally in the point E. The Rectangle conteined under the unequal ſegments C E aud E D, [b] with the ſquare of the mean Section F E, is equal to the ſquare of the halfe F D; that is to ſay, of F B: but the ſquare of F B [c] is equal to the two ſquares of F E and E B: therefore the Rectangle conteined under C E and E D, with the ſquare of F E, ſhall be alſo equal to the two ſquares of F E and E B; therefore if the common ſquare of F E be taken away, there will re-

a) 3. 3.
b) 5. 1.
c) 47. 1.

remain the Rectangle conteined under C E and E D, equal to the ſquare of E B; that is to ſay, to the Rectangle under A E and E B: For ſeeing that they are equal, the Rectangle conteined under the two is a ſquare, according to the Firſt Definition of the ſecond Book. (See the Second Figure.)

Thirdly, Let C D paſſing by the center F, divide A B in two parts unequally, in the point E, and let the ſame A B be divided in two equal parts in the point G, and let G F and F B be drawn; [d] F G ſhall be perpendicular to A B. Foraſmuch then as C D is divided into two equal parts at F, and unequally at E, the Rectangle conteined under C E and E D, with the ſquare of the mean Section F E, ſhall be equal to the ſquare of the half F D; that is to ſay, to the ſquare of F B its equal: But the ſquare of F B [e] is equal to the two ſquares F G and G B: Therefore the Rectangle conteined under C E and E D, with the ſquare of F E, is [f] alſo equal to the two ſquares of F G and G B, and the ſquare of F E is equal alſo to the two ſquares of F G and G E. Therefore the Rectangle under C E and E D, with the two ſquares of F G and G E, is equal to the two ſquares of F G and G B, taking away therefore the common ſquare of F G, there will remain the Rectangle conteined under C E and E D, with the ſquare of G E, equal to the ſquare of G B; but the Rectangle conteined under A E and E B, with the ſquare of G E, is alſo equal to the ſquare of G B: Seeing that A B is divided into two equal parts in the point G; and unequally in the point E. Therefore the Rectangle conteined under C E and E D, with the ſquare of G E, is equal to the Rectangle under A E and E B, with the ſame ſquare of G E, taking away therefore the common ſquare of G E, there will remain the Rectangle conteined under C E and E D, equal to the Rectangle conteined under A E and E B; which is propoſed. (See the third figure.)

d) 3. 3.
e) 47. 1.
f) 47. 1.

Fourthly, Let neither A B nor C D, (cutting one another in the point E,) paſſe by the center, and draw the diameter G H by the point E, (as in the fourth figure,) it ſhall be ſhewn as before, that the Rectangle conteined under A E and E B, is equal to the Rectangle under G E and E H, and the Rectangle under C E and E D, alſo equal to the Rectangle under G E and E H; and therefore equal to one another; whether the one of them A B and C D be divided in two equal parts or not: Therefore, If in a circle, &c. Which was to be demonſtrated.

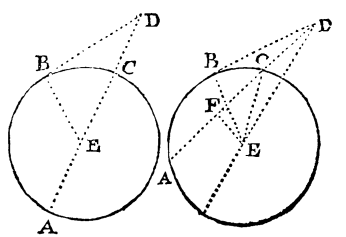
PROP. 36. THEOR. 30.

If there be taken ſome point D, *without a circle* A B C, *and from that point there fall two right lines to the circle, one of which* D A, *divides the circle, and the other* D B, *toucheth it; the Rectangle conteined under the whole line dividing (to wit,* A D*) and its part without* D C, *taken between the point* D, *and the convex circumference* C *is equal to the ſquare of the touching line* D B.

Demon-

Demonstration. IN the first place, Let D A passe by the center E, and then joyn the right line E B, [a] which shall be perpendicu-
a) 18. 3. lar to D B. Forasmuch as C A is divided into two equal parts in E, and
b) 6. 2. to it is added D C, the Rectangle conteined under A D and D C, [b] with the square of E C, that is to say, of E B, its equal, is equal to the square
c) 47. 1. of E D: But the square of E D [c] is equal to the two squares of D B and B E: Therefore the Rectangle conteined under A D and D C, with the square of B E shall be also equal to the two squares of D B and B E: Therefore taking away the common square of E B, there will remain the Rectangle under A D and D C, equal to the square of the touching line D B; which is proposed.

Secondly, Let the dividing line D A not passe by the center E, and divide A C into two equal parts in the point F, and joyn the right lines E B, E F, E C, and E D, the line E B shall be perpendicular to D B,
d) 6. 2. and E F to A C: Forasmuch then as A C [d] is divided into two equal parts in the point F, and to it is added D C, the Rectangle conteined under A D and D C, with the square of F C, is equal to the square of F D: Therefore adding the common square of E F, the Rectangle conteined under A D and D C, with the two squares of C F and F E, is equal to the two squares of D F and F E: But the square of E C, that is to say, of
e) 47. 1. E B its equal; [e] is equal to the two squares of C F and F E; therefore the Rectangle under A D and D C, with the square of E B, is equal to the two squares of D F and F E, and the square of D E is also equal to the two squares of D F



and F E; therefore the Rectangle under A D and D C, with the square
f) 47. 1. of B E, shall be equal to the square of D E, which is equal [f] to the two squares of D B and B E: Therefore the Rectangle under A D and D C, with the square of B E, is equal to the two squares of D B and B E, taking away therefore the common square of B E, there will remain the be Rectangle under A D and D C, equal to the square D B. Which was to demonstrated.

COROLLARIE I.

From this Proposition it is manifest that if from any point taken without the circle, there be drawn divers right lines, dividing the circle, the Rectangles comprised under each of the whole lines, and its exterior part, are equal to one another, for if from the point A *(in the first figure,) there be drawn* A C, A D,
g) 36. 3. *and* A E, *dividing the circle and* A B, *touching it, the Rectangle under* A C *and* A F, [g] *shall be equal to the square of* A B: *in like manner, the Rectangle under* A D *and* A G, *and the Rectangle under* A E *and* A H; *and therefore equal to one another.*

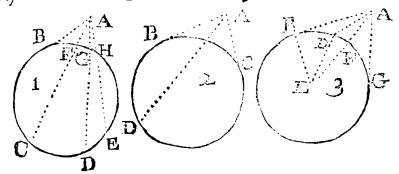
COROLLARIE II.

It is also manifest, that two right lines drawn from one and the same point, touching

the circle, are equal to one another, as are the two lines A B *and* A C, *which touch the circle in the points* B *and* C *(in the second figure;) I say they are equal to one another, for draw* A D, *dividing the circle in the point* E, *as well the square of* A B *as the square of* A C, [h] *shall be equal to the Rectangle con-* h) 36. 3.
teined under A D *and* A E; *and therefore equal to one another; and therefore the lines* A B *and* A C *likewise equal to one another.*

COROLLARIE. III.

It is likewise manifest, that from a point taken without the circle, there can only be drawn two right lines, touching the circle; for if besides the two A B *and* C D, *there may be a third* A D *drawn,*

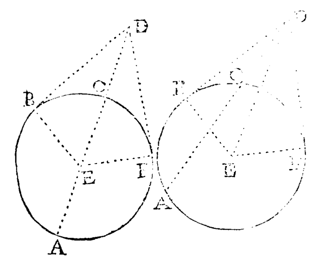


touching the same circle; having drawn the right lines E B, *and* E D, *from the center* E; [i] *the angles* A B E *and* A D E *shall be right* i) 18. 3.
angles; and therefore equal to one another, which is absurd: for if you draw the right line A E, [k] *the angle* A D E *shall be greater than the angle* A B E. k) 21. 1.

COROLLARIE IV.

It is in the last place evident, that if two equal right lines be drawn from some point without the circle to the convex circumference, and that the one of them touch the circle, the other shall also touch it, &c.

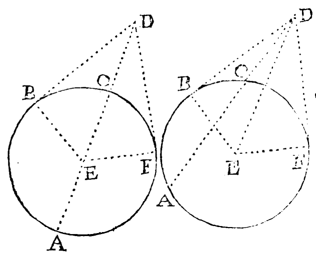
PROP. 37. THEOR. 31.



If without the circle A B C, *there be taken some point* D, *and from that point there fall to the circle two right lines* D B *and* D A, *one of which* D A, *cuts the circle, and the other reacheth the circle; and that the Rectangle contained under the whole dividing line* D A, *and its part without* D C, *taken between the point* D, *and the convex circumference* C, *be equal to the square of the line* D B, *which reacheth the circle, the same line* D B *shall touch the circle.*

a) 17. 3.
b) 18. 1.
c) 36. 3.
d) 15. def.

Demonstration. FOr, [a] Let D F be drawn, touching the circle, and let the Center E be found, and joyn E B and E F: and if D A passe not by the center E, let D E be also joyned, the angle [b] D F E shall be a right angle; and forasmuch as D F toucheth the circle, [c] the square of the same D F shall be equal to the Rectangle conteined under D A and D C: But the square of D B is equal to the same Rectangle under D A and D C, by Supposition; therefore the squares of D B and D F shall be equal to one another; and D B and D F also equal; But [d] E B and E F are equal; therefore the two sides D B and B E of the triangle D B E, are equal to the two sides D F and F E, of the triangle D F E, each to his correspondent, and the base D E common, and the angles B and F shall be equal; but D F E is a right angle; therefore D B E shall be also a right angle: Therefore by the Corollary of the sixteenth Proposition of this Book, D B shall touch the circle; which was proposed: Therefore if there be taken, &c. Which was to be demonstrated.



The End of the Third Element of EUCLIDE.

THE

THE FOURTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

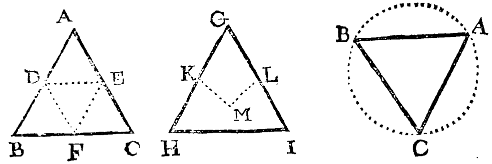
EUCLIDE treating in this Fourth Book of divers inscriptions of figures in Circles, and the descriptions of the same figures about the Circle. Also of the Inscriptions of the Circle in the same figures, and of the descriptions of the Circle about those figures, he exposeth in a few Definitions, what the meaning is, of a figure inscribed in a figure, or to be described about a figure, beginning with right lined figures.

DEFINITIONS.

I *A right lined figure is said to be inscribed in a right lined figure, when every one of the angles of the figure inscribed, doth touch each side of the figure in which it is inscribed.*

AS for Example, if each of the angles D, E, and F, of the interior triangle D E F doth touch each of the sides of the exterior triangle ABC, the triangle D E F shall be said to be inscribed in the triangle ABC.

ABC. But foraſmuch as the angle M of the triangle KLM, (a right line being drawn from K to L,) toucheth not the ſide HI of the triangle GHI, the triangle KLM ſhall not be ſaid to be inſcribed in the other, although it be all within, and that two of the angles of the one, do touch two ſides of the other.

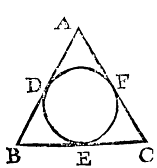


2 *In like manner, a figure is ſaid to be deſcribed about a figure, when each ſide of the figure circumſcribed, doth touch each angle of the figure about which it is deſcribed.*

Contrarily, the triangle ABC ſhall be ſaid to be deſcribed about the triangle DEF; Foraſmuch as each of the ſides of the one, doth touch each of the angles of the other. But the triangle GHI ſhall not be ſaid to be deſcribed about the triangle KLM: Seeing that the ſide HI doth not touch the angle M, and the ſame ought to be underſtood of the inſcriptions and circumſcriptions of other right lined figures, &c. and right lined figures are properly ſaid to be inſcribed or circumſcribed, when the number of the ſides are equal, and although that be not abſolutely neceſſary. (See the precedent figures.)

3 *A right lined figure is ſaid to be inſcribed in a circle, when each angle of the inſcribed figure doth touch the circumference of the circle.*

As if the angles A, B, and C, of the triangle ABC (See the precedent figure,) do touch the circumference of the circle ABC, the ſaid triangle ſhall be ſaid to be inſcribed in the circle; and if ſome one of the angles do not touch the circumference, the triangle ſhall not be ſaid to be inſcribed in the circle.



4 *But a right lined figure is ſaid to be deſcribed about a circle, when as each ſide of the figure circumſcribed doth touch the circumference of the circle.*

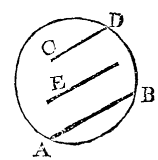
For

For Example, If the ſides of the triangle ABC, do touch the circumference of the circle DEF: The triangle ſhall be ſaid to be deſcribed about the circle.

5 *In like manner a circle is ſaid to be inſcribed in a right lined figure, when the circumference of the circle doth touch each ſide of the figure in which it is inſcribed.*

6 *But a circle is ſaid to be deſcribed about a figure, when the Circumference of the circle doth touch every angle of the figure about which it is deſcribed.*

In like manner, the Circle DEF (in the figure of the fourth Definition) ſhall be ſaid to be inſcribed in the triangle ABC. But the Circle ABC, in the figure belonging to the ſecond Definition, ſhall be ſaid to be deſcribed about the triangle ABC; and the ſame is to be underſtood of other right lined figures, which are ſaid to be inſcribed in the Circle, or deſcribed about the Circle, or in which the Circle is ſaid to be inſcribed or deſcribed about.



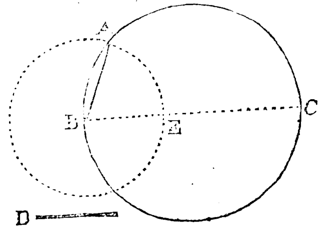
7 *A right line is ſaid to be apted or fitted in a Circle, when the extremities thereof are in the Circumference of the Circle. As appears by this figure.*

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. PROBLEM 1.



In a given circle ABC, *to fit a right line* AB, *equal to a given right line* D, *which may not be greater than the diameter* BC, *of the circle* ABC.

Construction. Draw the diameter BC; if D be equal to BC, then is BC fitted in the circle, equal to D; but if D be lesse than the
a) 3. 1. diameter BC: Let BE be cut off [a] equal to D, and on B as a center, with the distance BE describe the circle AE, cutting AB in the point A, and draw BA, which shall be the line fitted in the circle ABC, equal to D.

b) 25. def. *Demonstration.* For BA [b] is equal to BE, and D also equal to BE, by Construction: therefore BA and D shall be equal to one another, and the extremities of BA are in the circumference, being drawn from the extremity of the diameter, in the point of Section of the circles: Therefore we have, &c. Which was to be done.

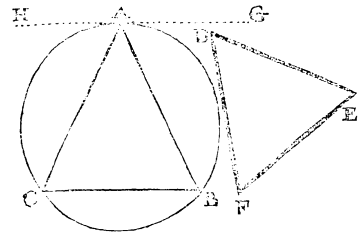
PROP. 2. PROBL. 2.

In a given circle ABC, *to inscribe a triangle equiangled to a given triangle* DFE.

Construction. Draw HG, touching the circle in the point A, and make
a) 17. 3. the angle GAB equal to the angle F, [a] and the angle HAC equal to the angle E, drawing the right lines AB and AC to the cir-

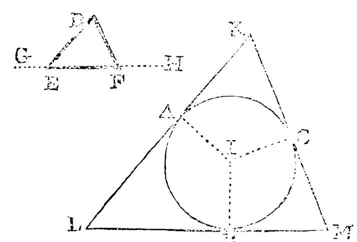
circumference, at the points B and C, and joyn the right line CB. I say, that the triangle ABC inscribed in the given circle, is equiangled to the triangle DEF.

Demonstration. For [b] seeing that HG doth touch the circle, and AB cuts b) 32. 3.
it, the angle GAB, which is made equal to the angle F, shall be equal to the angle C, in the alternate segment: therefore the angles C and F shall be equal to one another; In like manner, forasmuch as the angle HAC (which is equal to the angle E, by Constructi-



on,) is [c] equal to the angle B, in the alternate c) 32. 3.
segment, the angles B and E shall be equal: therefore seeing that the two angles B and C of the triangle ABC are equal
to the two angles E and F, of the triangle DEF, the two others [d] A and d) 32. 1.
D shall be likewise equal to one another: Therefore the triangle ABC is equiangled to the triangle DEF: Therefore, In a Circle, &c. Which was to be done.

PROP. 3. PROBL 3.



About a given circle ABC, *to describe a triangle equiangled to a given triangle* DEF.

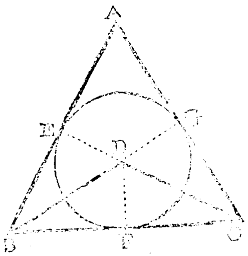
Construction. Having prolonged the side EF on both sides, to the points G and H: Let the center I be taken, and draw IA at the contingent point A, and in the point I of the same line AI, make the angle AIB equal to the angle DEG, and BIC equal to DFH, and from the extremities A, B, and C, draw at right angles the right lines KL, LM, and MK, which lines shall meet in the points K, L, and M: Forasmuch as if you joyn a right line from A to B, the two angles LAB and LBA, shall be lesse than two right angles, being no other than part of the two right angles IAL and
IBL; therefore [a] AL and LB shall meet in the point L, and so of the a) 11. def. 1.
rest: Therefore the triangle LKM is described about the circle ABC, and is equiangled to the given triangle DEF.

Demonstration. For [b] all the sides do touch the circle in the points A, B, b) C. 16. 3.
and C, and forasmuch as all the angles of the quadrilateral figure AIBL are equal to four right angles, as is shewn in the

3 Proposition of the first Book, and the two angles at the points A and
B are right angles; the two angles A I B and A L B will remain equal to
two right angles; and therefore equal to the two angles D E G and
c) 15. 1. D E F, which c are also equal to two right angles; therefore if the equal
angles A I B and D E G be taken away, there will remaine A L B equal to
D E F; even so the angle M will be shewn equal to the angle D F E:
Therefore the third angle K shall be equal to the third angle D: There-
fore the triangle K L M shall be equiangled to the triangle D E F: There-
fore, &c. Which was to be done.

PROP. 4. PROBL. 4.

In a given triangle A B C, *to inscribe a circle.*

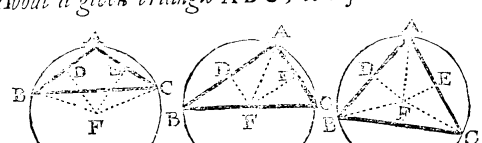


Construction DIvide a the two angles
B and C, each in two
a) 9. 1. equal parts, by the right lines B D and
C D, meeting one another in the point
D, and from that point draw the perpendiculars D G, D E, and D F, on
the three sides A B, B C, and C A.

Demonstration FOrasmuch as the two
angles D B F and D F B
of the triangle D F B, are equal to the two angles D B E and D E B of
the triangle D E B, each to its correspondent, and the side D B com-
b) 26. 1. mon; b the two sides D E and D F shall be equal, by the same reason,
the sides D F and D G shall be equal in the triangles D C F and D C G:
Therefore the three lines D G, D F, and D E, shall be equal to one ano-
ther, and the circle described on the point D as a center, at the distance
c) Co.16.3. D E, shall passe by the other points G and F, and shall c touch the sides
of the triangle in the points E, F, and G; seeing that the sides are at right
angles to the semidiameters D E, D F, D G: Therefore in a given
triangle, &c. Which was to be done.

PROP. 5. PROBL. 5.

About a given triangle A B C, *to describe a circle.*



Construction DIvide a the two sides A B and A C in two equal parts in the
a) 10. 1. points D and E, from which points draw D F and E F,
b) 11. 1. b at right angles to those sides, and meeting in the point F.

Demonstration FOr if a right line were drawn from D to E, the two
angles F D E and F E D should be lesse than two right
angles,)

angles,) and the point F shall be in the triangle, or in the side, or with-
out. Joyn then the right lines F A, F B, and F C: Forasmuch as the two
sides A D and D F of the triangle A D F, are equal to the two sides B D
and D F of the triangle B D F, and the angles at the point D right angles:
c the bases F A and F B shall be equal, by the same reason, F A and c) 4. 1.
F C shall be also equal; therefore the three sides F A, F B, and F C,
shall be equal to one another: Therefore the circle described on the point
F as a center, at the distance F A, shall also passe by the two other points
B and C: Therefore about the triangle, &c. Which was to be done.

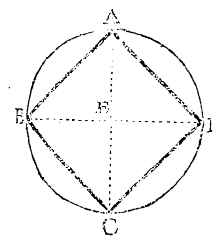
COROLLARIE.

*It is manifest from this Demonstration, that if the center fall within the trian-
gle, d that all the angles are acute, being in the greatest segment of the circle;* d) 31. 3.
but if it fall on the side, the angle e opposite to that side, shall be a right angle in e) 31. 3.
the semicircle: Lastly, if it fall without the triangle, the opposite angle f shall be f) 31. 3.
obtuse, being in the least segment of the circle.

*Contrarily, If the triangle be an Oxigon, the center will fall within it, if a Re-
ctangle, it will fall on the side which subtends the right angle, and if Ambligon,
it will fall without.*

PROP. 6. PROBL. 6.

In a given circle A B C, *to describe a square.*



Construction DRaw the two diameters
A C and B D, which
shall divide one another at right angles in
the center E, and joyn the right lines
A B, B C, C D, and D A: I say that
A B C D is the square inscribed in the
given circle.

Demonstration FOrasmuch as the two sides
E A and E B of the triangle
A E B are equal to the two sides E A and E D, of the triangle E A D,
being all drawn from the center, and the angles conteined of those sides
being right angles, a the bases A B and A D shall be equal; and in like a) 4. 1.
manner C D and D A, and D A and A B; therefore A B C D shall be
a square, having all the sides equal, and all the angles A, B, C, and D, right
angles, b each of them being in the semicircle: Therefore, In a circle, b) 31. 3.
&c. Which was to be done.

PROP. 7. PROBL. 7.

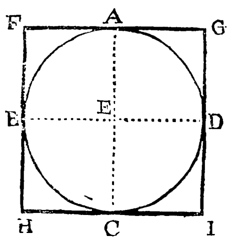
About a given circle A B C, *to describe a square.*

Construction DRaw two diameters, to divide one another at right angles
in the center E, and by the points A, B, C, and D, draw the
right lines F G, F H, H I, and I G, at right angles to the diameters, which
shall meet with one another in the points F, H, I, and G: Forasmuch
as if a right line be drawn from B to A, the two angles F B A and F A B,
shall be lesse than two right angles, and so of the others.

I say that F G I H is a square described about the given circle.

Demonstration. FOr seeing that the two angles A E B and F B E are right angles, F H and A C [a] shall be parallels, likewise G I and
a) 28. 1. A C; therefore F H and [b] G I shall be parallels, and by the same rea-
b) 30. 1. sons F G and H I shall be also parallel.

Now forasmuch as A C H F is a parallelogram, the [c] opposite sides A C and
c) 34. 1. F H shall be equal, and the opposite angles C and F equal: But E C H is a right angle; therefore A F H shall be a right angle; by the same reason, the angles H, I, and G, shall be shewn to be right angles; and the sides H I, I G, and G F, equal to the diameters A C and B D; and therefore equal to one another. Therefore F H I G shall have the four sides equal, and the four angles right angles; and therefore shall be a square, all whose sides



d) C. 16. 3. [d] do touch the circle: Therefore about a given circle, &c. Which was to be done.

PROP. 8. PROBL. 8.

In a given square F G H I, *to inscribe a circle.*

Construction. HAving divided the sides in two equal parts in the points B, C, D, and A, draw the right lines B D and C A, intersecting in the point E.

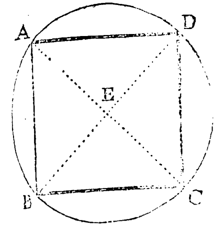
Demonstration. NOw [a] forasmuch as F G and H I are equal and parallel,
a) 34. 1. their halves F A and H C shall be equal and parallel; and therefore F H shall be equal and parallel to C A: In like manner, I G shall be equal and parallel to the same C A: Likewise F G and H I shall be equal and parallel to B D; therefore F E, E H, I E, and E G, shall be parallelograms; and therefore the lines E B, E C, E D, and E A, shall be equal to F A, B H, the equal lines G A, and F B; but those lines are equal to one another; being the halves of F G, F H, &c. Therefore E B, E C, E D, and E A, are equal; therefore the circle described from the point E as a center, and at the di-
b) C. 16. 3. stance E B: shall also passe by the points C, D, and A, and shall touch [b] all the sides, seeing that the angles in the points B, C, D, and A, are right angles, and shall be inscribed in the square F I: Therefore in a given square we have inscribed a circle: Which was to be done.

PROP. 9. PROBL. 9.

About a given square A B C D, *to describe a circle.*

Construction. DRaw the two diameters C A and B D, intersecting in the point E.

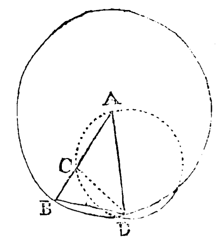
Demonstration. FOrasmuch as the sides A B and A D, of the triangle A B D are equal, [a] the angles A B D and A D B shall be
a) 5. 1. equal: But the angle B A D is a right angle in the square; therefore [b] A B D and A D B shall be the halves of right angles: In like manner, we



we shall shew that all the other angles, which are at the points A, B, C, and D, are the halves of right angles; and therefore equal to one another: Therefore seeing that the angles E A D and E D A are equal to one another, the lines E A and E D shall be equal, and by the same reason E A and E B shall be equal, and also E B and E C; likewise E C and E D: Therefore the circle described on the center E, at the distance E A, shall passe by the other points B, C, and D: Therefore about a given square, &c. Which was to be done.

PROP. 10. PROBL. 10.

To constitute an Isosceles triangle A B D, *which may have each of the angles* B *and* D, *which are at the base* B D, *double to the other* A.



Construction. ASsume any line, as A B, which divide in the point C, in such sort as the Rectangle conteined under [a] A B and B C, may be equal to a) 11. 2.
the square of A C; then on the center A, with the distance A B, describe a circle, [b] in which circle fit the right b) 1. 4.
line B D, equal to A C, and joyn A D: I say, that the triangle A B D is an Isosceles triangle, and hath each of the angles A B D and A D B double to the other angle A.

Demonstration. FOr having drawn C D, describe [c] about the triangle A C D c) 5. 4.
the circle D C A, forasmuch as the Rectangle conteined under A B and B C is equal to the square of C A, that is to say, of B D, its equal; and A B cuts the circle [d] A C D, the line B D shall touch it in d) 37. 3.
the point D: Therefore the angle B D C shall be [e] equal to the angle A in e) 32. 3.
the alternate segment C A D, adding the common angle C D A, the whole angle A D B shall be equal to the two angles C A D and C D A; but the exterior angle D C B is [f] equal to the same angles A and A D C; f) 32. 1.
therefore the angle B C D is equal to A D B; that is to say to [g] A B D, g) 5. 1.
its equal; therefore [h] C D and D B shall be equal; But B D is equal to h) 6. 1.
A C by Construction: Therefore C D shall be equal to C A, and therefore [i] the two angles C A D and C D A shall be equal: Therefore the i) 5. 1.
angle A D B, which is shewn equal to the two angles C A D and C D A, shall be double to the angle A; and therefore the angle A B D shall be also double to the same angle A: We have therefore constituted, &c. Which was to be done.

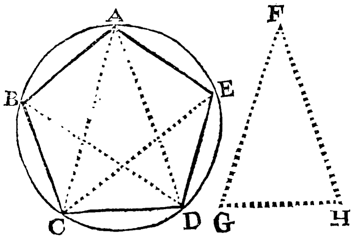
COROLLARIE.

Now forasmuch as the three angles [k] *of the triangle* A B D, *are equal to two* k) 32. 1.
right angles; that is to say, to five fifths of two right angles: It is manifest that the

the angle A *is the fifth part of two right angles, and each of the two angles* D *and* B *is the two fifth parts of two right angles: Likewise* A *is the two fifth parts of a*
l) 32. 1. *right angle, and either* B *or* D, *which of them you please is the four fifths;* [l] *forasmuch as all the three are equal to two right angles; that is to say, to ten fifths of a right angle.*

PROP. 11. PROBL. 11.

In a given circle ABCD, *to describe an equilateral and equiangled Pentagon.*



Construction Constitute the Isosceles triangle FGH; which may have each of the angles G and H double to the angle F, [a] and
a) 2. 4. in the circle ABCD, inscribe the triangle ACD equiangled to the tri-
b) 9. 1. angle FGH, [b] and divide both the angle ACD and ADC into two equal parts by the lines DB and CE, and joyn the right lines AB, BC, CD, DE, and EA: I say, that the Pentagon ABCDE inscribed in the circle is equilateral and equiangled.

Demonstration Forasmuch as each of the angles ACD and ADC, is double to the angle CAD, they being each divided into two equal parts, the five angles ADB, BDC, CAD, DCE, and ECA shall be equal; Therefore [c] the Arches AB, BC, CD, DE, and
c) 26. 1. EA, on which those angles are subtended, are equal: Therefore [d] the
d) 29. 1. right lines AB, BC, CD, DE, and EA, which subtend them are equal: Therefore the Pentagon ABCD is equilateral: Again forasmuch as the arches AB and ED are equal, if you adde the common angle BCD, the whole ABCD shall be equal to the whole EDCB: Therefore the
e) 27. 1. [e] angles AED and BAE, insisting on them shall be equal, in the same manner the other angles shall be equal, for they insist on equal arches, each of which is compounded of three equal arches: Therefore the Pentagon is equiangled, the which being also shewn to be equilateral: We have inscribed in a circle, &c. Which was to be done.

COROLLARIE.

Hence it follows that the angle of the equilateral and equiangled Pentagon, is the three fifths of two right angles, or the $\frac{6}{5}$ *of one right angle:* [f] *For seeing that*
f) 27. 3. *the three angles* BAC, CAD, *and* DAE *are equal, being they insist on the equal arches* BC, CD, *and* DE: *But* CAD *is* $\frac{1}{5}$ *part of two right angles, or* $\frac{2}{5}$ *parts of one right angle, the whole* BAE *shall be the* ^{3} *of two right angles, or six fifths of one right angle.*

PROP. 12. PROBL. 12.

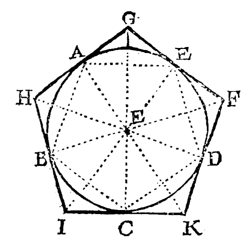
About a given circle ABC, *to describe an equilateral and equiangled Pentagon.*

Con-

Construction Inscribe [a] in the circle equilateral and equiangled Pentagon a) 11. 4.
ABCDE; and let the center E be taken, and draw the right lines FA, FB, FC, FD, and FE, by the extremities of which lines draw GH, HI, IK, KL, and LG, at right angles to them, meeting in the points G, H, I, K, and L.

For seeing that the angles GAE and GEA are lesse than two right angles, being part of the two right angles FAG and FEG, [b] the lines b) 11. c. f.
AG and EG shall meet with one another in the point G, and so of the others: And forasmuch as they [c] touch the circle, the Pentagon c) C. 16. 3.
GHIKL shall be described about the circle, which I say is equilateral and equiangled.

Demonstration For having drawn the right lines FG, FH, FI, FK, and FL, the two squares of FA and AH [d] shall be equal to d) 47. 1.
the square of FH; and in like manner, the squares of FB and BH shall be equal to the same square of FH: therefore the two squares of FA and AH shall be equal to the two squares of FB and BH, taking away then the equal squares of the equal lines FA and FB, the squares of AH and BH will remaine equal; therefore the lines AH and BH are equal: And forasmuch as the two sides AH and FH of the triangle AFH are equal to BF and FH of the triangle BFH, each to his correspondent, and the base AH equal to



the base BH, as is shewn, [e] the angle AFH shall be equal to the angle e) 8. 1.
BFH, therefore the angles [f] AHF and BHF shall be also equal: there- f) 4. 1.
fore the angle AFB is double to the angle BFH, and the angle AHB double to the angle BHF.

By the same discourse we shall shew that the angle BFC is double to the angle BFI, and the angle BIC double to the angle BIF: Seeing therefore [g] that the angles AFB and BFC are equal, as insisting on the g) 27. 3.
equal circumferences AB and BC, [h] being subtended by the equal right h) 28. 3.
lines AB and BC, their halves BFH and BFI, shall be equal.

Therefore seeing that the two angles BFH and HBF of the triangle BFH, are equal to the two angles BFI and IBF of the triangle IFB, and the side BF adjacent common, [i] the sides BH and BI shall be equal, i) 26. 1.
and the angles BHF and BIF equal: Therefore the line HI is double to the line HB.

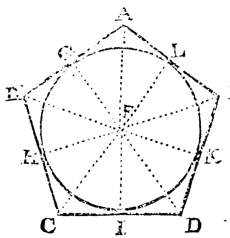
By the same reason, we shall shew that GH is double to HA; but HB and HA are shewn to be equal: Therefore their double, to wit, HI and HG shall be equal. In like manner, we shall shew, that the right lines IK, KL, and LG are equal to each of the right lines HI and HG: therefore the Pentagon GHIKL, is equilateral.

Again, forasmuch as it is shewn that the angles BHF and BIF are equal, and that they are the halves of the angles BHA and BIC; their doubles BHA and BIC shall be equal. By the same reason, the angles IKL, KLG, and LGH shall be equal to each of the angles BHA and BIC: Therefore the Pentagon GHIKL is equiangled:

There-

Therefore being shewn to be equilateral, we have described about a circle a Pentagon, equilateral and equiangled: Which was to be done.

PROP. 13. PROBL 13.



In a given Pentagon ABCDE, *which is equilateral and equiangled, to inscribe a circle.*

Construction Divide two of its angles BAE, and ABC, into two equal parts, by the right lines AF and BF, which shall meet with one another in the point F.

a) 11. c. f. *Demonstration* Seeing [a] that the two angles FAB and FBA are lesse than two right angles, being the halves of the two angles A and B, that are lesse than four right angles, as is demonstrated in the 32 Proposition of the first Book. Then draw the right lines FC, FD, and FE: Forasmuch as the two sides AB and BF of the triangle ABF, are equal to CB and BF of the triangle EBF, each to its correspondent, and ABF and
b) 4. 1. CBF conteined of them, equal by Construction, [b] the bases AF and CF, and the angles BAF and BCF are equal. And seeing that the angles A and C of the Pentagon are put equal, and BAF is the half of BAE by Construction, the angle BCF shall be the half of BCD: Therefore BCD divided into two equal parts, by the same reason, we shall shew that the two other angles D and E are divided into two equal parts: Now draw from the
c) 12. 1. point F the [c] perpendicular lines FG, FH, FI, FK, and FL, on the sides of the Pentagon: Forasmuch as the two angles FGA and FAG of the triangle FAG; are equal to the two angles FLA and FAL of the triangle FAL, and the side AF opposite to one of the equal common angles, FG and FL, shall be equal.

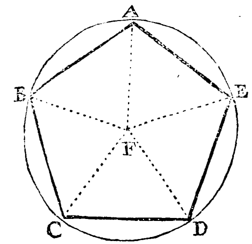
In like manner, it shall be shewn that the other perpendiculars FH, FI, and FK, are equal to each of the other FG and FL: Therefore the circle described from the center F, at the distance FG, shall likewise passe by all the points H, I, K, and L; and forasmuch as the sides of
d) C. 16. 1. o the Pentagon do [d] touch the circle, seeing that they make right angles with the semidiameters FG, FH, &c. We shall have described a circle in the given Pentagon. Which was to be done,

PROP. 14. PROBL. 14.

About a given Pentagon ABCDE, *which is equilateral and equiangled, to describe a circle.*

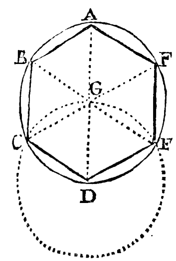
a) 9. 1. *Construction* Having divided the angles BAE and ABC, into [a] two equal parts, by the right lines AI and BF, which shall meet with one another in the point F within the Pentagon, as hath been demonstrated in the precedent Proposition, and having joyned the right lines FC, FD, and FE, we shall shew, as in the precedent Problem, that the other angles C, D, and E, are divided into two equal parts: Therefore all the half angles shall be equal to one another; seeing that the whole angles are equal.

Demon-



Demonstration Forasmuch then as in the triangle AFB, the two angles FAB and FBA, are equal, the lines [b] FA and FB shall be equal, by b) 6. 1.
the same reason, the other lines FC, FD, and FE, shall be equal to each of them: Therefore the circle described from the center F, at the distauce of FA, shall also passe by the points B, C, D, and E: Therefore, About a Pentagon, &c. Which was to be done.

PROP. 15. PROBL. 15.



In a given circle ABCDEF, *to inscribe an equilateral and equiangled Hexagon.*

Construction Having assumed the center G, and drawn the diameter AD; describe the circle CGE from the point D, which cutteth the given circle in the points C and E, from which points draw the right lines CF and EB, by the center G, and joyn the right lines AB, BC, CD, DE, EF, and FA. So shall the Hexagon ABCDEF be inscribed in the giuen circle, which I say is equilateral and equiangled.

Demonstration For by the definition of the circle, the three lines GC, CD, and DG shall be equal to one another, and the triangle CDG shall be equilateral; therefore [a] the three angles thereof a) 5. 1.
shall be equal to one another, which [b] being equal to two right angles, b) 32. 1.
which you please of the two, as CGD shall be the third part of two right angles: By the same reason, the angle DGE shall be the third part of two right lines: But the angles CGD, DGE, and EGF, are equal to two right angles: Therefore the other angle EGF shall be also the third part of two right angles: Therefore the three angles CGD,
DGE, and EGF, [c] are equal to one another, to which the angles c) 13. 1.
CGB, BGA, and AGB being [d] equal at the head, the six angles at d) 15. 1.
the center G, shall be equal to one another: Wherefore [e] the circumfe- e) 26. 3.
rences on which they insist are equal; [f] and therefore the right lines AB, f) 29. 3.
BC, CD, DE, EF, and FA, are equal. Wherefore the Hexagon ABCDEF shall be equilateral.

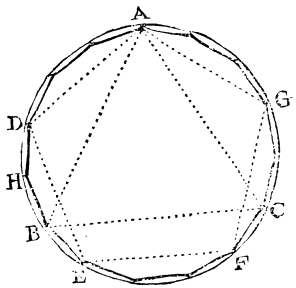
Again, forasmuch as the circumference BC is equal to AF, if you adde the common circumferences CDEF, the circumferences BCDEF
and AFEDC shall be equal: Therefore [g] the angles ABC and BAF, g) 27. 3.
which insist on them shall be equal: We shall shew in like manner, that the other angles of the Hexagon C, D, E, and F, are equal to each of the angles A and B, for that each of them insisteth as an arch compounded of four equal circumferences: Therefore the Hexagon is also equiangled: Therefore, In a given circle: Which was to be done.

CO-

COROLLARIE.

From this Proposition it is manifest, that the semidiameter of a circle is equal to the side of the Hexagon inscribed therein. For D G *the semidiameter, is equal to* D C *the side of the Hexagon, by the definition of the circle.*

PROP. 16. PROBL 16.



In a given cirtle A B C, *to inscribe an equilateral and equiangled Quindecagon.*

Construction INscribe in the circle, the equilateral triangle A B C, [a] the three arches A B, B C, and C A, shall be equal, because of the three sides, or the three equal angles A, B, and C.

a) 26, 28. 3.

Again, [b] Inscribe in the same circle an equilateral and equiangled Pentagon **A D E F G**, having one of the angles applyed at the point A, [c] the five arches A D, D E, E F, F G, and G A, shall be equal; and seeing that the circumference of the circle ought to be divided into 15 equal parts: the arch A B shall contein five of them, and the arch A D three, seeing the arch A B is the third part of the whole circumference, and A D the fifth part: Therefore the rest D B, shall contein two parts. [d] Therefore having divided the arch D B into two equal parts in the point H, B H shall be the fifteenth part of the whole circumference: Therefore drawing the right line B H, it will subtend the fifteenth part of the whole circumference: Therefore [e] if you fit in the circle fourteen other right lines, equal to B H, there will be inscribed in the circle an equilateral Quindecagon, [f] the which is also equiangled, seeing that each of the angles insist on equal arches, each of which is compounded of thirteen equal arches, as is manifest. Therefore, In a given circle we have described a Quindecagon. Which was to be done.

b) 11. 4.
c) 28. 3.
d) 30. 3.
e) 1. 4.
f) 27. 3.

The End of the Fourth Element of EUCLIDE.

THE

THE FIFTH ELEMENT OF EUCLIDE.

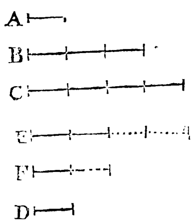
THE ARGUMENT.

IN the four precedent Books *E U-C L I D E* hath treated of continued quantity considered, absolutely: But in the two following Books he disputeth of the same quantity, not absolutely; but in as much as the one refers it self to the other, that is to say, in as much as it is compared with another, it hath some Reason. In this Book he teaches the proportions of continued quantities in general, not referring them to any kind of quantity, as to a Line, a Superficie, or to any Body. But in the Sixth Book he shewes especially what reason lines have to one another, the angles, the circumferences of Circles, Triangles, and other plain figures, and to keep his Method, he defineth first the terms needful in the Demonstration.

DEFINITIONS.

1 *A magnitude is part of another magnitude, the lesser of the greater, when as the lesser doth measure the greater.*

HE saith therefore that a lesser Magnitude which doth measure another greater Magnitude, is termed a part thereof; As for Example, forasmuch as the Magnitude A taken three times, doth measure the Magnitude B, A shall be termed a part of B, and taken four times measureth C, the Magnitude A shall be also termed a part of C. But forasmuch as D measureth neither E nor F, for taken twice, it exceeds F, and taken thrice it wanteth of E, and taken four times, it exceeds it; D shall not be termed a part of the Magnitudes E and F.



Now amongst Mathematicians, there are two sorts of parts, the one which measureth its whole, in such sort as being repeated, a certain number of times, it constituteth its whole, as 4 compared with 12, 16, 20, &c. is termed an aliquot part, the other doth not measure its whole, or being taken a certain number of times, it either exceeds or wants thereof, as 4 compared with 6, 7, 9, 18, 30, &c. is termed an aliquant part.

Therefore here *EUCLIDE* defineth only an aliquot part, as well because it measureth only its whole, (for the aliquant part is not said to measure its whole,) as for that (as in the Seventh Book,) the aliquant part in numbers is not by *EUCLIDE* called a part, but parts, for 4 is not a part of 6, but is the two third parts thereof, &c.

2 *A magnitude is multiplex of another lesser magnitude, when the greater is measured of the lesser.*

AS in the precedent Example, as well the Magnitude B, as C, is Multiplex of the Magnitude A; forasmuch as the Magnitude A measureth as well the one as the other; to wit, B and C: But neither the Magnitude E nor F, ought to be termed Multiplex of the Magnitude D; forasmuch as D measureth neither the one or the other: Therefore a part referreth it self to Multiplex, and Multiplex to a part, in such sort as a lesser quantity measuring a greater, is termed a part of the greater. But the greater, which is measured of the lesser, is termed Multiplex of the lesser.

It is then manifest enough, that the part heretofore defined, is that which exactly measures its whole, without any remainder.

Now when two lesser magnitudes do measure equally two other greater Magnitudes, that is to say, that if one of the least be conteined so many times exactly in one of the greater, as the other lesser is conteined times in the other greater, those two greater Magnitudes shall be termed equimultiplices of the two lesser, and the same is to be said, if divers lesser Magnitudes do equally measure divers other greater Magnitudes.

3 *Reason, is an habitude of two magnitudes of the same kind, compared the one to the other, according to quantity.*

When

WHen two quantities of the same kind, as two numbers, two lines, two superficies, two solids, &c. are compared to one another, according to quantity, that is to say, according as one is greater or lesse than the other, or equal thereto, such comparison or mutual habitude is called reason, or (as others will have it,) proportion. Wherefore if you compare a line with a superficie, or a number with a line, that comparison cannot be called Reason; forasmuch as, neither the line and the superficie, nor the number and the line are quantities of the same kind.

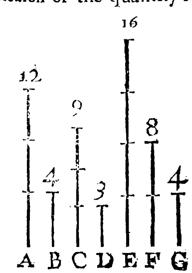
In like manner, if you compare a line with another line, or a superficie with another superfice, according to the quality, that is to say, according as the one is white, and the other black, or as the one is hot, and the other cold, &c. Although that both may be of one kind, neverthelesse such comparison shall not be termed Reason or Proportion, seeing that it is not made according to quantity.

Now although that Reason or Proportion is properly found betwixt quantities alone, neverthelesse the things which do receive in some manner the nature of the quantity, as Times, Sounds, Voices, Places, Motions, Weights, and Powers, are likewise said to have Reason, if you consider their habitude, according to quantity, as when we say that one time is greater than another time, or lesse, or that two times are equal, &c. this habitude shall be called Proportion or Reason; for then the times are considered according to the manner of some quantities.

It remains that in all Proportion, the quantity that refers it self to another, is termed by *EUCLIDE*, and other Geometricians, the Antecedent of the Reason. But that to which another refers it self, is termed the consequent of the Reason. As for Example, in the Reason of a line of 6 feet to a line of 3 feet, the line of 6 shall be termed the Antecedent of the Reason, and the line of 3 the consequent; and contrarily, if you compare the line of 3 to the line of 6, the line of 3 shall be termed the Antecedent, and that of 6 the consequent, otherwise the quantity put first shall be termed the Antecedent, and the second the consequent.

4 *Proportion is a similitude or likenesse of Reasons.*

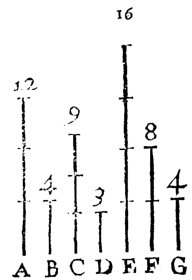
THat which I call here Proportion, the Greeks call ἀναλογία, Analogie, and many Latines Proportionality. Therefore even as the habitude of two quantities to one another is termed Reason; so the comparison of two or more Reasons to one another, is called Proportion, as if the Reason of the quantity A to the quantity B, be like the Reason of C to D, the habitude between the Reasons shall be called Proportion; in the same manner, if the Reason of E to F, be like to that of F to G, that similitude shall be called Proportion.



Now there are divers sorts of Proportion or habitude of Reasons, (and we shall call the habitude of two quantities Reason; but the habitude of Reasons Proportion,) described by Authors, principally by *Boetius* and *Iordanus*, of which those here have been

most

most in use amongst the Ancients; to wit, the proportion Arithmetical, Geometrical, Musical, or Harmonical. But *EUCLIDE* treateth only of Geometrical proportion in this Book. And of those there are two sorts, the one continued, in which the intermediate quantities are taken twice, in such sort as there is not made any interruption of Reasons. But each intermediate quantity is a consequent of the precedent quantity, and an antecedent to that which follows, as if you should say that there is the same reason of E to F, as of F to G, that proportion shall be called continued Proportion. But the other shall be termed discontinued proportion, in the which each of the intermediate quantities is taken only once, in such sort as there is made an interruption of Reasons, and not any quantity is Antecedent and Consequent, but Antecedent only, or Consequent only, as if you shall say that there is the same reason of A to B, as of C to D, this proportion shall be called Discontinual Proportion, or Proportion not continued.



Of the Division of the Reason.

Rational.

EUCLIDE divideth Reason in Rational and Irrational, Rational is that which may be expressed by numbers, as the reason of a line of 20 feet, to a line of 10 feet; for that reason is shewn by those two numbers 20 and 10. Irrational is that which cannot be expressed by numbers, as is the reason of the Diameter of a square, to the side of the same square, for that reason cannot be found in Numbers, as *EUCLIDE* doth demonstrate in his Tenth Book.

Irrational.

Others say that Reason Rational, is that which is between any two quantities commensurable; But Irrational is that which is between two quantities incommensurable; Now those quantities are said to be commensurable, which have one common aliquot part, or are measured by one and the same common measure, as a line of 20 feet, and a line of 8, for a line of 4 feet is an aliquot part of both of them: In like manner, a line of 2 feet is an aliquot part of them: and therefore as well 4 as 2, shall measure as well the line of 20, as the line of 8.

Commensurable quantities.

But Quantities incommensurable, are those which have no aliquot part, or any common measure that may measure them, as is the diameter of a square, and the side of the same square. For although that each of those lines have infinite aliquot parts, as the half, the third, and other parts: Neverthelesse, not any aliquot part of the one, be it never so little, can possibly measure the other, as *EUCLIDE* doth demonstrate in the Tenth Book and last Proposition, in which Book are shewn divers other incommensurable lines besides those two. Therefore only, Reason Rational is found in numbers: But as well Rational as Irrational, is found in continued quantity.

Incommensurable.

Reason is also divided into Reason of equality and Reason of inequality. Reason of equality is that which is between two equal quantities compared to one another, as between 20 and 20, between 100 and 100, be-

Reason of equality.

tween a line of 12 feet, and a line of 12 feet; but Reason of inequality is that which is between two quantities unequal, compared to one another, as between 20 and 10, between 8 and 40, between a line of 6 feet, and a line of 2, &c.

Now these two kinds of Reason have this in common with the first, that all reason of equality is necessarily rational, and not contrarily. In like manner, all Reason Irrational, is necessarily reason of inequality, and not contrarily, &c.

Reason of equality.

Again, Reason of inequality, (for we have left the reason of equality, considering that it cannot be subdivided; seeing that all equal quantities be they great or small, have alwayes the same reason of equality,) is subdivided into Reason of greater inequality, and of lesser inequality.

Inequality Major. Minor

Reason of greater inequality, is when the greatest quantity is compared to the least, as the reason of 20 to 10, also a line of 8 feet to a line of 6 feet, &c. Reason of lesser Inequality is when the lesser quantity is compared to the greater, as the reason of 10 to 20, also of a line of 6 feet to a line of 8 feet, &c. Now this reason is not vain and superfluous, as divers doubt: for there is not the same reason of 4 to 2, as of 2 to 4, but much different to one another, their use being divers, as is manifest to those that are any wise versed in Geometry and in Algebra.

These are the general Divisions of reason considering it as it conteins all the reasons, none excepted. Now we will subdivide as well the reason of the greater inequality, as of the lesser inequality, for that they only comprehend the Reasons Rational; forasmuch as we ought to speak in the Tenth Book of quantities which have Reason Irrational.

Therefore Reason Rational, of the greater inequality is divided into five Kinds, to wit, into Reason Multiplex, Super-particular, Super-partient, Multiplex Super-particular, and Multiplex Super-partient: In like manner, the reason of the lesser inequality is divided according to the same kinds, provided that to each term of the reason there be put before it this Præposition *(Sub)* that is to say, under, as in the Reason Sub-Multiplex, Subsuper-particular, &c.

Now of these five kinds, the three first are simple, but the two last are compounded of those three first, as is manifest.

Of Reason Multiplex.

Reason Multiplex is an habitude of a greater quantity to a lesser, when the greater conteineth the lesser, a certain number of times precisely, as 2 times, 3 times, 10 times, 100 times. In such sort as the lesser measureth the greater, as is the reason of this number 20 to the number 4, for 20 conteins 4 five times, also the reason of a line of 30 feet to a line of 5 feet, &c.

Now this Reason conteineth under it infinite kinds, for if the greater quantity of the Reason Multiplex doth contein the lesser only twice, it shall be called Reason Double: if thrice, Triple, if ten times, Decuple, if a hundred times, Centuple, &c.

From these things, we shall easily define all the kinds of Reason Multiplex; for the Reason Octuple shall be no other thing then an habitude of a greater quantity to a lesser, when the greater doth contein the lesser, 8 times. In the same manner, we may define the other Reasons Multiplices, as

as the Reaſon Quintuple, ſuch as is 40 to 8, of which Reaſon, the greater conteins the leſſer five times, alſo the Reaſon of a line of 10 feet, to a line of 5 feet, is that in which the greater quantity conteineth the leſſer twice, and ſo of the others.

Of Reaſon Super-particular.

Reaſon Super-particular is an habitude of a greater quantity to a leſſer, when the greater conteines the leſſer only once and over and above an aliquot part of the ſaid leſſer, to wit the half, the third or the quarter &c. As is the Reaſon of 3 to 2, for 3 conteins 2 once and over and above Unite which is the half of 2. So in like manner, a line of 12 feet hath Reaſon Super-particular to a line of 9 feet, for the firſt line 12 conteineth the laſt 9 once, and over above a line of 3 feet, which is the third part of 9 feet.

This Reaſon is divided in like manner into infinit kinds: for if the aliquot part conteined in the greateſt quantity, be the halfe of the leſſer quantity, the Reaſon conſtituted ſhall be called Seſquialtera, if it be the third part, it ſhall be called Seſquitertia: if the fourth Seſquiquarta, and if the Thouſandth part, Sequimileſima.

Therefore by this ſame term the definitions of all the Reaſons Super-particular ſhall be eaſie, as the Reaſon ſeſquioctava, is when the greater quantity conteins the leſſer, once and over and above the eighth part of the leſſer, as is the Reaſon of 9 to 8, alſo of 45 to 40, and the ſame is to be underſtood of the reſt.

Of Reaſon Super-partients.

Reaſon Super-partients is the habitude of a greater quantity to a leſſer, when the greater conteins the leſſer once only, and over and above ſome Aliquot parts of the ſame leſſer, which being taken together, do not make an Aliquot part, as is the proportion of 8 to 5, for 8 conteins 5 once, and over and above 3 Unites, each of which is an Aliquot part, to wit, the fifth part of this number 5, but the number 3 compounded of 3 Unites, is not an Aliquot part of 5.

I have ſaid that theſe Aliquot parts ought not to conſtitute an Aliquot part, becauſe of divers reaſons which at firſt ſeeme to be Super-partients, which neverthleſſe are Super-particulars, as is the reaſon between 10 and 8, for although that the number 10 conteineth 8 once, and over and above 2 Unites, each of which is the eighth part of 8, yet notwithſtanding, forasmuch as the number of 2, compounded of theſe 2 Unites, is the fourth part of 8: this reaſon ought not to be called Super-partients, but Super-particular, to wit Seſquiquarta: therefore, to the end that two quantities may be ſaid to have Reaſon ſuper-partients, it is neceſſary that the greater quantity do contein the leſſer once, and over and above divides Aliquot parts of the ſame leſſer, the which taken together, ought not to conſtitute an Aliquot part; which divers not conſidering do confound many kindes of Reaſons one among another.

Of Reaſon Multiplex Superparticular.

Reaſon Multiplex Super-particular, is the habitude of a greater quantity to a leſſer, when the greater doth contein the leſſer, a certain number of times, as 2, 3, 4, &c. and over and above an Aliquot part of the leſſer, as is the Reaſon of 9 to 4, for 9 conteins 4 twice, and over and above, (in which it agrees with the Multiplex as with the double) doth contein Unity, which is the fourth part of the leſſer number (and in that, this Reaſon is like to the Superparticular, to wit, to the Seſquiquarta) in ſuch ſort as this reaſon ſhall be rightly ſaid to be compounded of the Multiplex and of the Superparticular.

Now this Reaſon, even ſo as is the Multiplex, is divided into ſeveral kindes, as into double Superparticular triple Superparticular, &c. conſidering that the greater quantity conteineth the leſſer quantity twice or thrice and over and above an Aliquot part of the leſſer quantity.

Again each of thoſe kindes is ſubdivided into divers, having regard to the Reaſon Superparticular, for Example, the reaſon triple Superparticular, conteineth under it the triple Seſquialtera (to wit, when the greater quantity doth contein thrice the leſſer and over and above the halfe of the ſame leſſer) the triple Seſquitertia, the triple Seſquiquarta and ſo of others.

Of Reaſon Multiplex Superpartients.

Laſtly, Reaſon Multiplex Superpartients is the habitude of a greater quantity to a leſſer, when the greater doth contein the leſſe, a certain number of times, and over and above ſome Aliquot parts of the leſſer, the which being taken together do not make an Aliquot part; as is the reaſon of 11 to 3, I have ſaid [not making an Aliquot part] for the reaſon alleaged in the reaſon Superpartients. For if the ſame Aliquot parts ſhould make an Aliquot part, it ſhould not be Reaſon Multiplex Superpartients but Multiplex Superparticular.

As the reaſon of 20 to 6 ought not to be termed Multiplex Superpartients Sixths, although that 20 contein 6 three times and two Sixths, for as much as $\frac{2}{6}$ parts make $\frac{1}{3}$ part. Wherefore it ſhall be called reaſon triple Seſquitertia.

Now this reaſon is divided, firſt of all, having regard to the reaſon Multiplex; as in Multiplex double, Superpartients triple Superpartients, &c. Again, each of them, (regard being had to the number of the parts,) conteineth under it, infinite kindes: As under the triple Superpartients, is conteined the triple Superbipartients, &c.

Laſtly, each of them, in conſideration of the denomination of the Aliquot parts is again divided into infinite kindes: As the triple Supertripartients is divided into triple Supertripartients fourths and triple Supertripartients fifths, &c. Of all which the definitions and the Examples will be eaſie by what hath been ſaid.

Of Reaſons rational, of leſſer inequality.

All that hath been ſaid hitherto, of the 5 kindes of reaſons Rational, of the greater inequall, ought in like manner to be underſtood of the five Kindes correſponding, of the leſſer inequallity: Neverthelesse alwayes adding this Prepoſition (*Sub*) that is to ſay under as hath been ſaid.

For if in the Examples heretofore propoſed, we compare the leſſer quantities with the greater, we ſhall have the Reaſons of the leſſer inequality correſponding, for even ſo as the Reaſon of 100 to 1 is Centuple, ſo the Reaſon of 1 to 100, is Subcentuple, and as the Reaſon of 11 to 3, is Triple Superbipartient thirds, ſo the Reaſon of 3 to 11, is Subtriple, and Superbipartients thirds, and ſo of others.

Of

Of the Denominators of Reasons Rational.

Forasmuch as the use of the Denominators of Reasons Rational before exposed, is not a little profitable, it will not be amisse here to shew by what numbers they are denominated. The Denominator of any Reason, is the number which doth openly and distinctly expresse the habitude of one quantity to another, as the Denominator of the Reason Octuple is 8, for that the number 8 doth shew that the greater quantity of the Reason Octuple conteineth the lesser 8 times. In like manner, the Denominator of the Reason Sesquiquinta is $\frac{6}{5}$; forasmuch as this number doth signifie that the greater quantity of the Reason Sesquiquinta, conteineth the lesser once, and the fifth part of the same lesser: the same ought to be understood of the Denominators of the other Reasons.

Wherefore (in my opinion) EUCLIDE in the Sixth Book, and sundry other Mathematicians, do call the Denominator of any Reason whatsoever the quantity thereof: for the Denominator (as we have said) shews how great one quantity is in respect of another, to wit, of that with which it is compared, as appears by the Examples proposed.

Now from these things which we have said, may easily be gathered the Denominator of any Reason, for the Denominator of the Reason Multiplex whatsoever it be, is a whole number, which doth contein as many unites as the greater quantity doth contein times the lesser: As for Example, the Denominator of the Reason Double 2, of the Reason Octuple 8, the Centuple is 100, &c. but the Denominators of the Reasons Submultiplices, correspondent to the Multiplices, are aliquot parts, denominated of the Denominators of the Reasons Multiplices, to which the denominated do correspond. As the Denominator of the Reason Subdupla, is $\frac{1}{2}$, Subquintuple is $\frac{1}{5}$, Subcentuple is $\frac{1}{100}$, &c. and so we shall find the Denominators of the other Reasons Submultiplices. Therefore the Denominator of any Reason Submultiplex is a broken number, or a fraction of which the Numerator is alwayes an Unite: but the Denominator is a number denominating the Reason Multiplex corresponding, as appears by these Examples. Therefore it will not be difficult to find the Denominator of any Reason Multiplex, or Submultiplex, if what hath been said be well understood; forasmuch as the same prolation demonstrateth the Denominator of the Reason, as is manifest by the Examples proposed.

The Denominator of any Reason Super-particular is Unity with that aliquot part that the greater quantity ought to contein over and above the lesser: As the Denominator of the Reason Sesquialtera is 1 and $\frac{1}{2}$, Sesquioctava is 1 and $\frac{1}{8}$, Sesquicentesima is 1 and $\frac{1}{100}$ &c. and it will not be difficult to find the Denominator of any Reason Superparticular; seeing that the same prolation of Reason doth expresse the Denominator by its aliquot part, as is evident by the Examples proposed.

But the Denominators of the Reasons Subsuper-particulars correspondents, are fractions, whose Numerators are lesse than their Denominators by Unity only, each to his correspondent, as the Denominator of the Reason Subsesquialtera is $\frac{2}{3}$, Subsesquioctava is $\frac{8}{9}$, Subsesquicentesima $\frac{100}{101}$. Now the Denominator of any Reason Super-particular may be found, if for the Numerator of the fraction you take the Denominator of the aliquot part expressed in the reason, and for the Denominator of the same fraction, you take a number greater than Unity: As the Denominator of the

the Reason Subsesquidecimal is $\frac{10}{11}$; seeing that the Numerator of this fraction is the number denominating the tenth part, to wit 10, but the Denominator of the same fraction exceedeth the Numerator by Unity, the Denominator of the Reason Subsesquiseptimal is $\frac{7}{8}$, that of the reason subsesquiquinta is $\frac{5}{6}$, &c.

In like manner, we shall find the Denominator of any Reason Subsuperparticular, if we reduce the Denominator of the Reason Subsuperparticular, correspondent in a fraction, of the which fraction the Numerator shall exceed alwayes the Denominator by unity, the which is also of the aliquot part, whereof mention is made in the Reason proposed, seeing we transpose the terms of this fraction, in such sort as the Numerator may be made Denominator, and the Denominator Numerator; we shall have the Denominator of the reason Subsuperparticular proposed. As if the Reason Subsesquiseptimal be proposed; forasmuch as the Denominator of the Reason Sesquiseptimal which is correspondent thereto, is 1 and $\frac{1}{7}$, which is reduced into this fraction $\frac{8}{7}$, whose Numerator is greater than the Denominator by the aliquot part of Unity; wherefore if we transpose this fraction in this manner $\frac{7}{8}$, we shall say that the Denominator of the Reason Subsesquiseptimal is $\frac{7}{8}$, &c.

The Denominator of any Reason Superpartients is Unity with the aliquot parts, (not making an aliquot part) which the greatest quantity ought to contein over and above the lesser; As the Denominator of the Reason Supertripartient septimal is 1 and $\frac{3}{7}$, Supertripartient 20 is 1 $\frac{3}{20}$, &c. and is easie to be found, forasmuch as the prolation of the Reason doth shew the proper Denominator, as appeares by the Examples before mentioned.

But the Denominator of any Reason Super-partient, is a Fraction, whose Numerator is lesse than the Denominator by so many unites, as the greater quantity of the Reason proposed, doth contein aliquot parts, over and above the lesser, as the Denominator of the Reason Subsupertripartient septimal is $\frac{7}{10}$ of the Subsupertripartient 20 is $\frac{20}{23}$, &c. Now the Denominator of any Reason Subsupertripartient shall be found, if for the Denominator of the fraction, you take the Denominator of the aliquot parts conteined in the reason proposed, to which if you adde the number of the same parts, it will give you the Denominator of that fraction.

As the Denominator of the Reason Subsupertripartient 11, to wit 11, to which is added 4, the number of four aliquot parts, to the end that 15 be made the Denominator of the same fraction.

But the Denominator of the Reason Subsupertripartient fifths, is this fraction $\frac{5}{8}$, for the Numerator thereof is the number denominating the fifth parts, to wit 5: But the Denominator of the same fraction 8, is made of the Numerator 5, and of 3, the number of the three aliquot parts, and so of the other Denominators, which may be also found in this manner.

Let the Denominator of the Reason Subsuperpartient, correspondent in a fraction be reduced, whose Numerator may exceed the Denominator, which denominateth also the aliquot parts expressed, by so many unites as there are aliquot parts.

For the numbers of this fraction being transposed in such sort as the Numerator be made Denominator, you shall have the Denominator of the Reason proposed Subsuperpartient, as the Denominator of the Reason Subsupertripartient fifths is $\frac{5}{8}$, for that the Denominator of the

Reason Supertripartient fifths is $1\frac{3}{5}$, which is reduced into this fraction $\frac{8}{5}$, which makes $\frac{5}{8}$ in transposing the numbers, and the same ought to be done for finding the other numbers, &c.

The Denominator of any Reason Multiplex Superparticular, is a whole number denominating the Reason Muliplex proposed, with the aliquot part that the greater quantity ought to contein, over and above the lesser; as the Denominator of the Reason Triplesesquiseptimal is $3\frac{1}{7}$, the Quintuple sesquinoncuple is $5\frac{1}{9}$, &c, and is very easie to expresse the Denominator of any Reason Multiplex Superparticular, forasmuch as the prolation of the Reason sheweth distinctly both the Denominator of the Reason Multiplex, and the aliquot part as may be seen by the examples proposed. Now the Denominator of any Reason Submultiplex superparticular, is a fraction, whereof the Numerator is a number denominating the aliquot parts in the Reason, as the Denominator of the Reason Subtriple sesquiseptimal is $\frac{7}{22}$, the Subquintuple sesquinonal is $\frac{9}{46}$, &c.

Now this Denominator will be found, if for the Numerator of the fraction you take the Denominator of the aliquot part, which being multiplyed by the Denominator of the Reason Multiplex, if you adde unity to the product, you shall have the Denominator of the same fraction.

As for Example, the Denominator of the Reason subquadruple sesquisexta is $\frac{6}{25}$; forasmuch as the Numerator 6 doth denominate the sixth parts, and the same multiplyed by 4 the Denominator of the Reason quadruple, and having added to the product 24 unites, to the end the Denominator of the same fraction may be made 25, &c. otherwise you should find the same denominator, if you reduce into one fraction the Denominator of the Reason Multiplex superparticular, corresponding as is before said.

As if the Reason Subquadruple sesquisexta were proposed; forasmuch as the Denominator of the Reason quadruple sexta correspondent is $4\frac{1}{6}$, which shall be reduced to $\frac{25}{6}$; therefore if you change the order of the terms, of the same fraction you will have $\frac{6}{25}$, for the Denominator of the Reason Subquadruple sesquisexta, and so of the others.

The Denominators of any Reason Multiplex superpartients, is a whole number denominating the Reason Multiplex expressed therein, with the aliquot parts which the greater quantity ought to contein over and above the lesser, which said aliquot parts do not constitute an aliquot part, as the Denominator of the Reason Triple Superquinpartient octa is $3\frac{5}{8}$, of the quadruple superbipartient fifth is $4\frac{2}{5}$.

And to find the Denominators, there is no difficulty, seeing they are expressed distinctly and openly in each Reason proposed Multiplex superpartient, to wit, as well the Denominator of the Reason Multiplex conteined therein, as the aliquot parts, as is manifest by the Examples proposed. But the Denominator of any Reason Submultiple superpartient, is a fraction, of which the Numerator is a number denominating the aliquot parts of the same Reason.

As the Denominator of the Reason Subtriple superquintapartient is $\frac{8}{29}$, subquadruple superbipartient is $\frac{5}{22}$. Now the Denominator of those Reasons submultiplices superpartients will be found, if for the Numerator of the fraction you take the Denominator of the aliquot parts, which being multiplyed by the Denominator of the Reason Multiplex, and having added to the product the number of the aliquot parts, you will obtein the Denominator of the same fraction.

As the Denominator of the Reason subduple superoctupartient is $\frac{13}{34}$, for that the Numerator of this fraction 13 doth denominate the thirteenth part which multiplyed by 2, the Denominator of the Reason Duple, and to the number produced 26 there be added the number of 8 parts, it will make 34, the Denominator of the same fraction.

Otherwise there will be found the same Denominator, reducing the Denominator of the Reason correspondent in a fraction, and transposing the terms as is before said.

As for Example, Let the Reason Subquintuple supertripartient tenths be proposed, to find the Denominator, you shall reduce into a fraction the Denominator of the reason correspondent quintuple supertripartient tenth, which is $5\frac{3}{10}$, and you shall have this fraction $\frac{53}{10}$; and in changing the terms, that is to say putting 10 instead of 53, and 53 instead of 10, you will have $\frac{10}{53}$ for the Denominator of the reason subquintuple supertripartient tenth, and so of the rest.

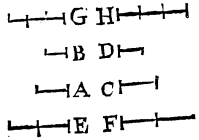
Lastly, the Denominator of the Reason of equality is alwayes unity; forasmuch as in this reason the quantities ought to be equal to one another, and therefore the one ought to contein the other once and no more, which is signified by unity.

5 *Magnitudes are said to have reason to one another, which being multiplyed may exceed one another.*

Forasmuch as EUCLIDE in the third Definition, hath termed the habitude of two magnitudes of the same Kind, Reason, he teacheth now in this fifth Definition, what thing is required, to the end that two quantities of the same kind may be said to have Reason to one another; saying that Magnitudes are said to have Reason to one another, of the which (which you please) being multiplyed, may be augmented in such sort as that it may exceed the other, as the diameter and the side of one and the same square, are said to have Reason, although it be Irrational, which cannot be expressed by any number; forasmuch as the side multiplyed by 2 or taken twice, doth exceed the diameter; for seeing that the two sides of the square, with the diameter do constitute an Isosceles triangle, the two sides of the square shall be greater than the diameter thereof.

So in like manner, the circumference of a circle and the diameter thereof shall have Reason, (although it be not as yet known;) forasmuch as the diameter multiplyed by 4, or taken 4 times, doth exceed the circumference, seeing that every circumference of a circle (as is demonstrated by *Archimedes*) doth contein only three times the diameter of the said circle, and over and above a particle; a little lesse than the seventh part of the diameter.

In the same manner, divers curviline figures will have Reason with right lined figures as *Proclus* hath shewn on the First of EUCLIDE. Now as divers Interpreters do say that neither all Lines, nor all Plain Angles have Reason to one another, according to this Definition: for say they, a finite line hath no Reason to an infinite; seeing that in what manner soever it be multiplyed, it cannot exceed the infinite line, neither hath a right lined angle Reason to an angle of contingence, for the angle of contingence cannot exceed a right lined angle.



6 *Magnitudes* A, B, C, *and* D, *are said to be in the same Reason, the first* A *to the second* B, *and the third* C *to the fourth* D, *when the equimultiplices* E *and* F, *of the first* A, *and the third* C, *to the equimultiplices* G *and* H *of the second* B, *and the fourth* D, *by whatever multiplication, are either deficient together, or are equal together, or do exceed together each to other, if those be taken which do answer one another.*

EUCLIDE shews in this place what conditions are required in Magnitudes, that they may be said to have the same Reason, which to expose, he is constrained to serve himself with their equimultiplices, to the end he may comprehend all the Reasons of Magnitudes, as well Rational as Irrational.

Now if you compare the one to the other, the magnitudes which have been taken equimultiplices, to wit, those which answer to one another; as the Multiplex of the first and the Multiplex of the second, which are E and G, also the Multiplex of the third, and the Multiplex of the fourth, to one another, to wit, F and H, and if it be alwayes found that they have such relation to one another, if E the Multiplex of the first Magnitude A, be lesse than G the Multiplex of the second Magnitude B, that F the Multiplex of the third Magnitude C, be also lesse than H the Multiplex of the fourth C; or if E be equal to G, also F is equal to H; or lastly, if E be greater than G, also F is greater than H, which is to be deficient together the one and the other, of the one and the other, or to be equal together, or to exceed together, in such sort as the contrary cannot be found in any kind of Multiplices; that is to say, that E be never lesse than G, but F also be lesse than H, and that E be never equal to G, if F also be not equal to H; and lastly, that E be never greater than G, if that F also be not greater than H.

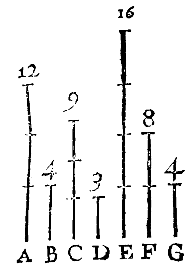
If therefore it be found that the equimultiplex taken in what manner soever, have relation perpetually to one another, as aforesaid, it will be said that there is the same Reason of the first Magnitude A, to the second B, as of the third magnitude C, to the fourth D; as if it were sometimes found, even in one only kind of Multiplices, that the Multiplex E wanteth of the Multiplex of G; but the Multiplex of F wanteth not of H, or that E is equal to G; but F is not equal to H: Or lastly, E doth exceed G, but F exceedeth not H, although in infinite other sorts of Multiplices, the above said condition be found in some sort, the Magnitudes shall not be said to have the same Reason to one another, but divers, as shall be made manifest by the Eighth Definition.

Contrarily, if there be the same Reason of the first to the second, as of the third to the fourth, it will follow that the equimultiplices of the first and

the third, shall be both equal to the Multiplices of the second and the fourth, or both lesse, or both greater in any Multiplication whatsoever.

And if there were not the same Reason of the first to the second, as of the third to the fourth, also the equimultiplices of the first and the third exceed not together the equimultiplices of the second and the fourth, or shall not be equal, or lesse, &c.

Now here is to be noted, that this Definition ought also to be taken of three Magnitudes, provided that the second be taken twice, to the end you may have 4, &c.



7 *Magnitudes which have the same Reason, are called proportionals.*

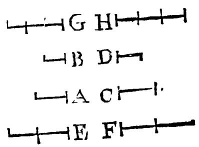
AS if of the Magnitudes A, B, C, and D, there is the same reason of A to B, as of C to D, those Magnitudes A, B, C, and D, shall be termed proportional. In like manner, if there be the same reason of E to F, as of F to G, the Magnitudes E, F, and G, shall be termed proportional. Now there be some Magnitudes continually proportional, between which the proportion continued is found, as are the Magnitudes E, F, and G; but some are not continually proportional, but seperately, as are the Magnitudes A, B, C, and D, for in these there is an interruption of Reasons, but in the others not, as is said in the fourth Definition.

8 *But when of the equimultiplices the multiplex of the first magnitude shall exceed that of the second. But the multiplex of the third magnitude shall not exceed that of the fourth, then the first magnitude shall be said to have greater Reason to the second, than the third to the fourth.*

EUCLIDE declareth in this Definition, what condition four magnitudes ought to have, to the end that the first may be said to have greater reason to the second, than the third to the fourth; saying that if the equimultiplices of the first and the third be taken, and likewise other equimultiplices of the second and the fourth, and that there be sometimes found (but not alwayes,) that the Multiplex of the first Magnitude be greater than the Multiplex of the second: but the Multiplex of the third is not greater than the Multiplex of the fourth, but lesse or equal, it will be said that there is greater reason of the first Magnitude to the second, then of the third to the fourth, as is manifest by the following figure proposed, in which are taken the double of the first Magnitude A, and of the third C, to wit, E and F; but of the second B and the fourth D, hath been taken the triples G and H. And forasmuch as E the Multiplex of the first, is greater than G the Multiplex of the second. But F Mul-

the Multiplex of the third, is not greater than H, the Multiplex of the fourth, but lesse, it may be said that there will be greater Reason of A the first Magnitude, to B the second Magnitude, then of C the third, to D the fourth.

Now it is not necessary (to the end that of four Magnitudes, the first be said to have greater Reason to the second, than the third to the fourth,) that the equimultiplices according to any Multiplication whatsoever, have such habitude, as the Multiplex of the first exceed the Multiplex of the second; But that the Multiplex of the third exceed not the Multiplex of the fourth. But it sufficeth that according to any Multiplication whatsoever, they have such an habitude, for it may happen sometimes that as well the Multiplex of the first is greater than the Multiplex of the second as the Multiplex of the third, of that of the fourth: In like manner, as the Multiplex of the first is lesse than the Multiplex of the second, that that of the third is also lesse than that of the fourth: Neverthelesse, seeing that, that happeneth not in every Multiplication; but that sometimes the Multiplex of the first exceeds that of the second, but the Multiplex of the third is lesse than, or equal to the Multiplex of the fourth, for this cause the first Magnitude shall be said to have greater Reason to the second, than the third to the fourth, and not the same as is seen in the Example proposed, where the third hath greater Reason to the second, than the fourth hath to the third.



15, 9, 13, 3, 2, 10, 18, 14.
20, 12, 16, 4, 3, 15, 12, 21.

Therefore to the end that four Magnitudes may be said to be proportional, it is necessary that the equimultiplices of them taken, according to any Multiplication whatsoever, be deficient together, equal together, or do exceed together, as hath been exposed in the Sixth Definition.

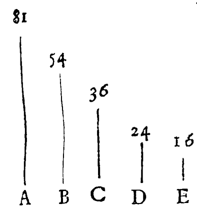
But to the end that the first may be said to have greater Reason to the second, than the third to the fourth, it sufficeth that according to some Multiplication, the Multiplex of the first exceed the Multiplex of the second; but that of the third exceedeth not that of the fourth, although that according to infinite other Multiplications, the equimultiplices of the first and of the third, exceed together the equimultiplices of the second, and of the fourth.

But if contrarily, the Multiplex of the first Magnitude wanteth of the Multiplex of the second, the Multiplex of the third wanteth not of the Multiplex of the fourth, the first Magnitude shall be said to have lesser Reason to the second, than the third to the fourth; although that according to divers other Multiplications, the equimultiplices of the first and of the third do want together, of the equimultiplices of the second and of the fourth, as may be seen in the same numbers of the Example proposed, the Reason of 2 to 3 shall be lesse than that of 3 to 4, &c.

9 *Proportion cannot be constituted in lesse than three terms.*

FOrasmuch as all Analogie which is termed Proportion, is a similitude or likenesse of Reasons, and seeing that in all Reasons there is the term ante-

antecedent, and the term consequent, it is necessary, that in all proportion there may be at least two terms antecedents, and two consequents: Wherefore if the proportion be not continued, there will be required at least four terms or Magnitudes: But if the Proportion be continued, there will be at least three terms; forasmuch as the middle term is taken twice; seeing that it is the consequent of one Reason, and the antecedent the other, and this here is the least number of the terms of proportion; for in any two terms is only found Reason, and not Proportion.



10 *When there are three magnitudes* A, B, C, *proportional, the first* A *to the third* C, *is said to have duplicate reason of the first* A *to the second* B: *But when four magnitudes are proportional, the first* A *to the fourth* D, *is said to have triplicate reason of the first* A *to the second* B, *and alwayes after the same order, one more, until that the proportion be accomplished.*

AS if the Magnitudes A, B, C, D, and E, are continually proportional in such sort, as there may be the same reason of A to B, as of B to C, and of C to D, and of D to E; the reason of the first Magnitude A, to the third Magnitude C, shall be termed the duplicate of the reason that the first Magnitude A hath to the second Magnitude B: Forasmuch as between A and C, there are two reasons, which are equal to the reason of A to B; to wit, the reason of A to B, and of B and to C, in such sort as the reason of A to B is doubled, that is to say, placed twice in order. But the reason of the first A to the fourth D, is said to be triplicate of the reason that the first Magnitude A hath to the second Magnitude B; forasmuch as between A and D are three reasons, which are equal to the reason of A to B; to wit, the reason of A to B, of B to C, and of C to D, insomuch as the reason of A to D encloseth or comprehendeth in some sort the reason of A to B triplicated, that is to say, put three times in order. So in like manner the reason of A to E, shall be said to be quadruplicate of the reason of A to B; forasmuch as four reasons are found between H and F, which are equal to the reason of A to B, &c.

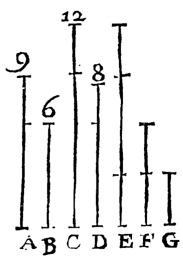
And if contrarily, there be the same reason of E to D, as of D to C, of C to B, as of B to A, the reason of E to C shall be termed the duplicate of the reason of E to D: But the reason of E to B shall be said to be the triplicate of the reason of E to D: So in like manner, the reason of E to A shall be said to be quadrupled of the reason of E to D.

11 *Magnitudes are said to be Homologal, or of like reason, the antecedents to the antecedents, and the consequents to the consequents.*

He

HE hath defined heretofore that proportion is a ſimilitude of reaſons, and now he teacheth that not only in ſome proportion the Reaſons are ſaid to be alike, but alſo the terms themſelves or quantities, are ſaid to be alike or homologal, ſaying that the Magnitudes antecedents are termed homologal, or alike to one another in reaſon, and the ſame of the conſequents to one another, to the end that it may be underſtood in ſeveral Demonſtrations what ſides of figures compared to one another, ought to be the antecedents of the reaſons, and which the conſequents, as ſhall be declared in the Sixth Book.

If there be then the ſame reaſon of A to B, as of C to D, the quantity A ſhall be ſaid to be alike to the quantity C and B alike to D, for becauſe of the ſimilitude of Reaſons, it is neceſſary that both the one and the other Magnitude antecedent be equal to one and the other conſequent, or in the ſame ſort, greater or leſſe, otherwiſe both the one and the other conſequent, or in the ſame ſort greater or leſſe, otherwiſe both the one and the other antecedent ſhould not have the ſame reaſon to the one and the other conſequent: Behold the Example.



In the Magnitudes propoſed, to the which the antecedents are in the ſame ſort greater than the conſequents, to wit, by the halfe: See another Example.

In the Magnitudes continually proportional E, F, and G, where as well E and F are homologal as F and G, as is manifeſt, and for this cauſe *EUCLIDE* in the ſixth and eighth Definition, enjoynes to take the equi-multiplices of the firſt and third Magnitude, that is is to ſay the antecedents.

In like manner, to take the other Multiplices of the ſecond and the fourth Magnitudes, to wit, the conſequents, for they are alike to the Magnitudes proportional, as is manifeſt by this Definition; but to the Magnitudes that are not proportional, they are unlike.

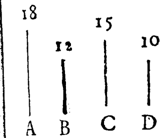
12 *Reaſon alternate, is to take the antecedent compared to the antecedent, and the conſequent to the conſequent.*

HE here unfoldeth ſome certaine kinds of arguing in proportions, whoſe uſe is very frequent amongſt Geometricians, which kinds of arguings are ſix in number; The firſt is termed alternate reaſon or reaſon by permutation, the ſecond is termed inverſe, or contrary reaſon, the third compoſition of reaſon, or conjunct proportion, the fourth diviſion of reaſon, or diſjunct or ſeparated proportion, the fifth converſion of reaſon, or reverſed proportion: Laſtly, the ſixth is termed reaſon of equality, or equal proportion.

Therefore, (ſaith he,) alternate reaſon, or reaſon by permutation, is when four proportional Magnitudes being propoſed, it be inferred that there is the ſame reaſon of the antecedent of the firſt reaſon, to the antecedent of the laſt, as of the conſequent of the firſt, to the conſequent of the laſt; as if we propoſe the ſame reaſon of A to B, as of C to D; and

and therefore conclude that there is the ſame Reaſon of A to C as of B to D; we are ſaid to argue according to Permutation of Reaſon.

The Greek Authours do uſe in this kinde of arguing; neer this manner of expreſſion; to wit, as A is to B, ſo C is to D: Therefore alternately, or contrarily, A ſhall be to C as B to D. Now we ſhall demonſtrate the truth of this Illation at the ſixteenth propoſition of this Book. And in this kinde of arguing, it is neceſſary that all the four Magnitudes be of one and the ſame kind, to the end they may have reaſon to one another, being taken two and two at pleaſure, for it is no arguing to ſay as the line A is to the line B, ſo the number C is to the number D; then alternately, as the line A is to the number C, ſo the line B is to the number D, conſidering that there is no proportion of a line to a number, and contrarily, as is manifeſt by the fifth definition.



But as to the other following kindes of arguing, the firſt Magnitudes may be of one kinde of quantity and the laſt Magnitudes of another, as ſhall be made manifeſt by the demonſtrations of this fifth Book.

13 *Inverſe or tranſpoſed Reaſon, is to take the conſequent as antecedent, to compare it to the antecedent, as if it were the conſequent.*

AS if (ſeeing that A is to B as C is to D) we ſhould inferre that B is to A as D is to C (ſee the preceedent figure) we ſhall be ſaid to argue according to reaſon Inverſe, in this kinde of arguing Authours expreſſe themſelves after this manner, to wit, as A is to B, ſo C is to D then inverſely B ſhall be to A as D to C, which manner of arguing ſhall be demonſtrated to be true by the Corollarie of the eighth Propoſition of this Book.

Laſtly, the two firſt Magnitudes, may be of one kinde and the latter of another, for it may be very well inferred, as the line A is to the line B, ſo the Triangle or the number C, is to the Triangle or number D, then inverſely, as the line B is to the line A, ſo the Triangle or the number D, is to the Triangle or number C, as ſhall be made evident by the Corollary of the fourth Propoſition of this Book.

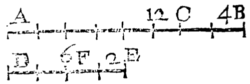
14 *Compoſition of reaſon is when the antecedent with the conſequent are taken together as one to be compared to the ſame conſequent.*

AS if there were the ſame reaſon of A C to C B, as of D F, to F E, and from that may be gathered, that there is the ſame reaſon, of the whole A B, to wit, of the antecedent with the conſequent, to C B the conſequent; as of the whole D E (to wit, the antecedent with the conſequent,) to F E the conſequent, this ſort of arguing ſhall be called Compoſition of reaſon for as much as, of the antecedent and of the conſequent there is compounded another new antecedent.

Now the Greek Authours uſe this manner of expreſſion in this kind of arguing, to wit, as A C is to C B, ſo D F is to F E: then in compounding; as

as A B shall be to C B, so D E to F E, this manner of arguing shall be demonstrated in the eighteenth Proposition of this Book.

To this, there may yet be added two other means of arguing, the first may be termed composition of Reason converse; to wit, when the antecedent and the consequent are taken as one, to be compared to the antecedent, as if (forasmuch as A C is to C B, so D F to F E) we inferre then as A B compounded of the antecedent, and of the consequent, is to the antecedent A C; so D E compounded of the antecedent, and of the consequent, is to the antecedent D F, which manner of arguing we shall shew to be of worth by the eighteenth Proposition of this Book, in which we shall use this manner of expression, when we shall speak of composition of converse reason.



The other manner of arguing, may be termed composition of contrary reason, which is done when you compare one and the same Magnitude the antecedent, to the antecedent, and to the consequent, as to one only; as if A C be to C B, so D F is to F E, and we inferre by composition of contrary reason: Therefore as A C antecedent, is to the whole A B, compounded of the antecedent, and of the consequent; so D F antecedent shall be to D E, compounded of the antecedent, and of the consequent; and we shall also demonstrate this kind of arguing to be of worth in the eighteenth Proposition of this Book.

15 *Division of Reason is when you take the excesse by which the antecedent surmounts the consequent, to compare it to the same consequent.*

As if you shall say, that there is the same reason of the whole A B to to C B, as of the whole D E to F E (see the precedent figure): Therefore there will be the same reason of A C, (the excesse by which the antecedent surmounts the consequent,) to C B consequent, as of D F, (the excesse, by which the antecedent surmounts the consequent,) to F E consequent.

Now in this Division of Reason Authors speak thus: [Therefore in dividing, &c.] But this Illation shall be demonstrated in the seventeenth Proposition of this Book.

Now to this manner of arguing there may be added two others, the first of which may be called Division of Converse Reason, to wit, when the consequent is compared to the excesse, by which the antecedent surmounts the consequent, as if A B be to C B as D E is to F E, and we conclude by Division of Reason converse; Therefore as C B consequent shall be to A C (the excesse by which the antecedent surmounts the consequent) so F E consequent shall be to D F the excesse by which the antecedent surmounts the consequent; but we shall demonstrate the certainty of this manner of arguing, in the seventeenth Proposition of this Book.

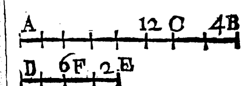
Now it is manifest that to the end that both the one and the other of those arguings may take place, (to wit, this, and that of EUCLIDE) that the antecedent ought to be greater than the consequent, otherwise the Definition cannot be made.

The other manner of arguing may be called Division of Contrary Reason, which is made when the antecedent is compared with the excesse by which the consequent surmounts the antecedent, as when we say, as A C is to A B, so D F is to D E: Then by Division of Contrary Reason A C the antecedent shall be in like manner to C B (the excesse by which the consequent surmounts the antecedent,) as D F the antecedent is to F E the excesse by the which the consequent surmounts the antecedent; which manner of arguing we shall also demonstrate in the seventeenth Proposition of this Book.

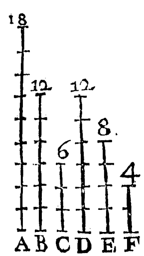
Lastly, it is manifest, that in this division of contrary Reason, the consequent ought to be greater than the antecedent, to the end the excesse may be taken, by which the consequent surmounts the antecedent.

16 *Conversion of Reason, is to take the antecedent to compare it to the excesse, by which the antecedent surmounts the same consequent.*

As if we say after this manner, as the whole Magnitude A B is to C B, so the whole D E is to F E; therefore also the same A B shall be to A C, the excesse by which the antecedent surpasseth the consequent, as D E to D F; we shall be said to argue according to conversion of Reason: Therefore Authors do expresse themselves almost after this manner: [Therefore by Conversion of Reason, &c.] But this manner of arguing shall be confirmed in the Corollary of the nineteenth Proposition of this Book. See the precedent figure.



¶ It is manifest also than in this manner of arguing by conversion of Reason, the antecedent ought to exceed the consequent; to the end the excesse by which the antecedent surmounts the consequent may be taken.

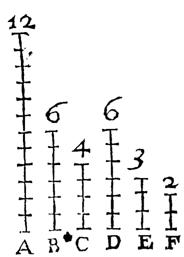


17 *Equal Reason, or Reason of equality, is when there be divers magnitudes, and other magnitudes equal to them in multitude, which may be taken two and two, and in the same reason: When as in the first magnitudes, the first is to the last of the same magnitudes; so in the second magnitudes, the first is to the last of the same. Otherwise, it is to take the extreams by the subtraction of the meanes.*

LEt A B C be several Magnitudes of one part, and D, E, and F, as many Magnitudes of the other part, which being taken two and two, in the same Reason; that is to say, as A is to B, so D is to E, and as B is to C, so E is to F, if therefore for this cause it be inferred, that there is the same Reason of A the first of the first Magnitudes, to C the last of the same Magnitudes, as of D the first of the second Magnitudes, to F the last of the same Magnitudes, this manner of arguing shall be called equal Reason or Reason of equality, to wit, in which the extream Magnitudes being substracted from the meanes, are said to have one and the same Reason to one another, as appears by the other Definitions.

But forasmuch as it is permitted to argue in proportions by two several manners, by the equality; to wit, in the one, when we take the Magnitudes two and two in the same Reason, in proceeding by order, and in the other, when the order is changed. *EUCLIDE* explaines in the two following Definitions, what is meant by Ordinate Proportion, and Perturbated or troubled Proportion.

18 *Ordinate proportion is, when as the antecedent is to the consequent, as the antecedent is to the consequent; and as the consequent is to some other, so the consequent is also to some other.*



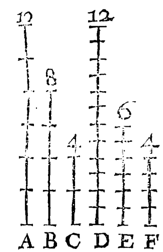
AS if A be to B, as D is to E; and again, as B the consequent is to some other, to wit, to C, so E the consequent is to some other, as to F; such manner of arguing shall be termed Ordinate: and forasmuch as the same order is observed, as well in the first Magnitudes, as in the second; seeing that on both parts the first is first of all compared with the second, and after that the second with the third. Therefore when in the manner of arguing by equality, the Ordinate Proportion is observed and kept, it is demonstrated in the two and twentieth Proposition of this Book, that this manner of arguing is good.

19 *Perturbated or troubled Proportion is, when as three magnitudes are proposed on one part, and other magnitudes, equal unto them in multitude, and as in the first magnitudes, the antecedent is to the consequent; so in the second magnitudes the antecedent is to the consequent: But as in the first magnitudes, the consequent is to some other, so in the second magnitudes, some other is to the antecedent.*

That

THat is to say, when three Magnitudes are proposed on the one part and as many on the other, the first of the first is to the second of the same; as the second of the seconds, is to the third of the same; and as the second of the first is to the third of the same; so the first of the seconds is to the seconds of the same.

As if there be the same Reason of A to B, as of E to F; then afterward in the first Magnitudes, as B consequent, is to some other, to wit C; so in the second Magnitudes, some other, to wit D, is to E the antecedent. And this sort of arguing shall be termed disturbed Proportion, forasmuch as the same order in the proportions of the Magnitudes is not observed; seeing that in the first magnitudes, the first is compared with the second: but in the second Magnitudes, the second is compared with the third: And again, in the first Magnitudes, the second is compared to the third, but in the second Magnitudes, the first is compared with the second. Therefore, when in the manner of arguing by equality, the proportion disturbed is kept and observed, it is demonstrated that this manner of arguing is good, as in the three and twentieth Proposition of this Book.

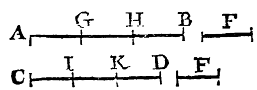


Lastly, as well the disturbed Proportion, as the Ordinate, inferre alwayes by equality, that there is the same proportion of the extreams, although more than three Magnitudes be proposed, as shall be made manifest by the two and twentieth and three and twentieth Propositions of this Book.

PRO-

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

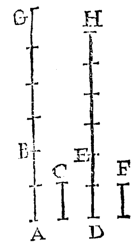


If there be a number of magnitudes AB *and* CD, *as many as you please, equal in multitude to other magnitudes* E *and* F, *and equimultiplices to them, each to his correspondent, as the one of the magnitudes* AB, *shall be multiplex of one* E, *so the whole* AB *and* CD, *shall be Multiplex of the whole* E *and* F.

Demonstration FOr seeing that A B and C D are equimultiplices of E and F,
if A B be divided according to the Magnitudes A G, G H,
and H B, equal each of them to E, and C D in like manner according
to C I, I K, and K D, equal each to F. Now each of them may be divided
into parts totally equal to E and F; seeing that A B and C D are equi-
multiplices of E and F; and therefore E shall be conteined exactly as many
times in A B, as F in C D, as appears by the second Definition of this
Book; the Magnitudes A G, G H, and H B, shall be as many in number
as C I, I K, and K D. But forasmuch as A G and E, are equal to one ano-
ther, if to them be added the equal Magnitudes C I and F, A G and
C I together, they shall be equal to E and F together. In like manner,
GH and I K together shall be equal to E and F together; and also H B
and K D to the same E and F; therefore as many times as E is contained
in A B, or F in C D, so many times E and F together shall be conteined
in A B and C D together; therefore as A B is Multiplex of E, so A B
and C D together, shall be Multiplex of E and F together, by the second
Definition of this Book: Therefore, If there be, &c. Which was to be
demonstrated.

PROP.

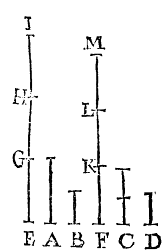
PROP. 2. THEOR. 2.



If the first AB, *be as much multiplex of the second* C, *as the third* DE *is of the fourth* F, *and that the fifth* BG *be also as much Multiplex of the second* C, *as the sixth* EH *is of the fourth* F. *The magnitude* AG, *compounded of the first* AB, *and of the fifth* BG, *shall be as much multiplex of the second* C, *as the magnitude* DH, *compounded of the third* DE, *and of the sixth* EH, *is of the fourth* F.

Demonstration FOr seeing that A B and D E are equimultiplices of C, and
of F, there will be in A B as many Magnitudes equal to
C, as there are in D E, equal to F: In like manner, there will be in B G
as many Magnitudes equal to C, as there are in E H, equal to F. If there-
fore to the equal Multitudes A B and D E, the equal Multitudes B G
and E H, all the Multitudes A G and D H shall be equal in number:
Therefore C shall be conteined as many times in A G as F is conteined in
D H: Therefore A G (compounded of the first and fifth,) shall be as
much Multiplex of C the second, as D H (compounded of the third and
the sixth,) is of F the fourth: Therefore, If the first, &c. Which was to
be demonstrated.

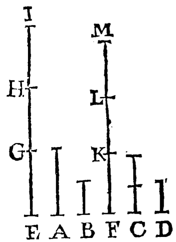
PROP. 3. THEOR. 3.



If the first A, *be as much Multiplex of the second* B, *as the third* C, *is of the fourth* D, *and there be taken the equimultiplices* E *and* F, *of the first* A, *and of the third* C, *in equal reason, the magnitude* E, *taken multiplex of the first* A, *shall be also so much multiplex of the second as the magnitude taken* F, *multiplex of the third* C, *shall be of the fourth* D.

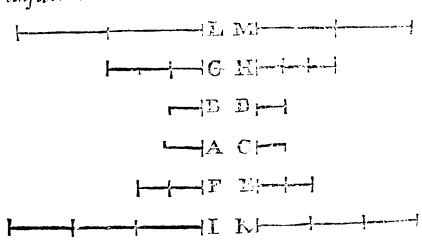
Demonstration FOr seeing that E and F are equimultiplices of A and C,
if E be divided into Magnitudes equal to A, as into E G,
G H, and H I, also F into Magnitudes, equal to C, as into F K, K L,
and

and L M; there will be in E as many parts equal to A, as there are in F, equal to D; forasmuch therefore as E G is equal to A, and F K equal to C: But A and C are equimultiplices of B and D, by supposition; E G and F K shall be equimultiplices of the same B and D: In like manner, G H, and K L, and H I and L M shall be equimultiplices of the same B and D: forasmuch therefore as E G the first magnitude, is so much Multiplex of the second B, as F K the third is Multiplex of the fourth D, also G H the fifth, is as much Multiplex of B the second, as K L the sixth is Multiplex of the fourth D; also G H the fifth is as much Multiplex of B the second, as K L the sixth is Multiplex of the fourth D; E H compounded of the first and of the fifth, shall be as much Multiplex of the second B, as F L compounded of the third, and the sixth is Multiplex of the fourth D. Again, forasmuch as E H the first is as much Multiplex of B the second, as F L the third, is of D the fourth, as we even now demonstrated: But H I the fifth is so much Multiplex of the second B, as L M the sixth is of D, the fourth E I, compounded of the first and fifth, shall be as much Multiplex of the second B, as F M compounded of the third, and of the sixth, is of D the fourth, and so of the others, if there be more parts in E and F: Therefore, If, &c. Which was to be demonstrated.



PROP. 4. THEOR. 4.

If the first A, *hath the same reason to the second* B, *as the third* C, *to the fourth* D, *also the equimultiplices* E *and* F, *of the first* A, *and of the third* C, *shall have the same reason to the equimultiplices* G *and* H, *of the second* B, *and of the fourth* D, *according to any Multiplication, if they be taken so as that they answer to one another.*



Demonstration FOr if you constitute E the first Magnitude, and G the second, F the third, and H the fourth, and that I and K any what ever equimultiplices of E and F, the first and third also L and M, some equimultiplices of G the second, and H the fourth; forasmuch

as

as E the first is as much Multiplex of A the second, as F the third, is of C the fourth, and that I and K be taken equimultiplices of E and F, the first and third, in equal reason, [a] I and K shall be also equimultiplices of A and C, the second and fourth. By the same reason L and M are equimultiplices of B and D; and forasmuch as A is to B, as C to D, and that I and K are taken equimultiplices of A and C, the first and third, [b] if I the Multiplex of the first A, wanteth of L the Multiplex of the second B; also K the Multiplex of the third C, shall want of M the Multiplex of the fourth D: if I be equal to L, also K shall be equal to M: and lastly, if I exceed L, also K shall exceed M, according to any whatsoever Multiplication they be taken. Therefore, seeing that I and K are equimultiplices of E the first, and F the third, also L and M equimultiplices of G the second, and H the fourth, and that it hath been demonstrated that if I the Multiplex of E the first, be lesse than L the Multiplex of G the second, that K the Multiplex of F the third, is lesse than M the Multiplex of H the fourth, and if I be equal to L, K is equal to M, and if I be greater than L, K is greater than M, and that this doth happen in every Multiplication, there [c] shall be the same reason of E the first, to G the second, as of F the third, to A the fourth; Therefore if the first, hath the same reason to the second, as the third to the fourth, &c. Which was to be demonstrated.

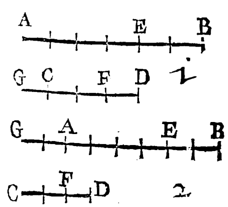
a) 3.
b) 6. def.
c) 6. def.

COROLLARIE.

By this Demonstration the proof of inverse reason explained by EUCLIDE, *is manifest;* [d] *which is if four Magnitudes are proportional, they shall be the same taken backwards, or contrarily, to wit, if* A *be to* B, *as* C *to* D, *in inverse reason as* B *shall be to* A, *so* C *to* D; *for seeing that as* A *is to* B, *so* C *is to* D; [e] E *and* F *equimultiplices of* A *and* C *first and third, shall be both greater, equal, or lesse than* G *and* H, *equimultiplices of* B *and* D, *the second and fourth, in any Multiplication whatever: Therefore contrarily,* G *and* H *shall be both lesse, equal, or greater than* E *and* F. *Therefore* [f] *there will be the same reason of* B *to* A, *as of* D *to* C.

d) 13. def.
e) 6. def.
f) 6. def.

PROP. 5. THEOR. 5.



If a magnitude AB *be as much Multiplex of a magnitude* CD, *as the part cut off* AE, *is of the part cut off* CF; *also the remainder* EB *shall be as much Muliplex of the remainder* FD, *as the whole* AB, *is of the whole* CD.

Demonstration ASsume CG, of which EB may be so much Multiplex, as AE is of CF, or the whole AB of the whole CD: forasmuch as AE and EB are equimultiplices of CF and GC, the [a] whole AB, shall be so much Multiplex of the whole GF, as AE is of CF; that is to say, the whole of the whole, as one only Magnitude, of one only Magnitude. But AB is proposed so much Multiplex of CD,

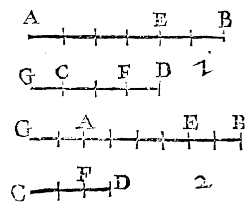
a) 1. 5.

 as

as A E of C F; therefore A B shall be as much Multiplex of G F, as of
b) 6. c. s. C D; therefore [b] G F and C D are equal; therefore taking away the
common C F, there remains D F, equal to G C; therefore E B shall be
as much Multiplex of F D, as of C G, but E B is as much Multiplex
of C G, by supposition, as A E is of C F; that is to say, as the whole
A B of the whole C D: Therefore the remainder E B shall be as much
Multiplex of the remainder F D, as the whole A B, of the whole C D.
Which was proposed.

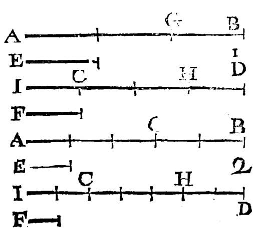
Otherwise, Let A B be as much Multiplex of the whole C D, as the
part cut off A E, of the part cut off C F. I say, that the rest E B, is as
much Multiplex of the rest F D, as the whole A B, of the whole C D.

For having taken G A as much Multiplex of F D, as A E is of C F,
or the whole A B of the whole C D: Forasmuch as A E and G A are
c) 1. 5. equimultiplices of G F and F D, [c] the whole G E shall be as much Multiplex of the whole C D, as A E of C F; But A B is in like manner, also as much Multiplex of C D, as A E of C F, by supposition: Therefore G E and A B
d) 6. c. s. are equimultiplices of C D; [d] and therefore equal to one another, taking away then the common A E, there will remain G A and E B equal; and therefore equimultiplices of F D; seeing that G A is taken as much Multiplex of F D as A E of C F, or the whole A B of the whole C D; therefore the remainder E B shall be as much Multiplex of the remainder F D, as the whole A B of the whole C D, which was proposed: Therefore, If a Magnitude, &c. Which was to be demonstrated.



PROP. 6. THEOR. 6.

If two magnitudes A B *and* C D, *are equimultiplices of two other magnitudes* E *and* F, *and there be cut off from them some equimultiplices* A G *and* C H, *of the same magnitudes* E *and* F, *either the remainders* G B *and* H D, *shall be equal to the same* E *and* F, *or equimultiplices of the same* E *and* F.



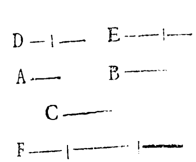
Demonstration FOr in the first place, Let G B be equal to E: I say, that H D shall be equal to F: Let C I be put equal to F; forasmuch as
A G the first, is as much Multiplex of E the second, as C H the third, is of F
the fourth, & that G B the fifth, is equal to E, and I C the sixth, equal to F the
a) 2. 5. fourth, [a] A B compounded of the first and of the fifth, shall be as much Multiplex of

of E the second, as I H compounded of the third and sixth, is of F the
fourth. But C D is as much Multiplex of F by Supposition, as A B is of E: b) 6. c. s.
therefore H I and C D are equimultiplices of F. Therefore [b] equal to one
another; therefore taking away the common part C H, there will remain
C I and H D equal: Now seeing that C I is put equal to F, H D shall be
likewise equal to F, which was proposed. In like manner we shall demonstrate, that if G B be Multiplex of E, that H D shall be as much
Multiplex of F, in putting I C as much Multiplex of F, as G B is of E, &c.

Otherwise, forasmuch as A B and C D are equimultiplices of E and
F, there will be in A B as many Magnitudes equal to E, as there are in C D
equal to F. Again, forasmuch as A G and C H are equimultiplices of the
same E and F, [c] there will be in like manner in A G as many magnitudes c) 1, 2. def.
equal to E, as there are in C H equal to F.

If therefore from the equimultiplices A B and C D, you take away the
equimultiplices A G and C H, there will remaine the equimultiplices G B
and H D equimultiplices. Therefore G B shall contein as many times E, as
H D shall contein F, if once only, G B and H D shall be equal to E F, if more
than once, they shall be equimultiplices. Which was proposed.

PROP. 7. THEOR. 7.



Equal magnitudes A *and* B, *have the same reason to one and the same magnitude* C, *and one and the same magnitude* C *hath the same reason to the equal magnitudes* A *and* B.

Demonstration FOr assume D and E the equimultiplices of the equal Magnitudes A and B, [a] the same D and E shall be equal to one a) 6. c. s.
another. Again, Let F be any Multiplex of C; forasmuch then as
D and E are equal, and equimultiplices of A and B, the first and third,
both the one and the other shall be lesse than F, or equal, or greater, after
what Multiplication soever they be taken.

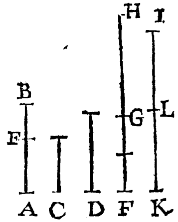
Therefore seeing that D and E equimultiplices of the first A, and of
the third B, are both greater than F, the Multiplex of the second and
of the fourth, (for C is put as for two Magnitudes) or equal, or lesse;
[b] there will be the same reason of A the first, to C the second, as of B the b) 6. def. 5.
third, to D the fourth.

In the same manner, we shall demonstrate that F is lesse than each of
the Magnitudes D and E, or equal, or greater: Therefore seeing that F the
Multiplex of the first, and of the third C, is lesse than either D or E,
equimultiplices of the second, and the fourth A and B, or equal, or greater, there will be likewise the same reason of the first C, to the second A,
as of the third C, to the fourth B. Which was proposed.

SCHOLIUM.

This second part may be more briefly demonstrated by Inverse Reason, it having been already demonstrated that there is the same reason of A *to* C, *as of* B *to* C; *alternately,* [c] *there shall be the same reason of* C *to* A, *as of* C *to* B. *Therefore the* c) Cor. 4. 5.
equal Magnitudes, &c. Which was to be demonstrated.

PROP. 8. THEOR. 8.



Of unequal magnitudes A, B *and* C, *the greater* A B, *hath greater reason to one and the same magnitude* D, *than the lesser* C, *and one and the same magnitude* D, *hath greater reason to the lesser* C, *than to the greater* A B.

Demonstration. FOr from the greatest A B cut off A E equal to C; and let A B be proposed the first Magnitude, C the third, and D the second and fourth, then take H G and G F, equimultiplices of A E and E B, in such sort as G H the Multiplex of A E be greater than D, and F G the Mutiplex of E B, be not lesse than the same D, but also greater.

Now forasmuch as H G and G F are equimultiplices of A E and E B; [a] the whole H F shall be as much Multiplex of the whole A B, as G H is of A E; that is to say of C, equal to A E. In like manner, Let I K be taken Multiplex of D the second and fourth, in such sort as that it be greater than H G, but lesse than the whole H F, which will be easily done; for seeing that D is lesse than F G, and also lesse than G H, you have only to adde D so many times as its product I K exceeds H G; by this means I K shall be lesse than the whole H F, and greater than G H, for having cut off K L equal to D, it shall be lesse than F G, (F G being taken greater than D,) and the remainder L I lesse than G H, or equal, (the same G H being put greater than D,) and D added only so many times as its product I K exceeds G H. Therefore I K shall be greater than G H, and lesse than the whole H F. Therefore seeing that F H and H G are equimultiplices of A B the first, and C the third, and I K Multiplex of D, put for the second and the fourth; but F H the Multiplex of the first, is not greater than I K the Multiplex of the fourth, but lesse by supposition, ([b] for I K the Multiplex of D, is taken greater than H G,) there shall be greater reason of A B the first, to D the second, then of C the third, to D the fourth.

a) 2. 5.

b) 8. def.

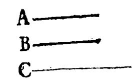
Secondly, forasmuch as I K the Multiplex of D the first; (Let D be taken for first, and third, and C the second, and A B fourth,) is greater than H G Multiplex of C the second: But I K the Multiplex of D the third, is not greater than F H Multiplex of A B the fourth, but lesse; F H being greater than I K, as hath been shewn, there will be greater reason of D the first, to C the second, than of D the third, to A B the fourth; which was proposed: Therefore, The Magnitudes, &c. Which was to be demonstrated.

PROP. 9. THEOR. 9.

Magnitudes A *and* B, *which have the same reason to one and the same magnitude* C, *are equal to one another: and those magnitudes* A B, *to which one and the same magnitude* C, *hath the same reason, are equal to one another.*

Demonstration. OTherwise, they should be unequal, and the one greater than the other. Let A then be the greater, and B the lesser, (if possible:) Therefore [a] there will be greater reason of A the greater, to C, then of B the lesser, to the same C, which is contrary to Supposition: Therefore A and B are not unequal, but equal.

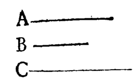
a) 8. 5.



Secondly, Let C have the same reason to A, as to B. I say again, that A and B are equal to one another: for if the one were greater, to wit A, and B the lesser, C [b] would have greater reason to B the lesser, then to A the greater, which is contrary to Supposition: Therefore A shall not be greater than B, but equal thereto. Therefore Magnitudes, &c. Which was to be demonstrated.

b) 8. 5.

PROP. 10. THEOR. 10.



Of magnitudes A, *and* B, *which have reason to one and the same magnitude* C, *that which hath the greatest reason* A, *is the greater; but that* B *to which one and the same magnitude* C, *hath the greatest reason, is the least.*

Demonstration. FIrst, Let A have greater reason to C, then B to the same C: I say that A is greater than B, for if A were equal to B, the same [a] A and B would have the same reason to C, and if A were lesse than B, [b] the greatest B would have greater reason to C, than the least A, which is contrary to supposition; therefore A is not equal to B, nor lesse, but greater.

a) 7. 5.

b) 8. 5.

Secondly, Let C have greater reason to B then to A. I say that B is lesse than A; [c] for then C would have the same reason to A as to B, which is contrary to supposition: B also shall not be greater then A, forasmuch as C should have greater reason to the least A, then to B the greater, which is contrary to supposition: Therefore B is lesse than A, which was proposed: Therefore the Magnitudes, &c. Which was to be demonstrated.

c) 7. 5.

PROP. 11. THEOR. 11.

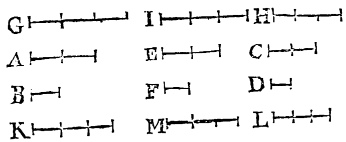
Reasons which are the same to one and the same reason, are also the same to one another.

LEt the reason of A to B, be as that of E to F, and E be to F, as C to D: I say that as A is to B, so C is to D.

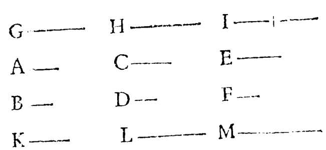
Demonstration. FOr let there be taken any equimultiplices whatsoever, of all the antecedents A, E, and C, which may be G, I, and H: In like manner, Let there be also taken any whatsoever equimultiplices K,

K, M, and L, of the consequents B, F, and D, forasmuch as A the first is to B the second, as E the third, to F the fourth, [a] if G the Multiplex of A the first, wanteth of K the Multiplex of B the second, I the Multiplex of E the third, shall also want of M the Multiplex of F the fourth, and if G be equal to K, I shall be equal to M, and if G be greater than K, I, shall be greater than M. [b] But (as shall be demonstrated in the same manner,) if I, be lesse than M, or equal, or greater: Likewise H shall be lesse than L, or equal, or greater; forasmuch as by Supposition there is the same reason of E the first, to F the second, as of C the third, to D the fourth. Wherefore if G the Multiplex of A the first, be lesse than K the Multiplex of B the second; H the Multiplex of C the third, shall be lesse than L the Multiplex of D the fourth, and if G be equal, or greater than K, also H shall be equal or greater than L, and shall be demonstrated that it will happen so in any whatever other equimultiplices: Therefore [c] there shall be the same reason of A the first, to B the second, as of C the third, to D the fourth. Therefore, Reasons which are the same, &c. Which was to be demonstrated.

a) 6. def. 5. b) 6. def. 5. c) 6. def. 5.



PROP. 12. THEOR. 12.



If as many magnitudes as you please A, B, C, D, E, *and* F, *are proportional, as one of the antecedents* A, *shall be to one of the consequents* B; *so all the antecedents* A, C, *and* E, *shall be to all the consequents* B, D, *and* F.

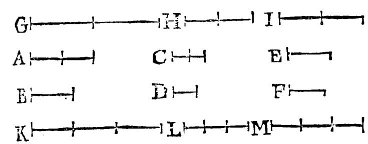
Demonstration FOr assume G, H, and I, equimultiplices of the antecedents A, C, and E, and K, L, and M, equimultiplices of the consequents B, D, and F, [a] the whole G, H, and I, together, shall be as much Multiplex of the whole A, C, and E, together, as one of them alone, is of one alone, to wit, as G is of A, and the whole K, L, and M together, shall be as much Multiplex of the whole B, D, and F, together, as one alone, is of one alone, to wit, as K is of B: but forasmuch as by supposition there is the same reason of A the first, to B the second, as of C the third, to D the fourth, and as another E the third, to F the fourth, it will happen that if G the Multiplex of A the first, doth want of K the Multiplex of B the second, In like manner H the Multiplex of C the third, will want of L the Multiplex of D the fourth, and I of M, and if G be equal to K, or greater. Likewise, H shall be equal to L, and I to M, or greater; and therefore if G be lesse or equal, or greater than K, the whole G, H, and I, together, shall be lesse than the whole K, L, and M, together, or equal, or greater. Wherefore as

a) 1. 5. 6. def. 5.

as A the first, is to B the second, so A, C, and E, the third, shall be to B, D, and F, the fourth: Therefore if there be, &c. Which was to be demonstrated.

6. def. 5.

PROP. 13. THEOR. 13.

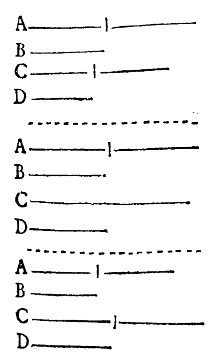


If the first A, *hath the same reason to the second* B, *as the third* C, *hath to the fourth* D; *But the third* C, *hath greater reason to the fourth* D, *than the fifth* E, *to the sixth* F, *also the first* A, *shall have greater reason to the second* B, *than the fifth* E, *to the sixth* F.

Demonstration FOr, assume G, H, and I, equimultiplices of the antecedents A, C, and E, and K, L, and M, equimultiplices of the consequents B, D, and F: forasmuch as there is the same reason of A the first, to B the second, as of C the third, to D the fourth, [a] it will happen, that if G, the Multiplex of A the first, exceed K, the Multiplex of B the second, that H the Multiplex of C the third, will exceed L the Multiplex of D the fourth, &c. But when H exceeds L, [b] it is not necessary that I exceed M, but shall be sometimes equal, or lesse; forasmuch as there is greater reason of C the first, to D the second, then of E the third, to F the fourth; therefore if G exceed K, of necessity I shall exceed M: and [c] therefore there is greater reason of A the first, to B the second, then of E the fifth, to F the sixth: Therefore, &c. Which was to be demonstrated.

a) 6. def. 3. b) 6. def. 5. c) 8. def. 5.

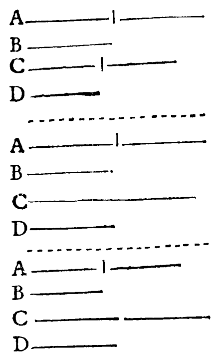
PROP. 14. THEOR. 14.



If the first A, *hath the same reason to the second* B, *as the third* C, *to the fourth* D, *and that the first* A, *be greater than the third* C, *the second* B, *shall be also greater than the fourth* D, *and if the first* A, *be equal to the third* C, *also the second* B, *shall be equal to the fourth* D, *and if lesse, lesse.*

Demonstration IN the first place, Let A the first, be greater than C the third,

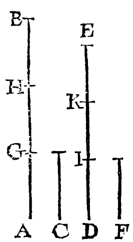
a) 8. 5. third, there [a] will be greater reason of A the greater, to B, then of C the lesser, to the same B: Therefore forasmuch as C the first, is to D the second, as A the third, to B the fourth: But the reason of A the third, to B the fourth, is greater than the reason of C the fifth, to B the sixth, as we have shewn;
b) 13. 5. [b] there will be therefore greater reason of C the first, to B the second, then of C the fifth, to B the sixth; therefore D shall be lesse than B, therefore B shall be greater than D, which was proposed.



Secondly, Let A be equal to C, there
c) 7. 5. [c] will be the same reason of A to B, as of C to B; forasmuch then as the reasons of C to D, and of C to B, are the same as the reason of A to B,
d) 11. 5. [d] the reasons of C to D, and of C to B, shall be the same to one another: Therefore B and D, shall be equal, which was proposed.

e) 8. 5. Lastly, Let A be lesse than C, [e] there will be greater reason of the greatest C, to B, then of the least A, to the same B. Forasmuch then as C the first, is to D the second, as A the third, to B the fourth. But the reason of A the third, to B the fourth, is lesse than that of C the fifth,
f) 13. 5. to B the sixth; there [f] will be likewise lesse reason of C the first, to D the second, than of C the fifth, to B the sixth: Therefore B shall be lesse than D, which was
g) 10. 5. proposed: [g] Therefore, If, &c. Which was to be demonstrated.

PROP. 15. THEOR. 15.



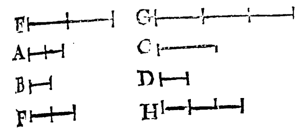
The parts C *and* F *are to one another, as are their equimultiplices* A B *and* D E, *to one another, if they be taken so as they mutually answer to one another.*

Demonstration FOr seeing that A B is as much Multiplex of C, as D E is of F, the Magnitude C shall be as oftentimes conteined in A B, as F is conteined times in D E.

Let therefore A B be divided according to the parts A G, G H, and H B, each equal to C, and D E according to the parts D I, I K, and K E, each equal to F. And forasmuch as A G and C are equal, and D I and F, also equal, A G [a] shall be to D I, as C shall
a) 7. 5. be to F, by the same reason, G H shall be to I K, and H B to K E, as C to F; and therefore A G, G H, and H B, shall have the same reason to D I, I K, and K E: Wherefore as A G shall be to D I, that is to say, as C to
b) 12. 5. F, so [b] A B shall be to D E, to wit, all the antecedents together, A G, G H, and H B, to all the consequents together D I, I K, and K E, which was proposed. Therefore, The parts, &c. Which was to be demonstrated.

PROP.

PROP. 16. THEOR. 16.



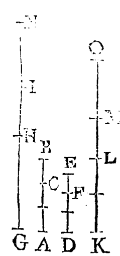
If four magnitudes A, B, C, *and* D, *be proportional, they shall be also proportional alternately.*

Demonstration FOr, Let A be to B, as C is to D; I say that alternately, or by permutation, A shall be to C, as B to D.

Assume E and F equimultiplices of A and B, the first and second; Also G and H equimultiplices of C the third, and D the fourth, E shall be
a) 15. 5. to F, as A is to B; [a] Seeing that E and F are equimulciplices of A, and by the same reason, G shall be to H, as C to D.

Now seeing that the reasons of E to F, and C to D, are the same as the
b) 11. 5. reason of A to B, [b] they shall be the same to one another. Again, forasmuch as the reasons of E to F, and G to H, are the same as the reason of
c) 11. 5. C to D, [c] they shall be also the same to one another: That is to say, as E the first, shall be to F the second, so G the third, to H the fourth:
d) 14. 5. Therefore [d] if E the first be greater than G, the third, F the second, shall be greater than H the fourth, and if E be equal or lesse than G, also F shall be equal or lesse than H, in whatsoever Multiplication they be ta-
e) 6. def. 5. ken E F and G H. Therefore [e] A the first, shall be to C the second, as B the third, to D the fourth; seeing that E and F, equimultiplices of the first and third A and B, are both greater than G and H, equimultiplices of the second and the fourth C and D, or both equals or both lesse, &c. Which was proposed. Therefore, If four Magnitudes, &c. Which was to be demonstrated.

PROP. 17. THEOR. 17.

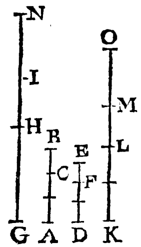


If Compounded Magnitudes A B, C B, D E, *and* F E, *are proportional, they shall be also proportional, being divided.*

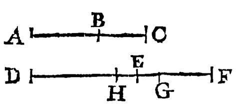
Demonstration FOr, Assume the equimultiplices of A C, C B, D F, and F E, of the same or-
a) 1. 5. der, to wit, G H, H I, K L, and L M, and [a] G I shall be as much Multiplex of A B, as G H is of A C;
b) 1. 5. that is to say, as K L is Multiplex of D F, [b] K M is likewise Multiplex of D E: Therefore G I and K M are equimultiplices of A B and D E. Again, assume I N and M O, equimultiplices of C B and F E. Forasmuch as H I the first, is as much Multiplex of C B the second, as L M the third, is of F E the fourth, also I N the fifth, as much multiplex of C B the second, as M O the sixth, is of F E the fourth, H N compounded of the first and the fifth, shall be as much multiplex of the second C B, as L O compounded of the third and sixth, is of F E the fourth. Therefore seeing that A B the first, is to C B the second, as D E the third, to F E the

fourth,

fourth, and that the equimultiplices of the firſt and of the third, AB and DE, have been aſſumed; to wit, GI and KM. In like manner, of the ſecond, and the fourth, CB and FE, the equimultiplices HN and LO,
c) 6. def. 5. c) it will happen that if GI the Multiplex of the firſt AB, wanteth of HN the Multiplex of the ſecond CB, alſo KM the Multiplex of the third DE, ſhall want of LO the Multiplex of the fourth FE, and if it be equal, equal; and if greater, greater. And if as well GI wanteth of HN, as KM of LO, having taken away the common magnitudes HI and LM: in the like manner, GH will want of IN, and KL of MO; and if GI be equal to HN, and KM to LO, having taken away the common magnitudes HI and LM; GH ſhall be equal to IN, and KL to MO. And if laſtly, GI exceed HN, and KM, LO, taking away the common parts HI and LM; GH ſhall likewiſe exceed IN, and KL, MO. Therefore ſeeing that GH and KL have been aſſumed equimultiplices of AC the firſt, and of DF the third. In like manner, IN and MO, equimultiplices of CB the ſecond, and FE the fourth, and it is ſhewn in any multiplication whatſoever, that the equimultiplices of the firſt and third, do want together of the equimultiplices of the ſecond and of the fourth, or together are equal, or do
d) 6. def. 5. exceed them. d) As AC the firſt, to CB the ſecond, ſo DF the third, ſhall be to FE the fourth. Which was propoſed. Therefore, If the magnitudes compounded, &c. Which was to be demonſtrated.



PROP. 18. THEOR. 18.



If Magnitudes divided AB, BC, DE, *and* EF, *are proportional, being compounded they ſhall alſo be proportional.*

Demonſtration. Let it be, as AB is to BC, ſo DE to EF: I ſay, that being compounded, they ſhall be proportional, that is to ſay, as AC is to BC, ſo DF is to EF.

For if AC be not to BC, as DF is to EF and DF, ſhall have the ſame reaſon to ſome other magnitude, leſſe than EF, or greater than the ſame EF, as AC to BC.

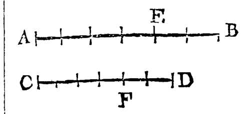
17. 5. Firſt, Let DF have the ſame reaſon to GF, leſſe than EF, as AC to BC, (if it may be done.)

Foraſmuch as AC is to BC, as DF to GF, by dividing, as AB is to BC, ſo DG ſhall be to GF: But as AB to BC, ſo DE to EF, by ſup-
11. 5. poſition. Therefore as DG the firſt, to GF the ſecond, ſo DE the third, to EF the fourth; and ſeeing that DG the firſt, is greater than DE the third; likewiſe GF the ſecond, ſhall be greater than EF the fourth, the part than the whole, which is impoſſible.

Secondly, Let DF have the ſame reaſon to HF, greater than EF, (if it may be) as AC hath to BC. Foraſmuch as AC is to BC, as DF to
17. 5. HF; by dividing, as AB ſhall be to BC, ſo DH to HF. But as AB to BC,

BC; ſo is DE to EF, by ſuppoſition: Therefore as DH the firſt, to HF the ſecond, ſo DE the third, to EF the fourth. And ſeeing DH the 11. 5.
firſt, is leſſe than DE the third. Likewiſe, HF the ſecond, ſhall be leſſe than EF the fourth, the whole than its part, which is abſurd: Therefore DF ſhall not have the ſame reaſon to GE the leſſer, as to EF, or to HF the greater, as AC to BC: Therefore DF ſhall be to EF, as AC to BC; Which was propoſed. Therefore, If the Magnitudes, &c. Which was to be demonſtrated.

PROP. 19. THEOR. 19.

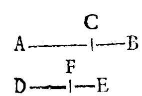


If as the whole AB, *is to the whole* CD, *ſo the part cut off* AE, *to the part cut off* CF, *then the remainder* EB, *ſhall be to the remainder* FD, *as the whole* AB, *to the whole* CD.

Demonſtration. For ſeeing that AB is to CD, as AE to CF, alternately, AB ſhall be to AE, as CD to CF: Therefore divided- 16. 5.
ly, EB ſhall be to AE, as CD to CF; therefore again alternately, EB ſhall be to FD, as AE to CF; that is to ſay, as the whole AB, to
the whole CD; ſeeing that AB is put to CD, as AE to CF: Therefore, 17. 5.
If as the whole, &c. Which was to be demonſtrated.

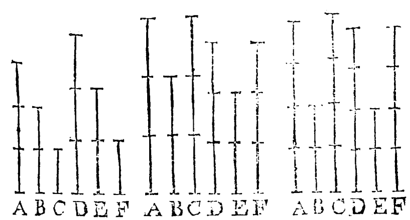
COROLLARIE.

Hence ſhall be demonſtrated the manner of arguing in proportions, which is taken from the converſion of reaſon, according to the ſixteenth Definition.



For, Let it be as AB *to* CB, *ſo* DE *to* FE. *I ſay, by converſion of reaſon, as* AB *is to* AC, *ſo* DE *to* DF: *For ſeeing that as* AB *is to* CB, *ſo* DE *is to* FE, *in dividing,* AC *ſhall be to* CB,
as DF *to* FE. *Therefore, alternately, as* CB *to* AC, *ſo* 17. 5.
FE *to* DF, *and therefore in compounding, as* AB *ſhall be to* AC, *ſo* DE *to* DF. *Which was propoſed.*

PROP. 20. THEOR. 20.

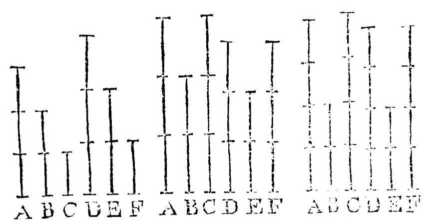


If there be three magnitudes A, B, *and* C, *and others equal to them in number* D, E, *and* F, *which being taken two and two, and in the ſame reaſon, as* A *to* B, *ſo* D *to* E, *and*

and B *to* C, *as* E *to* F, *and that in equal reason, the first* A, *be greater than the third* C, *also the fourth* D *shall be greater than the sixth* F, *if equal, equal, and if lesse, lesse.*

13. 5. *Demonstration* FOr seeing that A is greater than C, there will be greater reason of A to B, than of C to B. But as A is to B, so D is to E, there will be therefore greater reason of D to E, then of C to B. But as C is to B, so F to E; (for seeing that as B to C, so E to F; alternately, as C shall be to B, so F shall be to E,) there will be therefore in like manner, greater reason of D to E, than of F to E. Therefore D shall
10. 5. be greater than F. Which was proposed.
7. 5. Secondly, Let A be equal to C: I say that D shall be equal to F. For seeing that A is equal to C, as A shall be to B, so C to B; but as A is to B,
11. 5. so D is to E: Therefore D shall be to E, as C to B; but as C is to B, so F
9. 5. is to E, (by inverse reason, as before) therefore D shall be likewise to E, as F to E; therefore D and F shall be equal; which was proposed.
Thirdly, Let A be lesse than C: I say that D is lesse than F: For seeing that A is lesse than C, there will be lesse reason of A to B, than of C to B: but
8. 5. as A is to B, so D is to E; there will be therefore lesse reason of D to E,
15. 3. than of C to B; but by inverse reason as before, as C is to B, so F is to E: Therefore there is lesse reason of D to E, then of F to E; therefore D
10. 5. shall be lesse than F. Which was proposed. Therefore, If there be three magnitudes, &c. Which was to be demonstrated.

PROP. 21. THEOR. 21.

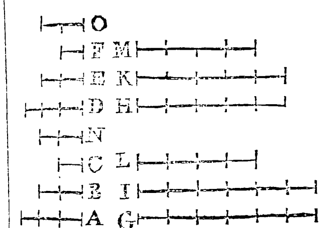


If there be three magnitudes A, B, *and* C, *and others equal to them in number,* D, E, *and* F, *which being taken two and two, and in the same reason, and that their proportion be perturbate, or without order, (that is, as* A *to* B, *so* E *to* F, *and as* B *to* C, *so* D *to* E:) *But that in equal reason, the first* A *be greater than the third* C, *the fourth* D, *shall be also greater than the sixth* F, *if equal, equal, if lesser, lesser.*

5. c. *Demonstration* FOr seeing that A is greater than C, there will be greater
13. 5. reason of A to B, than of C to B, but as A is to B, so E is to F; there will be therefore in like manner, greater reason of E to F, than of C to B. But seeing that as B is to C, so D is to E, by inverse reason, as

as C is to B, so E is to D. Wherefore there will be also greater reason of E to F, than of E to D; and therefore D shall be greater than F. 10. 5.
Which was proposed.
Secondly, Let A be equal to C; I say that D shall be equal to F. For 7. 5.
seeing that A is equal to C, as A shall be to B, so C shall be to B; but as
A is to B, so E is to F; Therefore as C is to B, so E shall be to F; but by 11. 5.
inverse reason, as C is to B, so E is to D, as before; therefore as E shall 9. 5.
be to F, so E shall be to D; and therefore D shall be equal to F. Which was proposed.
Lastly, Let A be lesse than C: I say that D shall be lesse than F: for
seeing that A is lesse than C, there will be lesse reason of A to B, than of 8. 5.
C to B: But as A is to B, so E is to F; therefore there will be lesser reason of E to F, than of C to B: But forasmuch as before, by inverse reason,
as C is to B, so E is to D, there will be lesser reason of E to F, then of E 10. 1.
to D. Wherefore D shall be lesse than F. Which was proposed. Therefore, If there be, &c. Which was to be demonstrated.

PROP. 22. THEOR. 22.



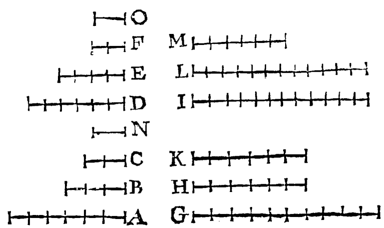
If there be as many magnitudes as you please, A, B, *and* C, *and others equal to them in number* D, E, *and* F, *which being taken two and two, and in the same reason, (that is as* A *to* B, *so* D *to* E, *and* B *to* C, *as* D *to* F,) *those magnitudes in equal reason, shall be proportional, that is as* A *to* C, *so* D *is to* F.

Demonstration FOr, assume G and H equimultiplices of A and D, and I and K, equimultiplices of B and E, and L and M of C and F:
And seeing that A the first, is to B the second, as D the third, is to E the 4. 5.
fourth. In like manner, G the Multiplex of A the first, shall be to I the Multiplex of B the second, as H the Multiplex of D the third, to K the Multiplex of E the fourth; by the same reason, seeing that B the first, is to C the 4. 5.
second, as E the third, to F the fourth, I the Multiplex of B the first, shall be to L the Multiplex of C the second, as K the Multiplex of E the third, shall be to M the Multiplex of F the fourth: Forasmuch then as there are
three magnitudes G, I, and L, and other three H, K, and M, which are 20. 5.
taken two and two, and in the same reason. It will happen, that if G the first, exceed L the third, that H the fourth, will exceed M the sixth, and if equal, equal, and if lesse, lesse. Wherefore seeing that G and H, the equimultiplices of the first A, and of the third D, either want together, of L and M equimultiplices of C the second, and of F the fourth, or together are equal, or together do exceed, according to whatsoever multiplications those equimultiplices be taken; A the first, shall be to C the second, as D the third, 6. def. 5.
is to F the fourth, which was proposed.

Again,

Again, Let them be more than three magnitudes, in ſuch ſort as C be alſo to N, as F to O, I ſay moreover, that as A ſhall be to N, ſo D ſhall be to O; for ſeeing it hath been already ſhewn in three magnitudes that A is to C, as D to F. But C is put to N, as F to O, there will be three magnitudes to A, C, and N, and other three D, F, and O, which are taken two and two, in the ſame reaſon: Therefore in equal reaſon demonſtrated in three magnitudes: Again, as A ſhall be to N, ſo D to O. In like manner, it may be ſhewn the ſame in five magnitudes by four, as it hath been demonſtrated in four magnitudes by three, and ſo of divers: Therefore, If there be as many Magnitudes, &c. Which was to be demonſtrated.

PROP. 23. THEOR. 23.

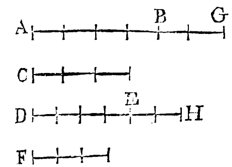


If there be three magnitudes A, B, *and* C, *and others equal to them in numbers* D, E, *and* F, *taken two and two, and in the ſame reaſon, and that their proportion be perturbed (that is to ſay, as* A *to* B, *ſo* E *to* F, *and* B *to* C, *as* D *to* E,*) in equal reaſon, they ſhall be alſo in the ſame reaſon (that is, as* A *to* C, *ſo* D *to* F.)

15. 5. *Demonſtration* ASſume G, H, and I, equimultiplices of A, B, and D. Likewiſe K, L, and M, equimultiplices of C, E, and F,
11. 5. as A ſhall be to B, ſo G ſhall be to H; ſeeing that G and H are equimultiplices of A and B. But as A is to B, ſo E is to F: Therefore as G is to H, ſo E is to F: But as E is to F, ſo L is to M; ſeeing that L and M are equi-
15. 5. multiplices of E and F. Therefore G ſhall be likewiſe to H, as L to M.
11. 8. Again, foraſmuch as B the firſt, is to C the ſecond, as D the third, to E the fourth; likewiſe H the Multiplex of the firſt B, ſhall be to K the Multiplex of the ſecond C, as I the Multiplex of D the third, is to L the Multiplex of E the fourth. Now foraſmuch as there are three magni-
4. 5. tudes G, H, and K, and other three I, L, and M, which taken two and
21. 5. two in the ſame reaſon, their proportion being perturbated: Seeing that it is ſhewn, that G is to H, as L is to M, and as H is to K, ſo I is to L; if G the firſt, exceed K the third, I the fourth ſhall likewiſe exceed M the ſixth, and if equal, equal, and if leſſe, leſſe, therefore ſeeing that G and I equimultiplices of the firſt A, and of the third D, are both greater, or equal, or leſſe, then K and M equimultiplices of the ſecond C, and of the fourth F, as A the firſt, ſhall be to C the ſecond, ſo D the third, ſhall be to F the fourth. Which was propoſed. Therefore, If there be three Magnitudes, &c. Which was to be demonſtrated.

PROP

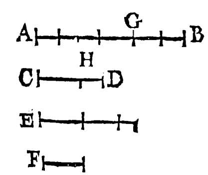
PROP. 24. THEOR. 24.



If the firſt AB, *hath the ſame reaſon to the ſecond* C, *as the third* DE, *hath to the fourth* F, *and that the fifth* BG, *hath alſo the ſame reaſon to the ſecond* C, *as the ſixth* EH, *hath to the fourth* F, *alſo* AG *the compound of the firſt* AB, *and the fifth* BG, *ſhall have the ſame reaſon to the ſecond* C, *as the compound* DH, *of the third* DE, *and of the ſixth* EH, *hath to the fourth* F.

Demonſtration FOr ſeeing that BG is to C, as EH to F; alternately, as C ſhall be to BG, ſo F ſhall be to EH; therefore foraſ-
much as AB is to C, as DE is to F, and that C is to BG, as F is to EH, 22. 5.
in equal reaſon, AB ſhall be to BG, as DE is to EH. Therefore by compounding, the whole AG ſhall be to BG, as the whole DH to EH, and BG to C, as EH is to F; in equal reaſon, AG ſhall be to C, as
DH is to F. Which was propoſed. Therefore, If the firſt, &c. Which 22. 5.
was to be demonſtrated.

PROP. 25. THEOR. 25.

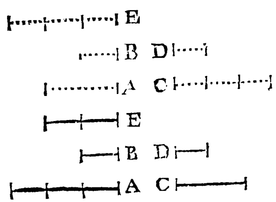


If four magnitudes are proportional, AB *to* CD, *as* E *to* F, *the greateſt* AB, *and the leaſt* F, *are greater than the two others* CD *and* E.

Demonſtration FOr, From the magnitude AB cut off AG equal to E, and from CD cut off alſo CH, equal to F. Then AG ſhall be to CH, as E to F; that is to ſay, as AB to CD. Therefore ſeeing that the whole AB, is to the whole CD, as the part cut off AG, is
to the part cut off CH. In like manner, as the whole AB, to the whole 91. 5.
CD, ſo the remainder GB, to the remainder HD. But AB (ſeeing that it is the greateſt of all,) is greater than CD. Therefore GB ſhall be alſo greater than HD. But foraſmuch as AG and E are equal, if you adde to them the equals F and CH, to wit, F to AG, and CH to E; AG and F together, ſhall be equal to E and CH together: Therefore adding the unequals GB and HD, AB and F together, ſhall be greater than E and CD together; ſeeing that GB is greater than HD. Which was propoſed. Therefore, If four Magnitudes, &c. Which was to be demonſtrated.

PROP

PROP. 26. THEOR. 26.



If the first A, *hath greater reason to the second* B, *than the third* C, *hath to the fourth* D, *by inverse reason, the second* B, *shall have lesse reason to the first* A, *than the fourth* D, *hath to the third* C.

Demonstration. FOr, Let it be understood that E is to B, as C is to D, the reason of A to B shall be likewise greater than that of E to B; and a therefore A shall be greater than E: Wherefore b there will be lesse reason of B to A the greater, than of B to E the lesser: But as B is to E, so by inverse reason, D is to C. Therefore the reason of B to A, is likewise lesse than that of D to C. Which was proposed. Therefore, If the first, &c. Which was to be demonstrated.

a) 10. 5.
b) 8. 5.

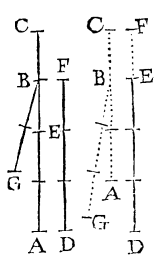
PROP. 27. THEOR. 27.

If the first A, *hath greater reason to the second* B, *than the third* C, *hath to the fourth* D; *also alternately, the first* A, *shall have greater reason to the third* C, *than the second* B, *shall have to the fourth* D.

Demonstration. FOr, Let E be put to B, as C to D, A shall likewise have greater reason to B, than E to B. Wherefore a A shall be greater than E, and b therefore there will be greater reason of A to C, than of E to C; but forasmuch as alternately as E is to C, so B is to D; (E being put to B, as C to D.) Therefore A shall also have greater reason to C, then B hath to D, which was proposed. Therefore If the first, &c. Which was to be demonstrated. (As by the precedent figure is manifest.)

a) 10. 5.
b) 8. 5.
c) 6. 5.

PROP. 28. THEOR. 28.



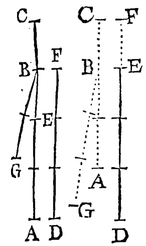
If the first AB, *hath greater reason to the second* BC, *than the third* DF, *hath to the fourth* EF, *the compounded* AC, *of the first* AB, *with the second* BC, *shall have also greater reason to the second* BC, *than the compounded* DF, *of the third* DE, *with the fourth* EF, *hath to the fourth* EF.

Demon-

Demonstration. FOr, Let GB be put to BC, as DE to EF, there will be also greater reason of AB to BC, then of GB to BC: Therefore a AB shall be greater than GB, adding therefore the common part BC, AC shall be greater than GC; and therefore AC shall have greater reason to BC, than GC to BC; but in compounding, GC is to BC, as DF to EF; (seeing GB is put to BC, as DE to EF;) therefore AC shall have greater reason to BC, than DF to EF. Which was proposed. Therefore, &c. Which was to be demonstrated.

a) 10. 5.
17. 5.

PROP. 29. THEOR. 29.

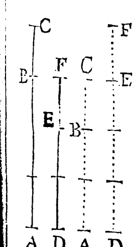


If the compound AC, *of the first* AB, *with the second* BC, *hath greater reason to the second* BC, *than the compound* DF, *of the third* DE, *with the fourth* EF, *hath to the fourth* EF. *Individing, the first* AB, *shall have also greater reason to the second* BC, *than the third* DE, *to the fourth* EF.

Demonstration. FOr, Let GC be put to BC, as DF to EF, the reason of AC to BC, shall be also greater than of GC to BC: Wherefore a AC shall be greater than GC, Therefore taking away the common part BC, there will remain AB, greater than GB. b Therefore there will be greater reason of AB to BC, than of GB to BC. But in dividing, as GB is to BC, so DE is to EF, (GC being put to BC, as DF to EF,) there will be therefore also greater reason of AB to BC, than of DE to EF. Which was proposed. Therefore, If the compound, &c. Which was to be demonstrated.

a) 10. 5.
b) 17. 5.

PROP. 30. THEOR. 30.



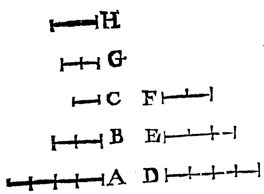
If the compound AC, *of the first* AB, *with the second* BC, *hath greater reason to the second* BC, *than the compound* DF, *of the third* DE, *with the fourth* EF, *hath to the fourth* EF, *by conversion of reason, the first* AB, *with the second* BC; *(that is* AC) *shall have lesser reason to the first* AB, *than the third* DE, *with the fourth* EF, *(that is* DF,) *hath to the third* DE.

Demon-

a) 29. 5.
b) 26. 5.

Demonstration FOr A C having greater reason to B C, than D F to E F; [a] in dividing, A B shall have greater reason to B C, than D E to E F. Therefore by conversion of reason, there shall be lesse reason of B C to A B, than of E F [b] to D E, and in compounding, there will be lesser reason of the whole A C to A B, than of the whole D F to D E. Which was proposed. Therefore, If the compound, &c. Which was to be demonstrated.

PROP. 31. THEOR. 31.



If there be three magnitudes A, B, *and* C, *and other three* D, E, *and* F, *equal to them in number, and that there be greater reason of the first* A *of the first, to the second* B, *then of the first* D, *of the last, to the second* E. *Likewise that there be greater reason of the second* B, *of the first, to the third* C, *than of the second* E, *of the last, to the third* F. *In equal reason, there will be also greater reason of the first* A, *of the first, to the third* C, *than of the first* D, *of the last, to the third* F.

a) 10. 5.
b) 8. 5.

Demonstration FOr, Let G be put to C, as E to F: Therefore there shall be also greater reason of B to C, than of G to C: Wherefore [a] B shall be greater than G: Therefore [b] there will be greater reason of A to G the lesser, then of A to B the greater: But the reason of A to B, is put greater than of D to E: Therefore there will be yet greater reason of A to G, then of D to E.

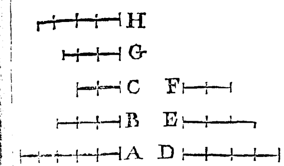
c) 10. 5.
d) 8. 5.
e) 22. 5.

Again, Let H be put to G, as D to E, there will be also therefore greater reason of A to G, then of H to G: Therefore [c] A shall be greater than H: Therefore [d] A the greatest, shall have greater reason to C, then H the lesser, shall have to the same C. But as H is to C, so D is to F, in equal reason, (seeing that as D is to E, so H is to G, and as E is to F, so G is to C;) Therefore there shall be also greater reason of A to C, then of D to F. Which was proposed. Therefore, If there be three Magnitudes, &c. Which was to be demonstrated.

PROP. 32. THEOR. 32.

If there be three magnitudes A, B, *and* C, *and others equal to them in number* D, E, *and* F, *and that there be greater reason of the first* A, *of the first, to the second* B, *then of the second*

E,



E, *of the last, to the third* F; *likewise that there be greater reason of the second* B, *of the first, to the third* C, *then of the first* D, *of the last, to the second* E: *In equal reason, there will be also greater reason of the first of the first* A, *to the third* C, *then of the first of the last* D, *to the third* F.

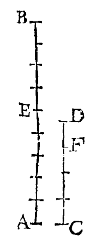
a) 10. 5.
b) 8. 5.

Demonstration FOr put G to C, as D to E; therefore also B shall have greater reason to C, than G to C; therefore [a] B shall be greater than G. Wherefore [b] there will be greater reason of A to G the lesse, than of the same A to B the greater. But there is greater reason of A to B, than of E to F: Therefore there will be yet greater reason of A to G, than of E to F.

c) 10. 5.
d) 8. 5.
e) 23. 5.

Again, put H to G, as E to F; there will be then greater reason of A to G, than of H to G. Therefore [c] A shall be greater than H. Wherefore [d] A shall have greater reason to C, than H the lesser, shall have to the same C: But [e] as H is to C, so D is to F, in equal reason, (D being to E, as G to C, and as E to F, so H to G:) there is therefore also greater reason of A to C, than of D to F. Which was proposed. Therefore, If there be three Magnitudes, &c. Which was to be demonstrated.

PROP. 33. THEOR. 33.

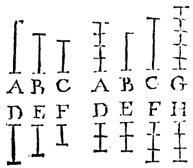


If there be greater reason of the whole A B, *to the whole* C D, *then of the part cut off* A E, *to the part cut off* C F; *there will be also greater reason of the remainder* E B, *to the remainder* F D, *then of the whole* A B, *to the whole* C D.

a) 27. 5.
b) 30. 5.
c) 27. 5.

Demonstration FOr seeing that there is greater reason of A B to C D, than of A E to C F; [a] by permutation, there shall be also greater reason of A B to A E, than of C D to C F; therefore [b] by conversion of reason, there will be lesser reason of A B to E B; than of C D to F D. Therefore [c] alternately, there shall be also lesser reason of A B to C D, than of E B to F D; to wit, the remainder E B, shall have greater reason to the remainder F D, than the whole A B, to the whole C D. Which was proposed. Therefore, If there be greater reason, &c. Which was to be demonstrated.

PROP. 34. THEOR. 34.



If there be as many magnitudes as you please A, B, *and* C, *and others equal to them in number* D, E, *and* F, *and that there be greater reason of the first of the first* A, *to the first of the last* D, *than of the second* B, *to the second* E; *and of the second* B, *to the second* E, *then of the third* C, *to the third* F, *and so on; all the first* A, B, *and* C *together, shall have greater reason to all the last* D, E, *and* F *together; then all the first* B *and* C, *the first* A *being taken away, to all the last* EF, *the first* D *also taken away; But lesse reason than the first of the first* A, *to the first of the last* D; *and lastly, also greater reason than the last of the first* C, *to the last of the last* F.

Demonstration. For seeing that there is greater reason of A to D, than of B to E; [a] by permutation, there shall be greater reason of
a) 27. 5. A to B, then of D to E; therefore [b] in compounding, there shall be greater rea-
b) 28. 5. son of A and B together to B, then of D and E together to E; and [c] again, by
c) 27. 5. permutation there will be greater reason of A & B together, to D and E together, than of B to E. Therefore the whole A & B having greater reason to
d) 33. 5. the whole D & E; than the part cut off B, to the part cut off E; [d] the remainder A shall also have greater reason to the remainder D, than the whole A & B, to the whole D & E. In like manner, there will be greater reason of B to E, than of the whole B and C, to the whole E and F; therefore there will be yet greater reason of A to D, than of the whole B and C, to the whole E and
e) 27. 5. F. Therefore [e] by permutation, there will be greater reason of A to B and
f) 28. 5. C, than of D to E and F. Therefore [f] in compounding, there will be greater reason of the whole A, B, and C, to B and C, than of the whole D, E, and F,
g) 27. 5. to E and F, and again, [g] by permutation, there will be greater reason of the whole A, B, and C, together, to the whole D, E, and F, together, then of B and C to E and F. Which was in the first place proposed.

Therefore there being greater reason of the whole A, B, and C, to the whole D, E, & F than of the part cut off B and C, to the part cut off E and F;
h) 33. 5. There [h] will be also greater reason of the remainder A, to the remainder D, then of the whole A, B, and C, to the whole D, E, and F. Which was in the second place proposed.

But forasmuch as there is greater reason of B to E, than of C to F; al-
i) 28. 5. ternately, [i] there will be also greater reason of B to C, than of E to F, and in compounding, greater reason of the whole B and C to C, than of the whole

whole E & F to F; and again, by permutation, greater reason of B and C, to E & F, than of C to F. But there is greater reason of A, B and C, to D, E, & F, as hath been shewn, than of B and C to E and F: Therefore there will be yet greater reason of the whole A, B, and C, to the whole D, E, and F, than of the last C, to the last F. Which was in the third place proposed.

Let there be afterward proposed four magnitudes on both parts, by the same supposition; that is to say, that there be greater reason of C the third, to F the third, than of G the fourth, to H the fourth; I say that the same thing will follow, for (as hath been already shewn in three Magnitudes, there is greater reason of B to E, than of B, C, and G, to E, F and H. Therefore there will be yet greater reason of A to D. Then of B, C, and G, to E, F, and H, Therefore [k] alternately, there will be greater reason k) 27. 5.
of A to B, C, and G, than of D to E, F, and H. And [l] in compounding, l) 28. 5.
greater reason of A, B, C, and G, to B, C, and G, than of D, E, F, and H, to E, F, and H; [m] and alternately, greater reason of A, B, C, and G, m) 27. 5.
to D, E, F, and H, than of B, C, and G, to E, F, and H. Which was in the first place proposed.

Therefore having greater reason of the whole A, B, C, and G, to the whole D, E, F, and H, than of the part cut off B, C, and G, to the part cut off E, F, and H, [n] the remainder A, shall have greater reason to the n) 33. 5.
remainder D, than the whole A, B, C, and G, to the whole D, E, F, and H. Which was in the second place proposed.

But (as is demonstrated in three magnitudes,) forasmuch as there is greater reason of B, C, and G, to E, F, and H, than of G to H; and greater of A, B, C, and G, to D, E, F, and H, than of B, C, and G, to E, F, and H, as hath been shewn; there will be much greater reason of A, B, C, and G, to D, E, F, and H, than of the last G, to the last H. Which was in the third place proposed. The same may be shewn in five Magnitudes, by four, and in six by five, &c. as hath been shewn in four by three. Therefore, If, &c. Which was to be demonstrated.

The End of the Fifth Element of EUCLIDE.

THE

THE SIXTH ELEMENT OF EUCLIDE.

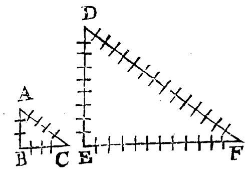
THE ARGUMENT.

THis Sixth Book treateth of the proportion of figures among themselves, of like and reciprocal figures, of proportional right lines, of the application of Parallelograms to right lines, which may either want or exceed by like Parallelograms, and how a terminated right line may be divided by extream and mean proportion, and of the proportions of circumferences and angles; also of Sectors in equal circles.

DEFINITIONS.

1 *Like right lined figures are those which have the angles equal each one to his correspondent, and the sides which are about the equal angles proportional.*

AS the triangles ABC, and DEF, shall be said to be alike, if they be equiangled; that is to say, that the angle A, be equal to the angle D, and the angle B to the angle E, and the angle C to the angle F; and in



in like manner, that the sides about the equal angles be proportional; to wit, as AB is to AC, so DE may be to DF, and as AB to BC, so DE to EF; and as AC to CB, so DF to EF.

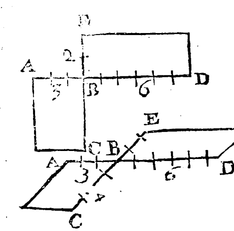
And if each of the angles of one of the figures be equal to each of the angles of the other. But the sides about the equal angles are not proportional; or contrarily, such figures shall not be said to be alike; as are the square and the oblong, or long square; for those figures have the angles equal; to wit, right angles, but the sides of the one, are not proportional to the sides of the other; the sides of the square having reason of equality; and those of the oblong about the right angle, reason of inequality; from whence it appears that all right lined figures equiangled, and equilateral, having the sides and the angles equal in number, are alike, although they be unequal.

2 *Reciprocal figures are such, when the reasons antecedents and consequents are in both the figures.*

Or thus,

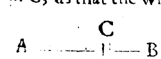
The figures are Reciprocal, when the terms, antecedents and consequents of the reasons, are in both the figures.

FOr by these reasons, antecedents and consequents ought to be understood, the terms antecedent and consequent of the proportion; as if there be two figures rectangled, or not rectangled, AC and ED; and that as AB is to BD, so EB is to BC, those figures shall be termed reciprocal: Forasmuch as in the one is the term antecedent of the first reason, to wit, AB, and the consequent of the second reason, to wit, BC, and in the other is the term consequent of the first reason BD, and the antecedent of the second EB, such figures are also said to be equal to one another, as shall be hereafter demonstrated.



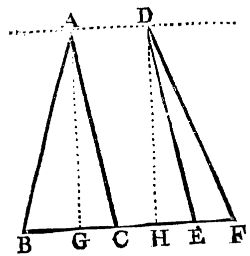
3 *A right line is said to be divided according to mean and extream reason, when as the whole is to the greatest segment, as the greatest segment is to the lesser.*

IF any right line, as AB, be divided, in such sort unequally in the point C, as that the whole AB, be to the greatest segment AC, as the greatest segment AC, is to the least segment CB, it shall be said to be divided according to mean and extream reason, which said division EUCLIDE teacheth at the thirtieth Proposition of this Book.



4 *The height of any figure is the perpendicular line drawn from the top to the base.*

IF from the vertex A, of the triangle ABC, there be drawn AG, perpendicular, to the base BC, the quantity of that perpendicular shall be the height or altitude of the triangle ABC. So also the perpendicular DH, drawn from the vertex D, of the triangle DEF, on the base EF, prolonged towards E, shall be the height of that triangle DEF. Therefore if the perpendiculars of the two figures, drawn from their tops to their bases, (be the bases prolonged, or not,) are equal; such figures shall be said to have the same height. Now such perpendiculars shall be equal, when the bases of the figures and the tops shall be constituted between the same parallels, as are AG and DH.



For seeing that the interior angles AGH, and DHG, on the same part, are equal to two right angles, or to expresse it better, are right angles;
a) 28. 1. [a] AG and DH shall be parallels, but AD and GH are also parallels; forasmuch as the triangles are presupposed to be constituted between the same parallels; Therefore AD and HG, shall be a parallelogram,
b) 34. 1. [b] and therefore the sides AG and DH, shall be equal: Therefore those triangles shall be said to have the same height the one as the other. And if in the same triangles, C and F be put for the tops, and AB and DE for the bases, those triangles shall not be said to have the same height; for the perpendicular drawn from F to the base DE, is not equal to the perpendicular drawn from from C, to the base AB, those triangles therefore cannot in any wise be constituted between the same parallels, as is manifest.

5 *A reason is said to be compounded of reasons, when the quantities of the reasons multiplyed in one another, do make some reason.*

FOrasmuch as the denominator of any reason whatsoever, doth expresse the quantity of the Magnitude antecedent, in respect of the consequent, (as the denominator of the reason quadruple; to wit 4, shewes that in all reason quadruple, the Magnitude antecedent, conteineth four times the consequent. But the denominator of the reason subquadruple, to wit $\frac{1}{4}$, demonstrateth that the Magnitude antecedent is the fourth part of the consequent, &c. (Geometricians use to term the denominator the quantity of the reason; so as that the quantity of the reason, and the denominator of the reason, is the same thing;) therefore this definition teacheth us that a reason is said to be compounded of two or more reasons, when the denominators or quantities of those reasons multiplyed in one another, shall make that reason, or (according to *Zambert*,) shall make the quantity or denominator of that reason: As the reason duodecuple is said to be compounded of the double and the sextuple: Forasmuch as the denominator of the reason duodecuple, to wit 12, is produced of the multiplication of the denominator of the reason double, to wit of 2, by the denominator of the sextuple, that is to say, by 6. So the same reason duodecuple is said to be compounded of the triple and of the quadruple; For of the Multiplication of 3 by 4, is produced the denominator 12, of the reason duodecuple, In like manner, the reason trigecuple is said to be compounded of the double, triple, and quintuple, for the denominators of that reason 2, 3, and 5, multiplyed in one another, produce 30, the denominator of that reason trigecuple. So the raason double is said to be compounded of the sesquialtera and sesquitertia, forasmuch as the denominator of the sesquialtera, which is $1\frac{1}{2}$, multiplied by $1\frac{1}{3}$, denominator of the sesquitertia is produced of 2, the denominator of the double.

Again, the same reason double, is compounded of the sesquiseptuple, and the supertripartient fourths; for the denominator of those reasons $1\frac{1}{7}$, and $1\frac{3}{4}$, multiplyed in one another, do produce two denominators of the double: In like manner, the same reason double shall be said to be compounded of the subsesquiquarta, and of the double sesquialtera; forasmuch as their denominators $\frac{4}{5}$, and $2\frac{1}{2}$ multiplyed in one another, do produce in like manner the same denominator of the reason double, to wit 2, &c.

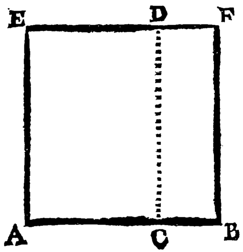
Therefore if you propose as many Magnitudes as you please in order, as A, B, C, and D, to wit, A to B in reason double, B to C triple, and C to D quintuple, the reason of the extreams A and D, shall be trigecuple, which shall be said to be compounded of the mean reasons, for if you multiply the denominator of A by B, which is 2, by that of B to C 3, you shall have 6 for the denominator of the reason of A to C.

A.	B.	C.	D
60	30	10	2

Therefore A to C, hath reason sextuple. Again, multiplying 6 the denominator of the reason of A to C, by that of C to D 5, you shall have 30, for the denominator of the reason of A to D. Wherefore A shall be to D, in reason trigecuple, &c.

And thus much may be said of the five definitions of *EUCLIDE*, on this Sixth Book, to which shall be added this following definition; which will render as well the 27, 28, 29, and 30th. Propositions of this Book, as also divers others of the Tenth Book, more easie and intelligible, and is as followeth.

6 *A parallelogram being applyed according to some right line is said to want thereof by a parallelogram, when it occupieth not all that right line. But is said to exceed it, when it occupieth a greater line than that according to which it is applyed: In such sort neverthelesse, that the parallelogram exceeding, or wanting, hath the same height as the parallelogram of which it wanteth, or which it exceedeth, and doth constitute therewith one only parallelogram.*



Let the line AB, be the line on which the Parallelogram A D is to be constituted, not imploying the whole line AB, but leaving C B, having finished the parallelogram A F, the parallelogram A D, applyed according to the line A B, shall be said to want thereof by the parallelogram C F, so as that C F shall be called the want.

Again, Let A C be the line on which the parallelogram A F is to be constituted, having the side A B, greater than A C, to which let C D be drawn parallel to B F, the parallelogram A F, applyed according to A C, shall be said to exceed A C, by the parallelogram C F, in such sort as C F shall be termed the excesse.

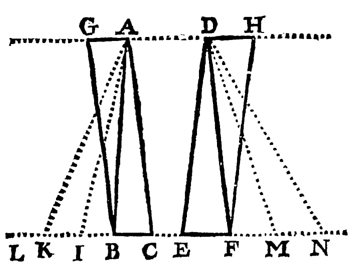
Now this want, or excesse in rectangles may be a square, or an oblong, and in parallelograms not rectangled, a Rhombe or Rhomboides, as is manifest.

PROPOSITIONS,

PROBLEMES, and THEOREMES.

PROPOSITION I. THEOREM I.

Triangles A B C *and* D E F, *and parallelograms* CG *and* E H, *which have the same height, are to one another, as their bases* B C *and* E F.



I Say that the triangle A B C, is to the triangle D, E, and F, and the parallelogram CG, to the parallelogram EA, as the base BC is to the base E F; that is to say, that if the base B C be taken for the first magnitude, and

and the base E F for the second. But the triangle A B C, or the parallelogram C G for the third, and the triangle D E F, or the parallelogram E H, for the fourth, the equimultiplices of the first and the third Magnitudes shall be both greater, or both lesser, or equal to the equimultiplices of the second and fourth, according to the meaning of the sixth definition of the fifth Book.

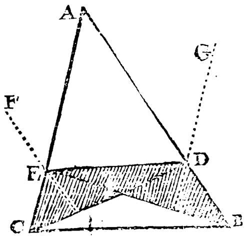
Demonstration. For let there be constituted as well the triangles, as the parallelograms, between the same parallels G H and LN, (then [a] they will have one and the same height, for that being constituted between the same parallels, the perpendiculars drawn from the tops, on the bases, shall be equal, as is said in the fourth definition,) and from B L take BI, I K, and K L, each equal to the base B C. a) 4. def.

In like manner, from F N, take F M and M N, each equal to E F; and from the points A and D, draw the right lines A I, A K, A L, D M, and D N. [b] the triangles A B C, A I B, A K I and A L K, being constituted on equal bases, and between the same parallels, shall be equal to one another; by the same reason, the triangles D E F, D F M, and D M N, shall be also equal to one another. b) 38. 1.

Now forasmuch as C L conteineth as many parts equal to B C, as the triangle A C L doth contein triangles equal to the triangle A B C, as hath been shewn, C L shall be as much Multiplex of C B, as the triangle ACL is of the triangle A B C. Likewise, forasmuch as E N doth contein as many parts equal to E F; as the triangle D E N conteineth triangles equal to the triangle D E F, as hath been also shewn, E N shall be as much Multiplex of E F, as the triangle D E N is of the triangle D E F. But forasmuch as if the base C L be equal to the base EN, [c] of necessity, the triangle A C L shall be equal to the triangle D E N, being between the same parallels; and therefore if C L be greater than E N, the triangle A C L shall be greater than the triangle D E N, and if lesse, lesse, as appears by the ninth Common Sentence. Therefore the right line C L, and the triangle A C L equimultiplices of the first Magnitude BC, and of the third A B C, will both want of the right line E N, and of the triangle D E N, equimultiplices of the second Magnitude E F, and of the fourth D E F, or shall be both equal, or both greater, the Magnitudes being so taken, as that they answer one another; [d] wherefore there will be greater reason of the first B C, to the second E F, than of the third A B C, to the fourth D E F, to wit, of the triangle to the triangle: Therefore as the base is to the base, so is the triangle to the triangle. Which was proposed. c) 38. 5. d) 6. def. 5.

For the second part; Seeing that as [e] the triangle A B C is to the triangle DEF, so the parallelogram C G [f] (which is double to the triangle ABC,) is to the parallelogram E H; (which [g] is double to the triangle DEF,) it is manifest that the parallelogram shall [h] be also to the parallelogram, as the base to the base, which may be proved by the same discourse which we have used in the Demonstration of triangles, if first of all, from the points I, K, and L, you draw parallels to B G, and also from M and N, to F and H: Therefore the triangles, &c. Which was to be demonstrated. e) 15. 5. f) 34. 1. g) 34. 1. h) 11. 5.

PROP. 2. THEOR. 2.



If to one of the sides B C, *of the triangle* A B C, *be drawn some parallel right line* D E, *it will divide the sides* A B *and* A C, *of the triangle* A B C, *proportionally; and if the sides* A B *and* A C *are divided proportionally, the right line* D E, *joyning the points of the sections shall be parallel to the other side* C B, *of the triangle* A B C.

a) 37. 1. *Demonstration* FOr having drawn the right lines C D and B E, [a] the triangles D E B and D E C, constituted on the base D E, and between the same parallels D E and B C, shall be equal to one another: Therefore as the triangle A D E, is to the triangle D E B; so the
b) 1. 6. same A D E shall be to the triangle D E C. But [b] as A D E is to D E B, so the base A D is to the base D B; (seeing that those triangles are of the same height, as appeareth, if by the point E, be drawn E F, parallel to A B,) and by the same reason, as the triangle A D E, is to the triangle D E C, so the base A E, to the base E C, (A E D and D E C, being be-
c) 11. 5. tween the same parallels A C and D G,) therefore [c] as A D is to D B, so A E is to E C, (seeing that these two reasons are the same to the reason of the triangle A D E) so the triangle D E B, and of the same triangle A D E, to the triangle D E C. Which was proposed.

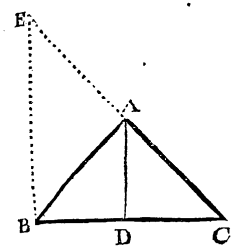
Secondly, Let the right line D E divide the sides A B and A C proportionally: I say that D E is parallel to the other side C B.

d) 16. 5. For again, having drawn B E and C D, as the [d] base A D shall be to the base D B, so the triangle A D E, to the triangle D E B; seeing they are both of one height; but as A D is to D B, so is A E to E C, by supposition. Therefore as the triangle A D E, shall be to the triangle D E B, so A E shall be to E C; but again, as the base A E is to the base E C, so the triangle A D E, to the triangle D E C, being of the same height. There-
e) 11. 5. fore [e] as the triangle A D E, is to the triangle D E B, so the same trian-
f) 9. 5. gle A D E, is to the triangle D E C; therefore [f] the triangles D E B and D E C shall be equal; and therefore seeing they are constituted on the
g) 39. 2. same base, [g] they shall be also between the same parallels: Therefore D E is parallel to B C. Which was proposed. Therefore, If to one of the sides, &c. Which was to be demonstrated.

PROP.

PROP. 3. THEOR. 3.

If an angle B A C, *of a triangle* A B C, *be divided into two equal parts, and that the right line* A D, *which divideth the angle, doth also divide the base* B C, *the segments of the base* B D *and* D C, *shall have the same reason to one another, as the other sides of the triangle* B A *and* A C, *and if the segments of the base* B D *and* D C, *have the same reason to one another, as the other sides of the triangle* B A *and* A C, *the right line* A D, *drawn from the top* A, *to the point of section* D, *divideth the angle* B A C, *of the triangle* A B C, *into two equal parts.*



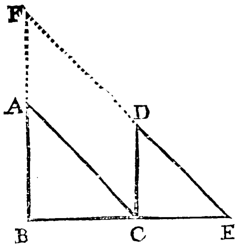
Demonstration FOr, Let B E be drawn parallel to D A, meeting C A prolonged towards E. Now they will meet, seeing that the angles C and C B E, are lesse than two right angles, for [a] C and a) 17. 1.
C D A, are lesse than two right angles, and [b] C D A is equal to C B E, b) 20.
the exterior angle to the interior, C and C E B, shall be likewise lesse than two right angles, and the angle E B A [c] shall be equal to its alter- c) [illegible]
nate angle B A D, and the angle E, equal to the exterior angle D A C; therefore seeing that the two angles B A D and D A C, are equal by supposition; the angles E B A and E, shall be equal to one another; [d] there- d) 6. 1.
fore the lines A B and A E equal to one another [e]. e) 7. 5.

Therefore as E A shall be to A C, so B A shall be to the same A C: But [f] as E A is to A C, so B D is to D C, seeing that in the triangle B E C, f) 2. 6.
the right line D A is parallel to the side B E. Therefore as B A shall be to A C, so B D shall be to D C. Which was proposed.

Secondly, Let it be as B A is to A C, so B D to D C; I say that the right line A D doth divide the angle B A C, into two equal parts: For again, draw the right line B E, by the point B, parallel to D A, meeting C A prolonged in the point E; forasmuch therefore as B A is to A C, so B D is to D C, by supposition; but [g] as B D is to D C, so E A is to A C; g) 2. 6.
(seeing that in the triangle B C E, the line A D is parallel to the side B E,) as B A [h] shall be to A C, so E A to the same A C. Therefore [i] B A h) 11. 5.
and A E shall be equal to one another; [k] therefore the angles A B E and i) 9. 5.
E, are equal: [l] Therefore seeing that the angle A B E is equal to its alter- k) 5. 1.
nate angle B A D, and the angle E equal to its exterior angle D A C, l) 29. 1.
the two angles B A D and D A C, shall be equal to one another. Which was proposed. Therefore, If the angle, &c. Which was to be demonstrated.

PROP.

PROP. 4. THEOR. 4.



Of equiangled triangles A B C *and* D E C, *the sides* A B *and* B C, D C *and* C E *which are about the equal angles* A B C *and* D C E, *as also* B C *and* C A, C E *and* E D, *which are about the equal angles* A C B *and* E, *and* A B *and* A C, *and* D C *and* D E, *which are about the equal angles* A *and* D, *are proportional, and the sides* A C *and* D E, *which subtend the equal angles* B *and* D C E, A B *and* D C *which subtend the equal angles* A C B *and* E, *as also* B C *and* C E, *which subtend the equal angles* A *and* D, *are homologal, or of the same reason.*

Demonstration. Constitute B C and C E, in a right line, in such sort as the exterior angle D C E may be equal to the interior angle A B C. Likewise the exterior angle A C B equal to the interior D E C; and forasmuch as the two angles A B C and A C B, are lesse than two right angles; but D E C is equal to A C B, the two angles B and E shall be lesse than two right angles. Wherefore [a] B A and E D, prolonged towards A and D, shall meet, let them then be prolonged, and they shall meet in the point F.

a) 11. c. 5.

But forasmuch as the exterior angle D C E is equal to the interior and opposite angle A B C, [b] C D and B F shall be parallels; and by the same reason C A and E F shall be also parallels, the exterior angle A C B being equal to the interior D E C. Therefore A C D F is a parallelogram: Therefore [c] A F is equal to C D, and C A to D F. Therefore forasmuch as in the triangle B E F, the line A C is parallel to E F, [d] A B shall be to A F, that is to say to D C, its equal, as B C to C E: Therefore alternately, as A B to B C, so D C to C E.

b) 28. 1.
c) 34. 1.
d) 2. 6.

Again, seeing that in the same triangle B E F, the line C D is parallel to the side B F, as B C shall be to C E, so F D, that is to say C A, (which is equal to F D,) to E D. Therefore [e] alternately, as B C to C A, so C E to E D: Therefore seeing that as A B to B C, so D C to C E, and as B C to C A, so C E to E D: In equal reason, A B shall be to C A, as D C to E D. Which was proposed. Therefore the triangles, &c. Which was to be demonstrated.

e) 22. 5.

COROLLARIE. I.

From hence it followes, that if in a triangle there be drawn a line parallel to one of the sides, it will take away a triangle resembling the whole, as in the triangle F B E, *where the line* C D *is drawn parallel to* B F. *I say that the triangle* D C E *is alike to the triangle* F B E, *for they are equiangled;* [f] *seeing that the angles* E D C *and* E C D *are equal to the exterior angles* F *and* B, *each to his correspondent angle, and the angle* E

f) 29. 1.

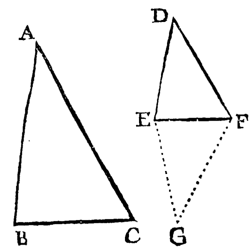
E common; therefore [g] *(as hath been demonstrated) they have the sides about the equal angles proportional; therefore according to the Definition they are proportional.*

g) 4. 6.

COROLLARIE II.

It follows moreover, that if two triangles equiangled, that is to say, which have the sides proportional, are disposed according to one angle, in such sort, as that two sides of the same reason may be parallels, the other sides will directly meet with one another, as in the triangles before mentioned; where A B *and* D C, *and* A C *and* D E, *being parallels, the other sides* B C *and* C D, *will directly meet with one another, as is manifest by the fourteenth Proposition of the First Book; all the angles at the point* C *being shewn to be equal to the three angles of the triangle* A B C; *that is to say, to two right angles, by the 32 Proposition of the First Book; which* EUCLIDE *demonstrateth again in the 32 Proposition of this Book.*

PROP. 5. THEOR. 5.



If two triangles A B C, *and* D E F, *have the sides proportional; that is,* A B *to* B C, *as* D E *to* E F, *and as* B C *to* C A, *so* E F *to* F D, *and as* A B *to* A C, *so* D E *to* D F; *those triangles shall be equiangled, and shall have the angles equal, under which the sides of the same reason are subtended.*

Demonstration Let the angle F E G be made equal to the angle B, and E F G equal to C, the two lines E G and F G, shall meet in the point G; and [a] the other angle G shall be equal to the other angle A. Therefore the triangles A B C and G E F are equiangled: Therefore [b] as A B to B C, so G E to E F; but as A B to B C, so D E to E F, by supposition: Therefore [c] as G E to E F, so D E to the same E F. Wherefore G E and D E shall be equal.

a) 32. 1.
b) 4. 6.
c) 11. 5.

Again, seeing that B C is to C A, as E F to F G; but [d] as B C is to C A, so E F to F D, by supposition, as E F shall be to F G; so the same E F shall be to F D, therefore [e] F G and F D, shall be equal: Therefore seeing that the sides G E and G F are equal to the sides D E and D F, each to its correspondent, and the base E F [f] common, the angles G and D shall be equal. Therefore [g] the other angles G E F and G F E shall be equal to the other angles D E F and D F E, each to his correspondent. Wherefore seeing that the angle G is equal to the angle A, also D its equal, shall be equal to A, by the same reason, D E F equal to B, and D F E equal to C. Which was proposed. Therefore, If two triangles, &c. Which was to be demonstrated.

d) 4. 6.
e) 9. 5.
f) 8. 1.
g) 4. 1.

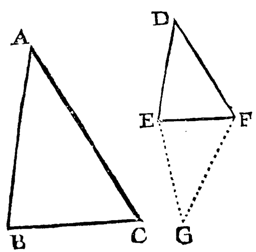
PROP. 6. THEOR. 6.

If two triangles A B C *and* D E F, *have an angle* B, *equal to*

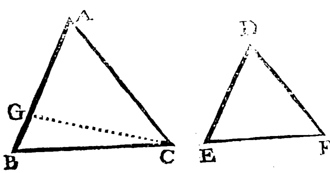
to an angle E, *and the sides about those equal angles proportional; that is, as* AB *to* BC, *so* DE *to* EF, *those triangles shall be equiangled, and shall have the angles equal, under which the homologal sides, or sides of the same reason are subtended; that is, the angle* A *equal to the angle* D, *and the angle* C *to the angle* F.

Demonstration. For, Let the angle FEG be made equal to B, and EFG equal to C, the triangle GEF shall be equiangled to the triangle ABC, as is shewn in the precedent Proposition. Therefore [a] as AB to BC, so GE to EF. But as AB to BC, so DE to EF by supposition: Therefore [b] as DE to EF, so GE to the same EF; therefore DE and GE shall be equal: Wherefore seeing that the sides DE and EF are equal to the sides GE and EF, and the angles conteined of those sides also equal, (for the angle B, to which the angle GEF is made equal, is equal to DEF by supposition: therefore the two angles at the point E shall be equal,) [c] the other angles D, and EFD shall be equal to the two others G, and EFG: Therefore seeing that the angle G is equal to the angle A, and EFG equal to C, the angles D and EFD, shall be also equal to the angles A and C; and therefore the triangles ABC and DEF, shall be equiangled. Which was proposed. Therefore, If two triangles, &c. Which was to be demonstrated.

a) 4. 6.
b) 11. 5.
9. 5.
c) 4. 1.



PROP. 7. THEOR. 7.



If two triangles ABC, *and* DEF, *have an angle* A, *equal to an angle* D, *and about another angle* ACB, *the sides proportional; that is,* AC *to* BC, *as* DF *to* EF. *But the other angles* B *and* E *together, the one and the other be lesse or not lesse than a right angle: The triangles* ABC *and* DEF *shall be equiangled, and shall have the angles equal, about which the sides are proportional; that is to say, the angles* ACB *and* F, *and* B *and* E.

De-

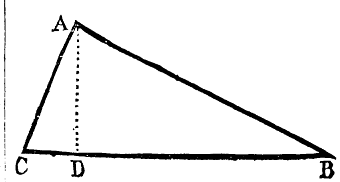
Demonstration. For first of all, Let as well B as E, be lesse than a right angle, that being, if the angles ACB and F are equal, the Proposition is manifest: But if ACB and F are not equal; Let ACB be greater than F, and let ACG be made equal to F: Therefore seeing that A is put equal to D, [a] the other angle AGC shall be equal to E. Therefore the triangles AGC and DEF shall be equiangled. Wherefore [b] as AC shall be to CG, so DF to EF. But as DF to FE, so AC to CB, by supposition: Therefore [c] as AC to CG, so the same AC to CB; and therefore [d] CG and CB shall be equal; [e] and the angles CBG and CGB shall be equal.

Therefore seeing that the angle B is put lesse than a right angle, CGB shall be also lesse than a right angle; and therefore AGC shall be greater than a right angle; seeing that [f] AGC and CGB are equal to two right angles. Now the angle AGC is shewn to be equal to the angle E. Therefore the angle E shall be also greater than a right angle. But it is lesse than a right angle by supposition, which is impossible.

Secondly, Let as well the angle B, as the angle E, not be lesse than a right angle, as before, the angle B shall be equal to the angle CGB; and therefore CGB shall not be lesse than a right angle; and so the angles CBG and CGB, in the triangle BCG, shall not be lesse than two right angles, but greater, or equal to two right angles, which is absurd; [g] for they are lesse than two right angles. Therefore the angles ACB and F are not unequal, but equal; and therefore [h] the other angles B and E are equal. Which was proposed. Therefore, If two triangles, &c. Which was to be demonstrated.

a) 32. 1.
b) 4. 6.
c) 11. 5.
d) 9. 5.
e) 5. 1.
f) 13. 1.
g) 17. 1.
h) 32. 1.

PROP. 8. THEOR. 8.



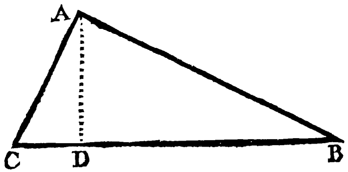
If in a rectangled triangle ABC, *there be drawn a perpendicular line* AD, *from the right angle* BAC, *on the base* CB, *the triangles* ADB, *and* ADC, *which are on both sides of the perpendicular, are alike to the whole, and alike to one another.*

Demonstration. For seeing that in the triangles ABC and DBA, the angles BAC and ADB are right angles, and ADB are right angles, and the angle B common, [a] the other angles ACB and DAB shall be equal: Therefore the triangle DBA is equiangled to the triangle ABC; and [b] therefore they will have the sides about the equal angles proportional, &c, that is to say, as CB shall be to BA, so BA to BD, and as BA to AC, so BD to DA, and as BC to CA, so BA to AD; for so the sides having the same reason, are opposite to the equal angles, as by the fourth Proposition of this Book. Wherefore the triangle ADB is alike to the whole triangle ABC, by the same reason it may be shewn that

a) 32. 1.
b) 4. 6.

 that

that the triangle ADC is alike to the ſame triangle ABC; for the angles BAC and ADC, are right angles, and the angle C common: Therefore [c] c) 32.1. the other angles ABC and CAD are equal; [d] d) 4.6. wherefore as BC to CA, ſo CA to CD, and as CA to AB, ſo CD to DA, and as CB to BA, ſo CA to AD; for ſo the ſides having the ſame reaſon, are oppoſite to the equal angles, as by the fourth Propoſition of this Book.



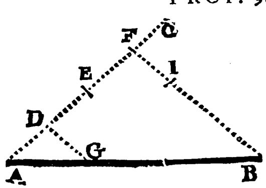
Even ſo it will be demonſtrated that the triangles ADB and ADC are alike to one another, ſeeing that the angles ADB and ADC are right angles, and ABD and CAD are ſhewn to be equal, and alſo the c) 4.6. angles BAD and ACD equal; and [c] therefore as BD to DA, ſo DA to DC; and as DA to AB, ſo DC to CA; and as AB to BD, ſo CA to AD. Therefore, If in a triangle, &c. Which was to be demonſtrated.

COROLLARIE.

From this Propoſition it is evident, that the perpendicular drawn from the right angle on the baſe, in a rectangle triangle, is a mean proportional between the two ſegments of the baſe. Likewiſe each of the ſides which contein the right angle, is a mean proportional between the whole baſe and the ſegment of the baſe, which is adjacent, or toucheth the ſame ſide.

For it is demonſtrated that as BD *is to* DA, *ſo* DA *is to* DC; *and therefore* DA *is a mean proportional between* BD *and* CD. *Likewiſe as* CB *is to* BA, *ſo* BA *to* BD; *and ſo* BA *is a mean proportional between* CB *and* BD: *Laſtly, it is demonſtrated that as* BC *is to* CA, *ſo* CA *is to* CD; *and therefore* CA *is a mean proportional between* BC *and* CD. *Which was propoſed.*

PROP. 9. PROBL. 1.



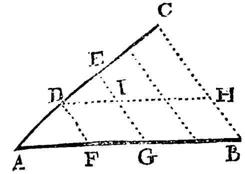
From a given right line AB, *to take away a required part, as* AG.

Conſtruction FRom A draw AC, making any angle whatſoever with AB, as CAB, and from AC take ſo many equal parts of ſuch a Magnitude as you would that the part to be ſubſtracted ſhall denote; (for AC ought to be ſo great as need requires,) as in the example propoſed, you ought to take three equal parts, AD, DE, and EF, then joyn FB, to which by the point D, draw DG, parallel to FB: I ſay that AG is the third part of AB required.

Demonſtration FOr ſeeing in the triangle ABF, DG is parallel to the ſide FB; [a] as FD to DA, ſo BG to GA; [b] therefore in com-
a) 2.6. b) 18.5. pounding, as FA to DA, ſo BA to GA; but FA is triple to AD, by Conſtruction, therefore BA ſhall be alſo triple to AG; and therefore AG ſhall be the required third part of AB. Therefore, From a given right line, &c. Which was to be done.

PROP.

PROP. 10. PROBL. 2.



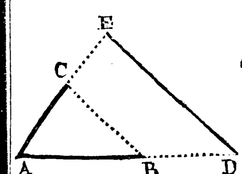
To divide a given right line undivided AB, *alike to a given right line* AC, *which is divided, as in* D *and* E.

Conſtruction JOyne the two given lines in the point A, making any angle whatſoever, as BAC, and joyn the right line BC, then from D and E, draw DF and EG, parallel to BC. I ſay that AB is divided alike at F and G, as AC is divided in D and E, [a] that is to ſay, as AD a) 2.6.
to DE, ſo AF to FG. Therefore the parts AF and FG are proportional to AD and DE, and if you draw DH parallel to FB, dividing EG in the point I.

Again, [b] as DE ſhall be to EC, ſo DI to IH; that is to ſay, ſo FG b) 2.6.
to GB [c]. c) 34.1.

Demonſtration FOraſmuch as FG is equal to DI, and GB to IH. Wherefore the parts FG and GB ſhall be alſo proportional to DE and EC. And ſo by the ſame reaſon if there were more parts. Therefore, We have divided a line, &c. Which was to be done.

PROP. 11. PROBL. 3.



To two given right lines AB *and* AC, *to find a third proportional line* CE.

Conſtruction DIſpoſe the right lines AB and AC, ſo as to make any angle A, to which a third proportional ought to be found; that is to ſay, as AB is to AC, ſo AC is to a third.

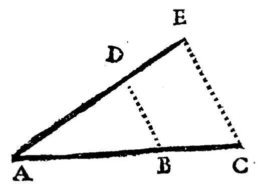
Prolong AB the antecedent line propoſed to D, and aſſume BD, equal to AC the conſequent or mean; then having drawn BC; by D draw DE, parallel to BC, meeting AC prolonged in E; I ſay that CE is the third proportional; that is to ſay, that as AB is to AC, ſo AC is to CE.

Demonſtration FOr ſeeing that in the triangle ADE, BC is parallel to DE, as [a] AB ſhall be to BD, ſo AC ſhall be to CE. a) 2.6.
But [b] as AB to BD, ſo the ſame AB to AC, equal to BD: Therefore b) 7.5.
as AB to AC, ſo AC to CE. Which was propoſed. Therefore, To two given right lines, &c. Which was to be done.

PROP. 12. PROBL. 4.

To three given right right lines AB, BC, *and* AD, *to find a fourth proportional; that is, as* AB *to* BC, *ſo* AD *to a fourth* DE.

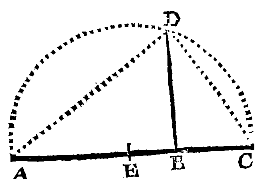
 Conſtru-



Construction Dispose the two first according to one right line, which let be A C; but that the third A D be so disposed as to make any angle with A B the first as A, then joyne B D, to which by C draw C E parallel thereto, meeting A D prolonged at E; I say that D E is the fourth proportional.

a) 2.6. *Demonstration* Seeing that in the triangle A C E, B D is drawn parallel to the side C E; as [a] A B shall be to B C, so A D to D E: Therefore D E is the fourth proportional. Wherefore to three given right lines, &c. Which was to be done.

PROP. 13. PROBL. 5.

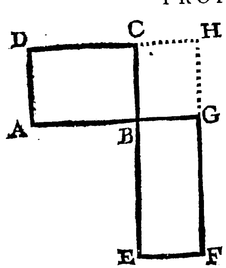


To two given right lines A B *and* B C, *to finde a mean proportional.*

Construction Having disposed the right lines A B and B C, according to one right line A C, divide A C into two equal parts in the point E, and from the point E as a center, and the distance E A or E C, describe the semicircle A D C; then from the point B, in the line A C, raise the perpendicular B D, until it touch the circumference at D. I say that B D is a mean proportional between A B and B C.

a) 31. 1. *Demonstration* For having drawn A D and C D, [a] the angle A D C shall be a right angle in the semicircle; and seeing that from the right angle of the rectangle triangle A D C, the perpendicular D B, is drawn on the base A C, the same B D shall be a mean proportion [b] between A B and B C. Therefore, To two right lines, &c. Which was to be done.
b) Cor. 8. 6.

PROP. 14. THEOR. 9.



The sides A B, B G, E B, *and* B C, *which are about the equal angles of equal parallelograms* A C *and* B F, *which have an angle* A B C, *equal to an angle* E B G, *are reciprocal; and the parallelograms which have an angle equal to an*

angle, and the sides which are about the equal angles reciprocal, are equal.

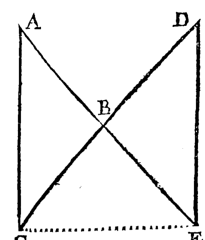
Demonstration For joyn those parallelograms according to the equal angles, in such sort as that A B and B G may make one right line E B and B C [a] shall also make a right line, as is evident; and let D C and F G be prolonged, untill they meet with one another in H. a) 2. c. s. 13, 14. 1.

Forasmuch then as the parallelograms D B and B F are equal; as [b] D B shall be to B H, so B F to the same B H; but as D B to B H, so the base [c] A B to the base B G; forasmuch as the parallelograms are of the same height. Likewise as B F is to B H, so the base E B to the base B C: Therefore [d] as A B to B G, so E B to B C. Which was proposed. b) 7.5. c) 1.6. d) 11.5.

Contrarily, Let the sides about the said equal angles be reciprocal; to wit, as A B to B G, so E B to B C: I say that the parallelograms D B and B F are equal.

Use the same Construction, seeing that A B is to B G, as E B to B C But [e] as A B to B G, so D B to B H; and as E B to B C, so B F to the same B H. Likewise [f] D B shall be to B H, as B F to the same B H; and therefore [g] the parallelograms D B and B F shall be equal. Therefore, The sides, &c. Which was to be demonstrated. e) 1.6. f) 11.5. g) 9.5.

PROP. 15. THEOR. 10.



The sides which are about the equal angles of the equal triangles A B C *and* D B E, *which have an angle* B *equal to an angle* B, *are reciprocal; that is, as* A B *to* B E, *so* D B *to* B C. *And the triangles which have an angle equal to an angle, and the sides which are about the equal angles reciprocal, are equal.*

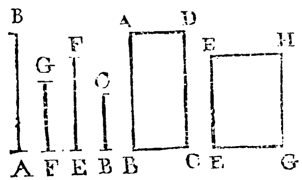
Demonstration Joyne those sides according to the equal angles, in such sort as that A B and B E may make one right line, that being D B, B C shall also make one right line; as is said in the precedent Proposition; and let C E be drawn; forasmuch as the triangles A B C and D B E are equal, as A B C shall be to B C E, so D B E shall be to the same B C E; but as the triangle A B C, to the triangle B C E; [a] so the base A B to the base B E, they being of one and the same height: And in like manner, as D B E to B C E, so is the base D B, to the base B C; therefore [b] as A B to B E, so is D B to B C. Which was proposed. a) 1.6. b) 11.5.

Now let the sides about the equal angles in the point B, be reciprocal; to wit, as A B to B E, so D B to B C; I say that the triangles A B C and D B E, are equal.

Let the same Construction be made, forasmuch as A B is to B E, as D B to B C; but as A B to B E, [c] so the triangle A B C to the triangle c) 1.6.
B C E;

BCE, and as DB to BC, ſo the triangle DBE to the ſame triangle
d) 11.5. BCE: Therefore [d] as ABC ſhall be to BCE, ſo DBE ſhall be to the ſame BCE; and therefore the triangles ABC and DBE ſhall be equal. Therefore the ſides, &c. Which was to be demonſtrated.

PROP. 16. THEOR. 11.



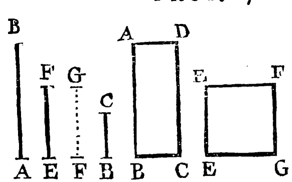
If four right lines AB, FG, EF, *and* BC, *are proportional; the rectangle* AC, *conteined under the extreams* AB, *and* BC, *is equal to the rectangle* EG, *conteined under the means* EF *and* FG, *and if rectangle conteined under the extreams, be equal to the rectangle conteined under the means, thoſe four lines ſhall be proportional.*

Demonſtration. FOr ſeeing that the angles B and F are equal, being right angles, and that AB is to FG, as EF to BC; thoſe ſides
a) 14.6. which are about the equal angles B and F, are reciprocal: Therefore [a] the Parallelograms AC and EG ſhall be equal. Which was propoſed.

Contrarily, Let the rectangles AC and EG be equal; I ſay that the four lines AB, FG, EF and BC are proportional; to wit, as AB to FG, ſo EF to BC.

For ſeeing that the rectangles AC and EG are equal; having the an-
b) 14.6. gles B and F equal; to wit, right angles, [b] the ſides which are about thoſe angles ſhall be reciprocal; to wit, as AB to FG, ſo EF to BC: Therefore, If four right lines, &c. Which was to be demonſtrated.

PROP. 17. THEOR. 12.



If three right lines AB, EF, *and* BC, *be proportional: The rectangle* AC *conteined under the extreams* AB *and* BC, *is equal to the ſquare* EG, *of the mean* EF *and if the rectangle conteined under the extreams, be equal to the ſquare of the mean: thoſe three right lines ſhall be proportional.*

Demonſtration. FOr aſſume the line FG, equal to EF, the four right lines AB, EF, FG, and BC ſhall be proportional; to wit, as AB to EF, ſo FG to BC; and the ſquare EG ſhall be conteined under the

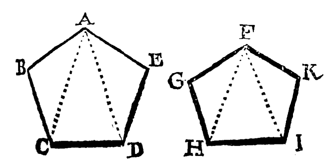
a) 16.6. the means EF and FG, ſeeing that EF and FG are equal: Wherefore [a] the rectangle AC, conteined under the extreams AB and BC, is equal to the ſquare EG; that is to ſay, to the rectangle conteined under the meanes EF and FG. Which was propoſed.

But let the rectangle AC, and the ſquare EG be equal; I ſay that as AB is to EF, ſo EF is to BC; for the rectangles AC and EG being equal;
b) 16.6. as AB ſhall be to EF, ſo FG to BC; but [b] as FG to BC, ſo EF [c] equal to the
c) 7.5. ſame FG, is to the ſame BC: Therefore as AB to EF, ſo EF to BC. Therefore, If three right lines, &c. Which was to be demonſtrated.

CORLLARIE.

From the latter part of this Theorem it follows, that every right line is a mean proportional between two other, whatſoever right lines, which do contein a rectangle equal to the ſquare of the ſame lines: For from that, that the right lines AB *and* BC, *do contein a rectangle equal to the ſquare of the right line* EF, *it hath been ſhewn that as* AB *is to* EF, *ſo* EF *is to* BC. *Wherefore* EF *is a mean proportional between* AB *and* BC.

PROP. 18. PROBL. 6.



On a given right line CD, *to deſcribe a rectiline figure* ABCDE, *alike, and alike poſited to a given rectiline figure* FGHIK.

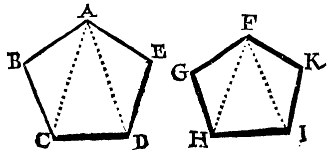
Conſtruction. FRom either of the angles, (as from F,) draw to each of the oppoſite angles, the right lines FH and FI, which lines do divide the rectiline figure into three triangles FGH, FHI, and FIK; then make the angle DCA equal to the angle IHF, and the angle CDA equal to HIF. Now AC and AD ſhall meet with one another at A, the angles ACD and ADC being leſſe than two right angles, be-
a) 17.1. ing made equal to FHI and FIH, ([a] which are leſſe than two right an-
32.1. gles,) and the other angle CAD ſhall be equal to the other angle HFI, and the triangle CAD equiangled to the triangle HFI.

Again, Let the angle ADE be made equal to FIK, and DAE
b) 17.1. equal to IFK; foraſmuch [b] as the two angles KIF and KFI, are leſſe than two right angles, the two angles EAD and EDA, equal unto them, ſhall be in like manner leſſe than two right angles; and therefore AE and DE ſhall meet in the point E, and the triangle AED ſhall be equiangled to the triangle FKI by the ſame reaſon.

Moreover, Make the triangle ABC equiangled to the triangle FGH, by the ſame reaſon, and ſo of the others, if there be any more triangles; I ſay that the rectiline figure ABCDE is alike, and alike poſited to the rectiline figure FGHIK.

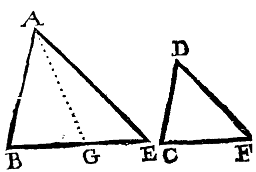
Demonſtration. FOr ſeeing that the angle ACD is made equal to FHI, and ACB to FHG, the whole BCD ſhall be equal to GHI; and by the ſame reaſon CDE is equal to HIK, and the other angles

angles equal to the other angles, as appears by the Construction; seeing that each part of the one is made equal to each of the parts of the other. c) 4. 6. Therefore the rectiline [c] figure A B C D E shall be equiangled to F G H I K. Now forasmuch as C D is to D A, as H I to I F, and as A D d) 22. 5. to D E, so I F is to I K, [d] in equal reason, C D shall be to D E, as H I to I K; therefore the sides about the equal angles C D E and H I K are proportional; even e) 4. 6. so [e] the sides about the equal angles E and K are proportional; because of the equiangled triangles A E D and F K I.



f) 4. 6. Again, [f] E A is to A D, as F K to F I, and A D to A C, as F I to F H, and A C to A B, g) 22. 5. as H F to F G: Therefore [g] in equal reason, E A shall be to A B, as K F to F G: Wherefore the sides about the equal angles E A B and K F G are proportional, and so of the others; therefore seeing that the rectiline figures are equiangled, and have the sides about the equal angles proportional, they are alike, and alike described. Therefore, On a given, &c. Which was to be demonstrated.

PROP. 19. THEOR. 13.



Like triangles A B E *and* D C F, *are to one another in a duplicate reason of their sides of the same reason.*

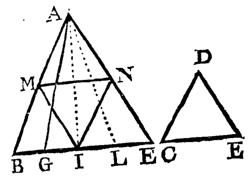
Construction LEt the triangles A B E and D C F, have the angles B and C equal; also E and F, and as A B to B E, so D C to C F: I say they are to one another in a duplicate reason of their sides of like reason B E and C F, or A B and D C, or A E and D F; that is to say, if you find the third proportional B G, the triangle A B E shall be to the triangle D C F, as B E is to B G; seeing that by the tenth Definition of the Fifth Book, the reason double is such.

Demonstration LEt therefore B G be a third proportional, and let A G be joyned to it; seeing that as A B is to B E, so D C to C F; alternately, as A B shall be to D C, so B E to C F; but as B E to C F, so
a) 11. 5. C F to B G by Construction: Therefore [a] as A B to D C, so C F to B G; therefore seeing that the triangles A B G and D C F have the sides about
b) 15. 6. the equal angles B and C reciprocal, [b] they shall be equal to one another:
c) 7. 5. Therefore as the triangle A B E shall be to the triangle D C F, so [c] the same triangle A B E shall be to the triangle A B G. But as the triangle
d) 1. 6. A B E is to the triangle A B G, so [d] the base B E to the base B G, they being of the same height. Therefore as the triangle A B E is to the triangle D C F, so B E is to B G.

Now seeing that the three lines B E, C F, and B G, are continually proportional, the reason of the first B E to the third B G, shall be said to be

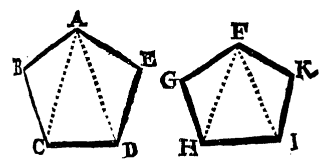
be double to the reason of B E the first, to C F the second; Therefore also the triangle A B E shall be to the triangle D C F, in reason double of the side B E to the side C F: Therefore, Like triangles, &c. Which was to be demonstrated.

COROLLARIE.



Whence it follows that if there be three lines proportional, as the first is to the third, so the triangle described on the first, shall be to the like triangle, and alike described on the second, &c. which is easie to be conceived by this last figure, where the third proportional line B G *being the quarter part of the base* B E; *or else* B E *being quadruple of the third proportional* B G, *the triangle* A B E *is also quadruple of the triangle* A B G, *as appeares by the eighteenth of the first,* G I, I L, *and* L E, *being equal to* B G; *and therefore also quadruple of the triangle* D C F, *as is shewn, and which is also manifest, the triangle* A B E *being divided into four other triangles* A M N, M B I, M I N, *and* I N E, *each of which is equal to the triangle* D C F, *which is easie to be understood by these figures which I thought good here to adde, to facilitate the sence and meaning of this Proposition, which had need be well understood, to the end the next following and the others after, may be the better comprehended.*

PROP. 20. THEOR. 14.



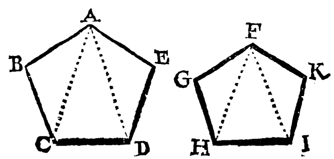
Like Polygons divide themselves into an equal number of like triangles, and proportional to their whole, and the Polygons are the one to the other in duplicate reason of their sides of the same reason.

LEt the Polygons be A B C D E and F G H I K, having the angles B A E and G F K equal, also the angles B and G, and so following, and having also the sides proportional about those angles; to wit, as A B to B C, so F G to G H, and as B C to C D, so G H to H I, &c. I say first of all that those Polygons may be divided into an equal number of like triangles.

Demonstration FOr from the angles A and F, draw A C, A D, F H, and F I, to the opposite angles C, D, H, and I, they shall be divided into an equal number of triangles; and forasmuch as the angles B and G are put equal, and the sides about them proportional; [a] the triangles A B C and F G H shall be equiangled, having the angles B A C, a) 6. 6.
G F H, B C A, and G H F, opposite to the equal Homologal sides; and
[b] therefore shall have the sides about the equal angles proportional, and b) 4. 6.
so shall be alike; by the same reason, the triangles A E D and F K I shall be

be alike; having the angles E A D, K F I, A D E, and F I K equal. And furthermore, c) 4. 6. [c] seeing that as A C is to C B, so F H to H G, because of the similitude of the triangles A B C and F G H: But as B C to C D, so G H to H I, by supposition, by reason of the similitude of the Polygons d) 22. 5. [d] in equal reason, as A C to C D, so F H to H I; and forasmuch as the angle B C D e) 6. 6. is put equal to G H I; but the angle cut off B C A, is shewn to be equal to the angle G H F, the remainder A C D shall be equal to the remainder F H I. Therefore seeing that the triangles A C D and F H I have the angles A C D and F H I equal, and the sides about them proportional; they shall be equiangled, and therefore alike; and so of the others, if there were more.

Secondly, I say that the triangles are proportional to their whole, that is to say, that each of the triangles in one of the Polygons, hath such reason to its triangle correspondent in the other Polygon, as the whole Polygon to the whole Polygon: Forasmuch as the triangles A B C and F G H are alike, they shall be to one another in double reason of their Homologal sides A C and F H, and by the same discourse the triangles A C D and F H I are in reason double of the same sides A C and F H. Wherefore as the triangle A B C is to the triangle F G H, so the triangle A C D shall be to the triangle F H I; seeing that the reason of the one and the other, is the duplicate reason of the side A C to F H. Likewise it may be concluded that the triangle A D E is to the triangle F I K, as the triangle A C D to the triangle F H I, and so following, if there were other triangles: Therefore the triangles of the one of the Polygons are proportional to the triangles of the other; in such sort as the triangles of the one are antecedents of the proportions, and the triangles of the other consequents. Now as one of the antecedents is to one of the consequents, f) 12. 5. [f] so all the antecedents are to all the consequents: Therefore as each of the triangles of one of the Polygons is to its correspondent triangle of the other Polygon, so the whole Polygon shall be to the whole Polygon: Therefore the triangles shall be proportional to their whole.



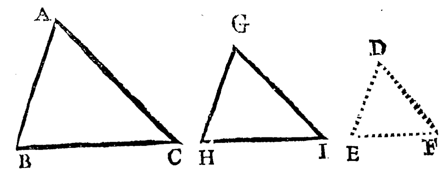
Lastly, I say that the Polygons are to one another in a duplicate reason of their sides, of the same reason, that is to say, if to the Homologal sides (for Example A B and F G,) there be found a third proportional, that the Polygon A B C D E shall be to the Polygon F G H I K, as the line A B the first, is to third proportional found. For seeing that as the triangle A B C is to the triangle F G H, so the Polygon A B C D E, to the Polygon F G H I K. But the triangle A B C is to the triangle F G H in a duplicate reason g) 19. 6. [g] of the side A B to the side F G. Likewise h) 11. 5. [h] the Polygons shall be the one to the other in duplicate reason of the same sides A B and F G. Therefore, The Polygons, &c. Which was to be demonstrated.

COROLLARIE.

From this Demonstration it is manifest that if there be three lines proportional, as the first shall be to the third, so the Polygon described on the first, shall be to the like Polygon, and alike described on the second, or else the Polygon described on the second, shall be to a like Polygon, and alike described on the third, as the first is to the third, which is evident, it having been demonstrated that the Polygon is to the Polygon, as the first A B is to the third proportional.

PROP. 21. THEOR. 15.

Rectiline figures A B C *and* D E F, *alike to one and the same rectiline figure* G H I, *are also alike to one another.*

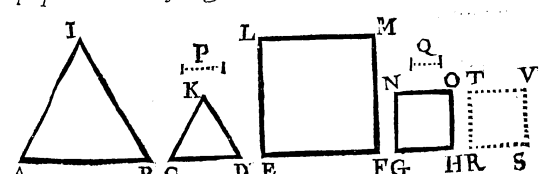


Demonstration. For seeing that the angles of A B C, are equal to those of a) 1. def. 6. [a] G H I, because of their similitude, and those of D E F to those of G H I; for the same cause, b) 1. c. 5. [b] the angles of A B C, shall be equal to the angles of D E F.

Again, because of the same similitude, the sides of A B C are proportional to those of G H I; to wit, those which are about the equal angles; also the sides of D E F are proportional to the sides of G H I; for the same cause, therefore c) 11. 5. [c] the sides of A B C shall be also proportional to the sides of D E F; to wit, those which environ equal angles. Therefore according to the Definition, A B C and D E F shall be alike: Therefore, Rectiline figures, &c. Which was to be demonstrated.

PROP. 22. THEOR. 16.

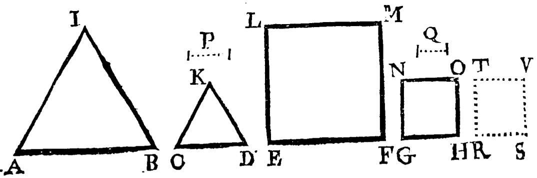
If four right lines A B, C D, E F, *and* G H, *are proportional, the rectiline figures* A B I, C D K, E M *and* G O, *alike, and alike described on them shall be proportional, and if the rectiline figures alike, and alike described on such right lines are proportional, those right lines shall be also proportional.*



Demonstration. For a) 11. 6. [a] to A B and C D, find the third proportional P, and to E F and G H, the third proportional Q: b) 22. 5. [b] In equal reason, as A B shall be to P, so E F to Q: But as A B is to P, so the recti-

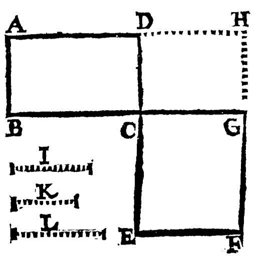
c) Co.20.6. rectiline figure c A B I is to the rectiline figure C D K alike, and alike deſcribed; by the ſame reaſon, as E F is to Q, ſo the rectiline figure E M;
d) 11. 5. is to the rectiline figure G O; therefore d as A B I is to C D K, ſo E M is to G O. Which was propoſed.

Secondly, Let the rectiline figures A B I, C D K, E M, and G O, be proportional; I ſay that the four right lines A B, C D, E F, and G H, are proportional; to wit, as A B to C D, ſo E F to G H.



e) 12. 6. For to the three lines A B, C D, and E F, let e there be found a fourth proportional R S, on which deſcribe the rectiline figure R V, alike, and
f) 21. 6. alike poſited to the rectiline figure E M; and therefore f to the rectiline figure G O; foraſmuch as A B is to C D, as E F is to R S: Likewiſe (as hath been already ſhewn,) as A B I ſhall be to C D K, ſo E M to R V;
g) 11. 5. but as A B I to C D K, ſo E M to G O: Therefore g as E M to R V, ſo E M
h) 9. 5. to G O; h therefore R V and G O are equal, which being alike, and alike poſited, of neceſity they ſhall be conſtituted on the equal right lines
i) 7. 5. R S and G H: Therefore i as E F ſhall be to R S, ſo E F to G H; but E F
k) 11. 5. is to R S, as A B to C D by ſuppoſition: Therefore k as A B to C D, ſo E F to G H: Therefore, If four lines, &c. Which was to be demonſtrated.

PROP. 23. THEOR. 17.



Equiangled Parallelograms A C *and* C F, *are the one to the other in a reaſon compounded of that of their ſides, (that is, of* B C *of* C G, *and of* D C *to* C E.

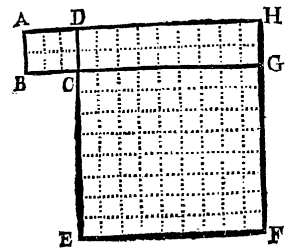
THat is to ſay, if you take three lines I, K, and L, in ſuch ſort as I may be to K, as B C to C G, and K to L, as D C to C E, the Parallelogram A C ſhall be to the Parallelogram C F, as I to L, which is the reaſon compounded of I to K, and of K to L, by the fifth Definition of this Book.

Demonſtration FOr, Diſpoſe the Parallelograms according to the equal angles in the point C, in ſuch ſort as B C and C G may make one only right line, which being ſo, ſeeing that the angles B C D and E C G are equal, and E C and C D ſhall alſo make one right line, as hath been ſhewn in the fourteenth Propoſition; prolong A D and F G to the point H, making the Parallelogram C H, and aſſume any right line, as

as I, and to B C, C G, and I, find a fourth proportional K; alſo to the three right lines D C, C E, and K, the fourth proportional L: Foraſmuch as a B C is to C G, as A C to C H: But as B C to C G, ſo I to K, as hath been ſhewn; likewiſe b as A C to C H, ſo I to K. a) 1. 6. b) 11. 5.

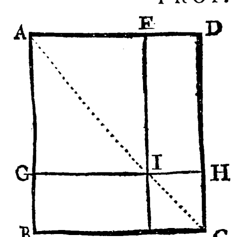
By the ſame reaſon it ſhall be ſhewn, that as H C is to C F, ſo K is to L; for as D C to C E, ſo c H C to C F: Therefore ſeeing that K is put to L, as D C to C E; ſo likewiſe H C ſhall be to C F, as K to L; Therefore d in equal reaſon, A C ſhall be to C F, as I to L; but the reaſon of I to L, is compounded of the reaſons of B C to C G, and D C to C E, by the fifth Definition of this Book: Therefore the reaſon of the Parallelogram A C to the Parallelogram C F, is compounded of the ſame reaſons. Therefore, The Parallelograms, &c. Which was to be demonſtrated. c) 1. 6. d) 22. 5.

SCHOLIUM.



This Propoſition will be eaſily underſtood by the annexed figure, thus divided into equal parts, where the ſides B C *is* 3, C G 9, *and* C E 8, *and in dividing* G C *by* B C, *to wit,* 9 *by* 3, *the quotient will give* 3, *for the denominator or quantity of the reaſon of* C G *to* C B; *and dividing alſo* C E 8 *by* D C 2, *the quotient will give* 4, *for the denominator or quantity of the reaſon of* C E *to* C D, *which denominators* 4 *and* 3, *multiplyed the one by the other, will produce* 12, *for the reaſon of the Parallelogram* C F, *to the Parallelogram* C A; *that is to ſay, that* C F *contiues* C A 12 *times, or elſe that* A C *is the twelfth part of* C F, *as is manifeſt, as well by the figure, as by the Demonſtration.*

PROP. 24. THEOR. 18.



Parallelograms G E *and* F H, *which are about the Diameter* A C, *of every Parallelogram, are alike to their whole* B D, *and alike to one another.*

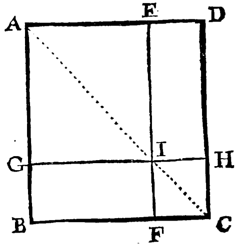
Demonſtration FOr they are equiangled to the whole, foraſmuch as the angle A is common to the Parallelogram G E, and to the whole B D; and a the exterior angle A E I equal to the interior angle D, and the exterior angle A G I equal to the interior angle B, and E I G the exterior angle, equal to the interior angle I F B; that is to ſay, to B C D, equal to the ſaid I F B: Wherefore the Parallelogram E G is equiangled to B D, by the ſame reaſon F H is equiangled to the ſame B D. a) 29. 1.

Now let E G and F H have alſo the ſides above the equal angles proportional to the ſides of the whole B D: It ſhall be thus demonſtrated, ſeeing

b) 29. 1. or Cor. 4. 6. seeing [b] that the triangle A G I is equiangled to A B C, and the triangle A F I equiangled to the triangle A D C, as A B shall be to B C, so A G to G I, and so the sides about the equal angles B and G shall be proportional; Again, [c] as B C to C A, so G I to I A: Likewise, as C A to C D, so I A to I E: Therefore [d] in equal reason, as B C to C D, so G I to I E; and therefore the sides about the equal angles B C D and G I E are proportional.

c) 4. 6.

d) 22. 5.

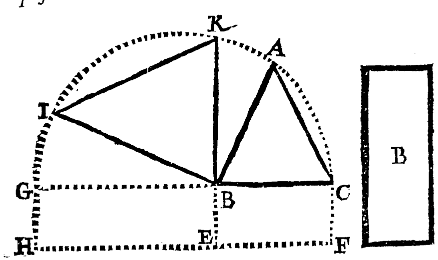


It may be likewise shewn that the sides about the other equal angles are proportional; Therefore by the first Definition of this Book, the Parallelogram E G shall be alike to the whole Parallelogram B D; and by the same reason, the Parallelogram F H alike to the same B D; and therefore also alike to one another:

e) 21. 6. Therefore, Parallelograms, &c. Which was to be demonstrated.

PROP. 25. PROBL. 7.

To describe a rectiline figure K I B, *alike to a given rectiline figure* A B C, *the which may be equal to another rectiline figure proposed* B.



Construction. On C B the side of the rectiline figure A B C, (to which another ought to be made alike,) constitute [a] the Parallelogram C E, in any angle whatsoever, equal to the rectiline A B C, and on

a) 44, 45. 1. B E make the Parallelogram B H, equal to B, having the angle E B G equal to B C F, and the lines C B and B G shall make one only right line, A B C, as by the 45th. Proposition of the first Book.

b) 13. 6. Now [b] finde the mean proportional B K, between C B and B G, on which constitute the rectiline figure B I K, alike, and alike posited to the rectiline A B C. I say that B I K is equal to the other rectiline figure B.

Demonstration. For seeing that C B, B K, and B G, are proportional, as C B [c] the first is to B G the third, so the rectiline figure on

c) Cor. 19, or 20. 6. the first C B, to the rectiline figure B I K, on the second B K; alike, and alike described; But [d] as C B to B G, so the Parallelogram C E, to the

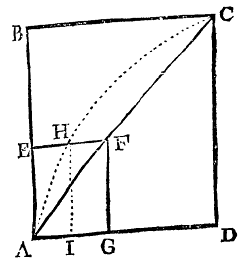
d) 1. 6. Parallelogram B H, of the same height: Therefore [e] as C E to B H, so

e) 11. 5. A B C

A B C to B I K; But [f] as C E to B H, so A B C to B: Forasmuch as A B C and C E are equal, and B H and B also equal: Therefore [g] as A B C to B, so A B C to B I K: Therefore [h] B and I B K shall be equal: But K I B is alike, and alike posited to A B C, by Construction: Therefore, We have described, &c. Which was to be done.

f) 7. 5.
g) 11. 5.
h) 9. 5.

PROP. 26. THEOR. 19.



If from a Parallelogram B D, *there be cut off a Parallelogram alike to the whole, and alike posited* E G, *having an angle* E A G *common with the whole; it doth consist about one and the same diameter* A C, *with the whole* B D.

Demonstration. Draw A F and C F, which lines if they make one right line, it is evident that E G shall be about one and the same diameter with B D. But if they make not a right line, draw A C, the diameter of the whole Parallelogram B D, cutting the side E F in the point H, by which point draw H I, parallel to F G. Now forasmuch as the Parallelograms B D and E I are about one and the same diameter A H C, [a] they shall be alike, and alike posited; Therefore [b] as B A to A D, so E A to A I; but as B A to A D, so E A is to A G; forasmuch as B D and E G are alike by supposition, and alike posited: Therefore [c] as E A to A I, so E A to A G; and therefore A I and A G shall be equal, the part to the whole, which is absurd: Therefore A F and F C do make one only right line; that is to say, the Parallelograms B D and E F do consist about one and the same diameter. Which was to be demonstrated.

a) 24. 6.
b) 1. def. 6.
c) 11. 5.

Now if it should be said that the right line A H C doth cut the side F G, there would happen the same absurdity.

PROP. 27. THEOR. 20.

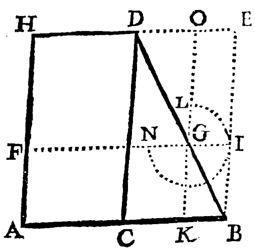
Of all Parallelograms applyed according to one and the same right line A B, *and being deficient by some Parallelogram figures, alike, and alike posited to that which is described on the halfe* B C; *the greatest is that which is applyed on the other half, and alike to the deficient.*

Divide the line A B into two equal parts in C, and on the half C B, constitute any Parallelogram whatsoever, as C E, whose diameter is B D: Therefore if the Parallelogram A B E H be finished, the Parallelogram A D constituted on the half A C, shall be applyed according to A B, and wanting of the Parallelogram C E, and alike to the deficient I say that the Parallelogram A D applyed to the half A C, and being defici-

deficient of the Parallelogram C E, is the greatest of all those which are applyed according to the line A B, and deficient by like Parallelograms, and alike posited to C E.

Demonstration FOr having assumed the point contingent G, in the diameter B D, by which point having drawn the right lines F G I and K G parallel to A B and B E, the Parallelogram F K shall be applyed according to A B, and wanting by the Parallelogram K I, [a] which is alike to C E, and alike posited; being about one and the same diameter with C E. But [b] forasmuch as the complements C G and G E are equal, if you adde the common Parallelogram K I, C I and K E shall be equal: But [c] C I is equal to C F, being on equal bases A C and C B: Therefore C F and K E shall be also equal, and having added the common part C G, the Parallelogram A G, and the Gnomon L N shall be equal. Therefore seeing that C E is greater than the Gnomon L N, (for it containeth the Parallelogram G D more than the Gnomon,) A D [d] equal to C E because of the equal bases A C and C B, shall be also greater than A G, by the same Parallelogram D G; In like manner, it may be shewn that A D is greater than all other Parallelograms which are applyed according to A B, in such sort as the point G be between the points B and D; that is to say, which imployeth or occupieth a line greater than the half A C; but have lesser altitude than A D, provided that the deficients be alike to C E.

a) 24. 6.
b) 43. 7.
c) 36. 1.
d) 36. 1.



SCHOLIUM.

Otherwise it may be demonstrated that A D *is greater than* A G, *in this manner*; [e] *The Parallelograms* F D *and* D I *are equal, seeing that the bases* H D *and* D E *are equal: But* D I *is greater than* G E, *that is to say, then the complement* C G, *(which is equal to* G E,*) by the Parallelogram* D G: *Therefore* F D *shall be also greater than* C G, *by the same parallelogram* D G; *and therefore adding the common* C F, A D *shall be greater than* A G, *by the same Parallelogram* D G.

e) 36. 1.

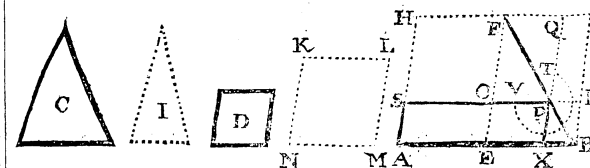
PROP. 28. PROBL. 8.

To a given right line A B, *to apply a parallelagram* A P, *equal to a rectilire figure given* C, *wanting by a parallelogram figure* P B, *which is alike to another given parallelogram* D, *but it behoveth that the given rectiline figure to which an equal rectiline figure ought to be applyed, be not greater than that which is applyed to the half* A E *of the given line, the deficients being alike to that which is applyed on the halfe, and to that which ought to be deficient by a like parallelogram.*

Con-

Construction THe line A B being divided into two equal parts in the point E, on the half E B [a] describe the Parallelogram E G alike, and alike posited to D, and so finish the whole Parallelogram B H; If then A F be equal to C, being applyed to A B, deficient by the Parallelogram E G, which is made alike to D, you have your desire. But if A F be greater than C, (for it ought not to be lesse, seeing that by the precedent Proposition it is the greatest of all the applyed, the Deficiences being alike, you cannot apply any one to A B, equal to C, but they shall all be lesse; Wherefore *EUCLIDE* hath added [but it behoveth that the given Rectiline, &c.] E G equal to the same, shall be also greater than C; let it then be greater by I, and constitute K M alike, and alike posited to D, or to E G, but equal to the excesse I, in such sort as E G be equal to C, and to K M together; and therefore greater than K M: Therefore seeing that as G F to F E, so N K to K L, because of their resemblance, the sides G F and F E shall be also greater than N K and K L; for if they were equal to them, or lesse, E G should be likewise equal to N L, or lesse, as appears.

a) 18. 6.



Having therefore cut off F O and F Q equal to K N and K L, and having made the Parallelogram O Q, it shall be equal to K M; and alike, and alike posited thereto; and therefore to E G, and [b] also about one and the same diameter with E G, which diameter is B F; and having prolonged Q P and O P, the Parallelogram A P shall be applyed to the line A B deficient of P B, [c] which is alike, and alike posited to E G; and therefore also to D: I say that A P is equal to the rectiline C.

b) 26. 6.
c) 24. 6.

Demonstration FOr [d] seeing that P G is equal to P E, the complement to the complement, if you adde the common P B, the Parallelogram B Q shall be equal to E R; that is to say, to E S, which is equal to E R, the bases A E and E B being equal: Wherefore if to the equal parallelogram A O and B Q, you adde the common part E P, A P shall be equal to the Gnomon T V: But the Gnomon T V is equal to the rectiline C; (for seeing that the Parallelogram E G is equal to C and N L together, if you take away the equal Q O and L N, the Gnomon T V shall remaine equal to C:) Therefore A P shall be also equal to the same C: Therefore, To a given right line A B, &c. Which was to be done.

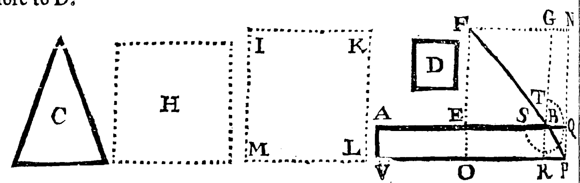
d) 36. 1.

PROP. 29. PROBL. 9.

To a given right line A B, *to apply a parallelogram* A P, *equal to a given rectiline figure* C, *exceeding the same given line* A B, *by a parallelogram* R Q, *alike to another parallelogram given* D,

Con-

Construction. Divide A B into two equal parts in E, and on the halfe E B,
describe the Parallelogram E G alike to D, and alike po-
a) 14. 2. sited, then [a] make the square H equal to the rectiline figure C, and
b) 25. 1. Parallelogram E G, [b] to which make the Parallelogram I L equal, but
alike to E G, and alike posited: Wherefore I L shall be greater than E G,
seeing that it is equal to the square H, which is made equal to C, and to
E G together; Therefore seeing that because of the similitude of M K
and E G, as M I to I K, so E F to F G; the sides M I and I K shall be also
greater than E F and F G; for if I M and I K were equal to E F and F G,
or lesse, M K should be equal to E G, or lesse, as is manifest: Therefore
having prolonged F E and F G, in such sort, as F O and F N may be
equal to I M and I K, and having finished the Parallelogram O N, the
same shall be alike, and alike posited to E G; seeing that it is equal to
c) 26. 6. M K, and alike, and alike posited: Wherefore [c] O N and E G are consti-
tuted about one and the same diameter F P; having prolonged A B and
G B to Q and R, and P O until they meet with A V, parallel to F O, to
the point V the Parallelogram A P shall be applyed to the line A B, ex-
d) 24. 6. ceeding it by the Parallelogram Q R, which [d] is alike to E G, and there-
fore to D.



Demonstration. I say then that AP is equal to the rectiline figure C, for [e] seeing
e) 36. 1. that AO and E R are equal; and [f] E R is equal to the comple-
f) 43. 1. ment B N, A O shall be also equal to B N: Therefore adding the common
part O Q, A P shall be equal to the Gnomon S T; but the said Gnomon is e-
qual to the rectiline C; (for seeing that M K, that is to say O N, is equal to C
and EG together, if you take away the common part EG, there will remain
the Gnomon S T, equal to C:) Therefore A P shall be also equal to the
rectiline figure C: Therefore, To a given right line, &c. Which was
to be done.

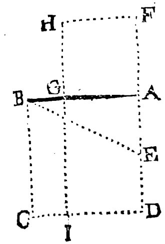
PROP. 30. PROBL. 10.

To divide a terminated right line proposed A B, *according to extream and mean reason in* G.

Construction. Having described on the proposed right line A B, the
a) 29. 6. square A C, [a] to the side D A apply the rectangle D H,
equal to the square A C, exceeding D A by the Parallelogram A H, alike
to the same square, in such sort as A H shall be likewise a square, consi-
dering that a square only is alike to a square: But let I H divide
A B in the point G, I say that A B is divided according to extream and
mean reason.

Demon-

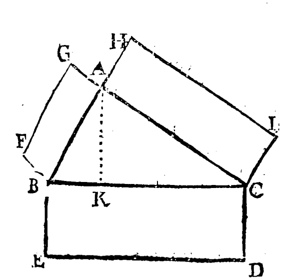
Demonstration. Seeing that the rectiline figure D H, and the square A C,
are equal, if you take away the common part A I, F G
and G C will remain equal, the which having the angles A G H and
B G I equal, to wit, right angles, [b] the side which are about those equal angles b) 14. 6.
shall be reciprocal, to wit I G, that is to say, A B
its equal, is to G H, that is to say, to A G, equal
to G H, as A G is to G B: Wherefore the whole
A B being to the greatest segment A G, as the
said segment A G is to the lesser segment G B;
A B is divided according to extream and mean
reason, by the Definition.



Otherwise, we may demonstrate that A B is
divided in the point G, according to extream
and mean reason; seeing that there are three lines
given, A B, A G, and G B, and that the rectan-
gle G C conteined under the first A B, and the
third G B, is equal to the square of the mean
A G; [c] those lines shall be proportional, to wit, as A B the first is to A G c) 17. 6.
the second, so A G the second shall be to G B the third; Therefore by
the Definition, A B is divided in the point G, according to extream
and mean reason.

Yet otherwise, to finish the whole Problem; Let [d] A B be divided in d) 11. 2.
the point G, in such sort as the rectangle conteined under the whole A B,
and the segment G B be equal to the square of the other segment A C:
I say that A B is divided at G, according to mean and extream reason:
For [e] again, as before, A B, A G, and G B, shall be proportional, and e) 17. 6.
so the Proposition manifest: Therefore, We have divided, &c. Which
was to be done.

PROP. 31. THEOR. 21.

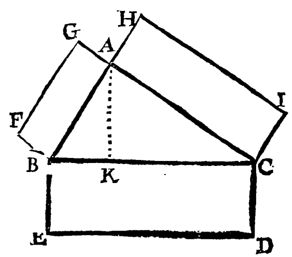


In rectangled triangles, as A B C, *the figure* B D, *described on the side* B C, *which subtendeth the right angle* B A C, *is equal to the two figures* A F *and* A I, *alike and alike described on the sides* B A *and* A C, *which contein the right angle* B A C.

Demonstration. For having drawn the perpendicular A K, from the
point A on B C, [a] as B C shall be to C A, so a) Cor. 8. 6.
C A shall be to C K: Wherefore as B C the first, to C K the third,
so the figure B D described on the first, shall be to the figure C H,
alike, and alike described on the second, [b] and by conversion of reason, b) Cor. 19. or 20. 6.
as

as C K to B C, ſo the figure C H to the figure B D: In like manner we ſhall ſhew that as B K is to B C, ſo the figure B G is to the figure B D, ſeeing that the three lines B C, B A, and B K are proportional.

Therefore ſeeing that as C K the firſt magnitude, is to B C the ſecond, ſo C H the third, is to B D the fourth; alſo as B K the fifth magnitude
c) 24. 5. is to B C the ſecond, ſo B G the ſixth, is to B D the fourth: [c] As the firſt C K, with the fifth B K, ſhall be to the ſecond B C; ſo the third C H, with the ſixth B G, ſhall be to B D the fourth; but C K the firſt, and B K the fifth together, are equal to the ſecond B C: Therefore the third C H, and the ſixth B G together, ſhall be likewiſe equal to the
d) 8. 6. fourth B D. Which was propoſed.



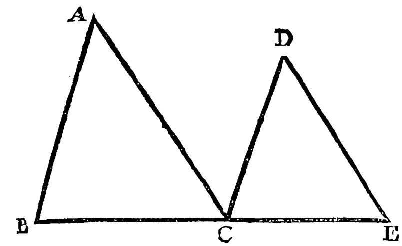
Otherwiſe, [d] ſeeing that the triangle K A C is alike to the triangle A B C, and that the ſides B C and C A are Homologal; (for as B C to C A, in the triangle
e) 19. 6. A B C, ſo A C to C K, in the triangle A C K;) [e] the triangle K A C ſhall be to the triangle A B C in duplicate reaſon of C A to B C:
f) 19, 20. 6. But [f] the figure C H is to the figure B D, alſo in a duplicate reaſon of C A to B C; There-
g) 11. 5. fore [g] the triangle K A C ſhall be to the triangle A B C, as the figure C H to the figure B D, by the ſame reaſon it may be ſhewn that as the triangle K A B is to the triangle A B C, ſo the figure B G is to the figure B D.

Again, foraſmuch then as K A C the firſt magnitude, is to A B C the ſecond, ſo C H the third is to B D the fourth. Alſo as K B A the fifth, to
h) 24. 5. A B C the ſecond, ſo B G the ſixth, to B D the fourth; [h] as K A C the firſt, with K B A the fifth, ſhall be to the ſecond A B C; ſo C H the third, and B G the ſixth together, ſhall be to the fourth B D. But K A C and K B A, the firſt and fifth together, are equal to the ſecond A B C: Therefore C H and B G, the third and ſixth together, ſhall be likewiſe equal to B D the fourth. Which was propoſed.

Otherwiſe, As the ſquare of A C the firſt magnitude, is to the ſquare of B C the ſecond magnitude, ſo the figure C H the third magnitude,
i) 19, 20. 5. to the figure B D the fourth magnitude; ſeeing [i] that they are all in duplicate reaſon of their ſides A C and B C; Likewiſe, as the ſquare of A B the fifth magnitude, is to the ſquare of B C the ſecond magnitude, ſo the figure B G the ſixth magnitude, is to the figure B D the fourth magni-
k) 24. 5. tude: Therefore [k] as the firſt magnitude with the fifth, to wit, the ſquare of A C, with the ſquare of A B, ſhall be to the ſecond, that is to ſay, to the ſquare of B C, ſo the third magnitude with the ſixth, to wit, the figure C H, with the figure B G, is to the fourth, to wit, to the figure B D: But the ſquares of A C and A B together, are equal to the ſquare of B C: Therefore the figures C H and B G together, ſhall be alſo equal to the figure B D. Which was propoſed: Therefore, In rectangled triangles, &c. Which was to be demonſtrated.

PROP.

PROP. 32. THEOR. 22.



If two triangles A B C *and* D C E, *which have two ſides* A B *and* A C *proportional to two ſides* D C *and* D E, *are diſpoſed (compounded) according to an angle* A C D, *in ſuch ſort as their Homologal ſides* A B *and* D C, *and* A C *and* D E *may be alſo parallels, the other ſides* B C *and* C E *of thoſe triangles ſhall be placed in one and the ſame right line.*

Demonſtration. For ſeeing that A B and D C are Parallels, [a] the angle A ſhall be equal to the alternate angle A C D, and by the ſame reaſon, the angle D ſhall be equal to the ſame angle A C D: And therefore A and D ſhall be equal to one another; foraſmuch then as the triangles A B C and D C E have the ſides about the equal angles A and D proportional, they ſhall be equiangled, and ſhall have the angles B and D C E equal: Therefore adding the equal angles A and A C D, the two angles B and A ſhall be equal to the two angles D C E and A C D, that is to ſay, to the whole angle A C E: Again, adding the common angle A C B, the two angles A C E and A C B ſhall be equal to the three angles of the triangle A B C: But [b] theſe three angles are equal to two right angles: Therefore the two angles A C E and A C B ſhall be equal to two right angles; and [c] therefore B C and C E ſhall make one right line: Therefore, If two triangles, &c. Which was to be demonſtrated.

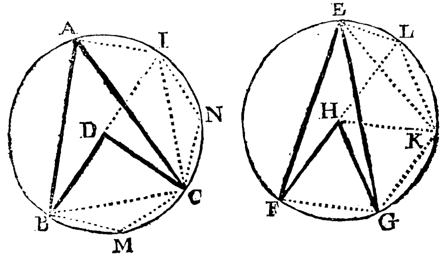
a) 29. 1.
b) 32. 1.
c) 14. 1.

PROP. 33. THEOR. 23.

In equal circles A B C *and* E F G, *the angles have the ſame reaſon to one another as the circumferenes* B C *and* F G, *on which they inſiſt, whether they inſiſt being conſtituted at the Centers* D *and* H, *or at the circumferences at* A *and* E; *And alſo the Sectors* B D C *and* F H G *are the ſame to one another, foraſmuch as they are conſtituted at the Center.*

Demon-

a) 1.4. *Demonstration.* FOr having drawn the right lines BC and FG, [a] apply in the circles the line CI, equal to BC, and GK and KL, each equal to FG, and draw the right lines ID, KH, and LH: Then forasmuch as BC and CI are equal, [b] the arches BMC and CNI
b) 28.1. shall be likewise equal; and therefore [c] the angles BDC and CDI
c) 27.1. are equal; by the same reason, the arches FG, GK, and KL, and the angles FHG, GHK, and KHL, are equal: Therefore the angle BDI shall be as much Multiplex of the angle BDC, as the arch BCI is Multiplex of the arch BC; by the same reason, the angle FHL shall be as much Multiplex of the angle FHG, as the arch FGL is of the arch FG; seeing that the angles BDI and FHL are divided each into as many equal parts as the arches BI and FL, on which they insist: Therefore, if the arch BCI be equal to the arch
d) 27.3. FGKL, [d] of necessity the angle BDI shall be equal to the angle FHL, and if the arch be greater than the arch, the angle shall be greater than the angle, and if lesser, lesser: Wherefore the arch BCI, and the angle BDI, equimultiplices of the first and third Magnitudes,



BC and BDC shall be together either greater, equal, or lesser than the arch FGKL, and the angle FHL equimultiplices of the second Magnitude FG, and of the fourth FHG; if they be
e) 6. def. 5. taken so as to answer one another: Wherefore [e] as the arch BC the first Magnitude, is to the arch FG the second Magnitude, so the angle BDC the third Magnitude, to the angle FHG the fourth Magnitude, the same may be shewn of the angles at the circumference.

Secondly, I say that as the circumference is to the circumference, so the sector is to the sector: Constitute in the segments BC and CI, the angles BMC and CNI, which [f] shall be equal, insisting on the equal
f) 27.3. arches BAC and CBAI: Wherefore the segments BMC and CNI shall be alike; [g] and therefore equal to one another, being constituted on
g) 24.3. the equal right lines BC and CI: Therefore adding the triangles BDC
h) 4.1. and CDI, [h] which are likewise equal, the sectors BDC and CDI shall be made equal; wherefore the sector BDI shall be as much Multiplex of the sector BDC, as the arch BCI is of the arch BC: Likewise the sector FHL shall be shewn to be as much Multiplex of the sector FHG, as the arch FGKL is of the arch FG; but forasmuch as if the arch BCI be equal to the arch FGKL, also the sector BDI is equal to the sector FHL, (as hath been shewn in the sectors BDC and CDI,) and if greater, greater, and if lesse, lesse; therefore the arch BCI and the

the Sector BDI, equimultiplices of the first Magnitude BC, and of the third Magnitude BDC, shall be deficient together, of the arch FGKL, and of the sector FHL, equimultiplices of the second Magnitude FG, and of the fourth FHG, or together shall be equal, or shall exceed, if they be taken so as they answer to one another: Therefore [i] as the arch BC the first Magnitude, is to the arch FG, the second i) 6. def. 6.
Magnitude; so the sector BDC, the third Magnitude, to the sector FHG the fourth Magnitude: Therefore, If in equal Circles, &c. Which was to be demonstrated.

COROLLARIE. I.

It is manifest from this, that the Sector is to the Sector, as the angle is to the angle; for the reason of the one to the other, to wit, of the angle BDC, to the angle FHG, and of the Sector BDC to the Sector FHG, is the same as the reason of the arch BC to the arch FG: Wherefore [k] they shall be the same to one another. k) 11.5.

COROLLARIE. II.

It is likewise manifest, that as the angle at the Center is to four right angles, [l] so the arch which subtends the same angle, is to the whole Circumference; and l) 33.6.
contrarily, as four right angles are to the angle which is at the Center, so the whole Circumference is to the arch which subtends the said angle.

For as the angle at the Center is to a right angle at the Center, so the arch which subtends the same angle, is to a quarter of the Circumference of the Circle, or to the Quadrant which subtends the said right angle: Wherefore as the angle at the Center, shall be [m] to the quadruple of the right angle; that is to say, m) 33.6.
to four right angles; so the arch subtended of the same angle, shall be to the quadruple of the Quadrant, that is to say, to the whole Circumference, by what hath been demonstrated in the two and twentieth Proposition of the Fifth Book. Which was first proposed.

Forasmuch then as the angle at the Center is to four right angles, so the arch which subtends that angle, is to the whole Circumference; by conversion of reason, as four right angles to the angle at the Center, so the whole Circumference shall be to the arch which subtends the said angle at the Center. Which was in the second place proposed.

But it may also be thus demonstrated, [n] seeing that as the right angle at the Cen- n) 33.6.
ter is to the angle at the Center, so the Quadrant or fourth part of the Circumference, which subtends the same right angle, is to the arch which subtends the same angle at the Center not being a right angle. Likewise as the quadruple of the right angle, that is to say, four right angles, shall be to the angle at the Center, by what hath been shewn in the two and twentieth Proposition of the Fifth Book; so the quadruple of the Quadrant, that is to say, the whole Circumference shall be to the arch which subtends the same angle. Which was proposed.

The End of the Sixth Element of EUCLIDE.

THE SEVENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

Hitherto, *Euclide* hath treated of the first part of *Geometry*, to wit, of that which concerneth *Planes*; there remaines the other part which treats of *Solids*, but first of all it hath been fonnd necessary to treat of *Lines commensurable* and *incommensurable*, for as much as the knowledge of these *Lines* is requisite for the explication and demonstration of the proprieties of divers *Solid Bodies*, and principally of those which are named *Regulars*; for that, without the knowledge of those *Lines* the tract of *Solids* would be imperfect; Moreover, without those *Lines*, divers sides, as well *Planes* as *Solids* (if you will reduce *Geometry* to use and practice) cannot be expressed or understood, for divers of the sides are those *Lines* which the Greeks call ἄλογοι, that is to say, *irrational*, if they be not irrational, they are incommensurable in length amongst themselves, and therefore are not expressed by *Numbers*.

Now seeing that the explication and understanding of those *Lines*, is as it were wrapped up, and joyned with *Numbers*, in such sort as without them they cannot be known, it is found needful in the first place to treat of *Numbers*; Where-

fore, in this ſeventh, and the two following Books, *Euclide* unfoldeth the affections and proprieties of *Numbers*, in as much as they may ſerve in *Geometry*, to the end that in the Tenth Book he may make the Demonſtrations of *Lines commenſurable* and *incommenſurable*, more amply, and with greater facility.

DEFINITIONS.

1 UNITY *is that according to which every thing, of thoſe which are, is ſaid to be one.*

BEginning then, according to his Cuſtome, by the Principles, he defineth firſt of all Unity: For according to unity we uſe to call one body, one body: one animal, one animal: one ſtone, one ſtone: and ſo of other things. And finally, Unity receiveth no diviſion in Numbers, even as a point in Magnitudes.

2 NUMBER, *is a Multitude compoſed of Unities.*

THat is to ſay, if you take as many unities as you pleaſe together, the collection of them ſhall be called number, from whence it is manifeſt, that in every number there are ſo many parts, as there are unites that make it: So as unity is part of each number named by the ſame number of which it is a part, as the number 8, compounded of 8 unites, divideth it ſelf into ſo many parts, to wit in 8 unites, each of them being ſaid to be the 8th. part of the number 8, &c. from whence it follows, that all the numbers are commenſurable amongſt themſelves, ſeeing they are all meaſured by one and the ſame meaſure, to wit, by unity, as is before ſaid, which cannot agree with all Magnitudes, as ſhall be ſhewn in the Tenth Book.

3 *A Number is a part of another number; the leſſer of the greater, when the leſſer doth meaſure the greater.*

THis Definition reſembles that of the 5th. Book, by which *Euclide* defineth the part of a quantity continued, for as there, ſo here, he defineth the aliquot part, which is ſaid exactly to meaſure his whole. As 6 is ſaid to be part of 18, foraſmuch as 6 meaſureth 18 exactly, and leaves no remainder, in like manner, each of theſe nnmbers 3, 4, 6, 8, is a part of 576. Seeing that each of them doe meaſure it exactly, for every part taketh his name from the number by which it meaſureth another. As 6 is ſaid to be the ſeventh part of 42, becauſe 6 meaſureth 42 by 7, that is to ſay, is conteined in 42, 7 times exactly, and ſo of the reſt.

4 But

4 *But a number is ſaid to be parts of another greater number, when the leſſer doth not meaſure the greater.*

AS 4 is not ſaid to be part of 6, for as much as 4 the leſſer number doth not meaſure 6 preciſely, but ſhall be called the two parts of 6, to wit $\frac{4}{6}$, and 5 ſhall be called parts of 18, to wit $\frac{5}{18}$, for as much as unity which is their common meaſure, is conteined 5 times in 5, and 18 times in 18.

5 *A number is ſaid to be Multiplex of another leſſer number, when the leſſer doth meaſure the greater.*

EVen as a number which doth preciſely meaſure a greater number, is ſaid to be part of that greater number; ſo alſo the greater number which is meaſured by the leſſer number, is ſaid to be Multiplex of the leſſer number, and ſo alſo, as a number which doth not exactly meaſure a greater number, is not ſaid to be a part but parts of the greater number, alſo the greater ſhall not be ſaid to be Multiplex of the leſſer number by which it is not meaſured, as 4 is a part of 24. Seeing that 4 doth meaſure 24 preciſely, and in like manner, 24 is ſaid to be Multiplex of 4, for that it is meaſured of 4, and 30 is Multiplex of 6, foraſmuch as 6 doth meaſure 30, &c.

6 *An even number is that which may be divided in two equal parts.*

AS all the following numbers 4, 10, 40, 100, 1000, are ſaid to be even numbers, foraſmuch as each of them may be divided in two equal parts, whereof their halves are 2, 5, 20, 50, 500.

7 *But an odde number is that which cannot be divided into two equal parts, or which differs from an even number by an unite.*

AS all theſe numbers 5, 11, 15, 37, 101, are called odde numbers, becauſe they cannot be divided in two equal parts, or becauſe they differ by unity from the even numbers 4, 10, 14, 36, 100, &c.

8 *A number evenly even, is that which an even number doth meaſure by an even number.*

AS 32 is ſaid to be evenly even, becauſe 8 an even number doth meaſure it by 4, which is alſo an even number, 24 is alſo ſaid to be evenly even; ſeeing that 4 an even number doth meaſure it by 6, alſo an even number, &c.

9 *A number evenly odde, is that which an even number doth meaſure by an odde number.*

THat is to ſay, that if an even number meaſure an even number by an odde number, the nnmber meaſured ſhall be called evenly odde. As

30, which 2 an even number doth measure by 15 an odde number, and 6 an even number doth measure the same 30 by 5, an odde number, &c.

10 *But a number oddely odde, is that which an odde number doth measure by an odde number.*

AS 45, which is measured of 9 an odde number, by 5 also an odde number, is said to be a number oddely odde, and also 15 which is measured of 5 by 3, and these 9, 21, 25, 27, 33, 35, &c. are said to be oddely odde nnmbers.

11 *A Prime or First number, is that which only unity doth measure.*

THat is to say, that if a number cannot be measured by any other number, but onely by unity, so as it cannot be said to be evenly even, or evenly odde, or unevenly odde, that number shall be called a Prime number, as these numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, &c. for onely unity measureth them.

12 *Numbers, Primes to one another, are those which have onely unity for their common measure.*

THat is to say, that if two or more numbers are proposed, and that there cannot be found any number which doth precisely measure each of them, save onely unity, those numbers are said to be Primes amongst themselves, as 15 and 8, or 3 and 5, are Primes amongst themselves, for as much as there is onely unity that may be a common measure to them.

13 *A Compound number, is that which some number may measure.*

THe Geometricians do call that a Compound number which some other number doth measure besides unity, as 15, which is measured by 5 and 3. Now it is manifest that all even numbers excepting onely the Binary, are Compound numbers, being all measured by 2, from whence it follows, that all the Prime numbers save onely the Binary are odde, the Binary being the first of even numbers.

14 *But numbers compounded amongst themselves, are those which are measured by some number, as their common measure.*

TWo or more numbers, which are measured by some number besides unity, as by their common measure, are said to be compounded amongst themselves, although that each be not compounded of himself; As 15 and 24, are compounded amongst themselves, for as much as 3 doth measure both, as their common measure: also 7, 21, and 35 are compounded amongst themselves, for the first of them doth measure it self, and the two others, although it be a Prime of it self.

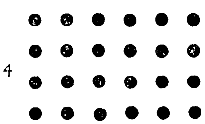
15 A

15 *A number is said to multiply another number, when there is produced another number, which is compounded as many times of the Multiplicator, as there are unites in the Multiplicand.*

AS 6 shall be said to multiply 8, when 8 shall be compounded as often times as there are unites in the Multiplicator 6, to wit, six times, and that there shall be produced another number, to wit 48, also by change, 8 shall be said to multiply 6, if you take 6 eight times, to wit, as often as there are unites in 8, and that it make the same 48; even so a number shall be said to be produced of two numbers, by multiplying the one in the other, as 63 of 7 and 9, &c.

16 *But when two numbers multiplying one another doe produce a number, the number produced shall be called a Plaine, and the numbers which multiply one anonher shall be said to be sides of the Plaine.*

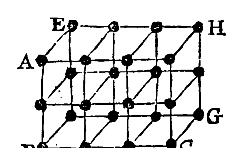
EVery number produced by the mutual Multiplication of two numbers is called a Plaine, forasmuch as his unites being disposed in length and breadth doe represent a rectangle Parallelogram, whose sides are the two numbers multiplying one another, as hath been said in the second Book, as 24 the product of 4 by 6, is said to be a Plaine number, whose sides are 4 and 6; Seeing that their unites disposed in length and breadth doe represent a rectangled Parallelogram.

4 

6

17 *When three numbers multiplying one another doe produce a number, the number produced shall be called a Solid, and the numbers which multiply one another shall be called the sides of the Solid.*

FOrasmuch as these three numbers 2, 3, 4, multiplying one another produce 24, for 2 by three produceth 6, and 6 by 4 produceth 24, otherwise 2 by 4 makes 8, and 8 by 3 makes 24, that number 24 shall be called a Solid; and the numbers 2, 3, 4, are the sides; for as much as the unites disposed in length, breadth and depth, doe represent a Solid figure, called Parallelepipedon, as shall be explained in the 11th. Book, whose three dimensions are expressed by the three numbers which are multiplied in one another; For if you multiply 2 by 4, you shall have 8 for the number of the Base BG of the Solid AG, whose length is 4 unites, and breadth 2; and if this Base be multiplied

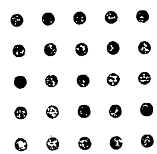


by

by 3, the whole number of the Solid, to wit 24, will be produced, being three unites high, &c.

18 *A Square number is that which is equally equal, or which is conteined under two equal numbers.*

THat is to ſay, produced of two equal numbers multiplied by one another. This Plaine number, which is equally equal, that is to ſay, whoſe unites diſpoſed in length and breadth, doe reſemble a rectangle Parallelogram, whoſe Length and Breadth are equal, and in ſuch ſort as all the ſides are equal, or that is produced of two equal numbers, and therefore is conteined under them, is called a ſquare, as 25, conteined under 5 and 5, that is to ſay, is produced by the mutual multiplication of the two fives the one by the other, for his unites being ranged in form of a Plaine, doth repreſent a perfect Square, having five unites on each ſide, and therefore equally equal; But either of the ſides of the Multiplication of which it is produced, is called the root of the Square.

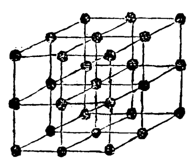
5 

5

19 *A Cube number is that which is equally equal equally, or which is conteined under three equal numbers.*

THat is to ſay, produced of three equal numbers multiplied by one another.

He calls this Solid a Cube, which is equally equal equally, that is to ſay, whoſe unites diſpoſed in length, breadth, and depth, doe repreſent a Geometrical Cube, in ſuch ſort as that all his Dimenſions, to wit, length, breadth and height, or depth, are equal; or which is produced of three equal numbers, as 27, which is conteined under theſe three numbers 3, 3, 3, for three times 3 makes 9, and three times 9 makes 27, for all his unites ranked in form of a Solid doe repreſent a Geometrical Cube, having 3 in every Dimenſion, and each of theſe three numbers is by the Geometricians called the Side of the Cube, and of divers Arithmeticians the Cubick Root.



20 *Numbers are proportional, when the firſt is as much Multiplex of the ſecond, or the ſame part or parts thereof that the third is of the fourth.*

OR elſe when the firſt doth contein as many times the ſecond, and over and above one and the ſame part or parts thereof, that the third doth contein of the fourth.

To the end you may the better comprehend all the numbers proportional (as all the unequal numbers are) we have added theſe words, as well becauſe you may underſtand what *Euclide* ſaith hereon, as alſo to render this Definition more plain and intelligible; (or elſe, when the firſt conteineth as many times the ſecond, and over and above one and the ſame part or parts thereof, as the third doth of the fourth.) For the vulgar definition of *Euclide*, (which *Clavius* thinks to be corrupted) comprehendeth onely the numbers proportional in the proportion multiplex and ſub-multiplex, and the other proportions of leſſer inequality. For in the proportion Multiplex, there are four numbers proportional, when the firſt is ſo much Multiplex of the ſecond, as the third is of the fourth; and in the ſub-multiplex when the firſt is the ſame part of the ſecond, that the third is of the fourth; and in the other proportions of leſſer inequality, when the firſt is the ſame parts of the ſecond that the third is of the fourth, as the Definition of *Euclide* will have it, but by that it cannot be comprehended, which are the numbers proportional in reaſon *Super-particular, Super-partient*; Multiply *Super-particular* and Multiply *Super-partient*, for in all theſe the firſt number is not ſo Multiplex of the ſecond, as the third is of the fourth, nor the ſame part or parts; but the firſt conteins the ſecond, and the third the fourth equally, to wit, once or more, and over and above one and the ſame part thereof, or ſame parts; as is manifeſt by Definition the 4th. of the Fifth Book.

Wherefore theſe numbers 12, 4, 9, 3, are proportional, the firſt being ſo Multiplex of the ſecond, as the third is of the fourth, to wit Triple; For 4 is ſuch a part of 12, (to wit one third) as 3 is of 9: Againe, 6, 8, 9, 12, are proportional, foraſmuch as 6 is the ſame parts of 8, as 9 is of 12, to wit, the three quarters; Laſtly, 7, 6, 14, 12, and 7, 4, 14, 8, and 11, 5, 22, 10 and 12, 5, 24, 10, are numbers proportional; for as in the firſt example, the firſt number conteins once the ſecond, and the third conteins once the fourth, and over and above, $\frac{1}{6}$ part, and in the ſecond example, the firſt conteins once the ſecond, and the third the fourth, and over and above $\frac{3}{4}$ parts, in the third example twice, and $\frac{1}{5}$ part, and in the laſt example the firſt conteins twice the ſecond, and the third the fourth, and over and above $\frac{2}{5}$ parts, for otherwiſe the numbers ſhould not be in any kind proportional: Therefore, whenſoever 4 numbers are put proportional, you muſt accord that if the greater numbers are compared to the leſſer, the firſt and the third ſhall be equi-multiplices of thé ſecond and the fourth, or elſe the firſt and the third ſhall equally contein the ſecond and the fourth, and over and above the ſame part or parts, and contrariwiſe, if the firſt and the third be put equi-multiplices, or elſe that the firſt be ſaid to contein ſo many times the ſecond, as the third doth contein the fourth, and over and above the ſame part or parts, you may gather that the numbers are proportional, as if you compare the leſſer with the greater, and that they are ſaid to have the ſame proportion, you muſt confeſſe that the firſt is the ſame part of the ſecond, as the third is of the fourth, or the ſame parts, and contrariwiſe, if the firſt be put the ſame part of the ſecond as the third is of the fourth, or the ſame parts, you may conclude that thoſe nnmbers are proportional.

But *Euclide* defineth onely thoſe numbers proportional there, which have the ſame proportion of inequality: For if there be queſtion of the proportion or reaſon of equality, it is manifeſt that the firſt ought to be equal to the ſecond, and the third to the fourth, to the end they may be ſaid to be proportional.

Now from this Definition may be gathered manifeſtly that the equal num-

numbers have the ſame reaſon to one and the ſame, and contrariwiſe one and the ſame to the equal, alſo that the numbers which have the ſame reaſon to one and the ſame, or to which one and the ſame number hath the ſame reaſon, are equal.

For ſeeing that equal numbers are equi-multiplices, or the ſame part, or the ſame parts; alſo ſeeing that one and the ſame number is equi-multiplex of the equal numbers, or the ſame part or parts, or conteineth them equally, and over & above the ſame part, or the ſame parts of them, it is manifeſt that equal numbers have the ſame reaſon to one and the ſame, or one and the ſame to equal numbers according to this Definition. Againe, foraſmuch as the numbers which have the ſame reaſon to one and the ſame, are equi-multiplices thereof, or of the ſame part or parts, or doth equally contein it, and over and above the ſame part or parts thereof: And a number which hath the ſame reaſon to certain other numbers is equi-multiplex of them, or of the ſame part or parts, or conteineth it equally, and over and above the ſame part or parts thereof; according to this Definition, it is manifeſt that the numbers which have the ſame reaſon to one, or to which one and the ſame, hath the ſame reaſon, are equal amongſt themſelves.

In like manner, it may be ſaid that there is greater reaſon of a greater number to one and the ſame, then of a leſſer, and contrariwiſe, one and the ſame hath greater reaſon to a leſſer, then to a greater number, alſo that number that hath the greateſt reaſon to one and the ſame, is the greateſt number, and that number to which one and the ſame hath the greateſt reaſon is the leſſer number; which things are eaſie, if this Definition be well underſtood.

Clavius ſaith here, that this Definition agreeth alſo in broken numbers, whether there be whole numbers joyned with them or not: For Example, theſe four numbers are proportional, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{1}{4}$, the firſt being ſo Multiplex of the ſecoud, as the third is of the fourth, to wit, double, which will appear, if you reduce the two firſt into the ſame Denomination, to wit to $\frac{6}{8}$ and $\frac{3}{8}$, and the two laſt to $\frac{2}{4}$ and $\frac{1}{4}$, in like manner theſe four are proportional $2\frac{3}{8}$, $4\frac{9}{12}$, $1\frac{1}{4}$, $2, \frac{5}{9}$, ſeeing that the firſt is ſuch a part of the ſecond, as the third is of the fourth, to wit, the half, which is manifeſt, reducing them into one Denomination, to wit, to $\frac{19}{8}$, $\frac{38}{8}$, and $\frac{5}{4}$, $\frac{10}{4}$, in which *Clavius* is much miſtaken, in not conſidering that to ſpeak properly, there are no broken numbers or Fractions, Unity being indiviſible, according to *Euclide*, and the Fraction making it ſelf in the thing numbred, and not in the Numbers, to wit, when you divide any whole number into other equal parts, and of another name, which parts being applyed to numbers, or the numbers applyed to them, they are then abſolute numbers, that is to ſay, whole numbers, as in the Examples above, where he ſaith, that $\frac{6}{8}$ are double to $\frac{3}{8}$, you muſt underſtand that theſe numbers 6 and 3, are taken abſolutely, and as whole numbers, without regarding the name, for you ought onely to compare the number to the number, without comparing the name to the name, that is to ſay, to the Denominator, and ſo you ought to underſtand as touching the reſt; for thoſe things thus reduced are no more Fractions; Seeing that Fractions are ſo called onely in compariſon to their whole, ſo divided into parts of another name: but theſe parts applyed to numbers, are whole numbers, and ſo *Euclide* doth underſtand.

21 *Like Plaine numbers and like Solid numbers, are ſuch as have their ſides proportional.*

TO the end that a Plaine number may reſemble another Plaine number, it is not neceſſary that two ſides of the one, (which you pleaſe,) be proportional to two ſides of the other, (which you pleaſe alſo;) but it ſufficeth that the one hath two ſides, (which ſo ever they be) proportional to any two ſides of the other, as the Plaine numbers 24 and 6 are alike; foraſmuch as the ſides of that of 6 and 4, are proportional to the ſides of the other of 3 and 2, although that the other ſides of the firſt 8 and 3, or 12 and 2, are not proportional to that of 3 and 2.

In like manner, it is not requiſite that to the end that two Solid numbers be alike, that three of the ſides of the one (which you pleaſe) be proportional to three of the other, (which you alſo pleaſe) but it ſufficeth that three ſides be proportional to any three ſides, for ſo they are reduced into a Solid forme, according to their unites, and their Lengths, Breadths, and Heighths, ſhall be proportional, as the Solid numbers 192 and 24 are alike, the ſides of the one 8, 6, 4, being proportional to the ſides of the other, 4, 3, 2, although the other ſides of the firſt, 12, 8, 2, or 16, 4, 3, are not proportional to the ſides of the other, 4, 3, 2, &c.

22 *A perfect number, is that which is equal to all his aliquot parts.*

THat is to ſay, ſuch a number to which his parts being put together are equal, (to wit all his aliquot parts,) and that number is called by the Mathematicians a perfect number, as 6, 28, 496, for the firſt conteineth theſe three aliquot parts 1, 2, 3, which together doe make 6, all theſe of the ſecond are 1, 2, 4, 7, 14, whoſe ſumme do make 28, and thoſe of the third 1, 2, 4, 8, 16, 31, 62, 124, 148, all which being added together do make 496, &c.

And if all the aliquot parts of a number taken together be greater then it, it ſhall be called abundant, if leſſer, deficient.

Whence it appeares that this word part, is taken by *Euclide* onely for an aliquot part, for otherwiſe every number ſhould be a perfect number, every number being equal to all his parts, if each leſſer number may be ſaid to be parts of a greater, whether it meaſure it or not.

To theſe Definitions of EUCLIDE we ſhall adde theſe which follow, according to ſome Interpreters.

23 *A number is ſaid to meaſure a number by that number, which multiplying or being multiplied, produceth the ſame number.*

AS 4, ſhall be ſaid to meaſure 12 by 3, foraſmuch as 4 multiplied by 3 produceth 12; again, 3 ſhall be ſaid to meaſure the ſame 12 by 4, ſeeing that 4 multiplying 3 produceth 12, &c.

24 *Proportion or Reason of numbers is an habitude of one number to another, according to what the one is Multiplex of the other, or part or parts, or else conteineth it once, or divers times, and over and above a cetaine part or parts thereof.*

AS if you compare 20, with 4 according to what 20 is Multiplex of 4, to wit, quintuple, this comparison or habitude shall be called reason, as hath been said in the third Definition of the 5th. Book, where *Euclide* hath spoken of Reason in general.

25 *Two numbers are said to be terms or Roots of a proportion to which three cannot be found two lesse numbers, in the same reason or proportion.*

26 *When three numbers are proportional, the first is said to have to the third, the double reason of that of the first to the second: and when there are four, the first is said to be to the fourth in triple reason of the first to the second, and so in one and the same order, one over and above, until the proportion be finished.*

27 *If there be so many numbers as you please in order, the reason of the first to the last, is said to be composed of the reasons of the second to the third, and of the third to the fourth, and so in order until the proportion be finished.*

PETITIONS.

It is required to be granted.

1 *That it is possible to take numbers equal or Multiplices to any number given.*

2 *That it is possible to take a greater number then any given number.*

ALthough that number cannot be infinitely diminished, but comes to unity indivisible, it may neverthelesse be augmented infinitely by the

the continual addition of unity, therefore any number being proposed, a greater may be found, to wit, that which is produced by the addition of a unite, or of divers unites to the number given.

COMMON SENTENCES.

1 *Numbers equi-multiplices of equal numbers, or of one and the same number are equal to one another.*

2 *Numbers of which one and the same number is equi-multiplex, or of which equal numbers are equi-multiplices, are equal one to another.*

3 *Numbers that are the same part, or the same parts of equal numbers, or of one and the same number are equal to one another.*

4 *Numbers, of which one and the same number or equal numbers, are the same part, or the same parts, are equal to one another.*

5 *Unity measureth every number, by the unites that are therein, that is to say, by the same number.*

FOr the unite being taken so many times as there are unites in the number proposed, doth constitute the same number, and therefore it is measured by the unites that are therein, that is to say, by the same number made of his unites.

6 *Every number measureth it selfe by unity.*

SEeing that every number once taken is equal to it self, it is manifest that every number measureth it self by unity.

7 *If a number multiplying a number, produce another number, the Multiplicand shall measure the product by the Multiplicator: but the Multiplicator shall measure the same by the Multiplicand.*

FOr let B multiplying A produce C, I say, that A measureth C by B, and B measureth the same C by A, for (as by the 15 Defin.) seeing that B being as

as many times compounded, as there are unites in A, doth make C: it is manifest that B doth measure C by A, and by the same reason, A doth measure the same by B; Seeing that B multiplying the same A doth produce the same C, as shall be demonstrated in the 16 Proposition of this Book.

A.... 4 B ... 3
C 12

8 *If a number measure another number, that number by which it measureth it, doth measure the same by the unites that are in the number measuring, that is to say, by the same number measuring.*

Forasmuch as the number 6 doth measure 18 by 3, the same 3 shall measure 18, by 6, that is to say, by the unites that are in 6, for seeing that 6 doth measure 18 by 3, the number 18 shall be (according to the 15 Defin.) produced by the multiplication of 6 by 3, or of 3 by 6, therefore by the afore-going Axiome 3 shall measure 18 by 6.

9 *If a number measuring a number, multiply that number by which it measureth it, or is multiplied thereby, it will produce the number which it measureth.*

Let the number A measure C by B, I say, that A multiplying B or multiplied of B produceth C, for (by the 25 Definition) A is said to measure C by that number, the which multiplying or being multiplyed, produceth C, therefore A being put to measure C by B, it is manifest that A multiplying B, or being multiplyed thereby, produceth C.

A....4 B...3
C............ 12

10 *The number that measureth as many other numbers as you please, doth also measure the numbers compounded thereof.*

Let the number A measure BC and CD; I say it doth also measure their compound BD; for seeing that A measureth BC and CD, as well

A....4
B....4 E....4 C....4 F....4 G....4 D 20

BC as CD shall be Multiplex of A, dividing therefore BC according to the parts BE and EC, equal to A and CD, according to CF, FG and GD, equal to A; BD compounded of all the parts BE, EC, CF, FG and GD, equal to A shall be Multiplex of A; therefore A measureth B, which was proposed.

11 That

11 *That number that measureth some other number, doth also measure every number which that number measureth.*

Let the number A measure B, and B measure CD. I say, that A doth also measure CD. For seeing that B doth measure CD, the same CD shall be Multiplex of B, dividing therefore CD into CE and ED, equal to B; A shall measure CE and ED, seeing that A is put to measure B, a number equal as well to CE, as to ED, (as by the 10th. Axiome) therefore A shall also measure CD, compounded of CE and ED, which was proposed.

A...3
B.........9
C.........9 E.........9 D

12 *That number that measureth the whole and the part cut off, measureth also the rest.*

Let A measure the whole BC, and the part cut off BD; I say it will also measure DC, the rest; For seeing that A measureth BC and BD, as well BC as BD shall be Multiplex of A, or BD shall be equal to A; therefore having divided as well BC as BD, in parts equal to A, the rest DC, shall be one onely part equal to A, or Multiplex thereof, wherefore A shall measure DC, which was proposed.

A...3
B.........9 D...3 C
B.........9 D......6 C
B...3 D......6

All that hath been here above added, besides the Definitions of *Euclide*, is done to render the Demonstrations hereafter mentioned more clear and intelligible, they being all founded on what *Euclide* hath before set down in his former several Books.

PRO-

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

A.........10 E..2 G.1 B
C...3 F..2 D
H.....

If from two unequal numbers propoſed A B, C D, *there be cut off ſtill alternately, the leaſt* C D, *from the greateſt* A B, *and that the remainder* E B *ſhall never meaſure his precedent* A B, *until the unite* G B *be taken; the numbers* A B, C D, *propoſed at the beginning, ſhall be Primes to one another.*

Demonſtration FOr if they be not Primes to one another, they ſhall be mea-
ſured by ſome number, which ſuppoſe to be H, their com-
mon meaſure over and above unity; foraſmuch then as H meaſureth CD,
and C D, A E, (C D being a part of A E, or equal thereto, ſeeing that it
being taken from A B, there doth remaine E B) H [a] ſhall alſo meaſure
a) 11.c.ſ. A E. But H doth meaſure the whole A B, and [b] therefore ſhall meaſure the
b) 12.c.ſ. reſt E B. But E B doth meaſure C F, [c] therefore H doth meaſure the ſame
c) 11.c.ſ. C F: and therefore ſeeing that it meaſureth the whole C D, it [d] will alſo
d) 12.c.ſ. meaſure the reſt F D, and ſeeing F D meaſureth E G, H [e] ſhall alſo mea-
e) 11.c.ſ. ſure E G, but H meaſured the whole E B, therefore [f] H ſhall alſo meaſure
f) 12.c.ſ. the unite reſting G B, the whole his part, which is abſurd, therefore any
number beſides unity, ſhall not meaſure A B and C D, and therefore they
ſhall be Primes to one another which was to be demonſtrated.

PROP. 2. PROBL. 1.

A...............16 E......6B
C......6 F..D
G......

To find the greateſt common meaſure F D, *of two given numbers* A B, C D, *that are not Primes to one another.*

Con-

Conſtruction LEt the leſſer number C D, be taken from A B the greater number, as often as may be, and there will reſt E B, which being ſubtracted from C D leaveth F D, and ſo following, alwayes taking the leſſer from the greater by an alternate ſubtraction, by which means you ſhall arrive to a number which will meaſure the precedent; For if you ſhould proceed to unity, [a] A B and C D, ſhould be Primes to one another, (a) 1.7.) which is contrary to the Hypotheſis; come we then to F D, the which being ſubtracted from the precedent, as often as may be, there will remaine nothing; I ſay then, that F D is the greateſt common meaſure of A B and C D.

Demonſtration FOr that it meaſureth both of them, we ſhall thus demon-
ſtrate, foraſmuch as F D meaſureth E B, and E B meaſureth
C F, [b] F D, ſhall meaſure the ſame C F, and ſeeing it meaſureth it ſelf, [c] it b) 11.c.ſ.
ſhall alſo meaſure the whole C D, compounded of the two numbers C F and c) 10.c.ſ.
F D. But C D meaſureth A E, [d] therefore F D ſhall alſo meaſure A E, and d) 11.c.ſ.
ſeeing that F D doth alſo meaſure E B, it [e] ſhall meaſure the whole A B, e) 10.c.ſ.
compounded of A E and E B, therefore F D doth meaſure the two numbers
A B and C D, and is alſo their greateſt common meaſure: For if it be not ſo,
let it be another, greater then F D, ſuppoſe G; Foraſmuch then as G mea-
ſureth A B and C D, being their common meaſure, and C D meaſureth
A E, [f] the ſame G ſhall alſo meaſure A E, [g] and therefore E B: but E B mea- f) 11.c.ſ.
ſureth C F, [h] therefore G ſhall alſo meaſure C F, and [i] therefore it ſhall alſo g) 12.c.ſ.
meaſure the reſt, F D the greater, the leſſer, which is impoſſible; There- h) 11.c.ſ.
fore a greater then F D ſhall not meaſure i) 11.c.ſ.
A B and C D, and therefore F D is their greateſt common meaſure. But if the leſſer number C D doth meaſure the greater A B, in ſuch manner, as being ſubtracted there-from as often as ſhall be poſſible, there reſt nothing, C D ſhall be the greateſt common meaſure of the two; Seeing it meaſureth it ſelf, as appears by this figure. Therefore, &c. which was to be demonſtrated.

A.........10 B
C.....5 D

COROLARIE.

From this Demonſtration it is manifeſt that that number which meaſureth two others, meaſureth alſo their greateſt common meaſure; This is gathered from that part, by which it is ſhewn that F D *is the greateſt common meaſure of* A B *and* C D, *for it is ſhewn that if* G *meaſure* AB *and* C D, *it meaſureth alſo* F D, *their greateſt common meaſure.*

PROP. 3. PROBL. 2.

A...............16 D....4
B...........12 E..2
C......6 F......

To find the greateſt common meaſure D, *of three given numbers* A, B *and* C, *not Primes to one another.*

Con-

Construction LEt D be found, the greatest common measure of A and B, if D doth also measure C, it is manifest that D is the greatest common measure of the three, A, B, and C.

Demonstration FOr if a greater number then D be said to measure the three A, B, C, the same [a] shall also measure D the greatest common measure of A and B, the greater, the lesser, which is impossible. But if D doth not measure C, at least D and C shall be compounded between themselves, for A, B and C, being compounded, any common measure of theirs shall also measure D, the greatest common measure of A and B; therefore, seeing that that common measure doth also measure C, D and C shall be compounded between themselves, and E be their greatest common measure; [b] I say, that E is the greatest common measure of the three, A, B and C, forasmuch as E measureth C and D, but D measureth A and B; E measureth the same A and B, and therefore it shall measure the three A, B and C.

a) Cor. 2. 7.

A . . . . 16 D . . . 4
B . . . . 12 E . . 2
C . . . . . . 6 F . . . . . .

b) 2. 7.

Now that E is the greatest common measure, it is evident: For suppose F to be their common measure, greater then E, forasmuch as F measureth A and B, [c] it shall also measure D, their greatest common measure: But F also measureth C, therefore F measuring D and C, shall also in like manner, measure E, their greatest common measure, the greater, the lesser, which is impossible: Therefore, a greater number then E will not measure A, B and C, and therefore E is their greatest common measure; Therefore, &c. which was to be demonstrated.

c) Cor. 2. 7.

COROLARIE.

It is manifest from this Demonstration, that one number that measureth three, will in like manner measure their greatest common measure.

This is also gathered from the last part of the Demonstration, where it is shewn that seeing that F *doth measure* A, B *and* C, *it measureth also* E, *their greatest common measure: In like manner, if there be more then three numbers proposed, not Primes to one another, this Probleme sheweth how to find their greatest common measure: For if there be four numbers, you must first find the greatest common measure of three, and if five, of four, &c. and you must work, in the rest, as hath been shewn of three numbers.*

PROP.

PROP. 4. THEOR. 2.

A 7
B 10

A 5
B 10

A . . 2 D . . 2 E . . 2 F
B 10
C . . 2

Every lesser number A, *is a part or parts of every greater number* B.

Demonstration LEt A and B in the first place be Primes to one another, forasmuch as each unite of A, is a part of B, it is manifest that A is parts of B, that is to say, so many parts as there are unites in A.

Secondly, let A and B not be Primes to one another, but compounded; and that A measureth B, that being [a] it appears that A is part of B.

a) 3. d. 7.

But if A doth not measure B, [b] find C their greatest common measure; and let A be divided according to the parts A D, D E, and E F, each equal to C [c], forasmuch as C is part of B; seeing that it measureth it, A D shall be also part of B, in like manner DE and EF; and therefore all the number A is parts of B, to wit, as many parts as C is conteined times in A E. Therefor every Lesser, &c. Which was to be demonstrated.

b) 2. 7.

c) 3 diff.

PROP. 5. THEOR. 3.

A . . . . . 6 D 4
B . . . . . . 6 G . . . . . . 6 C E . . . . 4 H . . . . F

If one number be such part of one number as another number is of another number, the one and the other together, shall be such part of the one and the other together, as one single number to a single number.

THat is to say, if of four numbers, the first be such a part of the second, as the third is of the fourth, the first and third together, shall be such part of the second and the fourth together, as the first is of the second.

Demonstration FOr having divided the numbers B C and E F, according to the parts B G, G C, E H and H F, equal to A, to D, there will be in B C so many parts equal to A, as in E F, equal to D; seeing that A is such part of B C, as D is of E F.

Forasmuch then as A and B G are equal, if there be added to them the equal numbers D and E H, A and D together, shall be equal to B G and E H together. in like manner, A and D together, shall be equal to G C and H F together, and so on, if there were more parts in B C and E F: and therefore the one and the other, A and D together, shall be the same part of the one and the other, B C and E F together, as A is of B C, or D of E F; Therefore if one number, &c. Which was to be demonstrated.

 PROP.

PROP. 6. THEOR. 4.

A...3 G...3 B
C.........9
D....4 H....4 E
F............12

If one number A B, *be ſuch parts of one number* C, *as another number* D E, *is of another number* F, *the one and the other* A B *and* D E, *together, ſhall be ſuch parts of the one and the other* C *and* F *together, as one ſingle number* A B, *of* C, *is to one ſingle number* D E, *of* F.

THat is to ſay, if of four numbers, the firſt be ſuch part of the ſecond, as the third is of the fourth, the firſt and third together, ſhall be ſuch parts of the ſecond and fourth together, as the firſt is of the ſecond.

Demonſtration. FOr having divided A B, and D E according to the parts A G, G B, D H, and H E, of the numbers C and F; A B ſhall contein as many parts of C, as D E of F; ſeeing that A B is ſuch parts of C, as D E is of F, then foraſmuch as A G is ſuch part of C, as D H is of F, the a one and the other A G, and D H, together, ſhall be ſuch part of the one and the other C and F together, as A G is of C, or D H of F; By the ſame reaſon G B, and H E, together, ſhall be the ſame part of C and F together, as G B is of C, or H E of F, and ſo following, if there be more parts in A B, and D E; Therefore, the one and the other, A B and D E together, ſhall be the ſame parts of the one and the other, C and F together, as A B is of C, or D E of F: Therefore if one number, &c. Which was to be demonſtrated.

a) 5.7.

PROP. 7. THEOR. 5.

A....4 E..2 B
G....4 C........8 F....4 D

If one number A B, *be ſuch a part of one number* C D, *as the part cut off* A E, *is of the part cut off* C F, *the reſt* E B, *ſhall be alſo ſuch a part of the reſt* F D, *as the whole* A B *is of the whole* C D.

Demonſtration. FOr let G C be taken, of which let E B be ſuch a part as A E is of C F, or the whole A B, of the whole C D, foraſmuch as A E is the ſame part of C F, as E B is of G C, a the one and the other A E and E B, together, ſhall be ſuch part of C F and G C, together, as A E is of C F, that is to ſay, as the whole A B, is to the whole C D: and therefore, A B being ſuch a part, as well of F G, as of C D, the ſame F G, and C D, ſhall be equal to one another; then taking away the common C F, G C, and F D, will remaine equal: therefore, E B ſhall be ſuch part of F D, as of G C, that is to ſay, as the whole A B, to the whole C D. Therefore, if one number, &c. Which was to be demonſtrated.

a) 5.7.

PROP.

PROP. 8. THEOR. 6.

A......6 K......6 E....4 B
C..................18 F......6 D
G......6 L..2 I......6 M..2 H

If one number A B *be ſuch parts of another number* C D, *as the part cut off* A E, *to the part cut off* C F, *the reſt* E B *ſhall be alſo ſuch parts of the reſt* F D, *as the whole* A B, *is of the whole* C D.

Demonſtration. FOr let G H be taken equal to A B; G H ſhall be then alſo the ſame parts of C D, as A B is of C D, that is to ſay, as A E of C F, having then divided G H according to G I and I H, parts of C D, and A E, according to A K and K E parts of C F, the number of the parts G I, and I H, ſhall be equal to the number of the parts A K and K E, and as well G I as I H, ſhall be ſuch part of C D, as A K, or K E is of C F: and ſeeing that C D is greater then C F, as well G I as I H, part of C D, ſhall be greater then A K, or K E part of C F.

Having therefore taken G L and I M, equal to A K and K E; G L ſhall be the ſame part of C F, as A K, of the ſame C F, or as G I of C D, and therefore the whole G I, being the ſame part of the whole C D, that the part cut off G L is of the part cut off C F a, the reſt L I, ſhall be ſuch part of the reſt F D, as the whole G I, is of the whole C D. So M H ſhall be ſhewn to be the ſame part of F D, as the whole G I, or I H, is of the whole C D, therefore, ſeeing that as well G I as I H, is the ſame part of C D, as L I and M H, of F D, the one and the other, G I and I H together, ſhall be ſuch parts of C D, as L I and M H together, of F D. But G H is the ſame parts of C D, as A B of the ſame C D, (A B, and G H, being equal,) therefore L I and M H together, ſhall be the ſame parts of F D, as A B of C D.

a) 7.7.

But foraſmuch as if from the equal A B and G H, there be taken the equal A K, K E and G L, I M, the reſt E B, ſhall be equal to the reſt L I and M H, together: and therefore the reſt E B, ſhall be the ſame parts of the reſt F D, as the whole A B of the whole C D: to wit, as L I and M H, together, of the ſame F D. Therefore if one number, &c. which was to be demonſtrated.

PROP. 9. THEOR. 7.

A....4
B....4 G....4 C
D......6
E......6 H......6 F

If one number A B, *be ſuch part of one number* B C, *as another number* D, *is part of another number* E F, *alſo alternately, the firſt* A, *ſhall be ſuch part or parts of the third* D, *as the ſecond* B C, *is of the fourth* E F.

That is to say, If of four numbers, the first be such part of the second, as the third is of the fourth, also alternately, the first shall be such part or parts of the third, as the second is of the fourth.

Demonstration. For having divided B C, and E F, according to the parts B G, and G C, each equal to A, and E H, and H F, each equal to D, the number of the parts of B G, shall be equal to the number of the parts of E F. But B G, and G C, being equal to one another, and lesse then E H, and H F, also equal to one another; forasmuch as the whole B C, is lesse then the whole E F, by supposition, B G shall be the same part of E H, or the same parts, as G C of H F [a], and therefore B G, and G C, together,

A....4
B....4 G....4 C
D......6
E......6 H......6 F

a) 5, or 6. 7. together, to wit, B C the second, shall be the same part or parts of E H, and H F, together, that is to say, E F the fourth, as B G is of E H, that is to say, as A the first is of D the third: Therefore, if one number, &c. Which was to be demonstrated.

PROP. 10. THEOR. 8.

A..2 G..2 B
C......6
D.....5 H.....E
F...............15

If a number A B, *be such parts of a number* C, *as another* D E, *is parts of another* F, *also alternately, the first* A B *shall be such part or parts of the third* D E, *as the second* C, *is of the fourth* F.

Demonstration. For having divided A B, and D E, according to A G, and G B, parts of C; and D H, and H E, parts of F, there will be as many parts in A B, as in D E, and as well A G, as G B, shall be such part of C as D H, or H E, of F, [a] therefore, alternately A G shall be the same part or

a) 9. 7. parts of D H, and G B of H E, as C is of F, and therefore, A G shall be the same

b) 5, 6. 7. part or parts of D H, as G B is of H E: [b] therefore A G, and G B, together, that is to say, A B the first, shall be the same part or parts of D H, & H E together, that is to say, of D E the third, as A G is of D H, that is to say, as C the second of F the fourth: Therefore, if one number, &c. Which was to be demonstrated.

PROP. 11. THEOR. 9.

A....4 E..2 B.
C........8 F....4 D

If as the whole A B, *is to the whole* C D, *so the part cut off* A E, *is to the part cut off* C F, *also the rest* E B, *shall be to the rest* F D, *as the whole is to the whole.*

a) 20. d. 7. *Demonstration.* For seeing that A B is to C D, as A E is to C F, [a] A B being lesse then C D, shall be the same part, or the same parts, of C D, as A E is of C F; therefore the rest E B, shall be the same part, or parts

b) 7, 8. 7. parts of the rest F D, as A B is of C D: [b] therefore, E B shall be to F D, as A B is to C D. Which was to be demonstrated.

PROP. 12. THEOR. 10.

A....4 C..2 E...3
B........8 D....4 F......6

If there be as many numbers as you please A, B, C, D, E, *and* F, *proportional, as one of the antecedents* A, *shall be to one of the consequents* B: *so all the antecedents* A, C, *and* E, *shall be to all the consequents* B, D, *and* F.

a) 20. d. *Demonstration.* Forasmuch then as because of the same proportion, A is [a] such a part, or such parts (A, C, and E, being lesse) of B, as C

b) 5, 6. 7. is of D, and E of F, [b] A and C together, shall be such part, or parts of B and D together, as A is of B, or E of F.

Againe, forasmuch as A and C, together as one, is the same part or parts of B and D together as one, as E is of F, the two A and C together, as one and F together, shall be the same part, or the same parts of the two B and

c) 20. d. 7. D together, as one, and F together, as A is of B: [c] Therefore, there will be the same proportion of A, C and E together, to B, D and F together, as of A to B.

PROP. 13. THEOR. 11.

A..2
B....4
C......6
D............12

If four numbers A, B, C, *and* D, *be proportional, also alternately they shall be proportional.*

a) 20, 10. 7. *Demonstration.* For seeing that A is to B, as C to D, [a] A is the same part, or parts of B, as C is of D, therefore, alternately, A shall be

b) 20. d. the same part or parts of C, as B is of D, [b] and therefore, as A shall be to C, so B shall be to D, which was to be demonstrated.

PROP. 14. THEOR. 12.

A............12 D.........9
B........8 E......6
C....4 F..2

If there be as many numbers as you please of the one part; A, B, *and* C, *and as many of the other part,* D, E, *and* F, *the which being taken two and two, and in the same reason; also in equality they shall be in the same reason.*

Demon-

Demonſtration. FOr ſeeing that as A is to B, ſo D is to E, [a] alternately, as
a) 13. 7. A ſhall be to D, ſo B ſhall be to E; in like manner, ſeeing that as B is to C, ſo E is to F, alternately, as B ſhall be to E, ſo C ſhall be to F: Therefore as A is to D, ſo C ſhall be to F, (for ſeeing that the one and the other reaſon of A to D, and C to F, is the ſame to the reaſon of B to E, as hath been demonſtrated, they ſhall be alſo the ſame one to another)
b) 13. 7. and [b] therefore, alternately, as A ſhall be to C, ſo D ſhall be to F. Which was to be demonſtrated.

A............12 D........9
B........8 E......6
C....4 F..2

PROP. 15. THEOR. 13.

A.1
B.1 G.1 H.1 C
D..2
E..2 I..2 K..2 F

If unity A, *meaſureth ſome number* BC, *and that another number* D *doth meaſure as oftentimes another number* EF, *alſo alternately, unity* A *ſhall meaſure the third number* D, *as oftentimes as the ſecond* BC *ſhall meaſure the fourth* EF.

Demonſtration. FOr having divided BC, according to the unites BG, GH, HC and EF, according to the parts EI, IK and KF, each equal to D, there will be in EF as many parts equal to D, as there are unites in BC: therefore BG ſhall be the ſame part of EI, as GH of IK, and
a) 20. d. 7. HC of KF: [a] therefore as BG, is to EI, ſo GH to IK, and HC to KF;
b) 12. 7. [b] therefore, as BG to EI, ſo is BG, GH and HC, to EI, IK, and KF, all the antecedents to all the conſequents, that is to ſay, BC to EF, therefore BC ſhall be the ſame part of EF, as BG is of EI, that is to ſay, as A is to D: Therefore if unity, &c. Which was to be demonſtrated.

PROP. 16. THEOR. 14.

Unity.1
A...3 B....4
C............12
D...........12

If two numbers A *and* B *multiplying one another, do produce other numbers* C *and* D, *the products* C *and* D, *ſhall be equal to one another.*

a) 15. d. 7. *Demonſtration.* FOraſmuch as A multiplying B produceth C [a], B ſhall be as often found in C, as unity in A, and therefore ſhall mea-
b) 15. 7. ſure C, as often as the unite ſhall meaſure A [b], therefore, by permutation, A ſhall meaſure C, as often times as unity ſhall meaſure B: Again, ſeeing
15. d. that B multiplying A produceth D, A ſhall be as often found in D as unity in

in B, and therefore ſhall meaſure D, as oftentimes as unity ſhall meaſure B; But A ſhall meaſure C alſo, as oftentimes as unity ſhall meaſure B, therefore, A doth meaſure C and D equally, and therefore C and D are equal to one another. Therefore, if, &c. Which was to be demonſtrated.

PROP. 17. THEOR. 15.

Unity.1
A...3
B..2 C....4
D......6 E...........12

If one number A, *by multiplying two others* B *and* C, *doth produce other numbers* D *and* E, *the products of thoſe numbers ſhall have the ſame proportion to one another as the numbers multiplied.*

Demonſtration. FOr ſeeing that A multiplying B, maketh D; B ſhall be as oftentimes conteined in D as unity in A: In like manner, 15. d.
C ſhall be as oftentimes conteined in E, as unity in the ſame A, and therefore B ſhall meaſure D as oftentimes as C ſhall meaſure E. Therefore B ſhall be ſuch part of D, as C of E [b], and therefore as B ſhall be to D, ſo C ſhall be to E [c], and alternately, as B is to C, ſo is D to E: Therefore if one b) 20. d.
number, &c. Which was to be demonſtrated. c) 13. 7.

PROP. 18. THEOR. 16.

A..2 B....4
C...3
D......6 E............12

If two numbers A *and* B, *multiplying ſome other number* C, *ſhall produce other numbers* D *and* E, *their products* D *and* E *ſhall have the ſame proportion as the numbers multiplying* A *and* B.

Demonſtration. FOr ſeeing that D is the product of C multiplyed by A [a], the a) 16. 7.
ſame D ſhall be the product of A multiplyed by C: in like manner E being the product of C by B, the ſame E ſhall be the product of B by C: Foraſmuch then, as the ſame number C multiplying A and B,
makes D and E, as A ſhall be to B, ſo D ſhall be to E: Therefore, if two 17. 7.
numbers, &c. Which was to be demonſtrated.

PROP.

PROP. 19. THEOR. 17.

A...3 B..2
C......6 D....4
E...........12
F...........12
G.................18

If four numbers A, B, C, *and* D, *are proportional, the product of the first* A, *and the fourth* D, *shall be equal to the product of the second* B, *and the third number* C: *and if the number produced of the first* A, *and the fourth* D, *be equal to the product of the second* B, *and the third* C, *those four numbers shall be proportional.*

Demonstration. AS A is to B, so C is to D, and as A the first, multiplying D the fourth, maketh E, and B the second, multiplying C the third, makes F, I say that E and F are equal to one another; for again, let A multiplying C make G; forasmuch as A multiplying C and D, makes G and E, as C shall be to D: that is to say, as A is to B, so G
17.7. shall be to E.
a) 18.7. Again, forasmuch as A and B multiplying C, makes G and F, as [a] A shall be to B, so G shall be to F: therefore, as G shall be to E, so the same G shall be to F, therefore, E F shall be equal by what followes after the 20th. Definition.

Now let E the product of A the first, by D the fourth, be equal to F, the product of B the second, by C the third: I say, the four numbers A, B, C, and D, are proportional, to wit, as A is to B, so C is to D.

Againe, let G be produced of A multiplyed by C; forasmuch as A multiplying C and D, makes G and E; as C shall be to D, so G to E, or to F, equal to E, for G hath the same proportion to E, as to F, as is shewn at the 20th. Definition.

Again, forasmuch A and B multiplying C, makes G and F, as A is to B, so the same G is to F, therefore, the proportions of A to B, and C to D; being the same with the proportion of G to F, they shall be so to one another, and so as A shall be to B, so C shall be to D: Therefore, if four numbers, &c. Which was to be demostrated.

PROP. 20. THEOR. 18.

A.........9
B......6 D......6
C....4

If three numbers A, B, *and* C, *are proprotional: to wit, as* A *to* B, *so* C *to* D, *the product of the extreames* A, *multiplyed by* C, *is equal to the product of the mean* B, *and if the product of the extreames* A *and* C, *be equal to the product of the mean, those three numbers shall be proportional.*

Demonstration. FOr let D be assumed equal to B, as A shall be to B, so D shall be to C [a], and therefore the product of A by C, shall be equal to the product of B by D; that is to say, of B multiplyed by it selfe. a) 19.7.

Now let the product of A the first, by C the third, be equal to the product of B the mean multiplyed by it selfe; I say, A, B and C, are proportional.

For again, let D be equal to B; as A shall be to B, so D to C, and the product of A by C shall be equal to the product of B by D, equal to B, that is to say, of B multiplyed by himselfe: Therefore, the three numbers A, B, C, are proportional: Therefore, if three numbers, &c. Which was to be demonstrated.

PROP. 21. THEOR. 19.

A...3 G..2 B
C..2 H.1 D
E.........10
F......6

The least numbers A B, *and* C D, *of all those which have the same proportion with them* E *and* F, *do equally measure the numbers which have the same proportion with them: to wit, the greatest* A B, *the greatest* E, *and* C D *the lesser*, F *the lesser.*

Demonstration. FOr seeing that A B is to C D, as E is to F [a], alternately, as A B to E, so C D to F, and therefore A B and C D, being a) 13.7.
lesse then E and F, A B [b] shall be such part or parts of E, as C D of F: But b) 20.d.
they cannot be parts, for (if possible) let A B, and C D, be divided in A G, and G B, and C H, and H D, parts of E and F; the parts A G, and G B, shall be equal in number to C H, and H D; and therefore, A G shall be the same part of E, as C H of F; therefore, as A G shall be to E, so C H shall be to F [c], and alternately, as A G shall be to C H, so E to F, or A B c) 20.d.
to C D: and therefore, A G, and G H lesse then A B, and C D, shall have the same proportion as A B, and C D, which is impossible: seeing that A B, and C D, are the least in their proportion by Supposition: Therefore, A B, and C D, shall not be the same parts of E and F, therefore, the same part. Therefore, A B, and C D, shall equally measure E and F. Therefore, the least numbers, &c. Which was to be demonstrated.

PROP. 22. THEOR. 20.

A....4 D...........12
B...3 E.......8
C..2 F......6

If there be three numbers of the one part, A, B, and C, and as many of the other part, D, E, and F, the which being taken from two to two, and in the same reason, and that their proportion be disturbed: also by equality they shall be in the same reason.

Demonstration FOr ſeeing that as A is to B, ſo E is to F [a]; the product of A the firſt, by F the fourth, ſhall be equal to the product of B
a) 19. 7. the ſecond, by E the third; and ſeeing that B is to C, as D to E; the product of C by D, ſhall be equal to the ſame product of B by E; and therefore the product of A the firſt, by F the fourth, ſhall be equal to the product
b) 19. 7. of C the ſecond, by D the third [b]: Therefore, there ſhall be the ſame reaſon of A the firſt, to C the ſecond, as of D the third, to F the fourth: Therefore, if there be three, &c. Which was to be demonſtrated.

PROP. 23. THEOR. 21.

A.......7 B.....5
C.... D.... E..

The numbers that are primes to one another A and B, are the least of all those that have the same reason with them.

Demonstration FOr if it be denied, ſuppoſe C and D to be leſſer in the ſame reaſon: Foraſmuch, as C and D are leſſer in the ſame reaſon of A and B [a], C ſhall meaſure A, and D ſhall meaſure B equally, and
a) 21. 7. therefore according to one and the ſame number, which let be E, in ſuch manner as C meaſureth as often times A and D, as often times B as Unity is conteined in E: Therefore, ſeeing that Unity meaſureth E and C, meaſureth A equally [b], alternately, Unity ſhall meaſure C and E, ſhall meaſure
b) 15. 7. A equally: Again, ſeeing that Unity meaſureth E and D, the number B equally: alſo alternately Unity ſhall meaſure B equally; therefore, ſeeing that E meaſureth A and B the ſame E ſhall be their common meaſure: wherefore they ſhall not be primes to one another, but compounds, which is impoſſible, and againſt the Suppoſition; therefore, A and B are the leaſt in the ſame reaſon: Therefore, the numbers, &c. Which was to be demonſtrated.

PROP.

PROP. 24. THEOR. 22.

A........8 B.....5
C...
D..... E.....

The least numbers A and B of all those which have the same reason with them, are primes to one another.

Demonstration FOr if they be not Primes, they ſhall have one common meaſure, which ſuppoſe to be C; therefore, let C meaſure A as often times as Unity is in D, and B as many times as Unity is in E. Foraſmuch then, as C is conteined as oftentimes in A, as Unity is in D, (Seeing that it meaſureth A according to D,) and as the ſame C is conteined as many times in B as Unity is in E [a]: D and E multiplying C, ſhall produce A and B: wherefore there ſhall be the ſame reaſon of A to B, as a) 9. c. ſ.
of D to E: therefore, ſeeing that D and E parts of A and B, are leſſer then the ſame A and B: A and B ſhall not be the leaſt of all thoſe which have the ſame reaſon with them, which is abſurd: Therefore, A and B are Primes to one another: and therefore the leaſt, &c. Which was to be demonſtrated.

PROP. 25. THEOR. 23.

A......6 B.....5
C...3 D....

If there be two numbers A and B, primes to one another, the number which measureth one of them, shall be prime to the other.

Demonstration FOr if B and C be not Primes to one another, they will have ſome number for their common meaſure, which ſuppoſe to be D.
Foraſmuch then, as D meaſureth C, and C meaſureth A, [a] D ſhall alſo meaſure A: But it alſo meaſures B: therefore, A and B are not Primes to a) 11. ax.
one another, having D for common meaſure, which is impoſſible, and againſt the Suppoſition: Therefore, C ſhall be Prime to B: Therefore, If there be two numbers, &c. Which was to be demonſtrated.

PROP. 26. THEOR. 24.

A.......7 B...3
C........8
D.....................21
E — F —

If two numbers A and B, are primes to some other number C, their product D, shall be also prime to that other C.

 Demon-

Demonstration. For if C and D are not Primes to one another, let E be their common measure, measuring D as often times as there are unites in F, forasmuch then, as E is as many times in D, as there are unites in F [a], F multiplying E shall produce D, and contrariwise, E multiplying F, shall produce the same D; But, D is the product of A by B; therefore, seeing that the number produced of E the first, by F the fourth, is the same as of A the second, by B the third; there shall be the same reason of E the first, to A the second, as of B the third, to F the fourth. But forasmuch as A and C are Primes to one another, and E is supposed to measure C [b], E shall be Prime to A, and therefore E and A being Primes to one another, shall be the least of their reason; Therefore, they shall equally measure B and F, which have the same reason: to wit, E shall measure B, and A shall measure F; Therefore, seeing that E measureth both the one and the other, B and C: B and C shall not be Primes to one another; which is absurd, and against the supposition; Therefore, D shall be Prime to C, Therefore, if two numbers, &c. Which was to be demonstrated.

a) 9. c. f. 16. 7.
19. 7.
b) 25. 7.
23. 7.
21. 7.

A.......7 B...3
C........8
D.....................21
E— F—

PROP. 27. THEOR. 25.

A....4 B.....5
C................16
D...4

If two numbers A *and* B, *are Primes to one another, the product of the one of them* A, *shall be Prime to the other.*

Demonstration. For let D be made equal to A, it shall be also Prime to B: therefore, A and D, being Primes to B [a], the product of A multiplyed by D, this is to say, of A by himself; that is to say, C shall be Prime to the same B: so the product of B multiplyed by it self, shall be shewn to be Prime to A; Therefore, if two numbers, &c. Which was to be demonstrated.

a) 26. 7.

PROP. 28. THEOR. 26.

A.....5 B...3
E..............15
C....4 D..2
F........8

If two numbers A *and* B, *are Primes to two other numbers* C *and* D, *the one and the other, to the one and the other, their Products shall be also Primes to one another.*

Demonstration. For seeing that as well A as B is Prime to C, E [a] their product shall be Prime to the same C.
Again, A and B, being Primes to D, E their product shall be also Prime to

a) 26. 7.

to the same D; forasmuch then, as C and D are Primes to E [b]; F their product shall be also Prime to E; Therefore, if two numbers, &c. Which was to be demonstrated.

b) 26. 7.

PROP. 29. THEOR. 27.

A...3 B..2
C.........9 D....4
E 27 F 8
G 81 H 16

If two numbers A *and* B, *are Primes to one another, and the one and the other, multiplying it selfe, shall make a number, their products shall be Primes to one another, and if the numbers first proposed, multiplying those products do make a number, the same shall be also Primes to one another, and this will alwayes happen about the extreams.*

Demonstration. For seeing that A and B are Primes to one another [a], C the product of A by himselfe, shall be Prime to the other B, and in like manner, B and C being Primes to one another, D the product of B by himselfe, shall be also prime to C: and therefore the two products C and D, Primes to one another. Again, forasmuch as A and B are Primes to one another [b], C product of A by it selfe, shall be Prime to B, and D the product of B by it selfe, prime to A: But C is also shewn to be Prime to B, therefore, the one and the other, A and C, shall be Prime to the one and the other, B and D [c], and therefore, E the product of A by C, shall be Prime to F, the product of B by D: And if again, G be the product of A by E, and H of B by F: Seeing that A and C are Primes to B [d]; E their product shall be also Prime to B: by the same reason F shall be Prime to A; Forasmuch then, as the one and the other, A and E, is Prime to the one and the other, B and F, the [e] number G product of A by E, shall be Prime to H, product of B by F, and so following, if there were more: Therefore, &c. Which was to be demonstrated.

a) 27. 7.
27. 7.
b) 27. 7.
c) 28. 7.
d) 26. 2.
e) 28. 7.

PROP. 30. THEOR. 28.

A........8 B....5 C
D...

If two numbers A B, *and* B C, *are primes to one another, the one and the other together,* A C *shall be prime to each of them: and if the one and the other together, be prime to one of them, the numbers first proposed, shall be also primes to one another.*

Demonstration. For if A C and A B be not Primes to one another, they shall be measured by some number, besides Unity, which suppose to be D: Therefore, seeing that D measureth the whole A C, and the

the part cut off A B, it ſhall alſo meaſure the reſt B C; therefore A B and B C ſhall not be Primes to one another, ſeeing that D meaſureth them, which is abſurd, and contrary to the ſuppoſition; therefore A C ſhall be Prime to A B, and to B C.

10. c. ſ. Now let A B and B C together, be Prime to one of them; to wit, to A B, I ſay A B and B C are Primes to one another, otherwiſe they ſhould be meaſured by ſome number beſides Unity, which let be D, therefore D meaſuring A B, and B C, ſhall alſo meaſure their Compound A C, and therefore A C, and A B are not Primes to one another, which is abſurd, and againſt the ſuppoſition; therefore A B, and B C are Primes to one another, and in like manner, if A C be Prime to B C: Therefore, if two numbers, &c. Which was to be demonſtrated.

A........8 B.....5 C
D...

COROLLARIE.

It follows hence, that the number compounded of two others, and is Prime to one of them, is alſo Prime to the other, for if A C *be Prime to* A B, A B *and* B C *ſhall be Primes to one another, by the ſecond part of this Propoſition; therefore* A C *ſhall be alſo Prime to* B C, *by the firſt part of the ſame Propoſition, which was propoſed.*

PROP. 31. THEOR. 29.

A.....5 B........8
C—

Every prime number A, *is prime to every other number which it meaſureth not.*

Demonſtration FOr otherwiſe they ſhould be meaſured by ſome number beſides Unity, which ſuppoſe to be C; Therefore C ſhall not be the ſame as A, A being ſuppoſed not to meaſure B; therefore A being meaſured by another number C; A ſhall not be Prime, which is impoſſible, and contrary to the Suppoſition; Therefore, every firſt number, &c. Which was to be demonſtrated.

PROP. 32. THEOR. 30.

A....4 B......6
C........................24
D...3 E........8

If two numbers A *and* B, *multiplying one another, do make ſome other* C, *and that ſome prime number* D, *doth meaſure their product; It ſhall alſo meaſure one of the numbers firſt propoſed* A *or* B.

Demon-

Demonſtration FOr as D doth not meaſure A, but it doth meaſure C as many times as there are unites in E, in ſuch ſort, as C may be produced of D by E, the which C is produced of A by B.

Foraſmuch as C the product of D the firſt, by E the fourth, is equal to the product of A, the ſecond by B the third [a]; there will be the ſame reaſon of D the firſt, to A the ſecond, as of B the third to A the fourth [b]: But D the firſt being prime to A, for that it meaſures it not [c], D and A ſhall be the leaſt of their reaſon [d], therefore ſhall meaſure equally B and E: to wit, D the leaſt, ſhall meaſure B the leaſt, and A the greateſt, E the greateſt: Therefore, if D meaſure not A, it ſhall meaſure B, in like manner, it appears that if D meaſure not B, at leaſt, it ſhall meaſure A: Therefore, &c. Which was to be demonſtrated.

a) 19. 7.
b) 31. 7.
c) 23. 7.
d) 21. 7.

PROP. 33. THEOR. 31.

A.................. 18
B......6 C...

Every compound number A, *is meaſured by ſome prime number.*

Demonſtration FOr being compounded, it [a] ſhall be meaſured by ſome number, which let be B: and if B be a Prime number, the Propoſition is manifeſt: But if B be compounded, it ſhall be alſo meaſured by ſome number, which let be C, which ſhall be either prime or compounded: if prime, ſeeing that it meaſureth B, and B meaſureth A, the ſame [b] C prime, ſhall alſo meaſure A: therefore A ſhall be meaſured by a prime number: But if C be compounded, it ſhall be meaſured by another number.

Now foraſmuch as one number cannot infinitely diminiſh it ſelfe, we ſhall arrive at laſt to a number, that no other number may meaſure, and therefore to a prime number meaſuring all the precedents, it [c] ſhall alſo meaſure the compound number A.

a) 13. d.
b) 11. c. ſ.
c) 11. c. ſ.

PROP. 34. THEOR. 32.

A.........9

Every number A *is a prime number, or elſe is meaſured by ſome prime number.*

Demonſtration FOr ſeeing that every number is a prime or compound number, if A be a prime number, the Propoſition is manifeſt, but if it be a compound number, it [a] ſhall be meaſured by ſome prime number: Therefore, every number is prime, or, &c, Which was to be demonſtrated.

a) 33. 7.

PROP.

PROP. 35. PROBL. 3.

A......6 B...4 C........8
D..2
E...3 F..2 G....4
H — I — K —
L —

As many numbers as you pleaſe, A, B, *and* C, *being given, to find the leaſt numbers which have the ſame reaſon with them.*

Demonſtration FOr A, B, and C, are either primes to one another, or are not; a) 23.7. if primes, [a] they ſhall be the leaſt of all thoſe which have the ſame reaſon; but if they be compounded, let D be found their greateſt common meaſure, which ſhall meaſure the ſame A, B, and C, by E, F, and G,: I ſay, that E, F, and G, are the leaſt of all thoſe that have the ſame reaſon of A, B, and C: foraſmuch as D meaſureth A, B, and C, b) 9.c.ſ. by E, F, and G [b], D multiplying E, F, and G, ſhall produce A, B, and C: therefore E, F, and G, ſhall have the ſame reaſon as A, B, and C.

I ſay alſo, that E, F, and G, are the leaſt: For if it be denied, let H, I, and K, be the leaſt in the ſame reaſon, the which ſhall meaſure A, B, and 21.7. c) 8.c.ſ. C, equally; Let them meaſure them then by L, that being ſo, L [c] multiplying H, I, and K, will produce A, B, and A, and alternately, L ſhall meaſure A, B, and C, by H, I, and K; foraſmuch then, as E the firſt, multiplying D the fourth, and H the ſecond, multiplyiug L the third, doth produce d) 19.7. the ſame A [d], as E the firſt, to H the ſecond; ſo L the third ſhall be to D the fourth: But E is greater then H, therefore, L ſhall be greater then D: and therefore, ſeeing that L doth meaſure A, B, and C; D ſhall not be the greateſt common meaſure of the numbers A, B, and C, which is contrary to the Hypotheſis: Therefore E, F, and G, are the leaſt in the reaſon of A, B, and C: Therefore, if, &c. Which was to be done.

COROLLARIE.

It is manifeſt from this, that the greateſt common meaſure of as many numbers as you pleaſe, doth meaſure them by the leaſt numbers of all thoſe which have the ſame reaſon with them, for it hath been ſhewn that E, F, *and* G, *by the which* D, *the greateſt common meaſure of* A, B, *and* C, *meaſureth the ſame* A, B, *and* C, *are the leaſt in the reaſon of* A, B, *and* C, *and ſo of all others.*

PROP. 36. PROBL. 4.

A....4 B.....5
C......................20
D------
E---- F----

Two numbers A *and* B, *being given, to find the leaſt number that doth meaſure them.*

Con-

Conſtruction FIrſt of all, let A and B, be primes to one another, and multiplying one the other make C, I ſay, C is the number required.

Demonſtration NOw that it meaſureth them is manifeſt; For, C being the product of A by B, or of B by A, [a] A ſhall meaſure C by a) 7.c.ſ. 9.c.ſ. B, and B the ſame C by A; therefore, A and B do meaſure C, and if C be not the leaſt; let A and B meaſure another leſſer, to wit, D, (if poſſible) and as A meaſureth D by E, and B the ſame D by F [b]; ſo D ſhall be the pro- b) 19.7. duct as well of A by E, as of B by F, and contrariwiſe; foraſmuch as the ſame D is produced of A the firſt, by E the fourth, and of D the ſecond, by F the third; as A the firſt, ſhall be to B the ſecond, ſo F the third, to E the fourth, therefore, A and B (being put primes to one another [c], and therefore c) 23.7. 11.7. the leaſt in their reaſon) they ſhall meaſure E and F the one as the other; to wit, A ſhall meaſure F: and B ſhall meaſure E.

But, foraſmuch as A multiplying B and E, makes C and D; C ſhall be to D, as B to E: and therefore, ſeeing that B meaſureth E, as is ſhewn, C ſhall alſo meaſure D, the greateſt, the leaſt, which is impoſſible; therefore, C is the leaſt of all thoſe that do meaſure A and B.

Secondly, let A and B not be primes to one another; and [d] let C and D d) 35.7. be found the leaſt in the ſame reaſon; to the end there be four numbers proportional; to wit, as A to B, ſo C to D, for ſo the product of [e] A the e) 19.7. firſt, by D the fourth, ſhall be equal to the product of C the ſecond, by B the third, the which product let be E; I ſay, that E is the leaſt number which meaſureth A and B, and that it meaſureth them, is manifeſt; for E being the product as well of A in D, as B in C, [f] as well A as B, ſhall f) 7.c.ſ. meaſure E, and if you deny it to be the leaſt, let A and B meaſure another, ſuppoſed leſſe then E; to wit F: Now as A meaſureth F by G, and B the ſame F by H, then [g] F ſhall be the product as well of A by G, as of B g) 9.c.ſ. by H.

A.... B......6
C..2 D...3
E............12
F-----
G---- H---

19.7. Foraſmuch then, as the ſame F is made of A the firſt, in G the fourth, and of B the ſecond, in H the third: As A the firſt, to B the ſecond, ſo H the third, to G the fourth: therefore, C and D being the leaſt in the reaſon of A to B, or of H to G, it [h] meaſureth equally H and G, to wit, C ſhall h) 21.7. meaſure H, and D ſhall meaſure G. But, foraſmuch as A multiplying D and G, makes E and F [i], E ſhall be to F, as D to G, and therefore D mea- i) 17.7. ſuring G as is ſhewn, E ſhall alſo meaſure F, the greateſt the leaſt, which is impoſſible: Therefore, E is the leaſt: Therefore, Two numbers, &c. Which was to be done.

COROLLARIE.

From hence it follows, that if two numbers do multiply, the leaſt, having the ſame reaſon, the greateſt, the leaſt; and the leaſt the greateſt, the product ſhall be the leaſt number they ſhall mea-

measure: For C and D being supposed the least in the reason of A to B, it hath been demonstrated that E the product of A the least, by D the greatest, and of B the greatest, by C the least, is the least, number measured by A and B.

PROP. 37. THEOR. 33.

A..2 B...3
C------ F----D
E......6

If two numbers A and B, do measure some other number C and D, the least it measureth E, shall also measure the same number.

Demonstration For if E doth not measure C D, having taken E from C D, as often as may be, there will remaine a number lesse then E; let there remaine F D, lesse then E, (if possible) in such sort as E may measure the part cut off C F: Forasmuch then, as A, as well as B, doth measure E, and E measureth C F, [a] as well A as B shall also measure C F. Therefore, seeing that A and B measureth the whole C D, and the part cut off C F, [b] it shall also measure the residue F D. Now F D is lesse then E: therefore, E is not the least number that A and B do measure, which is contrary to Supposition: Therefore, if two numbers, &c. Which was to be demonstrated.

a) 11. c. f.
b) 12. c. f.

PROP. 38. PROBL. 5.

A...3 B....4 C......6
D............12
E------

Three numbers A, B, and C, being given, to find the least number they measure.

A..2 B...3 C....4
D......6
E............12
F-----

Construction Let [a] D be the least number that A and B doth measure, C doth measure it, or doth not measure it: if it doth measure it: I say, that it is the least that the three A, B, and C do measure: otherwise, suppose E to be lesse then D measured by A, B, and C: therefore A and B measuring E, lesse then D: D shall not be the least measured by A and B, which is against the Supposition.

If C measure not D, suppose [b] E to be the least measured by C and D; I say, that it shall be the least that A, B, and C, do measure, for A B measuring D, and D measuring E, [c] A and B shall also measure E, but C in like manner, measureth E; therefore, the three A, B, and C, measure E; and if you say E is not the least, let F lesse then E, be measured by A, B, and

a) 36. 7.
b) 36. 7.
c) 11. c. f.

and C, (if possible. Seeing A and B do measure F, D the least, measured by them, [d] shall also measure F and C, D measuring F: (for A, B, and C, measures it,) E the least number measured by C D, shall measure in like manner, F the greatest, the least, which is absurd: Therefore E is the least that A, B, and C, measureth. Therefore, &c. which was to be done.

d) 37. 7.

COROLLARIE.

From hence follows, that if three numbers do measure a number that the least number they measure, measureth also the same number: For in the last part of this Demonstration, for that A, B, and C, are put to measure F, it hath been demonstrated that E which is the least number measured by them, doth also measure F.

PROP. 39. THEOR. 34.

A...........12
B....4 C...3

If one number B, measure another number A, that which it measureth A, shall have a part denominated of the number measuring B.

Demonstration For let B measure A as many times as there are unites in C, forasmuch then, as unity measureth C, and B measureth A equally: [a] alternately, unity shall measure B, and C shall measure A equally, and therefore unity shall be the same part of B, as C is of A [b], but unity is a part of B denominated of the same B: therefore, C shall be a part of A denominated by B. Therefore, if a number &c. Which was to be demonstrated.

a) 15. 7.
b) 2. d. 7.

PROP. 40. THEOR. 35.

A..............15
B...3 C.....5

If a number A, hath a part B, the number denominated of that part C, measureth it.

Demonstration For seeing that B part of A, is denominated of C, but unity is one part of C denominated of the same C, unity shall measure C, and B shall measure A equally [a]: Therefore, alternately, unity shall measure B; and C shall measure A equally. Therefore, if a number, &c. Which was to be demonstrated.

a) 15. 7.

PROP. 41. PROBL. 6.

To find the least number which hath the parts given A, B, *and* C.

D..2 A Second
E...3 B Third
F....4 C fourth
G............12
H-----

Construction LEt D, E, and F, be denominated of the parts A, B, and C, and let G [a] be the least that D, E, and F, do measure; I say, that G is the least, having the parts given, A, B, and C.

a) 38.7.

Demonstration FOr seeing that D, E, and F, do measure G, [b] G shall have the parts denominated by D, E, and F; that is to say, the parts A, B, and C, being denominated of D, E, and F; I say, that G is also the least, having such parts.

b) 39.1.

40.7. For if it be denied, let another lesse then G have the same parts A, B, and C, which suppose to be H; Forasmuch then as H hath the part A, B, and C, the numbers D, E, and F, denominated of the same parts A, B, and C, shall measure H; therefore, H being lesse than G; G shall not be the least number measured by D, E, and F, which is contrary to Supposition. Therefore, a lesse number then G, shall not have the parts given, A, B, and C, but G shall be the least: Therefore, &c. Which was to be demonstrated.

The End of the Seventh Element of EUCLIDE.

THE

EIGHTH ELEMENT OF EUCLIDE.

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

A........8 D—
B...........12 E—
C.................18 F—

If there be as many numbers as you please continually proportional: A, B, *and* C, *and that the extreams* A *and* C, *be primes to one another, they shall be the least of all those which have the some rate with them.*

Demonstration FOr if it be denied, suppose D, E, and F, lesse then they in the same reason: Forasmuch as there are as many numbers as you please of the one part A, B, and C, and as many of the other, D, E, and F, which taken two and two, are in the same rate [a]; in equal rate, as A shall be to C, so D to F [b]: But A and C are the least of their rate, being primes to one another, by Supposition, [c] therefore as well A, as C, shall measure D and F equally, the greatest, the least, which is impossible: Therefore, A, B, and C, are the least of their rate. Therefore, if, &c. Which was to be demonstrated.

a) 14.7.
b) 23.7.
c) 21.7.

PROP.

PROP. 2. PROBL. 1.

A 2 B 3
C 4 D 6 E 9
F 8 G 12 H 18 I 27

To find as many numbers as you please continually proportional, the least in a given rate.

Construction. LEt A multiplying it self produce C, and multiplying B produce D, and B by it self produce E; I say, that C, D, and E, are the least in the rate of A to B, and are continually proportional.

Demonstration. FOrasmuch as A multiplying A and B, makes C and D: as A is to B, so C is to D; again, seeing that B multiplying A, B
a) 17. 7. makes D and E, [a] as A is to B, so D is to E: therefore, C, D, and E, are continually proportional in the rate of A to B, and are also the least.

For seeing the extreams C and E, are products of A and B, multiplyed
b) 24. 7. each by himself [b]; But A and B are primes to one another, being the least
c) 29. 7. of their rate, [c] the extreams C and E shall be also primes to one another. Therefore C, D and E, are the least in the rate of A, B, and C.

Secondly, let A multiplying the three numbers C, D, and E, produce F,
1. 8. G, and H, and B multiplying E make I; I say, that the four numbers F, G, H, and I, are the least in the same given rate of A to B, and continually proportional.

Demonstration. FOr seeing that A multiplying C, D, and E, hath produced F, G, and H; F, G, and H, shall have the same rate as C, D, and E, that is to say, as A and B; again, seeing that A and B, multiplying
d) 18. 7. E, have made H and I, [d] as A is to B, so H is to I, therefore F, G, H, and I, are continually proportional in the given rate of A and B, and are also the least.

e) 24. 7. For A and B, being the least in their rate, [e] they shall be primes to one another: But C and E are products of A and B multiplyed by themselves; and F and I are also products of A and B, in C and E: to wit, F of A in C,
f) 29. 7. and I of B in F, [f] F and I the extreams shall be also primes to one another: Therefore, F, G, H, and I, are the least of their rate, which is the rate of A to B, by the same reason we shall find 5 or 6, &c. Therefore, we have
1. 8. found, &c. which was to be done.

COROLLARIE. I.

It follows hence that, if three numbers are continually proportional, and the least of their rate, the extreams shall be squares: and if 4 be continually proportional, the extreames shall be Cubes, for C *and* E *the extreams, are products of* A *and* B, *each by himself, and* F *and* I, *extreams of four proportionals are products of* A *and* B, *multiplyed each by his square* C *and* D, *&c.*

CO-

COROLLARIE II.

It follows also that the extreams of the numbers continually proportional, found to be the least in the given rate, (by this Proposition) are Primes to one another, [a] *it having been demonstrated that* C E *and* F I, *are Primes to one another.* a) 24, 29. 7.

COROLLARIE III.

It appears also that two numbers, the least in the given rate, do measure all the means whatsoever of the least in the same rate, being products of the multiplication of them by some other numbers, as D *the mean, which is produced of* A *by* B, *and* G H *means of* A, *by* D *and* E, *or of* B *by* C *and* D, *&c.*

PROP. 3. THEOR. 2.

A 8 B 12 C 18 D 27
E 2 F 3
G 4 H 6 I 9
K 8 L 12 M 18 N 27

If there be as many numbers as you please continually proportional A, B, C, *and* D, *the least of all those that have the same rate with them, the extreams* A *and* D, *are primes to one another.*

Demonstration. FOr [a] let E and F be found the least in the rate of A to B, or B to C, or C to D, [b] E and F, shall be primes to one another: again, let the three G, H, and I, be found the least in the rate of E to F: and also the four K, L, M and N, and so following, until that K, L, M, and N, be equal in number to A, B, C, and D: Forasmuch, as A, B, C, and D, are the least of their rate, and K, L, M, and N, equal in number to them, being also the least in the same rate, they shall be equal to them, each to his correspondent; to wit, A to K, and D to N, [c] now K and N are primes to one another: Therefore, A and D are also primes to one another. Therefore, if there be, &c. Which was to be demonstrated.

a) 35. 7.
b) 2. 8.
c) 2 Cor. 2. 8.

PROP. 4. PROBL. 2.

A 6 B 5 C 4 D 3
H 4 F 24 E 20 G 15
I — K — L —

A 6 B 5 C 4 D 3 E 5 F 7
H 24 G 20 I 15 K 21
L — D — N — O —

A 6 B 5 C 4 D 3 E 2 F 7
H 24 G 20 I 15
M 48 L 40 K 30 N 105
O — P — Q — R —

There being given as many rates as you please A *to* B, *and* C *to* D, *from their least numbers, to find as many numbers as you please continually proportional, the least according to the given rate.*

Con-

a) 36. 7. *Conſtruction* HAving [a] found E the leaſt number meaſured by B and C, 2 and 3; let F be found as oftentimes meaſured by A, as E by B, and G as oftentimes meaſured by D, as E by C: F, E, and G, ſhall be continually proportional, the leaſt according to the given rate.

Demonſtration FOraſmuch as A and B do equally meaſure F and E; that is to ſay, by one and the ſame number; let them meaſure it
b) 9. c. ſ. 18. 7. by H, that being, [b] A and B multiplying H, ſhall produce F and E; wherefore, as A is to B, ſo F is to E. In like manner, C and D, equally meaſuring E and G, as C to D, ſo E ſhall be to G; therefore, F, E, and G, are continually proportional, according to the rates of A to B, and C to D. I ſay, that they are alſo the leaſt. For if it be denied, ſuppoſe I, K, and L, to be leſſe in the ſame rates, each to his correſpondent. Foraſmuch then as A and B are the leaſt in their rate, [c] the ſame ſhall equally meaſure I and K; being in the ſame rate; to wit, B conſequent, ſhall meaſure K conſequent: by the ſame reaſon C and D ſhall equally meaſure K and L; to wit, C antecedent, K antecedent: Therefore, B and C, meaſuring K and E, [d] the leaſt, meaſured by B and C, ſhall alſo meaſure K, the greateſt, the leaſt, which is abſurd, therefore F, E, and G, are the leaſt.

c) 21. 7.

d) 37. 7.

A 6 B 5 C 4 D 3
H 4 F 24 E 20 G 15
I — K — L —

A 6 B 5 C 4 D 3 E 5 F 7
H 24 G 20 I 15 K 21
L — D — N — O —

A 6 B 5 C 4 D 3 E 2 F 7
H 24 G 20 I 15
M 48 L 40 K 30 N 105
O — P — Q — R —

Let there be three rates given to the leaſt numbers of A to B, C to D, and of E to F, to find four numbers, the leaſt continually proportional, according to the given rates, [e] having again found G the leaſt that meaſureth B,
e) 36. 7. and C 2 and 3; let H be found as often times meaſured by A, as G by B, and I as oftentimes meaſured by D, as G by C, that being ſo, E ſhall meaſure I, or ſhall not meaſure it. Let it meaſure it in the firſt place, and as oftentimes as I is meaſured by E, let K be as often times meaſured by F. I ſay, that H, G, I, and K, are continually proportional, and the leaſt, according to the given rates, as hath been ſhewn.

f) 36. 7. Now let E not meaſure I, and [f] having found K the leaſt that E and I do meaſure, let L be as often times meaſured by G, and M by H, as K by I; and let alſo N be as often times meaſured by F, as K by E; I ſay, that M, L, K, and N, are the leaſt according to the given rates.

Demonſtration FOr, H, G, and I, meaſuring M, L, and K, equally, as is before ſhewn, M ſhall be to L, as H to G; and L to K, as G to I: but by the ſame reaſon, H is to G, as A to B, and G to I, as C to D; (A and B meaſuring H and G, and C and D, G and I, equally,) therefore, as A to B, ſo is M to L, and as C to D, ſo is L to K. But as E is to F, K is alſo to N: (E and F equally meaſuring K and N,) therefore, M, L, K, and N, are continually proportional, according to the given rates.

They are alſo the leaſt; For if it be denied, let O, P, Q, and R, be ſuppoſed the leaſt according to the ſame rates. Foraſmuch as A and B are the
g) 21. 7. leaſt of their rate, [g] they ſhall equally meaſure O and P in the ſame rate; to wit, B conſequent, ſhall meaſure P conſequent, and by the ſame reaſon, C and D, ſhall meaſure P and Q equally; to wit, C antecedent, P antece-
h) 37. 7. dent: wherefore, B and C meaſuring P, [h] G the leaſt number meaſured by B and C, ſhall meaſure the ſame P.

Now having ſhewn, that as G is to I, ſo is L to K: that is to ſay, ſo is P to Q: alternately, as G to P, ſo I to Q: and therefore G meaſuring P, I ſhall meaſure Q: But E ſhall alſo meaſure Q, [i] ſeeing that E and F, the i) 21. 7.
leaſt of their rate, do equally meaſure Q and R, in the ſame rate; to wit, the antecedent E, the antecedent Q: therefore, I and E, meaſuring Q and K, the leaſt number meaſured by I and E, ſhall alſo meaſure Q, the greateſt, the leaſt, which is abſurd: Wherefore M, L, K, and N, are the leaſt: and ſo following, if there be more rates given to the leaſt numbers, &c. Therefore, being given, &c. Which was to be demonſtrated.

PROP. 5. THEOR. 3.

A 24 B 18
G 18
C 4 D 6 E 3 F 19

Plain numbers A *and* B, *are the one to the other in a rate compounded of their ſides.*

Demonſtration FOr D multiplying E, produce G, foraſmuch as D multiply- 17. 7.
ing C and E, hath produced A and G, A ſhall be to G, as C
to E: and ſeeing that E multiplying D and F, hath produced G and B; [a] G a) 17. 7.
ſhall be to B, as D to F: therefore, A, G, and B, are continually proportio-
nal, according to the rates of C to E, and of D to F the ſides. But [b] the b) 27. d. 7.
rate of A to B, is compounded of the rates of A to G, and of G to B: therefore, the ſame rate of A to B, ſhall be compounded of the rates of C to E, and of D to F, the ſides. Therefore, the numbers, &c. Which was to be demonſtrated.

PROP. 6. THEOR. 4.

A 16 B 24 C 36 D 54
F 4 G 6 H 9

If there be as many numbers as you pleaſe, continually proportional, A, B, C, *and* D, *and that the firſt* A, *meaſure not the ſecond* B, *alſo neither of the other, ſhall meaſure any of the other.*

Demonſtration FOr that neither of them doth meaſure his next following, is manifeſt, foraſmuch as A, B, C, and D, are continually
proportional, and as A meaſureth not B, [a] it ſhall be no part thereof, but a) 3, 4. d. 7.
parts: wherefore [b] B, which is the ſame parts of C, ſhall not alſo meaſure b) 20. d. 7.
C, nor C and D: I ſay alſo, that neither of the other, ſhall meaſure any
of the other, as A meaſureth not C. [c] For having found F, G, and H, the leaſt c) 2. 8. 35. 7.
in the rate of A to B, or of A, B, and C, in equal rate, as A to C, ſo is F to H: but ſeeing that as A is to B, ſo F to G, & as A meaſureth not B, F ſhall not meaſure G, wherefore F ſhall not be unity, otherwiſe F ſhould meaſure G, unity meaſu-
ring every number: therefore, [d] F and H, being primes to one another, & F be- d) 3. 8.
ing not unity, F ſhall not meaſure H, and therefore alſo A ſhall not meaſure

C: for it hath been shewn that as A is to C, so F is to H; in like manner, is shewn that as B measureth not D, taking four numbers in the rate of A to B, and the least, not any other shall measure any other. Therefore, &c. Which was to be demonstrated.

PROP. 7. THEOR. 5.

A 3 B 6 C 12 D 24 E 48

If there be as many numbers as you please, A, B, C, D, *and* E, *continually proportional, and that the first* A, *measureth the last* E, *it shall also measure the second* B.

a) 6. 8. *Demonstration* For if A be not said to measure B the second, [a] also neither of the other shall measure either of the other: therefore A the first shall not measure E the last, which is absurd, A being put to measure E. Therefore, if there be, &c. Which was to be demonstrated.

PROP. 8. THEOR. 6.

A 24 C 36 D 54 B 81
G 8 H 12 I 18 K 27
E 32 L 48 M 72 F 108

If between two numbers A *and* B, *there fall mean proportionals* C *and* D, *in continual proportion: as many as shall fall between those (of means continually proportional) there shall fall as many mean proportionals between two others that shall have the same rate.*

2. 8. *Demonstration* For having taken G, H, I, and K, the least in the rate of A to C, and equal in number to A, C, D, and B, in equal rate, as
a) 3. 8. A shall be to B: and therefore, as E to F, so G to K [a]: wherefore, G and K, being primes to one another: seeing they are the extreams of the least
b) 23. 7. numbers, [b] and so the least of their rate: [c] G shall measure E equally, and
c) 21. 7. K shall measure F: therefore, as often times as G and K shall measure E and F, let there be two other numbers L and M, as many times measured by H and I, in such sort as G, H, I, and K, may measure E, L, M, and F,
d) 9. c f. equally, each his correspondent; [d] wherefore, G, H, I, and K, multiplying the number by the which they measure E, L, M, and F, they shall produce E, L, M, and F: therefore, E, L, M, and F, shall be in the same rate as G, H, I, and K, but G, H, I, and K, are continually proportional: therefore E, L, M, and F, shall be so also, and being equal in number to A, B, C, and D, there shall fall as many mean proportionals between E and F, as between A and B. Therefore if, &c. Which was to be demonstrated.

PROP.

PROP. 9. THEOR. 7.

A 8 C 12 D 18 B 27
Unity 1
E 2 F 3
G 4 H 6 I 9
K 8 L 12 M 18 N 27

If two numbers A *and* B, *are primes to one another, and between them there fall mean numbers, continually proportional,* C, *and* D, *as many mean continual proportionals as shall fall between them, there shall also fall as many between each of them and unity.*

Demonstration For having proposed unity, let E and F be found the least in the rate of A to C, and let the three G, H, and I, be taken in the same rate, and then the four K, L, M, and N, and so on until the numbers taken be as many as A, C, D, and B. Now A and B the extreams, being primes to one another, A, C, D, and B, shall be the least in the rate of E to F: But K, L, M, and N, which are as many as A, C, D, and B, are also the least in the same rate, by the construction, therefore, K, L, M, and N, are equal to A, C, D, and B, each to his correspondent, that is to say, K to A, and N to B, to the end there be none lesser than the least. 2. 8. 1. 8.

But, (as is manifest by the demonstration of the second Prop. 8.) E multiplying it selfe, hath produced G, and multiplying G hath produced K; E shall [a] measure G by E, and G shall also measure K by E: but [b] unity measures E by E, therefore unity measures E, and E measures G, and G measures K equally; and therefore, unity is the same part of E, and E of G, as G of K; therefore, unity and E, G, and K, are continually proportional: in like manner, unity and F, I, and N, are continually proportional: Therefore, as well E, G, and K, as F, I, and N, with unity, being equal in number to K, L, M, and N, there shall fall as many numbers continually proportional between unity and K, or his equal A; and between unity and N, or his equal B, as between A and B. Therefore, if two, &c. Which was to be demonstrated. a) 7. c. f. b) 5. c. f. 20. d.

PROP. 10. THEOR. 8.

A 8 I 12 K 18 B 27
E 4 H 6 G 9
D 2 F 3
C 1

If between two numbers A *and* B, *and unity* C, *there fall numbers continually proportional, as many as shall fall of continual proportionals between each of them and unity, there shall as many continual proportionals fall between them* A *and* B.

Demonstration Let A be the product of D in F, and I the product of D in H, and K the product of F in H, C being to D, as D to E, and

 D

a) 5.c.f. D to E, as E to A: [a] and C measuring D by the unites that are in D, D
b) 9.c.f. shall measure E, also by the unites which are in D, [b] therefore, D by it self
shall produce E, and E by D shall produce A; in like manner, F by himself
shall produce G, and G by F shall produce B.

A 8 I 12 K 18 B 27
E 4 H 6 G 9
D 2 F 3
C 1

Forasmuch then as D multiplying
c) 17.7. D and F, produceth E and H, [c] as D is
to F, so E is to H: by the same reason, see-
ing that F multiplying F and D, hath made
H and G, as D is to F, so H is to G: therefore,
E, H, and G, are continually proportional.

d) 17.7. Again, D multiplying E and A, hath made A and I; therefore, as E [d]
shall be to H, so A to I: and seeing that D and F multiplying H, makes I
e) 18.7. and K, [e] as D shall be to F, so I to K: and in [f] like manner, K shall be to B,
f) 17.7. as H to G: therefore A, I, K, and B, are continually proportional: therefore
between A and B, there doth fall two meanes continually proportional:
to wit, as many as between A and unity, or B and the same unity. There-
fore, &c. Which was to be demonstrated.

PROP. 11. THEOR. 9.

A 9 E 21 B 49
C 3 D 7

Between two square numbers A, *and* B, *there is a mean proportional number: and the square* A, *is to the square* B, *in a double rate of the side* C, *to the side* D.

Demonstration FOr let E be the product of C in D, or of D in C, foras-
much as C by himself hath made A [a] a square nnmber, and
a) 18.d.
b) 17.7. by D hath made E, [b] as C is to D, so A is to E: again, seeing that D mul-
18.d. tiplying C hath made E, and multiplying it self, hath made B a square
number, as C shall be to D, so E to B: therefore, A, E, and B, shall be con-
17.7. tinually proportional in the rate of their sides C and D: Therefore, between
A and B there falls a mean proportional number E.

Secondly, seeing that A, E, and B, are continually proportional, [c] A the
c) 6.d.7. first, shall be to B the third in a double rate of A the first, to E the second,
10.p.5. which is the same rate of the side C. Therefore, between, &c. Which was
to be demonstrated.

PROP. 12. THEOR. 10.

A 27 H 36 I 48 B 64
E 6 G 12 F 16
C 3 D 4

Between two Cube numbers A *and* B, *there are two numbers mean proportional,* H *and* I, *and the Cube* A, *is to the Cube* B, *in a Triple rate of the side* C, *to the side* D.

Demon-

Demonstration FOr C multiplying it self, makes E; and D multiplying it self makes F; and C and D, multiplying one another makes G, and multiplying G makes H and I.

Forasmuch as C multiplying C and D, hath made E and G, [a] as C a) 17.7.
shall be to D, so E to G: In like manner, seeing that D multiplying D and C,
hath made G and F, [b] as C is to D, so G is to F: therefore, E, F, and G, b) 17.7.
are continually proportional in the rate of C to D.

Again, forasmuch as C multiplying E, hath made A, a Cube number, 19.d.
and multiplying G hath made H, by the construction, [c] A shall be to H, as c) 17.7.
E to G, that is to say, as C to D: in like manner, seeing that by the con-
struction, D multiplying G, hath made I, and multiplying F hath made the
Cube B [d], I shall be to B, as G to F [e], that is to say, as C to D. But, [f] H is d) 19.d.
also to I, as C to D: forasmuch as C and D multiplying G, have made H e) 17.7.
and I: Therefore, A, H, I, and B, are continually proportional in the rate of f) 18.7.
C to D: therefore, between the Cubes A and B, there do fall two mean
proportionals H and I.

Secondly, forasmuch as A, H, I, and B, are continually proportional, A the first shall be to B the fourth, in a Triple rate of A the first, to H the second, that is to say, of the side C, to the side D: for C is to D, as A to H. Therefore, between two Cubes, &c. Which was to be demonstrated.

PROP. 13. THEOR. 11.

If there be as many numbers as you please continually proportional, A, B, *and* C, *and that each multiplying it self, make others, their products* D, E, *and* F, *shall be proportional, and if the numbers first taken* A, B, *and* C, *multiplying their products* D, E, *and* F, *do make others* G, H, *and* I, *those shall also be proportional, and alwayes this happens about the extreams.*

A 2 B 4 C 8
D 4 N 8 E 16 O 32 F 64
G 8 P 16 Q 32 H 64 R 128 S 256 I 512

Demonstration FOr let A and B multiplying one another make N, and B and C make O: then let A multiplying N and E, make P and Q, and B multiplying O and F, make R and S.

Forasmuch as A multiplying A and B hath made D and N, [a] as A to B, a) 17.7.
so D to N, by the same reason, seeing that B multiplying A and B, hath
made N and E, as A is to B, so N to E: therefore, D, N, and E, are continu-
ally proportional in the rate of A to B.

Again, seeing that B multiplying B and C, hath made E and O, [b] as B b) 17.7.
is C, so E is to O: in like manner, seeing that C multiplying B and C,
hath made O and F, as B is to C, so O is to F; therefore, E, O, and F, are
continually proportional in the rate of B to C, or of A to B: therefore, D, N,

and

and E, being continually proportional in the same rate: in equal rate, as D to E, so E to F: therefore, D, E, and F, are continually proportional.

Moreover, seeing that A multiplying D, N, and E, hath made G, P, and Q; G, P, and Q shall be in the same rate as D, N, and E: that is to say, as A to B. And seeing that A and B multiplying E, have made Q and H,

c) 18. 7. [c] as A is to B, so Q is to H; therefore, G, P, Q, and H, are proportional in the rate of A to B: in like manner, seeing that B multiplying E, O, and F, hath made H, R, and S, H, R, and S, shall be proportionals in the rate of E, O, and F, that is to say, of B to C, or of A to B.

d) 18. 7. Lastly, seeing that B and C, multiplying F, have made S and I, [d] S shall be to I, as B to C, or A to B: therefore, H, R, S, and I, are proportionals in the rate of A to B: therefore, seeing that G, P, Q, and H, are also in continual proportion in the rate of H, R, and I, in equal rate, as G shall be to H, so H to I: therefore G, H, and I, are continually proportional. Therefore, if there be, &c. Which was to be demonstrated.

PROP. 14. THEOR. 12.

A 4	E 12	B 36
C 2	D 6	

If a square number A, *doth measure a square number* B, *also the side* C, *shall measure the side* D, *and if the side* C *doth measure the side* D, *also the square* A, *shall measure the square* B.

Demonstration. For let C and D multiplying one another, make E, forasmuch as C multiplying C and D, hath made A and E, [a] as

a) 17. 7. C shall be to D, so A to E: in like manner, seeing that D multiplying C

b) 17. 7. and D, hath made E and B, E [b] shall be to B, as C to D: that is to say, as A to E: therefore, A, E, and B, are continually proportional in the rate of

7. 8. C to D. But A the first, measureth B the last: therefore, A the first, shall also measure E the second: therefore seeing that C is to D, as A to E, and A measureth E, therefore the side C, shall measure the side D.

Secondly, let the side C, measure the side D: I say, that the square A,

c) 11. 8. measureth the square B: [c] (for as hath been demonstrated, A, E, and B, are continually proportional in the rate of C to D, but C measures D: there-

d) 11 c. s. fore A shall measure E, and E shall measure B: [d] therefore also A shall measure B. Therefore, if one number, &c. Which was to be demonstrated.

PROP. 15. THEOR. 13.

A 8	H 24	I 22	B 216
E 4	G 12	F 36	
C 2	D 6		

If a cube number A, *measure a cube number* B, *also the side* C, *shall measure the side* D, *and if the side* C, *measure the side* D, *also the cube* A, *shall measure the cube* B.

Demon-

Demonstration. For let C and D, each multiplying it self, make E and F, and multiplying one another make G, and lastly, multiplying G, make H and I: Forasmuch then, (as by the demonstration of the 12 Prop. 8.) as well E, G, and F, as A, H, I, and B, are continually proportional in the rate of C to D: But A the first, measureth B the last, [a] A the a) 7. 8.
first shall measure also H the second; therefore A being to H; as C to D: C the side shall also measure D the side.

Now let the side C, measure the side D: I say, that the cube A shall measure the cube B: For by the reason above, as C is to D: so A is to H, A, H, I, and B, being continually proportional in the rate of C to D; as hath been shewn in the 12 Prop. 8. Wherefore C the side, measuring the side D, also A, shall measure H: but H measures I, and I measures the cube B:
(being continually proportional:) [b] therefore A shall also measure the cube b) 11 c. s.
B. Therefore, if a number, &c. Which was to be demonstrated.

PROP. 16. THEOR. 14.

A 16	B 81
C 4	D 9

If a square number A, *doth not measure a square number* B, *also the side* C, *shall not measure the side* D, *and if the side measure not the side also, the square shall not measure the square.*

Demonstration. For suppose the side C doth measure the side D: [a] therefore a) 14. 8.
the square A shall measure the square B, which is absurd: for it is proposed not to measure it, wherefore C the side shall not measure the side D.

Now I say, that if the side C doth not measure the side D, that also the 14. 8.
square A shall not measure the square B: for if A be said to measure B, the side C shall also measure the side D, which is absurd, for it is proposed not to measure it: Therefore, the square A shall not measure the square B. Therefore, if a square number, &c. Which was to be demonstrated.

PROP. 17. THEOR. 15.

A 8	B 27
C 2	D 3

If a cube number A, *measure not a cube number* B, *also the side* C, *shall not measure the side* D, *and if the side measure not the side, neither shall the cube measure the cube.*

Demonstration. For suppose that the side C doth measure the side D, [a] therefore the cube A, shall measure the cube B, which is a) 15. 8.
absurd: for it is proposed not to measure it: therefore, C shall not measure D.

Now

Now, as the side C measureth not the side D; I say, that the cube A,
b) 15. 8. shall not measure the cube B. For if A be said to measure B, the side C b shall also measure the side D, which is absurd: For it is proposed not to measure it: Therefore the cube A, shall not measure the cube B. Wherefore if a cube number, &c. Which was to be demonstrated.

PROP. 18. THEOR. 16.

A 13 G 18 B 27
C 6 D 2 E 9 F 3

Between two Plain numbers alike, A *and* B, *there is a mean proportional number* G, *and the Plain* A *is to the Plain* B, *in a double rate of the Homologal sides (or sides of like rate)* C E, *and* D F.

Demonstration For let D and E multiplying one another, make G; forasmuch then as C is to D, so E is to F, alternately, C shall be to E, as D to F: and seeing that D multiplying C and E, hath made A
a) 17. 7. and G, a A shall be to G, as C to E: that is to say, as D to F: and seeing
b) 17. 7. that E multiplying D and F hath made G and B, b G shall be to B, as D to F: therefore A, G, and B, are continually proportional in the rate of C to E: or of D to F: therefore between A and B, there is a mean proportional G. But
c) 26. d. 7. forasmuch as A, G, and B, are continually proportional, c A is to B, in a double rate of A to G; that is to say, of C to E, or of D to F: Therefore, the Plain A, is to the Plain B, in a double rate of C to E, or of D to F, sides of one and the same rate: Therefore, between, &c. Which was to be demonstrated.

PROP. 19. THEOR. 17.

Between two Solid numbers alike A *and* B, *there are two mean proportional numbers* M *and* N, *and the Solid* A, *is to the Solid* B, *in a triple rate of the Homologal side* C, *to the Homologal side* F.

A 30 M 60 N 120 B 240
I 6 L 12 K 24
C 2 D 3 E 5 F 4 G 6 H 10

Demonstration For let C and D, multiplying one another, make I; and F and G, make K; and D and F, make L: Lastly, E and H, multiplying L, make M and N: Forasmuch then as C, D, and E, are proportional to F, G, and H: alternately, they shall be also proportional; to wit, as C to F, so D to G, and E to H: and seeing that D multiplying C
a) 17. 7. and F, hath made I and L, a as C is to F, so is I to L: in like manner, seeing that F multiplying D and G, hath made L and K; as D is to G, so is L to K; wherefore I, L, and K, are continually proportional in the rate of C to

F, D to G, or of E to H: And forasmuch as A solid, is produced from the mutual multiplication of the sides C, D, and E: and I is made of C in D, or of D in C; E multiplying I, shall make A; in like manner, B solid, being made of the mutual multiplication of F, G, and H; and K of F in G, or of G in F, H multiplying K, shall make B.

Wherefore, seeing that E multiplying I and L, hath made A and M; b A b) 17. 7.
shall be to M, as I to L: that is to say, as C to F, or D to G, or E to H, by the same reason, seeing that H multiplying L and K, hath made N and B,
c N shall be to B, as L to K: that is to say, as C to F, or D to G, or E to H: c) 17. 7.
But d M is to N, as E to H, seeing that E and H multiplying L, have made d) 18. 7.
M and N: therefore, A, M, N, and B, are continually proportional in the rate of C to F, or of D to G, or E to H: therefore between A and B, like solids, there falls two mean numbers continually proportional.

Secondly, seeing that A, M, N, and B, are continually proportional, e A e) 10. d. 5.
is to B, in a triple rate of A to M: But A is to M, as C is to F, or D to G, 26. d. 7.
or E to H: therefore A shall be to B in a triple rate of C to F, or of D to G, or of E to H, sides of like rate: Therefore, between two Solids, &c. Which was to be demonstrated.

PROP. 20. THEOR. 18.

A 18 C 24 B 32
D 3 E 4 F 6 G 8

If between two numbers A *and* B, *there fall a mean proportional number* C, *those numbers* A *and* B, *shall be like Plaines.*

Demonstration For having taken D and E, the least in the rate of A, C, and
B; a D and E shall equally measure A and C: Let them a) 21. 7.
measure them by F, they shall also equally measure C and B, in the same
rate, which let be by G: therefore, b F multiplying D and E, shall pro- b) 9. c. f.
duce A & C, and c G multiplying D & E, shall produce C and B: Forasmuch c) 9. c. f.
then as E multiplying F and G hath produced C and B, d as C shall be to B, d) 17. 7.
so F to G, but as C is to B, so was D to E: therefore, D shall be to E, as F to G, and by permutation, as D to F, so E to G: But forasmuch as F multiplying D, hath made A, A shall be a Plain number, and the sides shall be D and F: in like manner, seeing that G multiplying E, hath made B, B shall be also a Plain whose sides shall be E and G, and those sides, being shewn to be
proportional: to wit, as D to F; so E to G, e A and B shall be like Plaines: e) 21. d. 7.
Therefore, if between, &c. Which was to be demonstrated.

PROP. 21. THEOR. 19.

A 8 C 12 D 18 B 27
E 4 F 6 G 9
H 2 I 2 M 2 K 3 L 3 N 3.

If between two numbers A *and* B, *there fall two mean proportionals* C *and* D, *they shall be like Solids* A *and* B.

a) 2. 8.
b) 20. 8.

Demonstration. Let [a] E, F, and G, be the least in the rate of A, C, D, and B; forasmuch as between E and G, there falls a mean proportional F; [b] E and G shall be like Plaines.

c) 21. 7.
d) 21. 7.
e) 21. d. 7.
f) 18. 7.
g) 17. 7.
h) 21. d. 7.

Let H and I be sides of E and K, and L the side of G, and forasmuch as E, F, and G, doe measure A, C, and D, equally [c], being the least in the same rate: Let them measure them by M, and [d] seeing that E, F, and G, do also measure C, D, and B; by the same reason, let it be by N: in such manner, as M multiplying E, F, and G, may make A, C, and D; and N also multiplying E, F, and G, may make C, D, and B. Now E is produced of his sides H and I, multiplyed by one another: and G is produced of the sides K and L: therefore, A is the product of H, I, and M, multiplyed together, and B the product of K, L, and N: Therefore, [e] A is a solid number, having the sides H, I, and M, and B also a solid number, having the sides K, L, and N: But seeing that M and N, multiplying F, do make C and D, as is shewn, [f] C shall be to D, as M to N [g]; but C and D, are in the same rate as E and F: seeing that N multiplying E and F, hath made C and D, and also E and F, are in the same rate as H and K, or I and L: therefore, H shall be to K, and I to L, as M to N, and alternately, H shall be to I, as K to L, and I to M, as L to N: Therefore the sides H, I, and M, are proportional to the sides K, L, and M: [h] Therefore, the numbers A and B, are like Solids: Therefore, if between, &c. Which was to be demonstrated.

PROP. 22. THEOR. 20.

A 9 B 54 C 324

If three numbers A, B, *and* C, *be continually proportional, and that the first* A, *be a square, the third* C, *shall also be a square.*

a) 20. 8.

Demonstration. For seeing that between A and D [a], there fals a mean proportional, A and C shall be like Plaines; therefore A being a square, C resembling it, shall be also a square: Therefore, if there be three numbers, &c. Which was to be demonstrated.

PROP.

PROP. 23. THEOR. 21.

A 27 B 45 C 75 D 125

If four numbers A, B, C, *and* D, *are proportional, and that the first* A, *be a Cube, the fourth* D, *shall also be a Cube.*

a) 21. 8.

Demonstration. For seeing that between A and D, there falls two mean proportionals, B and C, [a] A and D shall be like Solids. Therefore, A being a cube, D resembling him, shall be a cube also: Therefore, if four numbers, &c. Which was to be demonstrated.

PROP. 24. THEOR. 22.

A 36 F 48 B 64
C 9 E 12 D 16

If two numbers A *and* B, *be in the same rate to one another, as a square number* C, *to a square number* D, *and if the first* A *be a square, the second* B, *shall also be a square.*

a) 11. 8.
b) 8. 8.
c) 22. 8.

Demonstration. Forasmuch as C and D, are squares, [a] there shall fall between them a mean proportional: which let be E: [b] there will also fall one between A and B, which are in the same rate, which let be F: Forasmuch as the three numbers A, F, and B, are continually proportional, and A the first is a square, [c] B the third shall be also a square. Therefore, if two numbers, &c. Which was to be demonstrated.

COROLLARIE.

d) 24. 8.
c) 24. 8.

It is manifest from what is above demonstrated that the proportion of every square number, to any other number whatsoever that is not square, cannot be exhibited in two square numbers: for if they could, [d] *then the two first numbers which have the same rate as the squares of the proportion exhibited, should be squares also: seeing that the first is proposed a square, which is absurd, for the second is proposed no square, whence it follows, that the numbers which are in a double rate, are not to one another as a square number, to a square number: for all these double numbers here* 4, 8, 16, 32, 64, 128, *&c. should be squares, for the first number* 4 *being a square,* [c] 8 *should be also a square, and also* 16, *and* 32, *&c. which is impossible.*

 For

For between 4 and 8, between 8 and 16, between 16 and 32, &c. there would fall a mean proportional, if they were squares; seeing that it hath been formerly shewn, that between any numbers that bear a double proportion to one another, there cannot fall a mean proportional.

In like manner, the numbers which are in a quintuple rate to one another, shall not have the same rate as a square number to square number, for if it should, f) 11. 8. 18. 8. *there would fall between them mean proportional.*

Therefore, there would also fall a mean proportional between 5 and 1, which are the least numbers in a quintuple rate: but we have shewn the contrary.

PROP. 25. THEOR. 23.

A 8	G 12	H 18	B 27
C 64	E 96	F 144	D 216

If two numbers A *and* B, *have the same rate the one to the other, as one cube number* C, *to another cube number* D, *and that the first* A, *be a cube, the second* B, *shall be a cube also.*

a) 12. 8. 8. 8. *Demonstration* FOrasmuch as A is to B, as C is to D, but [a] between the cubes C and D, there falls two mean proportionals E and F: therefore between A and B, there shall also fall two mean proportionals G and H; therefore, forasmuch as the four numbers A, G, H, and B, are continually proportional, and A the first is a cube, b) 23. 8. [b] B the fourth, shall be also a cube. Therefore, if two numbers, &c. Which was to be demonstrated.

COROLLARIE.

It is manifest by what is above shewn, that the rate of any cube number whatsoever, to any other number whatsoever, not cube, cannot be found in two cube numbers: for if you suppose 25. 8. *it could, the first numbers having the same rate as the cubes of the proportion found, should be also cubes: seeing the first is proposed a cube, which is absurd, for the second is proposed no cube.*

PROP.

PROP. 26. THEOR. 24.

A 20	C 30	B 45
D 4	E 6	F 9

Plain numbers A *and* B, *have the same rate to one another, as a square number* D, *to a square number* F.

Demonstration FOr seeing that A and B are like Plaines, [a] their shall fall between them a mean proportional which shall be C. a) 18. 8.

Therefore, having taken the three numbers D, E, and F, the least in the rate of A, C and B, the extreams D and F shall be squares: Therefore, seeing that 1 Cor. 2, 8 in equal rate A is to B, as D to F: it is manifest, that A shall be to B, as a square to a square; to wit, as the square number D, to the square number F. Therefore the Plains, &c. Which was to be demonstrated.

PROP. 27. THEOR. 25.

A 16	C 24	D 36	B 54
E 8	F 12	G 18	H 27

Like Solid numbers A *and* B, *have the same rate to one another, as a cube number* E, *to a cube number* H.

Demonstration FOrasmuch as A and B are like Solids [a] there will fall beween them two mean proportionals C and D: And having taken a) 19. 8. the four numbers E, F, G, and H, the least in the rate of A, C, D, and B, the extreams E and H shall be cubes: Therefore seeing that in equal rate A is to B, as E is to H, it appears that A shall be to B, as the cube number E, Cor. 1. 2, 8. to another cube number; to wit, to H: Therefore the, &c. Which was to be demonstrated.

COROLLARIE. I.

If two numbers be in the same proportion the one to the other that a square number is to a square number: those two numbers shall be like superficial numbers. And if they be in the same proportion the one to the other, that a cube number is to a cube number, they shall be like solid numbers.

B 72	D 64
A 18	C 16

First, let the number A, have unto the number B, the same proportion, that the square number C hath to the square number D: Then I say, that A and B are like superficial numbers. For, forasmuch as between the square numbers C and D, there falleth a mean proportional, [a] there shall also between A and B, (which have the a) 11. 8. same

b) 8.8. c) 20.8. same proportion with C and D) fall a mean proportional [b]. Wherefore A and B are like superficial numbers [c].

But if A be unto B, as the cube number C, is to the cube number D. Then are A and B like solid numbers. For, forasmuch as C and D are cube numbers, there falleth between them two mean proportional numbers [d]. And therefore, [e] between A and B, (which are in the same proportion that C is to D) there falleth also two mean proportional numbers. Wherefore [f] A and B, are like solid numbers.

d) 12.8.
e) 8.8.
f) 21.8.

B 16	C 8
A 56	D 27

COROLLARIE. II.

If a number multiplying a square number, produce not a square number, the number multiplying shall be no square number.

For If it should be a square number, then should it and the number multiplyed, being like superficial numbers (by reason they are square numbers) have a mean proportional [a]. And the number produced of the said mean, should be equal to the number contained under the extreams, which are [b] square numbers. Wherefore, the number produced of the extreams, being equal to the square number produced of the mean, should be a square number. But the said number by supposition, is no square number. Wherefore neither is the number multiplying the square number, a square number.

a) 18.8.
b) 20.7.

The first part of the first Corollarie is the converse of the 26 Proposition of this book, and hath some use in the tenth book. The second part of the same also, is the converse of the 27 Proposition of the same.

The End of the Eighth Element of EUCLIDE.

THE

NINTH ELEMENT OF EUCLIDE.

PROPOSITIONS, PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

A 6		B 54
D 36	E 108	C 324

If two numbers being like Plains A, *and* B, *multiplying one another, do produce some one* C, *the product* C *shall be a square.*

Demonstration LEt D be the product of A by it selfe, D shall be a square number: Therefore, forasmuch as A multiplying A and B, hath produced D and C; [a] as A shall be to B, so D shall be to C: But between A and B, like Plains, [b] there falls a mean proportional, therefore also [c] there shall fall one between D and C, which shall be E, to the end that D, C, and E, may be continually proportional: But D the first, is a Square by the construction; therefore [d] C the third, shall be also a square: Therefore if two plain numbers, &c. Which was to be demonstrated.

a) 17.7.
b) 18.8.
c) 8.8.
d) 22.8.

PROP. 2. THEOR. 2.

A 6	B 54
D 36	C 324

If two numbers multiplying one the other, A *and* B, *make a square* C, *they are like Plains.*

Demonstration FOr let D be the product of A by it self, D shall be a square, forasmuch then as A mul iplying A and B hath produced D and C, [a] as A is to B, so D is to C. But [b] between D and C square numbers there fals a mean proportional, wherefore [c] between A and B there shall also fall a mean proportional, therefore [d] A and B are like Planes. Therefore, if two numbers &c. Which was to be demonstrated.

A 6. B 54.
D 36. C 324.

a) 17. 7.
b) 11. 8.
c) 8. 8.
d) 20. 8.

PROP. 3. THEOR. 3.

A 8.
E 16. D 4.
F 32. C 2.
B 64. Unity.

If a Cube number A, *multiplying it self, doth produce any one* B, *the product* B *shall be a Cube.*

Demonstration LEt C be the side of the cube A, and let D be the product of C by it self, and so the cube A shall be the product of C by D; forasmuch then as C multiplying it self hath made D [a] the same C, shall measure D by C: but [b] unity shall also measure C, by C, therefore, unity shall be the same part of C denominated of C, as C of D: and therefore, [c] as unity is to C, so is C to D.

Again, Forasmuch as C, multiplying D, hath made A, [d] D shall measure A, by C: but C measures D by C, therefore C shall be the same part of D, as D is of A, therefore [e] as C is to D, so D is to A: but as C, is to D, so unity was to C, therefore as unity, is to C, so is, C to D, and D, to A, therefore between unity and A, there doth fall two mean proportionals C and D.

But [f] forasmuch as A measure B by A, (for multiplying it self makes B) and [g] unity measures A, by A, unity shall be the same part of A, as A of B, and therefore unity [h] shall be to A, as A to B, and therefore seeing that between unity and A, there falls two mean proportionals C and D, [i] there shall fall as many between A and B, which shall be E and F.

Therefore, seeing that the four numbers A, E, F, and B, are continually proportional, and A the first is a cube, B the fourth shall be in like manner a cube. Therefore, if a cube, &c. Which was to be demonstrated.

a) 7. c. s.
b) 5. c. s.
c) 20. d.
d) 7. c. s.
e) 20 d.
f) 7. c. s.
g) 5. c. s.
h) 20 d.
i) 8. 8.
23. 8.

PROP. 4. THEOR. 4.

A 8. B 27.
D 64. C 216.

If a cube number A, *multiplying a cube, number* B, *makes some one* C, *the product* C *shall be a cube.*

Demonstration LEt A multiplying it self make D, D [a] shall be a cube, and forasmuch as A multiplying A and B, hath made D and C, [b] as A shall be to B, so D shall be to C. But between the cubes A and B, there

a) 3 9.
b) 17. 7.

there fals two mean proportionals, [c] therefore, between D and C, there shall also fall two mean proportionals, therefore, [d] D being a cube, C shall be a cube also. Therefore if, &c. Which was to be demonstrated.

c) 12. 8.
8. 8.
d) 23. 8.

PROP. 5. THEOR. 5.

A 8 B 27
D 64 C 216

If a cube number A *multiplying some number* B *doth produce a cube* C, *the number multiplyed* B, *shall be a cube also.*

Demonstration FOr let D, be the product of A, multiplyed by it self: [a] D shall be a cube.

Forasmuch as A, multiplying A and B, hath made D and C, [b] as A, shall be to B, so shall D, be to C. [c] But between the cubes D and C, there falls 2 mean proportionals, [d] therefore, between A and B, there shall also fall 2 mean proportionals, [e] and therefore, A being a cube, B shall be also a cube. Therefore, if a cube number, &c. Which was to be demonstrated.

a) 3 9.
b) 17. 7.
c) 12. 8.
d) 8. 8.
e) 22. 8.

PROP. 6. PROBL. 6.

A 8 B 64 C 512

If a number A, *multiplying it self makes a cube* B, *the same* A, *shall be also a cube.*

Demonstration FOr let C, be the product of B by A, C shall be a cube number, as is manifest. Forasmuch then as B, cube multiplying another number, to wit, A hath produced C, a cube number, A shall be also a cube. Therefore, if a number, &c. Which was to be demonstrated.

19. d.
5. 9.

PROP. 7. THEOR. 7.

A 6 B 11 C 66 D 2 E 3

If a compound number A, *multiplying, some number* B, *makes some other* C, *the product* C, *shall be a Solid.*

Demonstration FOr seeing that A is compounded, some other number besides unity shall measure it: Therefore, let D measure A, by E, that being done, [a] D multiplying E, shall produce A. Therefore, seeing that B, multiplying the same A hath made C; C shall be produced by the mutual multiplication of the three numbers D, E, and B, there-

13. d.
a) 9. c. s.

therefore, C shall be a Solid, whose sides are D, E, and B. Therefore,
17. d. if a compound number, &c. Which was to be demonstrated.

PROP. 8. THEOR. 8.

If from unity, as many numbers as you please A, B, C, D, E, F, G, H, I, K, L, *and* M, *are continually proportional, the third* B *from unity is a square, and all the rest which intermit, or leave out one* D, F, H, K, *and* M, *But the fourth is a cube* C, *and all the rest in leaving two*, F, I, *and* M, *and the seventh*, F *is a cube, and a square together, and all the rest which leave five* M.

Unity. A 3 B 9 C 27 D 81 E 243 F 729.
G 2187 H 6561 I 19683 K 59049
L 177147 M 531441.

Demonstration. Forasmuch as unity, is to A, as A is to B, unity shall measure A, and A shall measure B equally. [a] But unity measures A,
a) 5. c. 5.
b) 9. c. s.
by A, therefore, A shall also measure B, by A: [b] therefore, A multiplying A, shall produce B, therefore, B is a square.

c) 22. 8. Now, forasmuch [c] as B, C, and D, are continually proportional, and B is a square D shall be also a square: in like manner seeing that D, E, and F, are continually proportional, and as D is a square, F shall be also a square, and all the others leaving one.

Again, Forasmuch as unity is to A, as B to C, unity shall measure A and B shall measure C equally. But [d] unity measureth A by A: therefore, B shall also measure C, by A, [e] and therefore, B multiplying, A shall produce C, therefore, seeing that A multiplying it self makes B, and multiplying B makes C, [f] C shall be a cube.
d) 5. c. s.
e) 9. c. s.
f) 17. d. 7.

But forasmuch as C, D, E, and F, are continually proportion, and as C the first is a cube, F the fourth [g] shall be also a cube, by the same reason seeing that F, G, H, and I, are continually proportional, and that F is a cube, I, shall be also a cube, and all the rest leaving or intermitting two. Lastly, forasmuch as F the seventh, from unity, is shewn to be a square and a cube; the same shall be a cube, and a square together, and in the same manner, M the seventh from F, leaving G, H, I, K, and L, shall be a cube, and a square together, and also all the others, which leave 5. Therefore, if from unity, &c. Which was to be demonstrated.
g) 23. 8.

PRO-

PROP. 9. THEOR. 9.

If from Unity there be as many numbers as you please A, B, C, D, E, *and* F, *continually proportional; and if that number which follows unity* A, *be a square, all the others also shall be squares, and if that which follows unity be a cube, also all the others shall be cubes.*

Unity A 4 B 16 C 64 D 256 E 1024 F 4096
Unity A 8 B 64 C 512 D 4096 E 32768 F 262144

Demonstration. For seeing that A, B, C, D, E, and F, are continually proportional from unity, [a] B the third from unity, is a square, and all the rest leaving one: to wit, D and F, and forasmuch as A, B, and C, are continually proportional and that A is a square by the Supposition, [b] C the third shall be also a square: in like manner, taking C, D, and E continually proportional, seeing that C the first is a square, E the third, shall be also a square: therefore, all the numbers A, B, C, D, E, and F, are squares, and so of the rest. a) 8. 9. b) 22. 8. 22. 8.

Now let A next to unity be a Cube; I say, that the others are Cubes.

Forasmuch as from unity A, B, C, D, E, and F, are continually proportional, [c] C the fourth from unity is a cube, and all the others leaving two; to wit F, now that the others B, D, and E, are also cubes, we shall thus demonstrate. c) 8. 9.

Forasmuch as unity is to A, as A to B, unity shall measure A, and A the number B, equally [d]; but unity shall measure A by the same A, therefore A shall measure B also by A [e]: therefore, A multiplying it self, shall produce B: But, [f] A is a cube, therefore B shall be also a cube; and forasmuch as A, B, C, and D, are continually proportional, and as A is a cube, so [g] D shall be also a cube: Moreover, seeing that B, C, D, and E, are continually proportional, and as B is a cube, so shall E [h] also be a cube: therefore A, B, C, D, E, and F, are all cubes, and so on, if there were more. Therefore, if from unity, &c. Which was to be demonstrated. d) 5. c. s. e) 9. c. s. f) 3. 9. g) 23. 8. h) 23. 8.

PROP. 10. THEOR. 10.

If from unity there be as many numbers as you please A, B, C, D, E, F, G, H, *and* I, *continually proportional; and if that number* A, *which follows after unity be no square; also not any one of the others shall be a square, except the third* B *from unity, and all the others who leave one* D, E, *and* F, *and if that which is after unity* A, *be not a Cube; also not any one of the others shall be a Cube besides the fourth from unity* C, *and all the others who leave two.*

 Unity

Unity A 2 B 4 C 8 D 16 E 32 F 64
G 128 H 256 I 512

Demonstration FOr if it be ſuppoſed that beſides thoſe there are other ſquares: Let E be ſuppoſed to be a ſquare; foraſmuch
a) 8. 9. then as [a] D is a ſquare, and that as D is to E, or E to F, ſo A is to B, and by converſion, as E is to D, or F to E, ſo B to A; B ſhall be to A,
8. 9. as the ſquare number E, to the ſquare number D, or as the ſquare F, to
b) 24. 8. the ſquare E: But B the third from unity, is a ſquare, therefore [b] A the firſt after unity, ſhall be a ſquare, which is abſurd, for by Suppoſition it is no ſquare: therefore E ſhall not be a ſquare, by the ſame reaſon we ſhall ſhew that not one other is a ſquare, beſides the numbers above mentioned.

Now let A next to unity be no cube; I ſay that not any other is a cube, beſides C the fourth from unity, and all thoſe that do leave two, to wit, F, and I, for if you ſay, that ſome other is a cube, ſuppoſe it to be D: there-
8. 9. fore ſeeing that F is a cube, and as in equality as D is to F, ſo A is to C: ſeeing that D, E, F, and A, B, C, are in the ſame rate, and by converſion as F to D, ſo C to A; C ſhall be to A, as the cube number F, to the cube
c) 8. 9. number D; but [c] C the fourth from unity, is a cube, therefore [d] A the firſt
d) 25. 8. after unity, ſhall be alſo a cube, which is againſt the Suppoſition: Therefore D is no cube, by the ſame reaſons may be ſhewn that none but the fore-mentioned can be cubes. Therefore, &c. Which was to be demonſtrated.

PROP. II. THEOR. II.

If from Unity there be as many numbers as you pleaſe A, B, C, D, E, F, *and* G, *continually proportional, the leaſt doth meaſure the greateſt*, C *meaſures* G *by ſome one of thoſe which are between the proportional numbers* A, B, C, D, E, F, *and* G.

Unity A 3 B 9 C 27 D 81 E 243 F 729 G 2187

Demonstration FOraſmuch as by equality, as C is to G, ſo unity is to D, (for C, D, E, F, and G, are in the ſame rate continued as unity,
a) 20. d. and A, B, C, and D) [a] unity ſhall meaſure D; and C the number G equal-
b) 5. c. f. ly: But [b] unity meaſureth D, by the ſame D, therefore C ſhall alſo meaſure G by D: by the ſame reaſon E ſhall meaſure F by A: ſeeing that as E is to F, ſo unity is to A, &c. ſo A ſhall meaſure G by F: ſeeing that by equality as A is to G, ſo unity is to F, &c. and ſo on: Therefore, if, &c. Which was to be demonſtrated.

PROP.

PROP. 12. THEOR. 12.

If from Unity there be as many numbers as you pleaſe A, B, C, *and* D, *all the prime numbers which meaſure the laſt number* D, *the ſame numbers ſhall alſo meaſure that which is next to Unity* A.

Unity A 6 B 36 C 216 D 1296 E 3

Demonstration FOr let E be a prime number, which ſhall meaſure the laſt D: I ſay, E ſhall alſo meaſure A: for if it ſhall not mea-
ſure it, [a] the ſame E ſhall be prime to A: Foraſmuch then as A and E are a) 31. 7.
primes to one another, and A multiplying it ſelf, makes B, as appears, [b] B b) 27. 7.
ſhall be prime to E: therefore, ſeeing that A and B, are primes to E, and C is
the product of A in B, [c] C ſhall be prime to E: again, ſeeing that A and C c) 26. 7.
are alike primes to E, and that D is the product of A in C, [d] D ſhall be d) 26. 7.
prime to E: therefore E meaſures not D, which is againſt the Suppoſition, therefore, E ſhall meaſure A: Therefore, &c. Which was to be demonſtrated.

COROLLARIE.

Therefore, any prime number, greater then that next Unity, nor any leſſer that meaſurtth not that which is next unity, ſhall not meaſure the laſt: for if it did meaſure it, it would alſo meaſure that which is next Unity, as hath been demonſtrated, which is againſt the Suppoſition,

PROP. 13. THEOR. 13.

Unity A 5 B 25 C 125 D 625
H---- G---- F---- E----

If from Unity there be as many numbers as you pleaſe continually proportional, A, B, C, *and* D, *and if that number next after Unity* A, *be prime, any other ſhall not meaſure the greateſt* D, *beſide thoſe* A, B, *and* C, *that are between the proportionals.*

Demonstration FOr (if poſſible) let ſome other, as E, meaſure D, E ſhall be 12. 9.
a prime, or a compound number; if a prime, and meaſuring D the extream, it ſhall alſo meaſure A, the prime next to Unity, which is ab-
ſurd; therefore, E is not prime, but compounded, and [b] therefore meaſu- b) 33. 7.
red by ſome prime number, which cannot be any other then A: for E meaſuring

suring D, every prime number measuring E, shall also measure D. But measuring D the greater, [c] it shall also measure A, the prime next to unity, which is absurd: Therefore, any other prime number then A, shall not measures E, for as A measures E by F: therefore A multiplying F, makes
c) 12. 9.

Unity A 5 B 25 C 125 D 625
H---- G---- F---- E----

d) 17. 7. E and D: and therefore [d] E shall be to D as F to C, wherefore E measuring D, F shall measure C; and in like manner, it shall be shewn, that F is no Prime, but a compound number, which shall be demonstrated, as is before said to be only measured by the prime A.

Let A then measure F by G, therefore A multiplying G and B, produceth F and C; wherefore [e] F shall be to C, as G to B; but F measureth C: therefore G doth also measure B.
e) 17. 7.

f) 12. 9. Again, if G be a prime number measuring B, it shall [f] also measure the prime number A, which is absurd: therefore G is compounded, which shall be only measured by A, as hath been demonstrated of E: Let A then measure G by H; wherefore A multiplying A and H, shall make B and G: Therefore as G is to B, so H is to A; but G measures B; therefore H shall also measure A the first: wherefore H is equal to the same A.

g) 20. 7. But A is a mean proportional between H and G: seeing that [g] B the product of the extreams G and H, is equal to the same B, the square of A the mean, which is impossible: Therefore any other number besides A, B, and
h) 11. 9. C, shall not measure D the greatest number, of which the least [h] doth alwayes measure the greatest: Therefore, if from unity, &c. Which was to be demonstrated.

PROP. 14. THEOR. 14.

A 30 B 2 C 3 D 5
E---- F---

The least number A, *measured by certaine prime numbers* B, C, *and* D, *shall not be measured by any other prime number then by those* B, C, *and* D, *which measured it at first.*

Demonstration For suppose that E a prime number differing from B, C, and D, doth measure A by F: forasmuch as E measures A by F,
a) 9. c. f. E [a] multiplying F, shall make A, wherefore [b] each of the numbers B, C,
b) 32. 7. and D, should measure one of the two E and F, they shall not measure E prime, other then by these; therefore, F which is lesse then A, shall measure it, which is absurd; for A is proposed the least, which may be measured by the prime numbers B, C, and D: Therefore, the least, &c. Which was to be demonstrated.

PROP.

PROP. 15. THEOR. 15.

A 9 B 12 C 16
D 3 E 4

If three numbers A, B, *and* C, *be continually proportional, and the least of all those which have the same rate with them the number* B, *compounded of any two of them shall be prime to the number resting.*

Demonstration For having taken D and E, the least in the same rate, it appears by the demonstration of 2 Prop. 8. that A is square to D: and C square to E, and B the product of D in E; and the number compounded of D and E multiplyed by D, shall produce a number equal to the number compounded of A and B: but [b] D and E, being primes to one b) 24. 7.
another, [c] the number compounded of them shall be prime to E: and c) 30. 7.
therefore [d] also prime to C, the square of E, and by the same reason D shall d) 27. 7.
be prime to C, and the number produced of the number compounded of D 26. 7.
and E, multiplyed by D, shall be also prime to C, which product is shewn to be equal to the number compounded of A and B: therefore, the number compounded of A and B, is prime to C.

Again, the number compounded of D and E, being prime as well to D, 27. 7.
as to E, the product of the same compound number multiplyed by E, shall 3. 9.
be also prime to D; but the same product is equal to C, the square of E, and to B together, the product of D in E: therefore, the numbers compounded of B and C, shall be also prime to D, and [e] therefore prime to A the e) 27. 7.
square of D.

Lastly, [f] seeing that D and E, are primes to one another, and [g] primes f) 24. 7.
to their compound, B their product, [h] shall be also a prime to their com- g) 30. 7.
pound; [i] and the product of the number compounded of D and E, multi- h) 26. 7.
plyed by it self, shall be prime to B: but the same product is equal to A and i) 27. 7.
C, and to twice B together; therefore, the compound of A and C, and of twice B, is prime to B; then taking away twice B, the remainder of the 10. c. f.
compound of A and C, shall be prime to B: For if it were not so, they should be compounded to one another, and their common measure measuring the compound of A, C, and B, should also measure the compound of A, and C, and of twice B. Wherefore the compound of A, and C, and twice B, should not be prime to B, which is absurd, they having been demonstrated primes to one another: Therefore the compound of A, and C, is prime to B: Therefore, if three numbers, &c. Which was to be demonstrated.

PROP. 16. THEOR. 16.

A 5 B 7 C----

If two numbers A *and* B, *are primes to one another: as the first* A, *is to the second* B, *so the second* B, *shall not be to any other.*

Demon-

Demonstration FOr (if possible) as A is to B, so let B, be to another, to wit, C.

a) 23. 7. Forasmuch then, as A and B, are primes to one another, and [a] the least
b) 21. 7. that are in the same rate, [b] they shall equally measure B and C being in the same rate: to wit, A shall measure B, and B shall measure C: But A measureth also it self, therefore A shall measure A and B, primes to one another, which is absurd: Therefore, as A is to B, so B is not to C.

In like manner, as B is to A, so A shall not be to another: Therefore, if two numbers, &c. Which was to be demonstrated.

PROP. 17. THEOR. 17.

If there be as many numbers as you please A, B, C, *and* D, *continually proportional; and that the extreams* A, *and* D, *are primes to one another, as the first* A, *shall be to the second* B, *so the last* D, *shall not be to some other.*

A 8 B 12 C 18 D 27 E----

Demonstration FOr if it be denied, suppose as A is to B, so D may be to some other; to wit, to E: Forasmuch then as A is to B, so D is to E, alternately, as A is to D, so B is to E: But A and D, being primes
a) 23. 7. to one another, are [a] the least in their rate; therefore [b] they shall measure
b) 21. 7. B and E equally; to wit, A shall measure B, and D shall measure E: But as A is to B, so B is to C: therefore seeing that A measures B, also B shall
c) 11. c. f. measure C, and [c] therefore A shall measure C also, and seeing that as B is to C, so C is to D: But B measureth C, and C shall also measure D: where-
d) 11. c. f. fore A measuring C [d], shall in like manner measure D, and A measureth it self; therefore A measureth A, and D, primes to one another, which is impossible, therefore as A is to B, so is not D, to some other number; to wit, to E: in like manner, as D is to C, so is not A to another: Therefore, if there be, &c. Which was to be demonstrated.

PROP. 18. PROBL. 1.

Two numbers A *and* B, *being given, to consider if it be possible to find a third proportional to them.*

A 4 B 7
A 4 B 6 D 9 C 36
A 6 B 4 D oo C 16

Demonstration THe numbers A and B, are primes to one another, or not primes to one another, if primes, then [a] there can be no
a) 16. 9. third proportional found; if they are not primes, let B multiplying it self, make C. Now either A measureth C, or doth not measure it: if it measure it, let it be by D. I say, there may be a third proportional found, which is D, for seeing that A, measureth C, by D, [b] C shall be the product of A, by D: but the same is the product of B, multiplyed in himself: therefore, C, contained under the extreams A and D, is equal to the product of the mean B, multiplyed by it self: therefore, [c] the three A, B, and C, are continually proportional, to wit, as A, is to B, So is B, to D: therefore, D shall be the third proportional. b) 9. c. f. c) 20. 7.

But if A doth not measure C, I say, that there cannot be found a third proportional, seeing that as A, is to B: so B, is to D, the number produced of A and D, the extreams, shall be equal to the product of B, the mean multiplyed in it self: that is to say, to C; therefore, seeing that C, is made of A, multiplyed by D, [d] A shall measure C, by D, which is absurd, being proposed not to measure it. Therefore, there cannot be found a third proportional to A and B, seeing that A, a Prime number measureth not C, the product of B, the second multiplyed in it self, &c. Therefore, two numbers, &c. Which was to be done. d) 7. c. f.

PROP. 19. PROBL. 2.

Three numbers A, B, *and* C, *being given, to consider if it be possible to find a fourth proportional to them.*

A 8 B 12 C 18 E 27 D 216.
A 4 B 8 C 9 E 18 D 72.

A 4 B 6 C 9 E---- D 54
A 3 B 14 C 10 E---- D 40

Construction LEt B, multiplying C, make D, A shall measure D, or shall not measure it: suppose it to measure it by E, I say, that E, is the fourth proportional.

Demonstration SEing that A, measures D, by E, [a] D shall be the product of A, by E: but D is also the product of B by C, by construction, therefore, the product of the extreams A and E, is equal to the product of the means B, and C, therefore, [b] as A, is to B: So is C, to E. a) 9. c. 5. b) 19. 7.

But if A, measures not D, I say, there cannot be found a fourth proportional, for otherwise suppose E, to be a fourth proportional. Forasmuch then as the four numbers A, B, C, and D, are continually proportional, [c] the number contained under the extreams A, and E, shall be equal to the number contained under the means B and C, to wit, to D. Therefore, seeing that D, is made of A, multiplyed by E, [d] the same A, shall measure D, by E, contrary to the supposition. In like manner we may examine, if as A, is to B, so C is to some other number. Therefore, three numbers, &c. Which was to be done. c) 19. 7. d) 7. c. f.

PROP. 20. THEOR. 18.

Prime numbers are in a greater multitude then any multitude of prime numbers proposed. A, B, C.

A 2 B 3 C 5
30 1
D------ E-F

A 3 B 5 C 7
105 1
D----- E-F
35
G-----

THat is to say; That if you propose as many prime numbers as you please there will yet be found others.

Demonstration FOr [a] having taken D, the least that may measure A, B, and C, and adding unity E F, thereto, the whole D F, is a prime number, or not: if it be a prime number, then you have what is required, for A, B, C, and D F, are in greater number then the multitude proposed A, B, and C.

a) 38.7.

But if D F, be not a prime number, it [b] shall be measured by some, prime number, which let be G. I say, that G is not one of the numbers proposed A, B, C, for if it were one of them, seeing that A, B, and C, measureth D E, G should also measure D E; Therefore, measuring the whole D F, and the part cut off D E, it should also measure the remainder E F; to wit, a number should measure unity, which is impossible.

b) 33.7.

11. c. s.

Therefore the prime number G, is none of the numbers proposed A, B, C: Therefore, the prime numbers A, B, C; and G, are in a greater number then those proposed, A, B, and C, and so *ad infinitum* may be found other prime numbers: Therefore, The prime, &c. Which was to be demonstrated.

PROP. 21. THEOR. 19.

6 4 8
A......B....C........D

If as many even numbers as you please A B. B C. C D. *be added; the whole* A D. *shall be even.*

a) 6.7.

Demonstration FOrasmuch as [a] every even number may be divided in two equal parts, the halves of A B, B C, and C D, shall make the halfe of A D: but all the halves together, are equal to all the other halves together; therefore A D is an even number, &c. Which was to be demonstrated.

PROP. 22. THEOR. 20.

3 5 7 9
A...B.....C.......D.........E

If as many odde numbers as you please A B, B C, C D, *and* D E, *be added, and that the number of them be even: the whole shall be even.*

Demonstration FOr seeing that A B, B C, C D, and D E, are odde, each of them shall differ from an even number, by unity, according to the Definition: Wherefore if from each you cut off unity, each of the remainders shall be even: wherefore [a] the number componnded of them shall be even: Therefore [b] the whole A and E compounded shall be in like manner even: Therefore, If as, &c. Which was to be demonstrated.

a) 21. 9.
b) 21. 9.

PROP. 23. THEOR. 21.

3 5 7
A...B.....C.......D
15

If as many odde numbers as you please A B, B C, *and* C D, *are added, and that the multitude of them be odde, the whole* A D, *shall be also odde.*

Demonstration FOr seeing that an odde number, differeth from an even number by unity; by the Definition: having cut off unity E D, from the odde number C D, the rest C E, shall be even: But [a] A C compounded of the odde number A B, and B C, even in multitude are even: therefore [b] also A E, compounded of the even numbers A C, and C E, shall be even, therefore if you adde unity E D, the whole A D, shall be odde; seeing that even and odde, differ by unity, by the Definition: Therefore, if as many, &c. Which was to be demonstrated.

a) 21. 9.
b) 21. 9.

PROP. 24. THEOR. 22.

1 5 4
A.D.....C....B

If from an even number A B, *be cut off an even number* C B, *the remainder* A C, *shall be also an even number.*

Demonstration FOr if A C be not an even number, let there be cut off unity A D, the rest D C will remaine even, seeing that an even and odde number differeth onely by unity) therefore [a] the componnded number D B, shall be an even number: Therefore adding unity A D, to the compounded number D B, an odde number, the whole A B, shall be uneven

a) 21. 9.

even; which is impossible, for it is even by Supposition: therefore AC, shall not be odde: Therefore if, &c. Which was to be demonstrated.

PROP. 25. THEOR. 23.

7 1 4
A.......C.D....B

If from an even number AB, *there be cut off an odde number* CB, *the rest* AC, *shall be also odde.*

Demonstration FOr from CB, having cut off unity CD, the rest DB, shall be even. Therefore, forasmuch as the whole AB, is pure-
a) 24.9. ven, the rest [a] AD, shall be even; having therefore taken away unity CD, the rest AC, shall be odde: Therefore, if from a number, &c. Which was to be demonstrated.

PROP. 26. THEOR. 24.

4 6 1
A....C......D.B

If from an odde number AB, *be cut off an odde number* CB, *the rest* AC, *shall be even.*

Demonstration FOr having cut off unity from the odde numbers AB, and CB, the remainders AD, and CD, shall be even numbers: forasmuch then as from the even number AD, the even number CD, is cut
a) 24.9. off, [a] the rest AC, shall be an even number: Therefore if from, &c. Which was to be demonstroted.

PROP- 27 PROP. 25.

1 4 6
A.D....C......B

If from an odde number AB, *be cut off an even number* CB, *the rest* AC, *shall be odde.*

Demonstration FOr, from the odde number AB, let unity AD, be cut, the rest DB shall be even: from which the even number CB,
a) 24.9. being cut, the [a] rest DC, shall be also even: Therefore adding unity AD, AC shall be odde: Therefore, if from an, &c. Which was to be demonstrated.

PROP.

PROP. 28. THEOR. 26.

3 4
A...B

12
C...........

If an odde number A, *multiplying an even number* B, *produce some one* C, *the product* C, *shall be even.*

Demonstration FOr seeing that C is the product of A multiplyed by B: the same C shall be compounded of so many numbers equal to B, as there are unites in A: but B is an even number, therefore C shall be compounded of as many even numbers equal to B, as there are unites in A; and therefore [a] C shall be an even number: Therefore, if an, &c. Which was to be demonstrated. a) 21.9.

COROLLARIE.

A....4 B....4
C................ 16

It follows also that an even number multiplyed by it self, produceth an even number, as is manifest if you put the even numbers A *and* B, *equal.*

PROP. 29. THEOR. 27.

3 5
A...B.....
15
C...............

If an odde number A, *multiplying an odde number* B, *produce one* C, *the product* C, *shall be odde.*

Demonstration FOr seeing that C is produced of A in B, the same C shall be compounded of so many numbers equal to B, as there are unites in A: Therefore seeing that as well A as B, is an odde number; C shall be compounded of as many odde numbers, equal to B, as there are unites in A, the odde number: Wherefore the multitude of them shall be odde, and therefore [a] C shall be an odde number. Therefore, a) 23.9.
if an, &c. Which was to be demonstrated.

COROLLARIE.

3 3
A...B...
C.........
9

It follows from this that an odde number multiplyed by it self, produceth an odde number, as is manifest if the number A *and* B, *are proposed to be equal.*

PROP.

PROP. 30. THEOR. 28.

A 3 ... C 4 ... B 12 ....

If an odde number A, *measureth an even number* B, *it shall also measure its halfe.*

Demonstration For let A measure B by C, the number C shall be even; and therefore shall have a half: Wherefore seeing that A measureth B by C, [a] C shall also measure B by A: and so C shall be a part of B, denominated of A, as is shewn in the third Definition of the Seventh Book, but seeing that as C is to his half, so is B to his half, and alternately, as C is to B, so the half of C; is to the half of B. But C is a part of B, denominated of A, as is shewn; also the half of C, shall be a part of the half of B, denominated of A: therefore [b] A shall measure the half of B: Therefore, if an, &c. Which was to be demonstrated.

a) 8. c. f.
b) 40. 7.

PROP. 31. THEOR. 29.

A 5 ..... B 8 .... C 16 .... D ---

If an odde number A, *be prime to some number* B, *it shall be prime to its double* C.

Demonstration For if A and C, are not primes to one another, they shall be measured by some number, which let be D, which of necessity shall be odde: For if it be even, measuring the odde number A: [a] A the product of D an even number, multiplyed by the number by which it measures it, shall be an even number, which is absurd; for A is proposed odde: Therefore D an odde number, measuring C an even number, (for C is an even number, seeing it hath an half B,) [b] shall also measure B his half: But it shall also measure A, therefore D shall measure A and B, primes to one another, which is absurd; Therefore, A is a prime to C: Therefore, if a number, &c. Which was to be demonstrated.

a) 28. 9.
b) 30. 9.

PROP. 32. THEOR. 30.

Of all the numbers B, C, D, *and* E, *which follow the binary* A, *in a double Progression, each of them is onely evenly even.*

Unity A 2 B 4 C 8 D 16 E 32

Demonstration That it is so, is manifest; For having proposed unity, seeing that A is a binary, and B, C, D, and E, doubles from the binary, they shall be continually proportional from unity; to wit, in double proportion: Therefore [a] A shall measure each of them, B, C, D, and E, and each lesser shall measure the greatest following, by some, one of these, A, B, C, D, and E, which being all even numbers, to wit, double from the Binary, an even number shall measure each of these B, C, D, and E, by an even number; therefore each of them, shall be evenly even, acccording to the Definition.

a) 11. 9.

Now that they are onely evenly even is manifest: For seeing that A, B, C, and D, are continually proportional from unity, and that A next to unity, is a prime number, to wit, a binary, [b] not any other number, shall measure each of them, except those A, B, C, D, and E, which being all even, an even number shall measure each of them only by an even number: Therefore, of all, &c. Which was to be demonstrated.

b) 13. 9.

PROP. 33. THEOR. 31.

A 10 .... B 5 ..... C 2 .. D---- E---

If a number A, *hath its half* B, *odde, the same* A, *is evenly odde.*

Demonstration Forasmuch as B an odde number, is the half of A, the binary C shall measure A an even number, by the same half B, an odde number; therefore [a] A is evenly odde: I say also, it is onely so: for let B be the half of A, and C a binary, if A be not evenly odde onely, it shall be also evenly even; therefore some even number shall measure it by an even number: Let D an even number, measure it by E, also an even number, and so [b] A shall be the product of D by E: but the same A is the product of C the binary, by B the half of A; therefore the number produced of C, the first, by B the fourth, is equal to that which is produced of D the second, by E the third: therefore [c] as C is to D, so E to B, but C the binary, measures D an even number; therefore E an even number, shall measure also B an odde number, which is absurd; therefore A is not evenly even, but is onely evenly odde. Therefore, if, &c. Which was to be demonstrated.

a) 9. d.
b) 9. c. f.
c) 19. 7.

PROP. 34. THEOR. 32.

A 20 ....

If an even number A, *be none of those that are double from unity, nor hath its half odde, it is evenly even, and evenly odde.*

Demonstration That it is evenly even, is evident: For seeing that its half is even, the binary which is even, shall measure it by the same half; therefore it shall be evenly even.

9. d.

It is also evenly odde, for the same A being divided into two equal parts, and its half again divided in two, and so on, you shall at last come to an odde number (before you come to the binary, seeing that it is none of those that are double from unity) which shall measure A by an even number. For if it should measure it by an odde number, (seeing [a] that an odde number multiplying

a) 29. 9.

plying an odde number, makes an odde number) A should be odde, which is impossible, seeing that it is proposed even: therefore seeing that an odde number measureth A by an even number, and that an even number doth also measure it by an even number, A shall be evenly even, and evenly odde. Therefore, if, &c. Which was to be demonstrated.

PROP. 35. THEOR. 33.

If there be as many numbers as you please, continually proportional, A, B C, D, *and* E F, *and if there be cut off a number* C G, *from the second* B C, *as also from the last* EF, *a number* H F, *equal to the first, as the excesse* B G, *of the second* B C, *shall be to the first* A, *so the excesse* E H, *of the last* E F, *shall be to all the antecedents* A, B C, *and* D.

A.........8

B....4 G........8 C

D..................18

E.........9 K......6 I....4 H........8 F

Demonstration. FOr let F I be cut off equal to B C, and F K equal to D; forasmuch as F I is equal to BC, and the part cut off F H, is equal to the part cut off C G, the rest I H shall be equal to the rest B G; but forasmuch as A is to B C, so B C is to D, and D to E F: and by conversion, as E F is to D, so D is to B, and B C to A: But K F is equal to D C, & I F, to B C, and H F to A: in like manner, as E F shall be to K F, so K F to I F, and I F to H F; therefore by dividing, as E K, to K F, so K I, to I F, and I H to H F: and [a] therefore also all the numbers E K, K I, and I H, shall be to all the numbers K F, I F, and H F, that is to say, the whole E H, to D, B C, and A, together, (to which K F, I F, and H F, have been taken equal) as I H, to H F; that is to say, as B G to A, for B G and A, are equal to I H and H F: Therefore, if, &c. Which was to be demonstrated.

a) 12. 7.

PROP. 36. THEOR. 34.

If from unity be taken as many numbers as you please, A, B, C, *and* D, *continually proportional in a double proportion, until the whole compound* E, *be made a prime number, and that whole* E *multiplyed by the last* D, *maketh some one* F, *the product* F, *shall be a perfect number.*

Demo.

Unity A..2 B....4 C....8 D..................16

E.31 G-----62 H---124 I---248 F----496

K 31 M31 L31

O---- P---- N 465

Demonstration. FOr, Let there be taken E, G, H, and I, in a double rate, from E, and equal in multitude to A, B, C, and D, forasmuch then as A, B, C, and D, and E, G, H, and I, are equal in multitude, and being taken two and two, are in the same rate, being all in a double proportion in equal rate, as [a] A shall be to D, so shall E be to I, and therefore, a) 14. 7.
[b] the product of A the first, by I, the fourth, shall be equal to the product b) 19. 7.
of D, the second, by E the third: But F is the product of D in E: therefore, F shall be also the product of A in I: therefore [c] I shall measure F by A c) 7. c. s.
the binary, wherefore F shall be double to I, and therefore E, G, H, I, and F, are continually proportional in a double rate.

Let there be cut off from G the second, and F the last, K and L, equal to E the first, whose excesses are M and N, as [d] M shall be to E, so N shall d) 35. 9.
be to all the antecedents E, G, H, and I, together: But M is equal to E, being the half of G double to E: therefore N shall be equal to E, G, H, and I, together. But L is equal to unity, and A, B, C, and D, together, being proposed equal to E or K, therefore the whole L N, that is to say, F is equal to unity, and to the numbers A, B, C, D, E, F, G, H, and I, together: wherefore [e] seeing that unity and each of the numbers A, B, C, D, E, F, G, H, and I, e) 7. c. s.
do measure F, (for seeing F is made of E in D, [f] D shall measure F, and f) 11. c. s.
therefore unity and A, B, and C, which measureth D, shall also measure F: and again, seeing that I, as hath been shewn, doth measure F, E, G, and H, which measureth I: [g] because of the double proportion shall also measure g) 11. c. s.
F,) and no other shall measure F, as we shall presently shew: Unity, and A, B, C, D, E, G, H, and I, shall be all the parts that F can have, to which F being shewn to be equal, [h] the same F shall be a perfect number.

Suppose some other number to measure F, besides the aforegoing; to wit, O by P, [i] that being, F shall be the product of O by P: but the same F is al- i) 9. c. s.
so the product of E by D, therefore the number produced of E the first, by D the fourth, is the same, with the product of P the second, by O the third, therefore [k] as E is to P, so O is to D. k) 19. 7.

But, [l] seeing that A, B, C, and D, are continually proportional from Unity, and that A next to unity, is a prime number, no other number shall measure D the last, besides those of the proportion A, B, and C. Now O is different from A B and C, by the Supposition; therefore O shall not measure D; but as O is to D, so E is to P, therefore also neither shall E measure P: therefore [m] E being a Prime: E and P, shall be Primes to one another; m) 31. 7.
and so [n] the least of their rate Wherefore E shall measure O, and P shall n) 23. 7.
measure D equally; being in the same rate: But, [o] no number doth mea- 21. 7.
sure D, except A, B, and C, therefore P shall be the same with one of o) 13. 9.
those A, B, and C.

Suppose it the same as B, and let E, G, and H, be taken double from E, in such multitude, as B, C, D, [p] in equal rate, as B to D, so E to H; [q] but p) 14. 7.
the product of B in H, is equal to the product of D in B, and the product of q) 19. 7.
D in E, hath been shewn equal to the product of P in O: therefore the num-

number made of P by O, is the same as that which is produced of B by H, therefore r as P is to B, so is H to O; but P was the same with B, therefore H
r) 19. 7. shall be the same with O, which is absurd; for O was put different from any of the numbers A, B, C, D, E, G, H, or I; Therefore, A, B, C, D, E, G, H, and I, do only measure the number F, the which they do compose: Therefore, if from Unity, &c. Which was to be demonstrated.

SCHOLIUM.

Euclide *in these three fore-going Books intends onely Arithmetical Numbers and not continued quantity in which there happens Fractions; but in numbers there happens none, but only in relation to the things numbered when they are divided into other parts, which are called Fractions, insomuch as they are compared to their whole divided into certain number of equal parts, the which are yet neverthelesse explained by whole numbers, that is to say, by the numbers of which* Euclide *speakes in these three fore-going Books; and then when such continued quantities are reduced to the same equal parts, they are no more Fractions, and numbers compared to such things, are compared the one to the other as whole numbers, and are taken absolutely, without referring them to any intire quantity, and all the demonstrations and operations, which may be performed by vulgar Fractions of common Arithmetick, are all performed by Arithmetical and whole numbers, concerning which* Euclide *speaks in these Seventh, Eighth, and Ninth Books.*

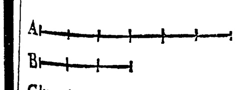
The End of the Ninth Element of EUCLIDE.

THE TENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

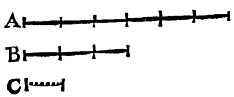
Euclide having finished those things which concern numbers in the three fore-going Books, so far forth as was needful for the understanding of Geometrical things: Now in this Tenth Book, he attempts the disputation of commensurable and incommensurable Lines, for whose cause, as we have said, those Books of numbers were undertaken by him. For without the knowledge of these Lines, divers Magnitudes, as well Solids, as Plaines, can neither be perfectly understood, nor brought into practice when it is required, because divers of their sides are incommensurable, which may be said likewise of Plains, and Solids themselves, for they also are often times incommensurable, as we shall demonstrate at the end of this Book.

DEFINITIONS.



I *Commensurable magnitudes* A *and* B, *are such as are measured by one and the same common measure* C.

Euclide, according to his cuſtom, begins here by the explication of the terms which he ought to uſe: and declareth in the firſt place what Magnitudes are ſaid to be commenſurable, and which incommenſurable, defining that thoſe are ſaid to be commenſurable, which are meaſured by one and the ſame common meaſure, or Magnitude: As the Magnitudes

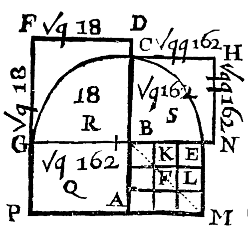


A and B; the which one and the ſame meaſure, as C doth meaſure: (for being repeated ſix times, it meaſureth A, and being repeated three times, it meaſureth B) ſhall be ſaid to be commenſurable.

After the ſame manner, a line of 20 palmes, and one of 13, are ſaid to be commenſurable; ſeeing that as well the line of one palme, as that of half a palme, &c. meaſureth both of them: In like manner, thoſe Superficies are ſaid to be commenſurable that are meaſured by one and the ſame Superficies, and ſolid bodies are alſo ſaid to be commenſurable; when they are meaſured by one and the ſame Solid.

2 *But Magnitudes incommenſurable, are thoſe which have no common meaſure.*

SUch Magnitudes are the Diameter of any ſquare, and the ſide of the ſame ſquare whereof it is the Diameter: foraſmuch as there is no common Magnitude that can meaſure them both, as ſhall be demonſtrated in the laſt Propoſition of this Book: There are alſo divers other lines incommenſurable, divers of which are unfolded in this Book; and the manner how to find them, is ſhewn: Again, Superficies and Solids, are ſaid to be incommenſurable, the which admit of no common meaſure.



3 *Right lines* AB, *and* BD, *are commenſurable in power, when their ſquares* BM, *and* BF, *are meaſured by one and the ſame Space or Superficies* E.

OF Right-lines, ſome are commenſurable in Length; to wit, thoſe which are meaſured by one and the ſame line, and are ſaid ſimply or abſolutely commenſurable, ſome others are ſaid to be incommenſurable; that is to ſay, ſuch as have no common meaſure, as hath been expounded in the firſt and ſecond Definitions.

Again, of Lines which are incommenſurable in Length, ſome of them are ſo, in ſuch manner as that their ſquares are commenſurable; as a line of two feet, and another, which ſhall be √ q. 8. (that is to ſay, of the which the ſquare ſhall contein 8 feet) are ſaid to be commenſurable in power, the ſquares 4, and 8, being meaſured by one and the ſame Superficies. So a line of three feet, and a line of 6 feet, ſhall be alſo commenſurable in power, ſeeing their ſquares 9, and 36, are meaſured by one and the ſame Superficies; to wit, by 3, or by 6.

Again,

Again, a line which ſhall be √qq of 20, and one that ſhall be √qq 125, are commenſurable in power: for their qq √ q 20, and √ q 125, are commenſurable, being as 5, to 2, and √ q 5, ſhall be in √ q 20, twice, and in √ q 125 5 times, which will be more eaſily underſtood by the figure, where the line AB is 3, and his q B M 9, whoſe Diameter is alſo B M, to which A B is incommenſurable, as hath been ſaid in the 2 Definition; and having drawn B D equal to the Diameter B M, and deſcribed the ſquare B F, the ſame ſhall be 18; it [a] being double the ſquare of B A, and his ſide ſhall be √ q 18 commenſurable in power to B A: for B meaſureth both the one and the other ſquares, to wit, B M 9 times, and B F 18 times, E tripled meaſureth alſo B F 6 times, and B M 3 times. a) 47. 1.

Now if you will have the ſquare E to be √ q 6, the q of the line A B, make √ q 486, and the q of B D, ſhall make √ q 1944, and √ q 6, ſhall meaſure both, & √ q 54, the triple of √ q 6, ſhall alſo meaſure thoſe two quantities.

4 *But Right Lines are incommenſurable in power, when the ſquares made of them cannot be meaſured by any common meaſure.*

THe other lines incommenſurable, whoſe ſquares are alſo incommenſurable, are ſaid to be incommenſurable in power, as a line of 2 feet, and a line which ſhall be √ √ 6, ſhall be incommenſurable in power, as not having any common ſpace that may meaſure their ſquares 4, and √ 6: So √ q 8, and √ qq 12, are alſo incommenſurable in power, foraſmuch as no Superficies can meaſure their Squares 8 and √ q 12.

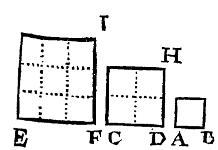
Again, as in the figure of the third Definition, the rectangled Superficies P B, is propoſed √ q 162, to the which [a] let the ſquare Superficies B H, be made equal, the line G B, or B D, his equal; ſhall be incommenſurable in power to B C, or B N; for B C is a mean proportional between G B, and B A, or B E, equal to B A: therefore G B, B C, and B E, being continually proportional, [b] G D the ſquare of G B, is to B H the ſquare of B C, as G B the firſt, to B E the third: But G B is incommenſurable to B E, as hath been ſaid in the ſecond Definition: therefore [c] the ſquare of G B, which is G D, ſhall be incommenſurable to the ſquare of B C; that is to ſay, to B H: therefore G B, and B C, or B N, his equal, are incommenſurable in power; that is to ſay, their ſquares are incommenſurable: and this may not be thought ſtrange, that this Demonſtration be built on the Propoſitions hereafter mentioned; for that it is not neceſſary for the Demonſtration of any of them.

a) 14. 1.

b) 20. 6.

c) 10.

Now all the right lines commenſurable in Length, are ſo alſo in power, ſeeing that the ſquare of their common meaſure, doth meaſure their ſquares, as appears by this figure: For as A B meaſureth C D, and E F, ſo the ſquare of A B, doth meaſure C H, and E I, not that all the right lines which are commenſurable in power, are ſo alſo in length, as appears by the figure of the third Definition, where the lines A B, and B D, are ſhewn to be incommenſurable in length, the which are ſhewn commenſurable in power, and as it ſhall be alſo demonſtrated in this Book.



5 *Theſe*

5 *These things being so, it is shewn, (that is to say, manifest,) that to every right line proposed, an infinite number of right lines are commensurable, and an infinite number are incommensurable, some in length and power, others onely in power; Now let that right line proposed, be called rational.*

IF there be proposed some right line, of a known, and determinated Magnitude, of all the lines compared with it, some shall be commensurable in length and power thereto, others in power onely, also some shall be incommensurable in length thereto, and some in length and power, as shall be shewn in this Book: But this line proposed, by reason of which, the others are said to be commensurable, or incommensurable, is named by the Greeks ῥητὴ, that is to say, rational; for that it is alwayes proposed of a known Magnitude, and that according thereto it is argued: But the others compared therewith, are not alwayes known: although according to the same lines taken apart, they may be known, each of them being divided into such equal parts as you please.

6 *And the right lines commensurable to this rational line, either in length and power, or in power alone, may be also called rationals.*

Euclide shews that the other lines commensurable, after what manner soever, to the line called *rational*, are also rationals, not by supposition, as the line it selfe: but as being compared therewith, they will be found commensurable thereto in length and power, or in power only.

7 *And the lines incommensurable to this rational line, may be called irrationals.*

SEeing that in the precedent Definition *Euclide* hath called the lines commensurable in length and power, or in power onely to the rational line proposed, Rationals, it is manifest by that Definition, that those lines which are incommensurable to the Rational proposed in length and power, and not in length onely, are Irrationals.

8 *And the square described of the rational line proposed, may be called rational.*

EVen as the line proposed to be known and determinated, is said to be rational: even so the square described thereof, is called rational, forasmuch as that also is known and certaine, and in comparison thereof, the other Superficies are said to be commensurables, or incommensurables.

9 *And the figures commensurable to this square rational, may be called rationals.*

ALL the Plain Superficies commensurable to the square of the rational proposed, are said to be rationals, yet not by position as the same, but as being compared to the same square, they are found to be commensurable thereto, or incommensurable, even so also as the lines in what manner soever commensurable to the line proposed rational, are called rationals.

10 *But the figures incommensurable to the rational square, may be called irrationals, or Surds.*

EVen so as the lines incommensurable to the line proposed rational, are said to be irrational, so also the plain Superficies incommensurable to the square of the rational line, are called irrationals.

11 *And the right lines which are in power as the same irrational figures, are said to be irrationals, and Surds; to wit, if they be squares, their sides may be irrational; but if they be other rectilines whatsoever, the lines which are in power, as the squares equal to those irrational figures may be irrational.*

HE calls the lines which are equal in power to the spaces, incommensurable to the square of the rational line, irrationals, in such sort, that if those spaces irrational are squares, their sides shall be said to be irrational, if they be not squares, but are other rectilines; the right lines which describe the squares, equal to these spaces incommensurable, shall be said to be irrational: for a right line is said to be equal in power to a figure, when the square described of the same line is equal to the same figure.

PETITIONS.

That any Magnitude whatsoever, may be multiplyed so many times, as that it may exceed any Magnitude whatsoever proposed of the same kind.

FOr two Magnitudes unequal, but of the same kind being proposed, the greatest being not infinite, and the least in power to be augmented infinitely, it is manifest, that the least may be multiplyed so many times, as that it may exceed the greatest, which appeares also by the fifth definition of the fifth Book.

COMMON SENTENCES.

1 *A magnitude measuring as many magnitudes as you please, doth also measure the magnitude compounded of them.*

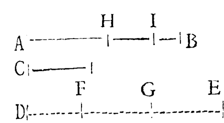
2 *A magnitude which measureth any magnitude whatsoever, doth in like manner, measure every other magnitude, which that magnitude doth measure.*

3 *A magnitude measuring a whole magnitude, and the part cut off there-from, doth also measure the rest.*

PROPOSITIONS,

PROBLEMES, & THEOREMES.

PROPOSITION 1. THEOREM 1.

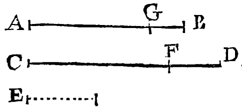


Two unequal magnitudes being proposed, A B, *and* C, *if from the greatest* A B; *there be cut off more then the halfe, and of the rest again, more than the halfe; and if that be done alwayes so, in continuing, there will remain in the end some magnitude lesse then the least magnitude* C *proposed.*

Demonstration. FOr let C be multiplyed so many times, as that the product may exceed A B, and let D E be the product, so as that D E may be multiplex of C, and next greater then A B; and let D E be divided according to the parts D F, F G, G E, each equal to C, and from A B, let there be cut off more then the half A H, and of the rest H B, also more then the half H I, and so alwayes until the parts of A B, may be equal in number to those of D E; Let the parts then A H, H I, I B, be as many in number, as D F, F G, G E; Forasmuch as D E is greater then A B, and that from D E there be taken D F, lesse then the halfe, (or the halfe, if D E were onely double to C,) and from A B more then the halfe A H, the rest F E shall be greater then the rest H B.

Again, F E being greater then H B, if from F E be taken the halfe F G, (or lesse then the halfe, if F E were greater then the double of C:) but from H B, more then the half H I, the rest G E shall be in like manner greater then the rest I B; But G E is equal to C, therefore C shall be also greater then I B, the last part of A B: Therefore, of A B there will remaine a magnitude lesse then C. The same will happen, if you take away the halfe of A H, and from the rest H B, the halfe H I, for in like manner, I B the rest shall be lesse than G E, that is to say, then C. Therefore, Two, &c. Which was to be demonstrated.

PROP. 2. THEOR. 2.



Two magnitudes unequal AB *and* CD, *being proposed, if you cut off still alternately, the least* AB, *from the greatest* CD, *and that the magnitude remaining, shall never measure his precedent magnitude, those magnitudes* AB *and* CD, *shall be incommensurable.*

Demonstration. FOr otherwise they should be measured by some common measure, which suppose to be E: Therefore, E shall be equal to AB, or lesse, and having taken AB from CD, as many times as may be, let there rest FD lesse then AB, in such sort as AB measures CF.

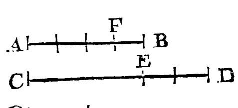
Again, having taken FD from AB, let there be left GB, lesse then the same FD, in such sort as that FD measure AG, and let this cutting off be still continued, till at last [a] there shall remaine of CD or AB some magnitude lesse then E. a) 1. 10.

Forasmuch as AB taken from CD leaves FD, lesse then it selfe, CF cut off shall be greater then the half CD: for if CF were the halfe, or lesse then the halfe of CD, AB might yet be cut off from CD: in like manner, AG, cut off from AB, shall be greater then the halfe of AB: and so it shall still happen, cutting off still therefore more then the halfe, there will remaine in the end some magnitude lesse then E. Let then GB the rest be lesse then E.

Forasmuch as E measures AB, and AB measures CF, [b] E shall also measure CF: but E measures the whole CD: therefore [c] E shall also measure the rest FD, and FD shall measure AG: therefore [d] E shall also measure AG: but E measures the whole AB; therefore [e] E shall also measure the rest GB the greatest, the least, which is absurd: Therefore no magnitude shall measure AB and CD; Therefore they are incommensurable: Therefore, If, &c. Which was to be demonstrated.

b) 2. c. s.
c) 3. c. s.
d) 2. c. s.
e) 3. c. s.

PROP. 3. PROBL. 1.



Two magnitudes commensurable AB, *and* CD, *being given, to find their greatest common measure.*

Construction. FOrasmuch as AB and CD are commensurable from the greatest CD: Let there be taken AB the least, as often as you can: and that there remaine ED, and from AB let there be taken ED also as often as you may, and there will rest FB, and doing so still

still alternatively, there will at last remaine a magnitude, which shall measure his precedent, otherwise AB and CD should be incommensurable, contrary to the Supposition. Let FB then measure ED: I say, that FB is the greatest common measure of AB and CD.

Demonstration. FOr seeing that FB measures ED, and ED measures AF [a] FB shall measure AF: but FB measures it selfe: therefore, it shall also measure the whole AB: therefore [b] FB shall also measure CE, which is measured of AB: therefore seeing that FB measures also ED, [c] the same FB shall measure the whole CD, it shall therefore measure AB, and CD therefore is their common measure.

a) 2. c. s.
1. c. s.
b) 2. c. s.
c) 1. c. s.

For if it be denied, suppose G to be the greatest common measure: Forasmuch then as G measures AB and CD, and that AB measures CE, [d] G shall also measure CE: but G measures CD, therefore [e] G shall also measure ED the rest; (seeing that G measures the whole CD, and the part cut off CE:) but ED shall measure AF: therefore [f] G shall also measure AF, and measuring the whole AB, it shall also measure the rest FB, lesse then it selfe, which is absurd; therefore FB is the greatest common measure of AB and CD.

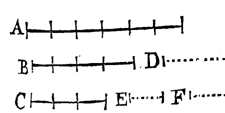
d) 2. c. s.
e) 3. c. s.
f) 2. c. s.
3. c. s.

Now if the least AB, measure the greatest, in such manner, as being cut off from CD, there shall remaine nothing, AB shall be the greatest common measure of the two, seeing that AB measures it selfe: Therefore, Two magnitudes, &c. Which was to be done.

COROLLARIE.

From this Demonstration it is manifest, that a magnitude which measureth two magnitudes, measureth also their greatest common measure, as may be gathered from that part of the Demonstration, by which it is shewn that FB *is the greatest common measure of* AB, *and* CD, *for there it is shewn that if* G *measure* AB *and* CD, *it shall also measure their greatest common measure* FB.

PROP. 4. PROBL. 2.



Three magnitudes commensurable A, B, *and* C, *being given, to find their greatest common measure.*

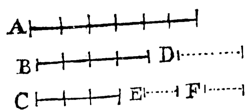
Construction. FInde [a] in the first place D, the greatest common measure of A and B, if D also measure C, it appears that D is the greatest common measure of the three A, B, and C. a) 3. 10.

Demonstration. IF a magnitude greater then D measure A, B, and C, the same [b] shall also measure D, the greatest common measure of b) Co. 3. 10.

of A and B, the greateſt, the leaſt, which is abſurd: but if D meaſure not C at leaſt, C and D ſhall be commenſurable.

For ſeeing that A, B, and C are commenſurable, any whatſoever, their common meaſure,[c] ſhall meaſure D the greateſt common meaſure of A and B: therefore the ſame common meaſure meaſuring alſo C, D and C ſhall be commenſurable: Let[d] E therefore be the greateſt common meaſure of C & D; I ſay, that E is the greateſt common meaſure of A, B, & C; Foraſmuch as E meaſures D and C, and D meaſures A and B, E[e] ſhall alſo meaſure A and B; But E alſo meaſures C: therefore E is the greateſt common meaſure of A, B, and C, I ſay, that E is alſo their greateſt common meaſure: For otherwiſe, ſuppoſe F a greater common meaſure then E.

c) Co. 3. 10.
d) 3. 10.
e) 2. c. f.



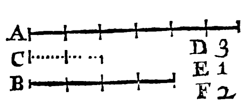
Foraſmuch then, as F meaſures A and B, it[f] ſhall alſo meaſure D their greateſt common meaſure: But F alſo meaſures C, it[g] ſhall then meaſure alſo E the greateſt common meaſure of C and D, the greateſt, the leaſt, which is abſurd: Therefore, Three, &c. Which was to be done.

f) Co. 3. 10.
g) Co. 3. 10.

COROLLARIE.

It is manifeſt alſo that a magnitude which meaſureth three magnitudes, doth alſo meaſure their greateſt common meaſure. For it hath been demonſtrated that ſeeing that F *meaſures* A, B, *and* C, *it ſhall alſo meaſure their greateſt common meaſure* E: *In like manner, if there be more then three magnitudes commenſurable given, their greateſt common meaſure may be found, and if there were four, the queſtion will be firſt of three, if of five, the queſtion will be of four, &c. and this ſame Corollarie ſhall take place.*

PROP. 5. THEOR. 3.



Magnitudes commenſurable A, *and* B, *have the ſame rate, the one to the other, as a number to a number.*

a) 3. 10.

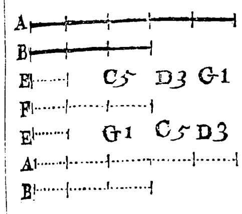
Demonſtration Let[a] C be their greateſt common meaſure, and as often times as C meaſures A, let unity E meaſure as often times a number, as D, and as often times as C meaſures B, let unity alſo meaſure the number F.

Foraſmuch then as the magnitude C, and unity E, meaſures the magnitude A, and the number D equally, the magnitude A ſhall contein C, as many times as the number D conteineth unity: Therefore, as A is to C, ſo the

the number D is to unity; But as C is to B, ſo unity is to F, (ſeeing that C meaſures B; and unity meaſures F equally,)[b] therefore in equal rate A ſhall be to B, as the number D, to the number F: Therefore, The, &c. Which was to be demonſtrated.

b) 22. 5.

PROP. 6. THEOR. 4.



If two magnitudes A, *and* B, *have the ſame rate to one another, as a number* G, *to a number* D; *they ſhall be commenſurable.*

Demonſtration FOr, Let the magnitude A be underſtood to be divided into as many equal parts as C conteins unites, and let one of the ſaid parts be equal to the magnitude E, and let there be another magnitude F meaſured as many times by E, as unity meaſures D.

Foraſmuch as E is as many times in A, as unity in the number C, the magnitude A, and the number C, ſhall contein E, and unity equally; therefore A ſhall be to E, as C to unity. But E is to F as unity to D, (E meaſuring F, and unity meaſuring D equally,) therefore[a] in equal rate, A ſhall be to F, as the number C, to the number D: but A is alſo to B, as the number C to the number D, by Suppoſition; therefore,[b] as A to B, ſo is A to F: therefore,[c] B and F are equal: therefore E meaſuring F ſhall alſo meaſure B; equal to F: but E ſhould alſo meaſure A: therefore E meaſures A and B; therefore[d] A and B are commenſurable: Therefore, If Two, &c. Which was to be demonſtrated.

a) 22. 5.
b) 11. 5.
c) 9. 5.
d) 1. d.

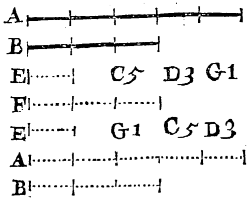
Otherwiſe, Let the magnitude A be ſuppoſed to be divided into as many equal parts as there are unites in the number C, to one of which parts let the magnitude E be equal.

Foraſmuch then, as E is to A, ſo is unity to the number C, (E meaſuring A, and unity the number C equally,) but A is propoſed to be to B, as the number C to the number D; therefore in equal rate, as E is to[e] B, ſo is unity to D: but unity meaſures the number D: Therefore E ſhall meaſure B, and E alſo meaſure A: therefore,[f] A and B, having A for common meaſure, ſhall be commenſurable, which is propoſed on the ſecond part of the figure.

e) 22. 5.
f) 1 def.

COROLLARIE.

From the firſt part of this Theorem it appears how you may finde a right line, to which any other right line propoſed, may be as a number to a number: For, (ſee the firſt figure,) let there be found a line, to which the propoſed line A *may be as the number* C *to the number* D; *divide* A *into as many equal parts*



parts as there are unites in C, *and take* F, *conteining as many of the ſame parts of* A *as there are unites in* D, *and ſo* A *ſhall be to* F, *as the number* C, *to the number* D, *as is demonſtrated.*

It alſo appeares by what meanes may be found a right line, to whoſe ſquare, the ſquare of another right line given may be, as a number to a number: As for example, let there be found a line, to whoſe ſquare, let the ſquare of the line A *be as the number* C, *to the number* D,; *in the firſt place, let* F *be found, to which let* A *be as* C *to* D, *as is ſaid, then* [a] *let there be found the mean proportional between* A *and* F, [b] A *the line found, and* F, *being continually proportional; the ſquare of* A *ſhall be to the ſquare of the mean, as* A *to* F: *But* A *is to* F, *as the number* C, *to the number* D: *therefore the ſquare of* A, *ſhall be to the ſquare of the mean proportional, as the number* C *to the number* D.

a) 13. 6.
b) Co. 20. 6.

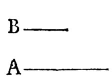
PROP. 7. THEOR. 5.

Incommenſurable magnitudes A, *and* B, *have not the ſame rate to one another, as a number to a number.*

Demonſtration For [a] if A and B, ſhould have the ſame rate to one another, as a number to a number, they ſhould be commenſurable, which is abſurd, being propoſed incommenſurable: therefore A and B are not to one another, as a number to a number: Therefore the Magnitudes, &c. Which was to be demonſtrated.

a) 6. 10.

PROP. 8. THEOR. 6.

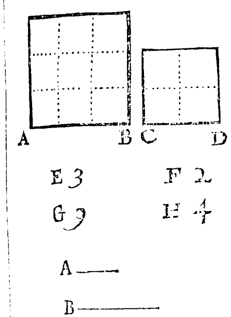


If two magnitudes A *and* B, *have not the ſame rate to one another, as a number to a number, thoſe magnitudes ſhall be incommenſurable.*

Demonſtration For if they were commenſurable, they [a] ſhould be to one another, as a number, to a number, which is abſurd, they being not to one another, as a number to a number, by Suppoſition: Therefore A and B, are not commenſurable: Therefore, If Two magnitudes, &c. Which was to be demonſtrated.

a) 5. 10.

PROP. 9. THEOR. 7.



The ſquares A B, *and* C D, *deſcribed on the right lines* A B, *and* C D, *commenſurable in length, have the ſame rate to one another, as a ſquare number to a ſquare number, and the ſquares which have the ſame rate to one another, as a ſquare number to a ſquare number, ſhall have alſo their ſides commenſurable in length. But the ſquares deſcribed of the right lines incommenſurable in length, have not the ſame rate to one another, as a ſquare number to a ſquare number: and the ſquares which have not the ſame rate to one another, as a ſquare number to a ſquare number, ſhall not have their ſides commenſurable in length.*

Demonſtration For A B and C D, being commenſurable in length, [a] A B ſhall be to C D, as a number to a number, which let be as the number E to the number F, and let the ſquares of E and F, be G and H. a) 5. 10.

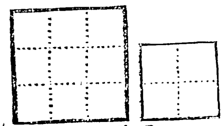
Therefore, ſeeing that A B. is to C D, as E to F: but [b] the ſquare of C D, in a double rate of the ſide A B to the ſide C D, [c] and the ſquare number G is to the ſquare number H, alſo in a double rate of the ſide E to the ſide F, the ſquare of A B ſhall be to the ſquare of C D, as the ſquare number G to the ſquare number H, foraſmuch as thoſe two rates are double to the equal rates, which is propoſed. b) 20. 6. c) 11. 8.

Secondly, Let the ſquare of A B be to the ſquare of C D, as the ſquare number G to the ſquare number H; I ſay, that A B and C D are commenſurable in length: for let E and F be the ſides of G and H.

Foraſmuch then, as the ſquare of A B, is to the ſquare of C D, as the ſquare number G, to the ſquare number H: but [d] the ſquare of A B is to the ſquare of C D, in a double rate of the ſide A B, to the ſide C D, and [e] the ſquare number G to the ſquare number H, alſo in double rate of the ſide E to the ſide F: Therefore A B ſhall be to C D, as E to F, their double rates being equal, therefore A B and C D, are commenſurable in length. d) 20. 6. e) 11. 8.

Thirdly,

Thirdly, Let A and B be incommensurable in length; I say, that their squares are not to one another, as a square number to a square number: For if the square of A were to the square of B, as a square number to a square number, A and B should be commensurable in length, as hath been shewn, which is contrary to the Supposition; therefore the the square of A is not to the square of B, as a square number to a square number, which is proposed.

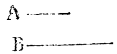


A B C D

E 3 F 2

G 9 H 4

Lastly, Let not the squares of A and B be to one another, as a square number; to a square number; I say, that A and B are incommensurable in length: For if it be not so, their squares are to one another, as a square number to a square number, as hath been shewn, which is contrary to Supposition: Therefore A and B are not commensurable in length, which is proposed: Therefore, The squares, &c. Which was to be demonstrated.



COROLLARIE.

From these things thus demonstrated, it is manifest, that right lines commensurable in length, are also commensurable in power: But that those which are commensurable in power, are not all so in length, and that those which are incommensurable in length are not all so in power: But those that are incommensurable in power, are all commensurable in length.

(a) 9. 10. *For the squares of lines* [a] *commensurable in length, being to one another, as a square number to a square number; that is to say,* (b) 6. 10. *as a number to a number simply;* [b] *they shall be commensurable,* (c) 5. d. *and* [c] *therefore their sides commensurable in power; therefore right lines commensurable in length, are also all of them commensurable in power.*

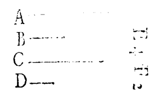
Again, seeing that lines whose squares are not to one another, as a square number to a square number, but simply, as (d) 5. d. *a number to a number,* [d] *are commensurable in power, (* [e] *their* (e) 9. 10. *squares being commensurable)* [f] *but not in length: It is mani-* (f) 9. 10. *fest that the lines that are commensurable in power, are not so alwayes in length, but onely the lines that are commensurable in power, whose squares are to one another as a square number to a square number are also commensurable in length, as appears by the second part of this Theorem.*

Again, forasmuch as those lines whose squares are not to one another, as a square number to a square number, but onely as

as a number to a number, are incommensurable in length, but commensurable in power, as hath been said; it appears that the lines incommensurable in length, are not all so in power, but onely the lines incommensurable in length, whose squares are not to one another, as a number to a number, are incommensurable in power, their squares being incommensurable.

Lastly, it is manifest that the lines incommensurable in power, are so also in length; for if they were commensurable in length, they should not be so in power, as is evident by the first part of this Corollarie; which is absurd, being proposed incommensurable in power.

PROP. 10. THEOR. 8.

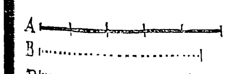


If four magnitudes A, B, C, *and* D, *be proportional, and that the first* A, *be commensurable to the second* B, *the third* C, *shall be also commensurable to the fourth* D, *and if the first* A, *be incommensurable to the second* B, *the third* C, *shall be also incommensurabe to the fourth* D.

Demonstration. Orasmuch as A and B, are commensurable; [a] they shall be to one another, as a number to a number: Let then A be to B, as the number E, to the number F: But as A is to B, so C is to D, by Supposition: Therefore C shall be also to D, as the number E, to the number F: therefore [b] C and D, are commensurable to one another. (a) 5. 10. (b) 6. 10.

Now let A be incommensurable to B; I say that, C is also incommensurable to D: For seeing that A and B are incommensurable: [c] A shall not be to B, as a number to a number, but as A is to B, so C is to D by Supposition: Therefore C shall not be to D, as a number to a number; [d] therefore C and D, are incommensurable to one another: Therefore, &c. Which was to be demonstrated. (c) 7. 10. (d) 8. 10.

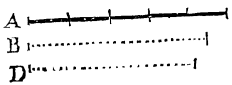
PROP. 11. PROBL. 4.



E 5 C 4

To find two right lines B *and* D, *incommensurable to a right line proposed* A; *to wit, the one* D *only in length, and the other* B *in length and power. That is to say, to the right line called Rational.*

Construction. LEt there be found two numbers E and C being not to one another, as a square number to a square number: then a finde the
a) Co.6.10. line D, to whose square let the square of A be as the number E, to the number C: I say, that D is incommensurable to A in length onely; For the square of A is to the square of D, as the number E to the number C: But E and C are not square numbers: Therefore the square of A shall not be to the square of D; as a square number to a square number; therefore, b A and D

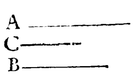


E 5 C 4

b) 9. 10. shall be incommensurable in length: and seeing that the squares of A and D are to one another, as a number to a number, they shall be commensurable:
c) 6. 10. therefore c A and D are commensurable in power, and therefore they are onely incommensurable in length.

Now to find a right line incommensurable in length and power to the
d) 13. 6. proposed line A: Let d B be found, a mean proportional between A and D; I say, that B is the line required: For A, B, and D, being continually pro-
e) Co.20.6. portional, e the square of A shall be to the square of B, as A is to D, but
f) 10.10. A is incommensurable in length to D; as is shewn: therefore f the square of A shall be incommensurable to the square of B; therefore A and B are
g) Co.9.10. incommensurable in power, by the Definition; and g by consequence also in length; wherefore we have found two right lines, &c. Which was to be done.

PROP. 12. THEOR. 9.



D 10 E 8
F 2 G 3
H 5 I 4 K 6

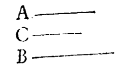
The magnitudes A *and* B, *commensurable to one and the same magnitude* C, *are also commensurable to one another.*

Demonstration. FOr seeing that A and C are commensurable, a A shall be to C, as a
a) 5. 10. number to a number, and let it be as the number D to the number E; and C and B being also commensurable, C shall be to B, as a number to a number.
b) 4. 8. Let it be then, as the number F is to the number G, and let b the three numbers H, I, and K, be found continually proportional, and the least in the rate of D to E, and of F to G; and H shall be to I, as D to E, and I to K, as F to G: that is to say, as C to B.

Therefore A being to C, as H to I, and C to B, as I to K: in equal rate, A shall be to B, as H to K: therefore A and B, shall be commensurable;
c) 6. 10. c being to one another, as the number H to the number K: Therefore, &c. Which was to be demonstrated.

PROP.

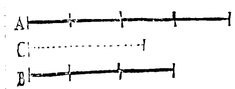
PROP. 13. THEOR. 10.



If there be two magnitudes A *and* B, *and that the one* A *be commensurable, and the other* B *incommensurable to one and the same magnitude* C, *those magnitudes shall be incommensurable to one another.*

Demonstration. FOr if B be said to be commensurable to A, C being proposed commensurable to the same A; B and C should be commensurable to one another, which is absurd, being proposed incommensurable to one another: Therefore A is not commensurable to B: 12. 10.
Therefore, If, &c. Which was to be demonstrated.

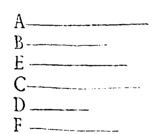
PROP. 14. THEOR. 11.



If there be two magnitudes commensurable A *and* B, *and that one of them* A, *be incommensurable to some magnitude* C, *the other* B *shall be also incommensurable to the same* C.

Demonstration. FOr if B be said to be commensurable to C, a A and C, should a) 12. 10.
be commensurable to one another, which is absurd, being proposed incommensurable: Therefore B and C are incommensurable to one another: Therefore, If there be, &c. Which was to be demonstrated.

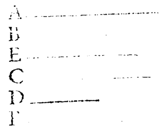
PROP. 15. THEOR. 12.



If four right lines A, B, C, *and* D, *are proportional, and the first* A *be more in power then the second* B, *by the square of a line* E, *which shall be commensurable in length thereto; the third* C *shall be also more in power then the fourth* D, *by the square of a line* F, *which shall be commensurable thereto in length; and if the first* A *be more in power then the second* B, *by the square of a line* E, *incommensurable in length thereto, the third* C *shall be also more in power then the fourth* D, *by the square of a line* F *incommensurable in length thereto.*

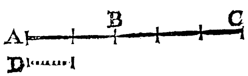
Demon-

a) 22. 6. *Demonstration.* For seeing that A is to B, as C to D: [a] also as the square of A shall be to the square of B, so the square of C to the square of D: But the square of A is equal to the squares of B and E, by Supposition, and the square of C, to the squares of D and F; therefore as the squares of B and E together, shall be to the square of B, so the squares of D and F to the square of D, and by dividing, as the square of E to the square of B, so the square of F to the square of D.



b) 22. 6. Therefore [b] as E is to B, so is F to D, and alternately, as B is to E, so D is to F; in equal rate as A shall be to E, so C shall be to F:
c) 10. 10. Therefore, [c] if A be commensurable in length to E, also C shall be commensurable in length to F, and if incommensurable, incommensurable: Therefore, if four lines, &c. Which was to be demonstrated.

PROP. 16. THEOR. 13.



If two commensurable magnitudes are compound, the whole A C *shall be also commensurable to each of them: And if the whole* A C *be commensurable to one of them, the magnitudes proposed at first,* A B, *and* B C, *shall be commensurable to one another.*

a) 3. 10.
b) 3. c. f. *Demonstration.* Let [a] D be the common measure of A B and B C: Forasmuch then as D measures A B and B C; it [b] shall also measure the whole A C: therefore seeing that D measures A C and A B, A C and A B shall be commensurable: In like manner, seeing that D measures A C and B C: A C and B C shall be also commensurable: Therefore A C shall be commensurable to each of them A B and B C.

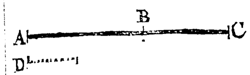
Now let the whole A C be commensurable to one of them A B or B C, of which it is compounded; to wit to A B: I say, that A B and B C are commensurable to one another.

c) 3. 10.
d) 3. c. f. For let [c] D be the common measure of A C and A B: Therefore D measuring the whole A C, and the part cut off A B: it [d] shall also measure the rest B C: Therefore A B and B C, are commensurrble to one another, D measuring both the one and the other: Therefore if two magnitudes, &c. Which was to be demonstrated.

COROLLARIE.

Hence it follows, that if a magnitude compounded of two, be commensurable to one of them, it shall be also commensurable to the other, as if A C *be commensurable to* A B, *it shall be also commensurable to* B C *the remainder, for as hath been shewn in the latter part of this Theorem:* D *common measure of the whole* A C, *and of* A B *the part cut off,* [c] *shall also measure the remainder* B C, *therefore* A C *and* B C *are commensurable.* c) 3. c. f.

PROP. 17. THEOR. 14.



If two magnitudes incommensurable A B *and* B C *are compounded, the whole* A C *shall be also incommensurable to each of them* A B *and* B C; *And if the whole* A C *be incommensurable to one of them, the magnitudes proposed at first,* A B *and* B C, *shall be incommensurable.*

Demonstration. For if it be denied, it should be commensurable at least to one of them: Suppose it then commensurable to A B, and [a] let D be their common measure: Therefore seeing that D measures the whole A C, and the part cut off A B; [b] it shall also measure the rest B C: Therefore A B and B C, shall be commensurable, having D for their common measure, which is absurd, for they are proposed incommensurable. a) 3. 10. b) 3. c. f.

Now suppose the whole A C to be incommensurable to one of them, to wit, to A B; I say, that A B and B C are incommensurable.

For if you say it is not so: [c] the whole A C shall be commensurable to each of them A B, and B C, which is absurd, and contrary to the Supposition: Therefore A B, and B C, are incommensurable, even so it might be demonstrated, that A C and A B, are incommensurable: Therefore, If, &c. Which was to be demonstrated. c) 16. 10.

COROLLARIE.

It follows from these Demonstrations, that if a magnitude compounded of two, be incommensurable to one of them, it is so also to the other, as if A C *be incommensurable to* A B, *it shall be so also to* B C: *For if* A C *were commensurable to* B C: [d] A C *should be also commensurable to the rest* A B, *which is absurd, for that they are proposed incommensurable: Therefore* A C *is not commensurable to* B C, *which is proposed.* d) C. 16. 10.

PROP. 18. THEOR. 15.

If there be two unequal right lines A B, *and* C, *and that to the greatest* A B, *be applyed a Parallellogram equal*

equal to the fourth part of the ſquare of the leaſt C, *wanting of* A B, *by a ſquare figure, & that it divide the ſame greater, in parts commenſurable in length, the greateſt* A B *ſhall be more in power then the leaſt* C, *by the ſquare of a line* F D, *which ſhall be commenſurable in length thereto; And if the greateſt* A B *be more in power then the leaſt* C, *by the ſquare of a line which ſhall be commenſurable in length thereto, and that there be applyed a Parallelogram upon the greateſt of them* A B, *equal to the fourth part of the ſquare of the leaſt* C, *wanting of* A B *by a ſquare figure; it ſhall divide the ſame in parts commenſurable in length.*

Demonſtration. FOraſmuch as A B is divided equally by E, and unequally by D, the Rectangle conteined under A D and B D, with the ſquare of E D, ſhall be equal to the ſquare of E B: And therefore four times the Rectangle under A D, and D B, and four times the ſquare of E D ſhall be equal to four times the ſquare of E B: But the ſquare of C is equal to four times the Rectangle under A D and D B, (the Rectangle under A D and D B, being propoſed equal to a quarter of the ſquare of C,) and the ſquare of F D is equal to four times the ſquare of E D, the halfe of F D, and the ſquare of A B equal to four times the ſquare of E B, the half of A B: Therefore the ſquares of C and F D, are equal to the ſquare of A B: Wherefore A B is more in power then C, by the ſquare of F D: I ſay alſo that F D is commenſurable in length to A B: For A D and D B being propoſed commenſurable in length,
a) 16. 10. [a] the whole A B ſhall be alſo commenſurable in length to his part D B, but the compounded of D B and A F, is alſo commenſurable in length to D B, (the compounded of A F and D B equals, being double to B D.) therefore A B, and the compounded of A F and D B, being commenſurable in length
b) 12. 10. each to D B, [b] A B and the compounded of A F and D B, ſhall be commenſurable in length to one another.

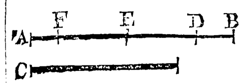
Therefore, A B compounded of A F and D B, as of one, and of F D, being commenſurable in length, to the compounded of A F and D B his
c) C. 16. 10. part, the ſame A B [c] ſhall be commenſurable in length to the remainder F D, therefore A B is more in power then C, by the ſquare of F D commenſurable in length to A B.

Now let A B be more in power then C by the ſquare of a line commenſurable in length thereto, and let there be applyed thereto a Parallelogram, as is ſaid before, making the ſegments A D and D B: I ſay, that A D and D B are commenſurable in length to one another: For the ſame conſtruction ſubſiſting, it may be demonſtrated in like manner, that A B is more in power then C, by the ſquare of F D: Wherefore A B being propoſed to be

be more in power then C, by the ſquare of a Right-line commenſurable in length thereto, A B ſhall be commenſurable in length to F D.

Therefore the whole A B, compounded of F D, and A F and D B as of one, being commenſurable in length to F D, the ſame A B [d] ſhall be alſo d) C. 16. 10.
commenſurable in length to the remainder, compounded of A F and D B; but the compounded of A F and D B, is alſo commenſurable in length to
D B, (being the double thereof,) [e] therefore A B and D B being commen- e) 12. 10.
ſurable in length to the compounded of A F and D B, they ſhall be commenſurable to one another: Therefore the whole A B compounded of A D and D B, being commenſurable in length to D B: A D and D B, ſhall be commenſurable in length to one another: Therefore, If there be, &c. Which was to be demonſtrated.

PROP. 19. THEOR. 16.

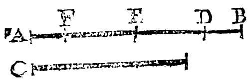


If there be two unequal right lines A B, *and* C, *and that to the greateſt* A B *there be applyed a Rectangle, equal to the quarter part of the ſquare of the leaſt* C, *wanting by a ſquare figure, and that it divide the ſame in parts incommenſurable in length* A D, *and* D B, *the greateſt* A B, *ſhall be more in power then the leaſt* C, *by the ſquare of a line, which ſhall be incommenſurable in length thereto: And if the greateſt* A B, *be more in power then the leaſt* C, *by the ſquare of a line incommenſurable in length thereto: And that there be applyed to the greateſt a Parallelogram equal to the quarter part of the ſquare of the leaſt, wanting a ſquare figure: It ſhall divide the ſame into parts incommenſurable in length.*

Demonſtration. FOr having made the Conſtruction as in the former, we ſhall ſhew as there is done, that A B is more in power then C by the ſquare of F D. And I ſay that F D is incommenſurable in length to
A B, for A D and D B, being propoſed incommenſurable in length, [a] the a) 17. 10.
whole A B ſhall be incommenſurable in length to his part D B: But D B is commenſurable in length to the compounded of A F and D B, the one being double to the other,) therefore ſeeing that of two lines commenſurable D B, and the compounded of A F and D B, the one, to wit, D B, is com-
menſurable in length to A B; [b] the remaining, compounded of A F and b) 14. 10.
D B, ſhall be alſo incommenſurable in length to A B, therefore A B compounded of A F and D B, as of one, and of F D, being incommenſura-
ble in length to the compounded of A F and D B, the ſame A B [c] ſhall c) C. 17. 10.
be alſo incommenſurable in length to the remainder F D: Therefore A B is more

more in power then C, by the square of F D, which is [illegible] in length [illegible].

Now let A B be more in power then C, by the square [illegible] shall be incommensurable [illegible] and let there be applyed to A B [illegible] Parallelogram equal to the [illegible] square of C, wanting a [illegible] figure, and making in A B, the parts A D and D B, I say, that A D and D B are incommensurable in length.



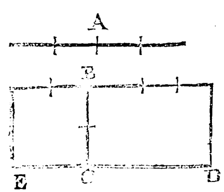
For [illegible] same construction, we shall shew, as before, that A B is more in power then C, by the square of F D, wherefore [illegible] A B is more in power then C, by the square of a line incommensurable in length thereto: A B shall be incommensurable in length to F D; [illegible] whole A B compounded of F D, and of A F and D B, [illegible] incommensurable in length to F D: [illegible] the same A B shall [illegible] incommensurable in length to the remainder, compounded of A F and D B: but [illegible] compounded of A F and D B, is commensurable in length to D B ([illegible] double to the other,) therefore seeing that the two lines [illegible] to wit, the compounded of A F and D B, and D B: the same compounded of A F and D B is incommensurable in length to A B,

d) C. 17. 10. [illegible]

e) 14. 10. the remainder D B shall be incommensurable in length to the same A B: Therefore the whole A B, compounded of A D and D B, being commensurable in length to D B: the same A D and D B, shall be incommensurable in length to one another: Therefore, If there be, &c. Which was to be demonstrated.

f) C. 17. 10.

PROP. 20. THEOR. 17.



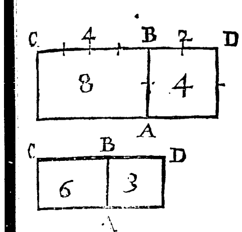
The Rectangle B D, *contained under two Rational Lines* B C *and* C D, *commensurable in length, according to some of the kinds aforesaid, is Rational.*

Demonstration. FOr of the one of them, as of B C: Let there be described the square B E: Forasmuch as B C is Rational, commensurable to the Rational proposed A, either in length and power, or in power onely, [a] his square B E shall be commensurable to the square of A: But the square of A is rational; [b] therefore B E commensurable thereto, shall be Rational: but B C, that is to say, E C and C D, being commensurable in length, (for they are proposed Rational, commensurable in length to one another,) and as [c] E C is to C D: so B E to B D: [d] B E and B D shall be also commensurable in length: wherefore B D commensurable to the space Rational B E, is also Rational: Therefore, &c. Which was to be demonstrated.

a) [illegible] d.
b) [illegible] d.
c) 1. 6.
d) 10. 10.

PROP.

PROP. 21. THEOR. 18.



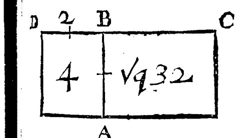
If a Rational Rectangle A C, *be applyed to a Rational line* A B, *it makes the breadth* B C *Rational, commensurable in length to the line* A B, *to which it is applyed.*

THat is to say, if a rational Rectangle have one side Rational, it will also have the other side rational, and those sides shall be commensurable in length to one another.

Demonstration. FOr on A B let there be described the square A D, [a] which shall be rational: But as the rectangle C A is to the square A D, so [b] C B is to B D, or to B A his equal. Now the rectangle A C and the square A D, are rational: and therefore commensurable: Therefore, [c] C B and B D, or B A, shall be commensurable in length: But A B is rational; therefore B C shall be also rational: Therefore, If, &c. Which was to be demonstrated.

a) 8. d.
b) 1. 6.
c) 10. 10.

PROP. 22. THEOR. 19.



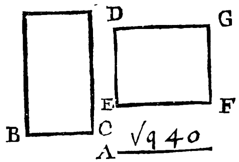
The Rectangle A C, *contained under two rational right lines* A B *and* B C, *commensurable in power onely, is irrational, and the right line equal in power to the said irrational, may be called Medial.*

Demonstration. FOr on A B rational, let be described the square A D, therefore [a] A D shall be rational; and forasmuch as A B, or D B, his equal, is incommensurable in length to B C; and that as D B is to B C, so [b] D A is to A C: [c] A D shall be also incommensurable to A C, but the square A D is rational: therefore [d] the Rectangle A C shall be irrational: Therefore the right line which is equal in power to the Rectangle A C that is to say, whose square is equal to A C is irrational: Let that line be called Medial, or mean proportional, between A B and B C: Forasmuch as his square [e] is equal to the Rectangle A C contained under A B and B C: Therefore the Rectangle, &c. Which was to be demonstrated.

a) 8. d.
b) 1. 6.
c) 10. 10.
d) 10. d.
e) 17. 6.

PROP.

PROP. 23. THEOR. 20.



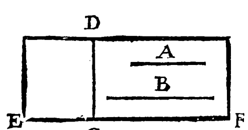
The square of a Medial Line A, *applyed to a rational Line* B C, *makes the breadth rational, and incommensurable in length to the Line to which it is applyed.*

a) 22. 10. *Demonstration* FOrasmuch as A is a Medial, it [a] is equal in power to a Re-
ctangle conteined under two rational lines commensurable
in power onely: the which let be E G, under E F and F G rationals: But
it is equal also in power to B D: therefore B D and E G are equal: and be-
b) 14. 6. ing [b] equi-angled, they will have their sides reciprocal about the equal an-
gles, to wit, as B C to E F, so F G to C D: Therefore as the square of B C
to the square of E F, so the square of F G to the square of C G, but
c) 22. 6. [c] the square of B C is commensurable to the square of E F; B C and
d) 10. 10. E F being rationals: therefore [d] the square of F G, is also commensura-
ble to the square of C D: therefore F G and C D are at least commen-
surable in power: But F G is proposed rational; therefore C D shall be
also rational.

I say also, that it is incommensurable in length to B C: Forasmuch as
E F and F G, are rational, commensurable in power onely, and as E F to
F G; so the square of E F to the Rectangle E G; [e] the square of E F shall
e) 10. 10. be incommensurable to the rectangle E G: and therefore also to the Rectan-
gle B D.

But the square of E F, is commensurable to the square of C D, (E F and
f) 13. 10. C D, being shewn rationals,) therefore [f] the square of C D, and the Re-
ctangle B D, are incommensurable: but the square of C D is to the Rectan-
gle B D, as C D to B C; therefore C D and B C are incommensurable in
g) 10. 10. length: therefore [g] C D is rational, commensurable in power onely to the
rational B C; that is to say, incommensurable in length: Which was to
be demonstrated.

PROP. 24. THEOR. 21.



The Right Line B *commensurable to a Medial Line* A, *is also Medial.*

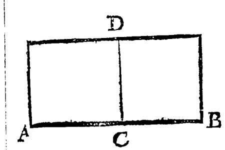
Demonstration FOr let C D a rational, be pro-
a) 45. 1. posed, to which let there [a]
be applyed the Rectangle D E equal to the
square of the Medial A, making the breadth
b) 13. 10. E C: Therefore [b] E C is rationa, l incommensurable in length to C D. Again,
to C D let there be applyed the Rectangle D F, equal to the square of B,
making the breadth C F: Then forasmuch as A and B are proposed com-
mensurable; their squares, that is to say, the Rectangles D E and D F,
equal

equal to them, shall be commensurable: But [c] as E D to D F, so the line c) 1. 6.
E C is to C F: Therefore [d] E C and C F are commensurable in length: But d) 10. 10.
E C is demonstrated rational, and incommensurable in length to C D;
[e] therefore C F is also rational, and incommensurable in length to C D: e) 14. 10.
Therefore D C, and C F, are rational, commensurable in power onely;
and [f] therefore the right line B, equal in power to the Rectangle D F, con- f) 22. 10.
teined under the same C D and C F, is Medial: Which was to be demon-
strated.

COROLLARIE.

From this Demonstration it is manifest that a space Medial, commensurable to a space Medial, is also Medial: For after it hath been demonstrated that D F *is commensurable to the medial Rectangle* D E, *from thence hath been shewn that* D F *is also a medial, conteined under the rationals* C D *and* C F, *commensurable in power onely.*

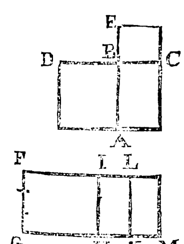
PROP. 25. THEOR. 22.



The Rectangle A D, *conteined under two medial right lines* A C *and* C D, *commensurable in length, is medial.*

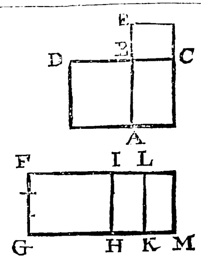
Demonstration FOr on the medial C D; let 1. 6.
there be described the square
B D, which shall be medial: And seeing that as A C is to C B; so A D
is to D B: But A C and C B, that is to say, A C and C D, are commen-
surable in length; A D and D B, shall be also commensurable: Therefore
the space A D, commensurable to the medial space D B, [a] is medial: There- a) C. 24. 10.
fore the Rectangle, &c. Which was to be demonstrated.

PROP. 26. THEOR. 23.



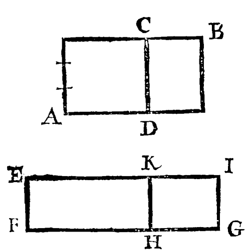
The Rectangle A C, *conteined under two medial right lines* A B *and* B C, *commensurable in power onely, is Rational, or medial.*

Demonstration FOr of A B and B C Medials,
let there be described the
squares A D and C E, which shall be Me-
dials; and let the rational F G, be exposed,
[a] to which let there be applyed the Rectan- a) 45. 1.
gles F H, H L, L M, equal to A D, A C, and C E: F M shall be a Rectan-
gle



gle alone; and A D and C E, being Medials,
their equal Rectangles F H, and L M, ſhall be
b) 34. 1. alſo Medials, which applyed to b F G & K L
c) 23. 10. rationals, (for K L is equal to F G,) & making
the breadths G H & K M, c the lines G H &
K M, ſhall be rational, incommenſurable in
length to F G, or K L; and foraſmuch as
A B, and B C, are commenſurable in power
to one another, their ſquares A D and
C E, and therefore their equal Rectangles
F H, and L M, ſhall be commenſurable:
d) 1. 6. But as d F H is to L M, ſo G H is to K M;
e) 10. 10. therefore e G H and K M, ſhall be commenſurable in length.
Therefore, ſeeing that it is ſhewn that G H and K M, are rationals, and
commenſurable in length to one another, and commenſurable in power
onely to the rational propoſed; being ſhewn incommenſurable in length
f) 20. 10. thereto; f the Rectangle under G H and K M, ſhall be rational. and g ſee-
g) 1. 6. ing that as D B is to B C, ſo A D is to A C, and as A B to B E, ſo A C to
C E: (D B and B C, being equal to A B and B E, each to his correſpon-
dent,) as A D ſhall be to A C, ſo A C to C E: therefore A D, A C, and
C E, ſhall be proportional: Wherefore F H, H L, and L M, their equal
h) 1. 6. Rectangles ſhall be continually proportional: But h G H, H K, and K M, are
in the ſame rate, as the Rectangles, F H, H L, and L M, they ſhall be then
i) 17. 6. proportional, i and the Rectangle conteined under G H and K M, is equal
to the ſquare of H K: but the Rectangle under G H and K M, is ſhewn ra-
tional, therefore the ſquare of H K, ſhall be alſo rational, therefore H K
k) 6. d. ſhall be rational: and k therefore commenſurable to F G, expoſed ratio-
nal, or to his equal H I, to wit, in length and power, or in power alone.
And if H K be commenſurable in length to H I; that is to ſay, to his equal
l) 20. 10. F G, l the Rectangle I K, or his equal, ſhall be rational, and if in power
m) 22. 10. onely, m the Rectangle I K ſhall be a Medial, or his equal A C: therefore the
Rectangle A C conteined under A B and A C Medials, commenſurable in
power onely, is rational or medial: Therefore, &c. Which was to be de-
monſtrated.

PROP. 27. THEOR. 24.

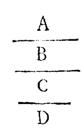


A ſpace or figure medial A B, *exceedeth not a figure medial* A C, *by a Rational figure* D B.

Demonſtration. FOr let D B be a rational (if
45. 1. poſſible.) and let the rational
E F be expoſed, on which let there be apply-
ed the Rectangle E G, equal to the Medial
A B, being equal to the Medials A B, A C, and
H I, rational, and the Rectangle E H equal
to the Medial A C, and ſo the reſt H I, ſhall
be equal to the rational D B, and E G, and E H, ſhall be Medials, being
equal

equal to the rational figure D B, and E G and E H being applyed on the
a) 23. 10. rational E F; they a ſhall have the other ſides F G, and F H rational, incom-
menſurable in length to E F.
b) 34. 1. Again, ſeeing that H I rational, is applyed to the rational E F: b that is to
c) 21. 10. ſay, to H K, his equal, the line c H G ſhall be rational, commenſurable in
length to the ſame E F, or H K; but F H is rational, incommenſurable in
e) 14. 10. length to E F: Therefore F H is incommenſurable in length to H G, e but
as F H is to H G, ſo the ſquare of F H is to the Rectangle conteined under
F H and H G; therefore the ſquare of F H is incommenſurable to the Re-
ctangle conteined under F H and H G: But the ſquares of F H and H G, are
commenſurable to one another: (For the one and the other, are deſcribed
f) 16. 10. on the rationals F H and H G;) and f therefore the two ſquares F H, and
H G, together, are commenſurable to the ſquare of F H; and twice the Re-
ctangle under F H, and H G, is commenſurable to the Rectangle F H, and
H G, the one being double to the other: Therefore the two ſquares of F H,
and H G, together, ſhall be incommenſurable to twice the Rectangle con-
teined under F H, and H G. Therefore the compounded of the ſquares
F G, and H G, and of the Rectangle twice under F H, and H G; that is to
g) 4. 2. ſay, the ſquare of F G, g equal to thoſe ſquares and Rectangle, h is incommen-
h) 17. 10. ſurable to the compounded of the ſquares F H, and H G, which is rational,
being commenſurable to the rational ſquare of F H.
Therefore ſeeing that the ſquare of F G is incommenſurable to the
i) 10 d. compounded of the two ſquares F H, and H G, which is rational, i it ſhall
k) 11. d. be alſo irrational, and k F G irrational, which is abſurd, for it hath been
ſhewn to be rational, commenſurable in length to E F: Therefore H I, or his
equal D B, is not rational: Therefore, a figure, &c. Which was to be de-
monſtrated.

PROP. 28. PROBL. 4.



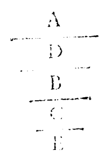
To find two medial lines A *and* B, *commenſurable in power onely, conteining a rational Rectangle.*

Conſtruction. LEt the two Rationals A, and B, be propo-
a) 13. 6. ſed, commenſurable in power onely; and a
let there be found C, a mean proportional between A and
b) 12. 6. B, and as A is to B, ſo let b C be to D: I ſay, that C and D are Medials; com-
menſurable in power onely, which do contein a rational Rectangle.

Demonſtration. FOr ſeeing that A and B are propoſed rational, commenſura-
c) 22. 10. ble in power onely; c the Rectangle conteined under A and B,
ſhall be irrational, called medial: But d C is equal in power to that Rectan-
d) 17. 6. gle, therefore it ſhall be medial: and ſeeing that as A is to B, ſo C is to D, and
e) 10. 10. A & B, are commenſurable in power onely, C and D e ſhall be alſo commen-
ſurable in power onely; and therefore D commenſurable to the medial C,
ſhall be alſo medial: Therefore we have found C and D medials, commen-
24. 10. ſurable in power onely: I ſay, they do contein a rational Rectangle: For
ſeeing that A is to B, as C to D; alternately, A ſhall be to C, as B to D:
But as A is to C, ſo is C to B: therefore C ſhall be alſo to B, as B to D:
Therefore B a mean proportional between C and D, is equal in power to
the

17. 6. the Rectangle conteined under C and D: But the ſquare of B a rational line, is rational: Therefore the Rectangle under C and D, ſhall be alſo rational: Therefore we have found C and D medials, commenſurable in power onely, which conteins a rational Rectangle: Which was to be done.

PROP. 29. PROBL. 5.



To find two medial lines commenſurable in power onely, which doe contein a medial Rectangle.

Conſtruction. LEt there be taken the three rational lines A, B, and C, commenſurable in power onely; and [a] between A and B, let there be found a mean proportional D; then let it be made as B is to C, ſo D to E: I ſay that D and E are medials, commenſurable in power onely, and do contein a medial Rectangle.

a) 12. 6. 12. 6.

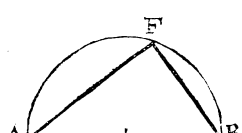
Demonſtration. FOr ſeeing that A and B are rationals, commenſurable in power onely; the Rectangle conteined under A and B, and therefore the ſquare of D equal [b] thereto, ſhall be irrational, called medial, and the line D equal in power to it medial: And ſeeing that B is to C as D to E, and B and C are rational, commenſurable in power onely, D and E ſhall be alſo commenſurable in power onely: Therefore ſeeing that D is a medial, [c] D and E commenſurable in power onely to D, ſhall be alſo medial: I ſay that D and E do contein a medial Rectangle.

b) 17. 6. 22. 10. c) 24. 10.

Foraſmuch as B is to C, as D to E, alternately, as B ſhall be to D: ſo C to E; But as B to D, ſo D to A: Therefore as D ſhall be to A, ſo C ſhall be to E: Therefore [d] the Rectangle conteined under D and E, is equal to the Rectangle conteined under A and C: But the Rectangle conteined under A and C rationals commenſurable in power onely, is irrational, and medial: therefore [e] the Rectangle under D and E, is alſo medial, therefore D and E are medials required: Which was to be done.

d) 16. 6. e) 22. 10.

PROP. 30. PROBL. 6.



C.... 4 E..... 5 D

To find two rational Lines commenſurable in power onely, in ſuch ſort as the greateſt may be more in power then the leaſt, by the ſquare of a right line commenſurable in length thereto.

Conſtruction. LEt the rational A B be expoſed, and let there be found two ſquare numbers C D and C E whoſe exceſſe E D may not be a ſquare; ſeeing [a] as the number C D, is to the number D E: ſo the ſquare of

a) Co. 6. 10.

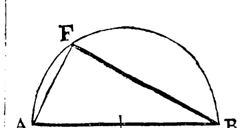
of A B, to the ſquare of ſome line A F: Let A F be fitted in the Semicircle deſcribed on A B, and let F B be joyned; foraſmuch as the angle F [b] is a right angle in the Semicircle, [c] the ſquare of A B ſhall be equal to the two ſquares of A F and F B; that is to ſay, that A B ſhall be more in power then A F, by the ſquare of F B: and the ſquare of A B being to the ſquare of A F as C D to D E, a number to a number; [d] the ſaid ſquares of A B and A F ſhall be commenſurable, and ſeeing that the ſquare of A B is rational, being deſcribed on the rational A B, the ſquare of A F ſhall be alſo rational: and therefore A F rational; therefore A B and A F are rational, and commenſurable at leaſt in power; and the ſquares of A B and A F, being not to one another, as a ſquare number to a ſquare number, the ſame A B and A F, ſhall be incommenſurable in length: But they are ſhewn to be commenſurable in power: Therefore A B and A F are rational, commenſurable in power onely.

b) 31. 3. c) 47. 1. d) 6. 10.

Now ſeeing that as C D is to D E, ſo the ſquare of A B, is to the ſquare of A F, by converſion of reaſon, as the ſquare number C D ſhall be to the ſquare number C E: ſo the ſquare of A B to the ſquare of B F: (For as C D exceedeth E D, by C E a ſquare number, ſo the ſquare A B exceeds the ſquare of A F, by the ſquare of F B:) Therefore the right lines A B and B F ſhall be commenſurable in length: therefore we have found A B and A F rationals, commenſurable in power onely, ſuch as A B the greateſt is more in power then A F by the ſquare of B F, commenſurable in length thereto: Which was to be done.

9. 10.

PROP. 31. PROBL. 7.



C 144 E 16 D C 160 D
C...... 6... E... 3 D

To find two rational lines commenſurable in power onely, in ſuch ſort as the greateſt be more in power then the leaſt, by the ſquare of a line incommenſurable in length thereto.

Conſtruction. LEt the rational A B be expoſed, and find the two ſquare numbers C E and E D, ſuch as that C D compounded of them be not a ſquare number: Or let C D a ſquare number be divided into two numbers not ſquare numbers, C E and E D, ſo the whole ſhall not be to any of them, as a ſquare number to a ſquare number; then [a] let there be found A F, to whoſe ſquare let the ſquare of A B be as C D is to D E, and let A F be fitted in the Semicircle deſcribed on A B, and let F B be joyned: Therefore [b] A B ſhall be more in power then A F, by the ſquare of F B and A B, A F ſhall be rational, commenſurable in power onely, as appears by the fore-going Demonſtration.

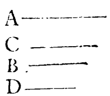
a) Co. 6. 10. b) 31. 3. 47. 1.

But foraſmuch as by converſion of reaſon, (as before) as C D is to C E: ſo the ſquare of A B is to the ſquare of B F: But C D and C E are not to one another, as a ſquare number to a ſquare number: in like manner, the ſquares of A B and B F, ſhall not be to one another, as a ſquare number to a ſquare number: Therefore [c] the lines A B and F B, are incommenſurable in length:

c) 9. 10.

length: Therefore we have found two rationall lines AB and AF, commensurable in power onely, such as AB the greatest, is more in power then AF the least, by the square of FB incommensurable in length thereto: Which was to be done

PROP. 32. PROBL. 8.



To find two medials commensurable in power onely, which contein a rational rectangle, in such manner, as the greatest may be more in power then the least, by the square of a line commensurable in length thereto.

a) 30. 10. *Construction* LEt [a] there be found A and [illegible]als, commensurable in pow-
er onely, such as A the grea[illegible]ay be more in power then B
b) 13. 6. the least, by the square of a line commensu[illegible]n length thereto; [b] and let
12. 6. C be a mean proportional between A and B, and as A is to B, so let C be to D.

Demonstration FOrasmuch as A and B are rationals, commensurable in power
onely: the [c] Rectangle conteined under A and B, is irratio-
c) 12. 10. nal, and the line C equal in power to the same medial; and A being to B
as C to D, and A and B commensurable in power onely: [d] C and D shall
d) 10. 10. be also commensurable in power onely; therefore [e] D commensurable to
e) 24. 10. the medial C, is a medial; therefore we have found two medials, C and
D commensurable in power onely: I say, that they comprehend a rational
rectangle.

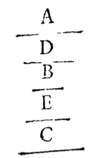
For seeing that A is to B, as C to D, also alternately A shall be to C, as
B to D: But as A is to C, so C is to B: C shall be in like manner to B, as
B to D: Therefore the rectangle under C and D, shall be equal to the
square of B: wherefore the square of B rational being rational; the rectan-
17. 6. gle under C and D, his equal shall be also rational: therefore C and D do
contein a rational rectangle.

But forasmuch as A is to B, as C to D; and A is more in power then B,
by the square of a line commensurable in length to the same; by the Con-
f) 15. 10. struction, [f] C shall be allo more in power then D, by the square of a line
commensurable thereto in length; therefore we have found two medials C
and D, commensurable in power onely, which do contein a rational
rectangle, and the greatest C is more in power then the least D, by the
square of a line commensurable in length thereto, which was to be done.

g) 31. 10. But if A and B [g] are found rational, commensurable in power onely; in such
sort as A may be more in power then B, by the square of a line incommensu-
rable in length thereto, and that the rest be done as before, it will be
shewn, in like manner, that C and D are medials, commensurable in pow-
er onely, which conteineth a rational rectangle: in such sort as C is more in
power then B, by the square of a line, which is incommensurable in length
thereto.

PROP.

PROP. 33. PROBL. 9.



To find two medial lines commensurable in power onely, the which doe contein a medial Rectangle: in such sort, as that the greatest may be more in power then the least, by the square of a line which shall be commensurable in length thereto.

Construction LEt there be found the three rationals A, B, and C, commensurable in power onely, in such sort, as A may be more in power then C, by the square of a line which shall be commensurable in length to it: Having found two rationals A and C, commensurable in power onely; in such sort as the greatest may be more in power then the least, by the square of a line which shall be commensurable thereto in length; let B be found commensurable in power onely, to the one and the other: then let D be taken a mean proportional between A and B, and as D is to B, so let C be to E.

Demonstration FOrasmuch as A is to C, as the Rectangle under A and B, is
to the Rectangle under B and C: but the square of D is
equal to the Rectangle under A and B: and the Rectangle under D and E,
is equal to the Rectangle under B and C: Seeing that D and B, C and E are
proportional; in like manner, as A is to C, so the square of D, shall be
to the Rectangle under D and E: But as the square of D is to the Rectangle
of D and E, so D is to E: Therefore as A is to C, so D is to E: but A and C
are commensurable in power onely: therefore D and E shall be so also: But 22. 10.
forasmuch as D is equal in power to the Rectangle of A and B rationals,
commensurable in power onely; is rational and medial, E commensurable 24. 10.
to D, shall be also medial: We have therefore found two medials D and
E, commensurable in power onely.

And forasmuch as the Rectangle under D and E is shewn to be equal to
the Rectangle of B and C medial; (seeing that B and C are rational, com- 22. 10.
mensurable in power onely,) the Rectangle under D and E shall be also
medial.

Lastly, seeing that we have shewn that as A is to C, so D is to E: But A
is more in power then C, by the square of a line commensurable in length 15. 10.
to it, by the Construction, D shall also be more in power then E by the square
of a line which shall be commensurable in length thereto.

We have therefore found two medials D and E, commensurable in power onely; which do contein a medial Rectangle; and the greatest D is more in power then the least E, by the square of a line which is commensurable in length thereto: Which ought to be done.

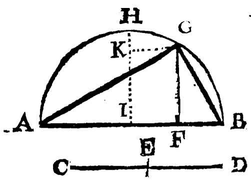
But if the Rationals before mentioned had been found, in such sort as A might be more in power then C, by the square of a line incommensurable in length thereto: the medial D would be more in power then E, by the square of a line incommensurable in length thereto; and having finished

the

the Construction, as before is said, it might be demonstrated in like manner, that D and E are found medials, commensurable in power onely, conteining a medial Rectangle, in such sort as the greatest may be more in power then the least, by the square of a line incommensurable in length thereto, &c.

PROP. 34. PROBL. 10.

To find two right lines incommensurable in power, which do make some compound of their rational, squares: But the Rectangle conteined under them a medial.



a) 31. 10. *Construction* Let [a] there be found two Rationals A B and C D, commensurable in power onely, such as the greatest A B may be more in power then the least C D; by the square of a line which shall be incommensurable in length thereto; and having divided C D in two equal parts in E: let there be applyed to A B a Rectangle equal to the square of C E, wanting by a square figure; and let it be that which is conteined under A F and F B, and having described the Semicircle A G B: let there be drawn F G at right angles, and let A G and G B, be joyned together.

Demonstration Forasmuch as A and B is more in power then C D by the square of a line incommensurable in length thereto; and that there is applyed to A B a Rectangle conteined under A F and F B, equal to the quarter part of the square of C D wanting by a square figure;
b) 19. 10. [b] A F shall be incommensurable in length to F B.

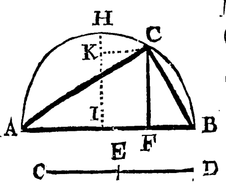
But as A F is to F B, so the Rectangle under A B and A F, to the Rectangle under A B and F B: but the Rectangle under A B and A F, is equal to the square of A G, and the Rectangle under A B and F B, equal to the
c) 31. 3. square of B G: seeing that the angle A G B, is right [c] being in the Semicircle: Therefore as A F to F B: so the square of A G to the square of G B: and
d) 10. 10. therefore seeing that A F and F B, are incommensurable in length, [d] the
e) 4. d. square of A G and G B shall be incommensurable: Therefore [e] the lines A G and G B, are incommensurable in power. And forasmuch as the square
f) 47. 1. of the rational A B is rational, and [f] equal to the squares of A G and G B, the compounded of the squares of A G and G B, is also rational.

Again, Forasmuch as the Rectangle under A F and F B, which is made
g) 17. 6. equal to the square of C E, [g] is equal to the square of F G: (Seeing that the angle A G B is right, and F G perpendicular to A B,) the squares of F G and C E, shall be equal; and therefore the lines F G and C E equal: wherefore C D the double of C E, shall be also double to F G: therefore the Rectangle under A B and C D, shall be double, the Rectangle under A B and F G: but the Rectangle conteined under the Rationals A B and C D, commensurable in power onely, is medial: therefore the Rectangle under

under A B and F G commensurable thereto, being its halfe, is [h] also medial: h) C. 24. 10.
But the Rectangle conteined under A G and G B is equal to the Rectangle conteined under A B and F G: Therefore the Rectangle conteined under A G and G B, is also medial, but the compounded of the squares of A G and G B, is shewn rational: Therefore we have found two right lines A G and G B, incommensurable in power, which make the compound of their squares Rational, and the Rectangle conteined under them medial: Which was to be done.

PROP. 35. PROBL. 11.

To find two right lines incommensurable in power, which do make the Compound of their squares medial, and the Rectangle conteined under them Rational.



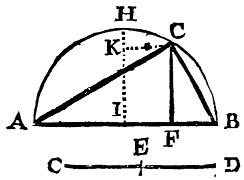
Construction Let [a] there be found two medials A B and C D, commensurable in power onely; conteining a Rectangle rational; in such sort as A B the a) 32. 10.
greatest, may be more in power then C D the lesser, by the square of a
line incommensurable thereto in length; and do for the rest as in the 34
Proposition is shewn, so as in the same 34 Prop. that A C and G B, are in-
commensurable in power, and the square of the medial A B, (equal [b] to b) 47. 1.
the squares of A G and G B,) being medial, the compounded of them, the
squares of A G and G B, shall be also medial.

Again, as in the 34 Proposition, the Rectangle under A B and C D is
shewn double, the Rectangle under A B and F G, and therefore com-
mensurable: Wherefore the Rectangle under A B and C D, being Ratio-
nal, by the Consttuction, the Rectangle under A B and F G, commensura-
ble thereto, shall be also rational: but the Rectangle under A G and G B,
is equal to the Rectangle uuder A B and F G: Therefore the Rectangle un-
der A G and G B, is also Rational.

Now the Compound of the squares of A G and G B, is shewn to be
medial; therefore A G and G B are found incommensurable in power, ma-
king the the Compound of their squares medial: But the Rectangle under
them Rational: Which was to be done.

 PROP.

PROP. 36. PROBL. 12.



To find two right lines incommensurable in power, making the Compound of their squares medial, and the Rectangle conteined under them medial, and incommensurable to the Compound of their squares.

a) 33. 10. *Construction* LEt [a] there be found A B and C D medials, commensurable in power onely; conteining a rectangle medial, in such sort as A B the greatest may be more in power then C D the least, by the square of a line incommensurable thereto in length: and having done the rest, as in the 34 Proposition: I say, that A G and G B are the lines required.

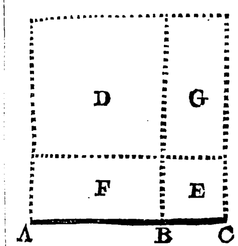
Demonstration FOr as in the 34 Proposition is shewn that A G, and G B, are incommensurable in power; and forasmuch as the square of b) 47. 1. A B medial is medial, (being [b] equal to the squares of A G and G B,) the Compound of the squares A G and G B, shall be also medial: Again, the Rectangle under A B and C D, is double the Rectangle under A B and F G, c) 34. 10. (as [c] is shewn,) and therefore commensurable: therefore by the Construction, the Rectangle under A B and C D, being medial, the Rectangle under A B and F G his equal; (that is to say, the Rectangle under A G and G B, his equal,) is also medial; but A B being proposed incommensurable in length to C D, and C E commensurable in length to d) 13. 10. the same C D, C D being double to C E: [d] A B and C E shall be incommensurable in length.

But as A B to C E, so the square of A B to the Rectangle under A B and C E: that is to say, to the Rectangle under A B and F G, that is to say, under A G and G B, (for the Rectangles under A B and F G, and under e) 10. 10. A G and G B, are equal:) Therefore [e] the square of A B is incommensurable to the Rectangle under A G and G B: Therefore A G, and G B, are found incommensurable in power, making the Compound of their squares medial, and the Rectangle under them medial, and incommensurable to the Compound of their squares: Which was to be done.

Here

Here begins the Senaries of Irrational Lines by Composition.

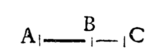
PROP. 37. THEOR. 25.



If two rational Lines A B, *and* B C, *commensurable in power onely, are compounded, the whole* A C *shall be irrational: and is called a Binomium, or a Binomial line.*

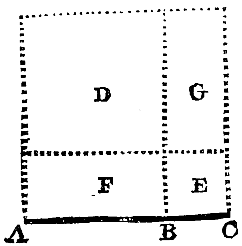
Demonstration FOr A B being incommensurable in length to B C; and as A B to BC, so the Rectangle under A B and B C to the square of BC; to wit [a] F to E, the Rectangle under AB and a) 16. 10. BC, shall be incommensurable to the square of BC. But twice the Rectangle under A B and B C is commensurable to the same Rectangle under A B and BC: to wit, F and G together, to G or F, and the square of A B, is commensurable to the square of B C; to wit, D to E, (being rationals,) wherefore [b] the Compound of the squares of A B and B C, is commensurable to b) 16. 10. the square of B C.

Therefore the Compound of twice the Rectangle under A B and B C, and the Compound of the squares of A B, and B C, are incommensurable; wherefore the square of A C, compounded of the squares of A B and B C, and of twice the Rectangle under A B and B C; (for [c] the square of A C c) 4. 2. is equal to the two squares of A B and B C, and to the two Rectangles under A B and B C, as appears,) is [d] incommensurable to the Compound of d) 17. 10. the squares of A B and B C: but the Compound of the squares of A B and B C, is rational, being shewn to be commensurable to the square of the Rational B C: therefore the square of A C incommensurable to the square of the Rational B C, is [e] irrational: Therefore A C shall be irrational: Now e) 10. d. let that line be called by two names, or as others expresse it *Binomial*: Forasmuch as it is compounded of two names, that is to say, of two rational lines A B and B C, commensurable in power onely: Therefore if two lines, &c. Which was to be demonstrated.



PROP.

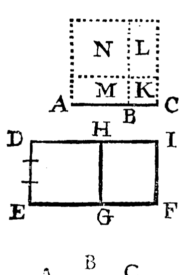
PROP. 38. THEOR. 26.



If two medial lines AB, *and* CD, *commensurable in power onely, conteining a rational Rectangle, are compounded, the whole* AC *shall be irrational: And is called a first Bimedial line.*

Demonstration. FOr AB being to BC, as the Rectangle conteined
a) 10. 10. under AB and BC, is to the square of BC, and [a] AB and BC incommen-
surable in length; the Rectangle under AB and BC shall be incommensu-
rable to the square of BC: to wit, as F to E: But twice the Rectangle un-
der AB and BC is commensurable to the Rectangle under AB and BC,
and the Compound of the squares of AB and BC is commensurable to the
square of BC, (for AB and BC, being commensurable in power, their
b) 16.10. squares shall be commensurable; and [b] therefore the Compound of them
shall be commensnrable to the square of BC:) Therefore twice the Re-
ctangle under AB and BC, and the Compound of the squares AB and
BC, are commensurable: Therefore the Compound of the squares of AB,
c) 4. 2. and BC and of twice the Rectangle under AB & BC, (which is [c] the square
d) 17. 10. of AC) is [d] incommensurable to the Compound of twice the Rectangle
under AB and BC: but the Rectangle under AB and BC is commensurable
to twice the Rectangle under AB and BC, being the half thereof: Therefore
e) 13. 1. [e] the square of AC is incommensurable to the Rectangle under AB and
BC: but the Rectangle under AB and BC, is proposed rational: there-
f) 10. d. fore the square of AC is irrational; and therefore [f] AC shall be irratio-
11. d. nal: Then let the line be called first of two medials, or a Prime *Bimedial*:
Therefore, &c. Which was to be demonstrated.

PROP. 39. THEOR. 27.



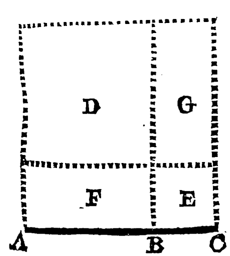
If two medial lines AB *and* BC, *commensurable in power onely, conteining a medial Rectangle, are compounded, the whole* AC *shall be irrational: And is called a second Bimedial line.*

Demonstration. FOr let the Rational DE be exposed, to which let there be
a) 45. 1. [a] applyed the Rectangle DF, equal to the square

square AC: and DG equal to the Compound of the squares of AB and BC; to wit NK, and the square of AC, (or DF, his equal Rectangle (being [b] equal to the two squares of AB and BC, and twice the Rectangle under AB and BC, the rest HF shall be equal to the Rectangle under AB and BC, twice, to wit, to ML: But the Rectangle under AB and BC, being proposed medial, the double of that which is commensurable thereto, to wit, [c] HF shall be also medial. b) 4. 2. c) C. 24. 10.

Again, seeing that the squares of AB and BC medials, commensurable in power, are commensurable; [d] their Compound, to wit, DG, shall be commensurable to each of them, and being medials, [e] DG shall be a medial; therefore seeing that the medials DG and HF, are applyed to the Rational DE; and HG [f] his equal, [g] the breadth EG and GF shall be Rational, incommensurable in length to DE: Again, AB and BC, being incommensurable in length, and as AB to BC, so the square of AB to the Rectangle under AB and BC: to wit, as N to L, the square of AB shall be incommensurable to the Rectangle under AB and BC: but the Compound of the squares of AB and BC is shewn to be commensurable to the square of AB, and twice the Rectangle under AB and BC, being the double thereof: Therefore the Compound of the squares of AB and BC, and twice the Rectangle under AB and BC, or their equal Rectangle DG and HF, are incommensurable: Wherefore [h] DG being to HF as EG to GF; [i] EG and GF shall be incommensurable in length, and being shewn Rationals, they shall be rational, commensurable in power onely; therefore [k] the whole EF shall be irrational: Wherefore the Rectangle DF, under DE Rational, and EF Irrational, shall be Irrational; and the square of AC equal to the same DF, is also Irrational: Wherefore AC is Irrational, which let be called second of two Medials, or a second Bimedial line: Therefore, If two, &c. Which was to be demonstrated. d) 16. 10. e) C. 24. 10. f) 34. 1. g) 23. 10. h) 1. 6. i) 10. 10. k) 17. 10.

PROP. 40. THEOR. 28.

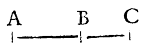


If two right lines AB *and* BC, *incommensurable in power, making the Compound of their Squares rational, and the Rectangle conteined under them Medial, are compounded, the whole shall be irrational: And is called the Greater, or Major.*

Demonstration. FOrasmuch as the Rectangle conteined under AB and BC, is proposed Medial, twice the Rectangle conteined under AB and BC, commensurable thereto, [a] shall be also Medial, and therefore Irrational: Now the Compound of the squares of AB and BC, is proposed rational: Therefore [b] twice the Rectangle under AB and BC, shall be incommensurable thereto: Therefore the square of AC compounded of the squares of AB and BC, and twice the Rectangle under AB and BC, shall a) C. 10. 10. b) 10. d. 17. 10.

ſhall be incommenſurable to the Compound of the ſquares of A B and B C: and this Compound of the ſquares of A B and B C being propoſed rational; [c] the ſquare of A C incommenſurable thereto, ſhall be Irrational, [d] and A C Irrational: Let this line be called Major: Therefore, If, &c. Which was to be demonſtrated.

PROP. 41. THEOR. 29.

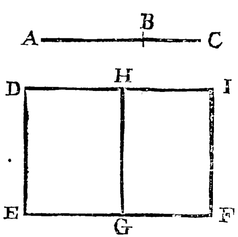


If two right lines A B *and* B C, *incommenſurable in power, are compounded, the which do make the Compound of their Squares medial; and the Rectangle conteined under them rational; the whole ſhall be Irrational: And is called a line equal in power to a rational, and a medial Superficies.*

Demonſtration. FOr the Compound of the ſquares of A B and B C, being medial: but twice the Rectangle under A B and B C rational, his half, to wit, the Rectangle once, under A B and B C, being propoſed rational, [a] the Compound of the ſquares of A B and B C, is incommenſurable to twice the Rectangle under A B and B C: therefore [b] the Compound of the ſquares of A B and B C, and twice the rectangle under A B and B C; that is to ſay, the ſquare of A C is incommenſurable to twice the rectangle under A B and B C, which being rational, as before ſaid, the ſquare of A C incommenſurable thereto, is Irrational, and therefore A C Irrational: And is equal in power to a rational and a medial: Therefore, &c. Which was to be demonſtrated.

a) 10. d.
b) 17. 10.
10. d.
11. d.

PROP. 42. THEOR. 30.



If two right lines A B *and* B C, *incommenſurable in power, are compounded, which make the Compound of their ſquares medial, and the Rectangle conteined under them medial, and incommenſurable to the Compound of their ſquares; the whole ſhall be irrational: And is called a line equal in power to two Medials.*

Demonſtration. FOr let the rational D E be expoſed, [a] to the which let there be applyed the rectangle D F, equal to the ſquare of A C, and D G equal to the Compound of the ſquares of A B and B C, and

a) 45. 1.

and the ſquare of A C [b] being equal to the Compound of the ſquares of A B and B C, and to twice the Rectangle under A B and B C, the reſt H F ſhall be equal to twice the reſt Rectangle under A B and B C.

b) 4. 2.

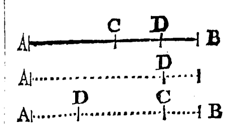
But ſeeing that as well the Compound of the ſquares of A B and B C; that is to ſay, the Rectangle D G, as the Rectangle under A B and B C: [c] therefore the Compound of twice the Rectangle under A B and B C: that is to ſay, H F (being commenſurable thereto,) is propoſed medial, D G and H F, applyed to the Rational D E, ſhall make the breadths E G and G F Rational, incommenſurable in length to D E.

c) C. 24. 10. 23. 10.

Again, the Compound of the ſquares of A B and B C; that is to ſay, the Rectangle D G being propoſed incommenſurable to the Rectangle under A B and B C, & twice the Rectangle under A B & B C; to wit H F, being commenſurable to the ſame Rectangle under A B and B C, (being double thereto,) [d] D G and H F ſhall be incommenſurable: Wherefore [e] E G being to G F, as D G to H F: [f] E G and G F ſhall be incommenſurable in length, and being ſhewn rational, the ſame E G and G F ſhall be rational, commenſurable in power onely: Therefore [g] the whole E F compounded of them is irrational: Therefore the Rectangle D F, conteined under D E rational, and E F irrational, is irrational; and therefore the ſquare of A C equal thereto, is alſo irrational, and the line A C equal in power to the ſame is irrational: And is called a line equal in power to two Medials: Therefore, If, &c. Which was to be demonſtrated.

d) 13. 10.
e) 1. 6.
f) 10. 10.
g) 37. 10.

PROP. 43. THEOR. 31.



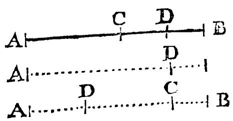
A Binomial line A B, *may be divided in its names onely in one point* C.

Conſtruction. LEt A B a *Binomial* be divided in the point C, in its names, in ſuch ſort as A C and C B may be rationals, commenſurable in power onely, according to the 37 Propoſition of the Tenth: I ſay the line A B cannot be divided into other names, at another point: that is to ſay, in other lines then A C and C B, which may be rational commenſurable in power onely.

Demonſtration. FOr (if poſſible) let it be againe divided in his names at D: It is manifeſt that A B is not divided at C in two equal parts, for A C and C B ſhould then be commenſurable in length, which is contrary to the Suppoſition, nor in like manner at D: Therefore A C is divided as well in C as in D unequally: Therefore A C and C B, are parts leſſe unequal then A D and D B, or more unequal: Wherefore the ſquares of A C and C B, ſhall be leſſe then the ſquares of A D and D B, or greater, (for the parts A D and D B, ſhall not be equal to the parts A C and C B, each to his correſpondent; to wit, the greateſt to the greateſt, and the leſſer to the leſſer, from what part ſoever the point D ſhall fall, (otherwiſe A B ſhould be divided at the ſecond Diviſion, at the ſame point as in the firſt.)

Put forasmuch as, if from equal things there be taken unequal; the excesse [a] of the remainders, is equal to the excesse of the parts cut off; but the squares of A C and C B, with twice the Rectangle under A C and C B, are equal to the squares of A D and D B, with twice the rectangle under A D and D B: [b] Seeing that as well the one as the other, are equal to the square of A B: It follows that the excesse of the Compound of the squares of A C and C B, and of the Compound of the squares of A D and D B, shall be equal, or the same with the Compound of twice the rectangle under A C and C B, and of twice the rectangle under A D and D B.

a) Cor. 2. 1.
b) 4. 2.

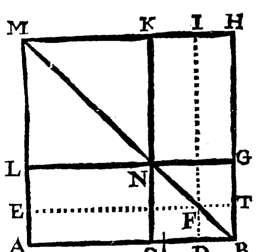


But the excesse of the Compounds of the squares of A C and C B, and A D and D B, is rational, (for A C and C B being rational, and therefore their squares rational, and their compound [c] being commensurable to each of them, shall be rational; by the same reason the compound of the squares of A D and D B shall be rational: therefore seeing that one rational exceeds another rational by a rational: It is manifest that the excesse of the compounds of the squares of A C and C B, and of A D and D B, is rational: Therefore the excesse of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, is also rational: But [d] the Rectangle under A C and C B being medial, twice the Rectangle under A C and C B commensurable thereto [e], is also medial: In like manner, twice the rectangle under AD & DB, is also medial: Wherefore [f] a medial that exceeds not a medial by a rational, the excesse of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, shall not be a space rational: But we have shewn that it is rational, which is absurd: Therefore A B cannot be divided in its names in any other point then C.: Therefore it is divided in its names onely in one point. Which was to be demonstrated.

c) 16. 10. 10. d.
d) 22. 10.
e) C. 24. 10.
f) 27. 10.

❡ This Demonstration is also easie to be understood by the Figure hereafter expressed.

PROP. 44. THEOR. 32.



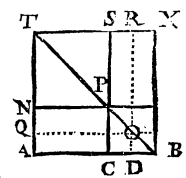
A first Bimedial line A B, *is divided in his names onely in one point* C.

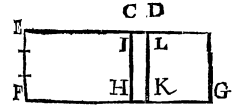
Demonstration FOr (if possible,) let the same be divided in D also, in such sort as A D and D B may be medials. commensurable in power onely, and conteining a rational Rectangle, from what part soever the point D shall fall, it will be shewn as in the precedent, that the excesse of twice the Rectangle under A C and C B, and twice the rectangle under A D and D B, is the same as the

the excesse of the Compounds of the squares of A C and C B, and of A D and D B: But the excesse of the said Rectangle is rational; (for the Rectangle under A C and C B, being rational, twice the Rectangle under A C and C B being commensurable thereto, is also rational, and in like manner, twice the Rectangle under A D and D B shall be rational, and a figure rational, exceeding a figure rational, by a figure rational; the excesse of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, is rational:) Therefore the excesse of the Compounds of the squares of A C and C B, and of the squares of A D and D B, is rational: A C C B being medials commensurable in power, their squares L K and C G shall be medials, and commensurable: Wherefore [a] their Compound shall be commensurable to each of them, and therefore medial; by the same reason, the Compound of the squares of A D and D B, which are E I and D I, is medial; and [b] a space medial not exceeding a medial by a rational; the excesse of the Compounds of the squares of A C and C B, and of A D and D B, shall not be rational, but we have shewn it rational; which is absurd: Therefore A B the first of two Medials, or a Prime Bimedial, cannot be divided in his names at any other point then C: And therefore is divided in his names onely in one point: Which was to be demonstrated.

a) 16. 10. C. 24. 10.
b) 27. 10.

PROP. 45. THEOR. 33.





A second Bimedial line A B, *is divided in his names, onely in one point* C.

Demonstration FOr if possible, let it be divided in D, in such sort as A D and D B, may be also medials commensurable in power onely, and conteining a medial rectangle: We shall shew as in the 43 Prop. that the squares of A C and C B, which are N S and C V, are greater or lesser then the squares of A D and D B, to wit, QR, and O B.

Let the rational E F be exposed, to which let there be [a] applyed the Rectangle E G equal to the square of A B, and E H, equal to the squares of A C and C B: Therefore the rest I G shall be equal to twice the Rectangle under A C and C B: Seeing [b] that the square of A B is equal to the said squares of A C and C B, and twice the Rectangle under A C and C B, as appeares by the figure.

a) 45. 1.
b) 4. 2.

In like manner, if to E F there be applyed E K, equal to the square of A D and D B, the rest L G, shall be equal to twice the Rectangle under A D and D B; and seeing that the squares of A C and C B, are unequal to the squares of A D and D B, the Rectangles E H and E K, which are equal unto them, shall be also unequal: and therefore F H and F K, shall be unequal.

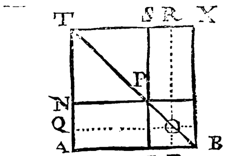
Again, the squares of A C and C B, being greater then twice the Rectangle conteined under A C and C B, E H shall be greater then I G, and therefore E H greater then the half of E G, and the line F H greater then the

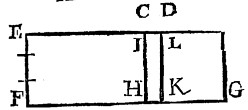
the halfe of F G, by the ſame reaſon, F K ſhall be ſhewn greater then the halfe of F G: Therefore the parts F H and H G, are unequal to the parts of F K and K G, each to his correſpondent: But foraſmuch as A C and C B, are medials, commenſurable in power, their ſquares are medials, and commenſurable: Therefore [c] the compound of them E H ſhall be commenſurable to every one of them; and [d] therefore medial: by the ſame reaſon, E K ſhall be medial; therefore [e] E H and E K applyed to the rational E F, make the breadths F H and F K rational, incommenſurable in length to E F.

c) 16.10.
d) C.24.10.
e) 23.10.

In like manner, ſeeing that the Rectangle under A C and C B is propoſed medial, twice the Rectangle under A C and C B commenſurable thereto; that is to ſay, [f] I G is alſo medial: Therefore being applyed to the rational H I, H G ſhall be rational, incommenſurable [g] in length to H I. In like manner, we ſhall ſhew L G to be a medial, and K G rational, incommenſurable in length to K L.

f) C.24.10.
g) 23.10.





Again, ſeeing that A C and C B are incommenſurable in length, and that as A C is to C B: ſo the ſquare of A C to the Rectangle under A C and C B, the ſquare of A C ſhall be incommenſurable to the Rectangles under A C and C B: But the Compound of the ſquares of A C and C B is commenſurable to the ſquare of A C: (for A C and C B being commenſurable in power; that is to ſay, their ſquares being commenſurable; the [h] Compound of them ſhall be commenſurable to each of them; to wit, to the ſquare of A C: But twice the Rectangle under A C and C B, is commenſurable to the Rectangle under A C and C B: Therefore the Compound of the ſquares of A C and C B; that is to ſay, the Rectangle E H is incommenſurable to twice the Rectangle under A C and C B, that is to ſay, to I G.

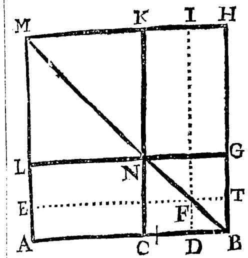
10. 10.
h) 16. 10.

Therefore ſeeing that E H [i] is to I G, as F H to H G: [k] F H and H G ſhall be incommenſurable in length: But they are ſhewn rationals, and therefore rationals commenſurable in power onely: [l] Therefore the whole F G is irrational, which is called by two names, and divided in his names in the point H: In like manner, we ſhall alſo demonſtrate that F G of two names, is divided into other names at another point K, [m] which is impoſſible: Therefore A B a Second Bimedial, is not divided in his names in any other point then C: And therefore is divided in his names at a point onely. Which was to be demonſtrated.

i) 1.6.
k) 10. 10.
l) 37. 10.
m) 43.10.

PROP.

PROP. 46. THEOR. 36.

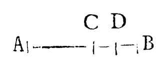


A major Line A B is divided in his names in one onely point C.

Demonſtration. For (if poſſible) let it be divided into other names qy D, [a] as it hath been demonſtrated, the exceſſe of the compound of the ſquares of A D and D B, is the ſame as that of twice Rectangle under A D and D B, but the exceſſe of the compounds of the ſaid ſquares is rational: (for ſeeing both the one and the other compound is propoſed rational, their exceſſe ſhall be alſo rational;) therefore the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B is rational: But foraſmuch as the Rectangle under A C and C B, is propoſed medial, twice the Rectangle of [b] A C and C B is alſo medial, being commenſurable: In like manner, the Rectangle under A D and D B twice, is medial: [c] Therefore ſeeing that a medial exceedeth not a medial by a rational, the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B ſhall not be rational: But we have ſhewn that it is alſo rational, which is abſurd: Therefore A B cannot be divided in his names in any other point then C: Therefore it is divided in his names in one onely point. Which was to be demonſtrated.

a) 43. 10.
b) C. 24.10.
c) 27. 10.

PROP. 47. THEOR. 35.



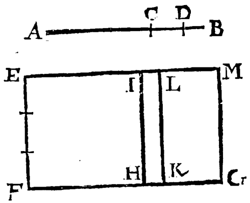
The line equal in power to a Rational and a Medial is divided its names onely in a point C.

Demonſtration. For, if poſſible, Let it be divided into other names at D, as we have ſhewn in the 43 Propoſition, that the exceſſe of twice the Rectangle under A C and C B, and of twice the Rectangle under A D and D B, is the ſame as the exceſſe of the compound of the ſquares of A C and C B, and the compound of the ſquares of A D and D B: but the exceſſe of the ſaid Rectangles is rational, [a] as hath been ſhewn: Therefore the exceſſe of the compound of the ſquares of A C and C B, and of A D and D B, is alſo rational: But the compounds of the ſaid ſquares are propoſed Medial; [b] therefore their exceſſe ſhall not be rational: But it is ſhewn rational, which is abſurd; Therefore A B is not divided in his names in any other point then C: And therefore onely in one point: Which was to be demonſtrated.

a) 44. 10.
b) 27. 10.

PROP.

PROP. 48. THEOR. 36.



The line A B *equal in power to two medials, is divided in his names in one point onely* C.

Demonstration. For, if possible, Let it be divided into other names by D; using the same Construction as in the 45 Proposition, we shall shew here, (as there) that the parts F H and H G, are unequal to the parts F K and K G, each to his correspondent: Forasmuch then as the compound of the squares of A C and C B is proposed Medial, E H his equal is also medial: and seeing that the Rectangle under A C and C B, is proposed medial also, I G equal
a) C.24.10. to the double thereof, [a] is medial, being commensurable: Therefore E H and I G medials, applyed to the rational E F, make F H and H G, rationals, incommensurable in length to E F; (for H I is equal to E F,) so we shall prove that F K, and K G, are rational, incommensurable in length to E F, but seeing that the compound of the squares of A C and C B, that is to say, E H is incommensurable to twice the rectangle under
b) 1.6. A C and C B, by the Supposition; that is to say, I G: and as [b] E H to I G,
c) 10.10. so F H to H G, F H and H G are [c] incommensurable in length: But they are shewn rational; therefore F H and H G are rational, commensurable in power onely: Therefore the whole F G is rational, called by two names; and divided in his names in H, but we shall shew that F G is also divided in other names at another point K, which is demonstrated to be impossible:
d) 43.10. [d] Therefore A B equal in power to two Medials, is not divided in his names at any other point then C: And therefore in one onely point: Which was to be demonstrated.

SECOND DEFINITIONS.

A Rational line being exposed, and a line of two names, (which is called Binomial,) divided in his names, of which the greatest name is more in power than the least, by the square of a right line which is commensurable in length thereto.

1 *If the greatest name be commensurable in length to the Rational exposed: the whole shall be called first of two names,* or A Prime Binomial.

2 *But if the least name be commensurable in length to the Rational exposed: Let it be called the second of two names, or* A Second Binomial.

3 *That if neither the one or the other of those names be commensurable in length to the Rational exposed: Let it be called the third of two names, or* A Third Binomial.

Again if the greatest name be more in power then the least, by the square of a right line which may be incommensurable in length thereto.

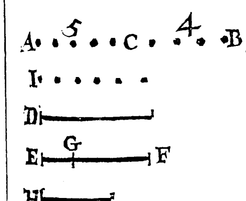
4 *If the greatest name be commensurable in length to the Rational exposed: Let it be called the fourth of two names, or* A Fourth Binomial.

5 *But if the lesser name be commensurable in length to the Rational exposed: Let it be called the fifth of two names, or* A Fifth Binomial.

6 *That if neither of those names be commensurable in length the Rational exposed: Let it be called the Sixth of two names, or* A Sixth Binomial.

PROP. 49. PROBL. 13.

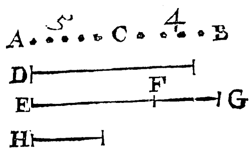
To find out a first Binomial line.

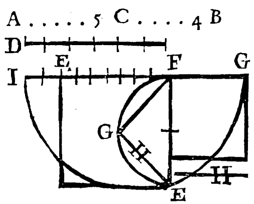


Construction. Having found two square numbers A B and C B, whose excesse A C may be no square number, in such sort as A B and C B may be to one another as a square number to a square number; but A B and A C may not be to one another as a square number to a square number; and let there be exposed a Rational line D, to the which let there be taken E F, commensurable in length; E F commensurable to
a) Co.6.10. D rational, shall be also [a] rational: then let there be drawn F G, to whose square

square let the square of E F be as the number A B to the number A C, (which will be easily done, by dividing E F into as many equal parts as there are unites in A B;) and taking F I of five such parts, and F G a mean proportional between them, as appeares by the Figure; I say, that E G the total, is a first Binomial.

Demonstration FOr seeing that the squares of E F and F G being to one another as the numbers A B and A C,[b] are commensurable at least in
b) 6. 10. power: But E F is shewn rational: Therefore F G is rational also: and A B being not to A C, as a square number to a square number, the squares
9. 10. of E F and F G shall not be to one another as a square number to a square number: Therefore E F and F G are incommensurable in length; and therefore Rational, commensurable in power onely; and the whole E G is irrational, called a Binomial: I say that it is also first, for the square of E F being to the square of F G as A B to A C, a number to a number; and A B being greater then A C, the square of E F shall be greater then the square of F G: Let it then be greater by the square of H: and as appeares by the Figure. Forasmuch as the number A B is to the number A C, as the square of E F to the square of F G; by conversion of reason, as A B shall be to C B; (to wit, to the excesse, therefore A B the antecedent, exceeds A C the consequent:) so the square of E F is to the square of H; to wit, to the excesse; therefore the antecedent square of E F exceeds the consequent square of F G: But A B and C B are square numbers; therefore the squares of E F and H, are to one another as a square number to a square number therefore E F and H are commensurable in length: therefore seeing that the greatest number E F is more in power then the least F G, by the square of H, commensurable in length thereto: and the same E F is also commensurable in length to D a rational exposed: E G shall be first of two names, or the first Binomial, according to the Definition: Therefore we have found, &c. Which was to be done.





PROP. 50. PROBL. 14.

To find out a second Binomial line.

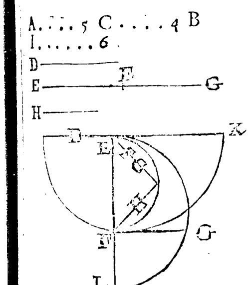
Construction HAving found two square numbers A B and C B, as in the fore-going Proposition, & the rational D being exposed; let F G be taken commensurable in length thereto; F G shall be then also rational:
a) C. 6. 10. and as the number A C is to the number A B: So [a] let the square of F G be to the square of F; which will be easie, as in the precedent Proposition, in taking a line as F I, of nine such parts as F G is five thereof, and F E a mean proportional between them, as appeares by the Figure; I say that E G is a second Binomial.

Demon-

Demonstration FOr the squares of F G and E F being to one another as the numbers A C and A B; [b] they shall be commensurable;
and the lines F G and E F, commensurable at least in power, and F G is shewn rational: therefore E F is also rational, But seeing that neither A C b) 6. 10.
nor A B are to one another as a square number to a square number, the squares of F G and E F also shall not be as a square number to a square number; and [c] therefore E F and F G incommensurable in length, and so c) 9. 10. 37. 10.
are rationals commensurable in power onely; and the whole E G irrational, which is called a Binomial: I say also that it is the second Binomial.

For seeing that the square of F G is to the square of E F, as the number A C to the number A B, and by conversion of reason, the square of E F to the square of F G as A B to A C: and as A B is greater then A C, the square of E F shall be greater then the square of F G: Let it then be greater by the square of H, we shall shew as in the fore-going that H is commensurable in length to E F: Now seeing that the greatest name E F is more in power then the least F G, by the square of H commensurable in length thereto, and the least name F G is commensurable in length to the Rational exposed D, E G by the Definition, shall be the second Binomial: Therefore we have found, &c. Which was to be done.

PROP. 51. PROBL. 15.



To find a third line of two names, or a third Binomial.

Construction HAving taken two numbers A B and C B, as in the 49 Proposition: Let there be taken another number I, which may not be to A B or to A C, as a square number to a square number: which may be done in taking I, the number not square, the next greater then A C: that being done, it shall not be to A B, as a square number to a square number.

Again, there being no neer greater then A C, it shall differ from A C by one or two: Therefore between I and A C there will not fall a mean proportional; therefore they shall not be like Plaines, nor shall be to one another as a square number to a square number: Now let the Rational D be
proposed, and as I is to A B, so the square of D may [a] be to the square of E F, a) Co 6. 10.
which will also be easie in taking E K, of nine such parts as D is six of them, and E F a mean proportional between the two D and E K: therfore the squares of D and E F shall be commensurable, and D and E F commensurable at least in power, and D being a Rational, E F shall be so also; and
forasmuch as I is not to A B; that is to say, the square of D is not to the square of E F as a square number to a square number [b] D, and E F shall be b) Co. 9. 10.
incommensurable in length.

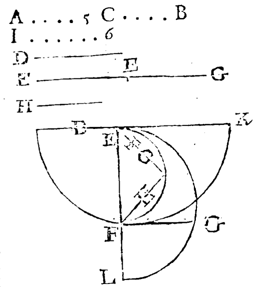
Again, let F G, to whose square let the square of E F be as A B to A C;
[c] which will be done by dividing E F in nine equal parts, and making F L of c) 6. 10.

five

five ſuch parts; then taking F G a mean proportional between E F and F L, as appeares by the figure, thoſe ſquares ſhall be commenſurable, and E F and F G, at leaſt commenſurable in power: Therefore E F being ſhewn rational, F G ſhall be alſo rational; and foraſmuch as A B is not to A C; that is to ſay, the ſquare of E F is not to the ſquare of F G as a ſquare number to a ſquare number, E F and F G [d] ſhall be incommenſurable in length: Therefore E F and F G are rational, commenſurable in power onely: Therefore [e] the whole E G is irrational, called a Binomial: I ſay, that it is the third.

d) 9.10.

e) 37.10.

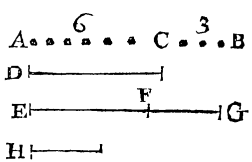


Demonſtration FOr ſeeing that as I to A B ſo the ſquare of D to the ſquare of E F, and as A B to A C; ſo the ſquare of E F to the ſquare of F G, in equal reaſon, as I is to A C, ſo the ſquare of D to the ſquare of F G: But I and A C are not to one another as a ſquare number to a ſquare number; therefore the ſquares of D and F G alſo, ſhall not be as a ſquare number to a ſquare number; therefore [f] D and F G are incommenſurable in length: But foraſmuch as A B is to A C as the ſquare of E F is to the ſquare of F G: But A B is greater then A C, therefore the ſquare of E F ſhall be greater then the ſquare of F G: Let it then be greater by the ſquare of H, we ſhall ſhew, as in the 49 Propoſition; that H is commenſurable in length to E F: Foraſmuch then as the greateſt name is more in power then the leaſt F G, by the ſquare of H, commenſurable in length thereto; and that neither E F nor F G is commenſurable in length to the Rational propoſed D, as is demonſtrated: E G by the Definition, ſhall be the third of two names: Therefore we have found, &c. Which was to be done.

f) 9.10.

PROP. 52. PROBL. 16.

To find a line fourth of two names, or a fourth Binomial.



Conſtruction HAving found two numbers A C and C B, ſuch as the Compound A B be not to either of them as a ſquare number to a ſquare number: Let the rational D be propoſed, and E F taken commenſurable in length thereto; therefore the ſame E F ſhall be alſo Rational; and having made the reſt as in the 49 Propoſition; we ſhall ſhew as there that the whole E G is a Binomial: I ſay, it is the fourth.

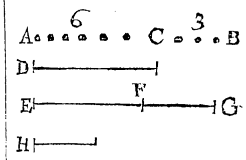
Demonſtration FOr the ſquare of E F, (as in the 49,) ſhall be greater then the ſquare of F G: Let it be ſo then, by the ſquare of H, and by converſion of reaſon, as in the 49, as A B is to C B: ſo the ſquare of E F is to the ſquare of H: But A B being not to C B as a ſquare number to a ſquare

ſquare number, the ſquare of E F ſhall not be to the ſquare of H, as a ſquare number to a ſquare number; therefore [a] E F and H are incommenſurable in length: and ſeeing that the greateſt name E F is more in power then the leaſt F G, by the ſquare of H incommenſurable in length thereto; and the greateſt name E F is commenſurable in length to the Rational D, E G by the Definition, ſhall be the fourth Binomial: Therefore, &c. Which was to be done.

a) 9.10.

PROP. 53. PROBL. 17.

To find a fifth line of two names, or a fifth Binomial.

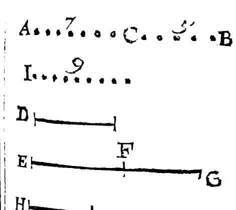


Conſtruction HAving found two numbers A C and C B, as in the precedent; Let the Conſtruction be made as in the 50th. that is to ſay; let F G be taken commenſurable in length to the expoſed rational D, it ſhall be ſhewn, as there, that E G is a Binomial: I ſay alſo that it is the fifth; for it ſhall be ſhewn as in the 50th. that the ſquare of E F ſhall be greater then the ſquare of F G: Let it then be greater by the ſquare of H.

Demonſtration BUt ſeeing by converſion of reaſon (as is demonſtrated in the 49, as A B is to C B, ſo the ſquare of E F, is to the ſquare of H,) it ſhall be ſhewn as in the precedent, that E F and H, are incommenſurable in length: therefore ſeeing that the greateſt name E F is more in power then the leſſer F G, by the ſquare of H, incommenſurable in length thereto; and the leaſt name F G is commenſurable in length to the Rational expoſed D, by the Definition E G ſhall be a fifth Binomial: Therefore we have found, &c. Which was to be done.

PROP. 54. PROBL. 18.

To find a line ſixth of two names, or a ſixth Binomial.

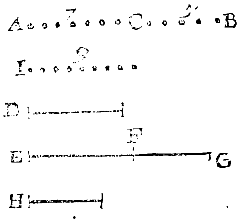


Conſtruction HAving found two numbers A C and C B Plaines, not alike, neither of them being a ſquare, and the Compound of them A B, alſo not a ſquare, nor being to one another as a ſquare number to a ſquare number; (which may be done by dividing ſome number not ſquare, into two numbers,) Primes to one another; [a] for ſo the whole ſhall be Prime to each of them; and therefore ſhall not be to either of them, as a ſquare number to a ſquare

a) [illegible]

number,) and let there be taken some other number I, that may not be to A B nor to A C, as a square number to a square number, and a rational D be exposed, and as I is to A B; so let the square of D be made [b] to the square of E F, and let the rest be done as in the 51th. We shall shew (as there,) that D and E F, are incommensurable in length, and that the whole E G is a Binomial: I say also that it is the sixth.

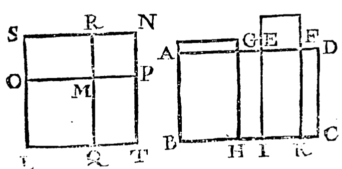
b)Co.6.10



Demonstration FOr it shall be demonstrated as in the 51 Proposition, that D and F G are incommensurable in length, and that the square of E F is greater then the square of F G: Let it then be greater by the square of H, we shall demonstrate also, as in the 49th. that by conversion of reason, as A B is to C B, so the square of E F is to the square of H: therefore seeing that A B is not to C B, and therefore the square of E F; to the square of H, as a square nnmber to a square number: [c] E F and H shall be incommensurable in length: Therefore seeing that the greatest name E F, is more in power then the least F G, by the square of H, incommensurable in length thereto; and neither of them E F nor F G is commensurable in length to the exposed Rational D, according to the Definition, E G shall be the Sixth Binomial: Therefore we have found, &c. Which was to be done.

c) 9. 10.

PROP. 55. THEOR. 37.



If a Superficies A C *be conteined under a Rational* A B, *and a first Binomial line* A D, *the right line being equal in power to the said Superficies, is irational; and is called a Binomial line.*

Demonstration FOr let A E be the greatest names of A D; therefore A E and E D, shall be rational, commensurable in power onely, ac-
cording to the Definition; and A E shall be more in power then E D, by the
square of a line commensurable in length thereto: and A E shall be commen-
surable to the Rational exposed AB: Let E D be divided in two equal parts
in F: Forasmuch then as A E is more in power then E D, by the square of a
line commensurable in length thereto: if there be applyed at A G a Re-
ctangle equal to a quarter of the square of E D; that is to say, to the square
of E F, conteined under A G and G E: and wanting by a square figure, it [a] shall
divide the same in parts commensurable in length: Therefore A G and
G E

a) 18. 10.

G E are commensurable in length to one another: Now let there be drawn G H, E I, and F K, parallel to A B and D C; and [b] let the square L M be equal to the Parallelogram A H, and the square M N equal to the Parallelogram G I, and let those squares be joyned together in the point M, in such sort as M O and M P may make one right line O P: Therefore Q M R shall also make a right line Q R, as [c] appeares: and having finished the Rectangle L N, O M and M P, being equal to Q M and M R; (because of the squares L M and M N:) and therefore the whole O P, to the whole Q R, and O P [d] being also equal; as well to S N as to L T, and Q R to L S and T N, the Rectangle L N shall be equilateral, and therefore a square.

b) 14. 2.

c) 15, 13, & 14. 1.

d) 34. 1.

But the Rectangle under A G and G E; being made equal to the square of E F; as [e] A G to E F: so E F is to G E; and [f] therefore as A H to E K, so E K to G I: Therefore E K is a mean proportional between A H and G I; that is to say, between their equal squares L M and M N: But T M is a mean proportional between L M and M N: Therefore T M is equal to E K: Therefore M S being equal to T M, and [g] F C equal to E K, M S shall be also equal to F C, wherefore the whole square L N shall be equal to the whole Rectangle A C: Therefore the whole O P is equal in power to the Superficies A C conteined under the Rational A B and A D the first Binomial: I say, that O P is irrational, called a Binomial: For seeing that A G and G E are shewn to be commensurable in length, the whole A E shall be commensurable in length to each of them: But A E (being the greatest name of A D the first Binomial,) is commensurable in length to the Rational A B; Therefore A G and G E are commensurable in length to the same A B, [h] as appeares, therefore A B being Rational, A G and G E shall be Rational: Therefore [i] the Rectangles A H and G I conteined under the Rational commensurable in length, are Rationals, as also their equal squares L M and M N: and therefore O M and M P shall be Rational.

e) 17. 6.

f) 1. 6.

g) 43. 1. 36. 1.

16. 10.

h) 12. 10.

i) 20. 10.

And A E being incommensurable in length to E D, and A G is shewn commensurable in length to A E, and E F commensurable in length to E D, being half thereof, A G and E F shall be commensurable in length to one another: Wherefore A H and E K being in the same rate as A G and E F; that [k] is to say, their equals L M and M T are incommensurable: Therefore O M and M P are incommensurable in length, ([l] having the same reason as L M to M T:) but O M and M P are shewn rationals; commensurable in power onely: Wherefore the whole O P equal in power to the Superficies A C is irrational called a Binomial: Therefore, If a Superficies, &c. Which was to be demonstrated.

k) 10. 10.

l) 37. 10.

PROP. 56. THEOR. 38.

If a Superficies A C *be conteined under a rational line* A B, *and a second Binomial line* A D, *the right line equal in power to the said Superficies is irrational, and is called a first Bimedial line.*

Demonstration FOr let A E be the greater name of A D; therefore A E and E D, are rational, commensurable in power onely, according to the Definition; and A E is more in power then E D, by the square of

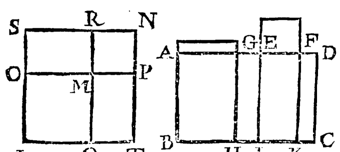
of a line commensurable in length thereto; and E D shall be commensurable in length to the Rational exposed A B, Let E D be divided in two equal parts in F; and having done the rest as in the precedent, it shall be demonstrated (as in that Proposition,) that O P is equal in power to the Superficies conteined under A B Rational; and A D a second Binomial: I say, that O P is irrational, and is called a first Bimedial.

For A E being incommensurable in length to E D, the lesser name of
a) 13. 10. A D a second Binomial, commensurable in length to A B rational, [a] A E
and A B are incommensurable in length, and A G and G E being shewn
b) 15. 10. [b] commensurable in length, [c] the whole A E shall be commensurable in
c) 6. 10. length to each of them: Wherefore A E the greatest name of A D being rational, A G and G E shall be also rational: Therefore seeing that as well A G as G E, is commensurable in length to A E; but A E incommensura-
d) 14. 10. ble in length to A B rational; [d] both A G and G E are incommensurable in length to the same A B: Therefore as well A B and A G as A B and G E
e) 22. 10. are rational, commensurable in power onely: Therefore [e] the Rectangles A H and G I, or their equal squares L M and M N are medials: and therefore O M and M P medials: But A G and G E being commensurable in
f) 15. 10. length, [f] A H and G I being in the same rate, and therefore their equal squares L M and M N shall be commensurable: Therefore O M and M P are commensurable at least in power.

And seeing that A E and E D are incommensurable in length, and A G is shewn commensurable in length to A E: and E F is commensurable in length to E D; being the halfe thereof, A G and E F shall be incommensu-
g) 10. 10. rable in length: Therefore A H and E K having the same rate, [g] their equal
h) 1. 6. Rectangles L M and M T are incommensurable; and [h] therefore O M and M P are incommensurable in length, having the same rate; therefore O M and M P being shewn medials, and commensurable, they shall be medial, commensurable in power onely.

Lastly, E D the least name being commensurable in length to A B; that
i) 12. 10. is to say[c] to E I: but E F is also commensurable in length to E D, [i] E I and and E F shall be commensurable in length; wherefore E I being rational,
k) 20. 10. E F shall be also rational: Therefore [k] E K is a rational Rectangle: But M T conteined under O M and M P, is equal to E K, therefore M T is rational. Therefore O M and M P are medials, commensurable in power
l) 38. 10. onely, conteining a Rectangle rational; and so [l] O P is irrational, and is called the first Bimedial: Therefore if a Superficies, &c. Which was to be demonstrated.

PROP. 57. THEOR. 39.



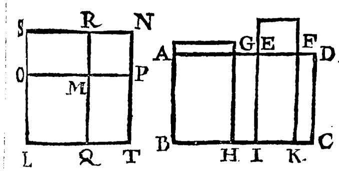
If a Superficies A C *be conteined under a Rational line* A B, *and a third Binomial line* A D, *the right line equal in power to the said Superficies* A C, *is irrational, and is called a second Bimedial.*

Demon-

Demonstration For let A E be the greatest name of A D; therefore A E and E D shall be rational, commensurable in power onely; according to the Definition; and A E shall be more in power then E D, by the square of a line commensurable in length thereto, and neither A E nor E D shall be commensurable in length to the rational exposed A B.

Let E D be divided in two equal parts in F, and do the rest as is shewn in the 55 Proposition, it shall be demonstrated (as there,) that O P is equal in power to the Superficies A C, and as in the precedent Proposition 56, that O M and M P are medials, commensurable in power onely: Seeing that A E is proposed incommensurable in length to A B rational; as the same A E was there incommensurable in length to A B, but E D and E F being commensurable in length, and E D incommensurable in length to the rational A B; that is to say E I: [a] E F shall be also incommensurable in a) 14. 10.
length to E I, but E F and E I are rationals, E F being the halfe of E D rational; and E I equal to the rational A B: Therefore E F and E I are rational, commensurable in power onely; therefore [b] E K is a medial: b) 22. 10.
Therefore M T his equal is also medial, conteined under O M and M P medials: Therefore O M and M P being medials, commensurable in power onely, conteining a Superficies medial, [c] O P shall be irrational, and is cal- c) 22. 2.
led a second Bimedial: Therefore, &c, Which was to be demonstrated.

PROP. 58. THEOR. 40.



If a Superficies A C *be conteined under a Rational line* A B, *and a fourth Binomial line* A D, *the right line equal in power to the Superficies* A C, *is irrational, and is called a Major line.*

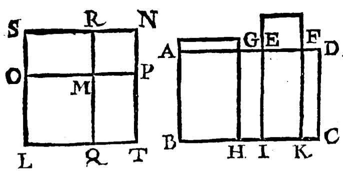
Demonstration For let A E be the greatest name of A D: Therefore A E and E D shall be rational, commensurable in power onely; according to the Definition, and A E shall be more in power then E D; by the square of a line incommensurable in length thereto, and A E shall be commensurable in length to A B: For having divided E D in two equal parts in F; Let the rest be done as by the 55 Proposition: Therefore [a] A G and a) 19. 10.
G E shall be incommensurable in length: Now we shall shew (as there,) that O P is equal in power to the Superficies A C: I say, that O P is irrational, called Major: For A G and G E being incommensurable in length: [b] A H b) 10. 10.
and G I having the same rate, are incommenserable in length, and therefore their equal squares L M and M N incommensurable: Wherefore O M and M P are incommensurable in power: But A E the greatest name being commensurable in length to A B rational, shall be also rational: [c] and the c) 20. 10.
Rectangle A I under them rational: which being equal to the Compound

of

of the ſquares L M and M N, the ſame Compound of L M and M N is Rational.

And foraſmuch as E D the leaſt name, is incommenſurable in length to the Rational A B; E F its halfe, ſhall be alſo incommenſurable in length to A B: and E F is rational, being commenſurable to E D rational: There-
d) 22. 10. fore E F and A B are rational commenſurable in power onely; [d] Wherefore E K conteined under them, and therefore M T his equal, conteined under O M and M P, is a medial; wherefore O P and M P being incommenſurable in power, and making the Compound of their ſquares L M and M N rational, and the Rectangle M T under the ſame medial, the whole [c] O P ſhall
c) 40. 10. be irrational, and is called Major: Therefore, &c. Which was to be demonſtrated.

PROP. 59. THEOR. 41.



If a Superficies A C *be conteined under a Rational line* A B, *and a fifth Binomial line, the right line equal in power to the ſaid Superficies* A C *is irrational, and is called a line equal in power to a rational and a medial Superficies.*

Demonſtration. FOr let A E be the greateſt name of A D: Therefore by the Definition A E and E D, are rational, commenſurable in power onely, and A E is more in power then E D, by the ſquare of a line incommenſurable thereto in length, and E D is commenſurable in length to A B the rational expoſed: Let E D be divided in two equal parts at F,
a) 19. 10. and let it be done as in the 55 Propoſition: Therefore [a] A G and G E ſhall be incommenſurable in length, and we ſhall ſhew (as there) that O P is equal in power to the Superficies A C: I ſay, that O P is irrational, and is called a line equal in power to a Rational and a Medial: For as in the precedent Propoſition, O M and M P ſhall be incommenſurable in power, and A E the greateſt name being rational, and incommenſurable in length to the rational A B: A E and A B ſhall be rational, commenſurable in pow-
b) 22. 10. er onely; [b] wherefore the Rectangle A I under them is medial: But A I is equal to the Compound of the ſquares L M and M N; therefore the ſame Compound ſhall be medial.

Again, E D being propoſed rational, commenſurable in length to the rational A B, being the leaſt name of A D a fifth Binomial, E F the halfe of E D ſhall be alſo commenſurable in length to A B, and rational: there-
c) 20. 10. fore [c] E K conteined under E I and E F rationals, commenſurable in length, and therefore his equal Rectangle M T under O M and M P is rational; therefore O M and M P being incommenſurable in length, and making the Compound of their ſquares L M and M N medial: but the Rectangle
d) 40. 10. M T under them rational; the [d] whole O P ſhall be irrational, and is called a line equal in power to a Rational, and a medial: Therefore, If a Superficies, &c. Which was to be demonſtrated.

PROP.

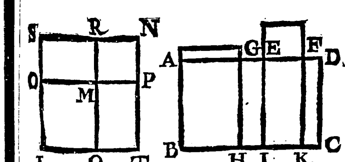
PROP. 60. THEOR. 42.

If a Superficies A C *be conteined under a rational line* A B, *and a ſixth Binomial line* A D, *the right line equal in power to the ſame Superficies* A C, *is irrational, and is called a line equal in power to two medials.*

Demonſtration. FOr let A E be the greateſt name: Therefore A E and E D are rationals, commenſurable in power onely, according to the Definition, and A E ſhall be more in power then E D, by the ſquare of a line which ſhall be incommenſurable thereto in length, and neither the one or the other of them A E and E D ſhall be commenſurable in length to the
Rational A B: Having done as in the precedent Propoſition, A G [a] and G E a) 19. 10.
ſhall be incommenſurable in length, and it ſhall be ſhewn as in the 58 Propoſition, that O P is equal in power to the Superficies A C, and that O M and M P are incommenſurable in power.

Again, as in the precedent Propoſition, the Compound of the ſquares L M and M N ſhall be medial; but as in the 58 Propoſition, M T under O M and M P ſhall be alſo medial: And foraſmuch as of A E and E F, the one A E is incommenſurable in length to E D, and the other E F is com-
menſurable to E D; [b] A E and E F ſhall be incommenſurable in length; b) 13. 10.
therefore [c] A I and E K are incommenſurable; having [d] the ſame rate as A E c) 10. 10.
and E F; wherefore the Compound of the ſquares L M and M N, equal d) 1. 6.
to A I, and M T equal to E K, are incommenſurable: Therefore O M and M P being commenſurable in power, and making the Compound of their ſquares L M and M N medial, and M T the Rectangle under them medial,
and incommenſurable to the Compound of their ſquares, the [e] whole O P e) 42. 10.
ſhall be Irrational, and is called a line equal in power to two Medials: Therefore, If, &c. Which was to be demonſtrated.

PROP. 61. THEOR. 43.



The Square of a Binomial line A B, *applyed unto a Rational line* D E, *makes the breadth* D G, *a firſt Binomial.*

Demonſtration FOr to D E Rational let there be applyed D H, equal to the ſquare of A C: and to H I let there be applyed I K, equal to the ſquare of C B: Therefore the reſt L F ſhall be equal to twice the
Rectangle under A C and C B, [a] the ſquares of A C and C B, and twice a) 4. 2.
the Rectangle under A C and C B; being equal to the ſquare of A B, even as the Rectangle D F is equal to the ſame ſquare of A B, by the Conſtruction.

Let

Let L G be divided in two equal parts at M, and let M N be drawn parallel to L K and G F, both the one and the other Rectangles L N and b) 37. 10. M F, shall be equal to the Rectangle under A C and C B: And [b] A C and C B being Rationals, commensurable in power onely, the whole A B being a c) 16. 10. Binomial, the squares of A C & C B shall be rational, & therefore commensurable, [c] and their compounds being commensurable to each of them, the said compound shall be also rational, to which D K being equal, by the costruction D K shall be a rational: & seeing that the same D K is applyed to the rational, it shall make the breadth D L rational, & commensurable in length to D E.

Again, seeing that A C and C B are rationals commensurable in power d) 22. 10. onely; the [d] Rectangle under them shall be medial, and therefore twice the Rectangle under A C and C B, commensurable thereto; that is to say, e) C. 24. 10. L E [e] shall be medial; therefore L F applyed to L K Rational, [f] makes the f) 23. 10. breadth L G rational, incommensurabe in length to L K, that is to say, to D E: but D L is shewn commensurable in length to D E; therefore [g] D L g) 13. 10. and L G are incommensurable in length: Therefore they are rational, commensurable in power onely; and [h] therefore D G is a Binomial, or of two h) 37. 10. names: I say that it is the first.

For the Rectangle under A C and C B, being a mean proportional between the squares of A C and C B, L N shall be also a mean proportional between D H and I K; therefore D I, L M, and I L having the same rate as D H, L N, I K, and L M, shall be also mean proportionals between D I and I L: i) 17. 7. [i] Therefore the Rectangle under D I and I L shall be equal to the square of L M, and the squares of A C and C B being commensurable, (seeing that A C and C B are proposed commensurable in power,) D H and I K equal to them, shall be commensurable: Therefore [k] D I and I L k) 10. 10. having the same rate, shall be commensurable in length.

But seeing that D K is greater then L F, the squares of A C and C B being greater then twice the Rectangle under A C and C B; D L shall be greater then L G: ([l] D L and L G having the same rate as D K and L F;) there- l) 1. 6. fore seeing that D L is greater then L G, and at D L is applyed the Rectangle under D I and I L, equal to the square of L M; that is to say, to a quarter of the square of L G, the lesser wanting a square figure, (for the Rectangle under D I and I L, is shewn equal to the square of L M,) and D L is divided at I, in D I and I L, commensurable in length, as hath been demonstrated: Therefore [m] D L the greatest, shall be more in power then m) 18. 10. L G the lesser, by the square of a line commensurable in length thereto; wherefore D G being shewn to be a Binomial, and that D L the greatest name is more in power then the lesser L G; by the square of a line which is commensurable in length thereto, and that the same D L the greatest, is commensurable in length to the Rational proposed D E, by the Definition D G shall be first Binomial: Therefore, &c. Which was to be demonstrated.

PROP. 62. THEOR. 44.

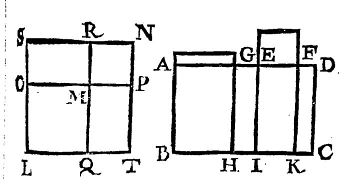
The square of a first Bimedial line A B, *applyed to a Rational line* D E, *makes the breadth* D C *a second Binomial line.*

Demonstration. Let there be done as in the precedent Proposition, in such sort as that D H and I K may again be equal to the squares of

of A C and C B, and L N and M F, equal each of them to the Rectangle under A C and C B, forasmuch as A C and C B compounding A B a first Bimedial, are [a] medials, commensurable in power onely; conteining a rationa Rectangle: a) 38. 10. the squares of A C and C B, and therefore their equal Rectangles D H and I K are commensurable and medials; and [b] therefore the whole D K being b) 16. 10. commensurable to each of them, D H and I K [c] shall be also medial, the c) C. 24. 10. which being applyed to the Rational D E, [d] its breadth D L shall be Ra- d) 23. 10. tional, incommensurable in length to D E.

Again, the Rectangle under A C and C B being Rational, the double thereof, to wit, L F shall be also Rational, which being applyed to the rational L K [e] its latitude L G shall be rational, commensurable in length to e) 21. 10. L K; that is to say, to D E; therefore seeing that L G is commensurable in length to D E, and D L incommensurable in length to D E, [f] D L and f) 13. 10. L G shall be incommensurable in length, and being shewn rational, they shall be rational, commensurable in power onely; and [g] therefore the whole g) 37. 10. D G is a Binomial, and as in the precedent we shall shew that D L is the greatest name, and is more in power then the least L G; by the square of a line commensurable thereto in length, and the least name L G being shewn commensurable in length to the rational exposed D E, according to the Definition, D G shall be second of two names, or a second Binomial: Therefore, &c. Which was to be demonstrated.

PROP. 63. THEOR. 45.



The Square of a second Bimedial line A B, *applyed to a Rational line* D E, *makes the breadth* D G, *a third Binomial line.*

Demonstration. For having made the Construction, as in the 61 Proposition; Forasmuch as A C and C B, making the whole A B a second Bimedial, [a] are medials, commensurable in power onely; conteining a a) 39. 10. medial; the squares of A C and C B; and therefore their equal Rectangles D H and I K, are commensurable, and medials: Wherefore the whole D K being commensurable to each of them, [b] shall be also medial, and be- b) C. 24. 10. ing applyed to the Rational D E; [c] shall make the breadth L D rational, c) 23. 10. commensurable in length to D E.

Again, the Rectangle under A C and C B being medial; the double thereof, to wit L F, shall be also medial: which being applyed to the Rational L K, [d] the other side or breadth L G shall be Rational, commensu- d) 23. 10. rable in length to L K; that is to say, to D E; and seeing that A C is incommensurable in length to C B; and as A C to C B, so the square of A C to the Rectangle under A C and C B: [e] the square of A C is also incommen- e) 10. 10. surable to the Rectangle under A C and C B. [f] But the Compound of the f) 16. 10. squares of A C and C B, is commensurable to the square of A C, (those

 squares

squares being commensurable, described of the lines proposed commensurable in power,) and twice the Rectangle under A C and C B is commensurable to the Rectangle under A C and C B, being double thereto: Therefore the Compound of the squares of A C and C B; that is to say, the Rectangle D K is commensurable to twice the Rectangle under A C and C B; that is to say, to the Rectangle L F; therefore D L and L G having the same
g) 10.10. rate as D K and L F, g are incommensurabre in length: But they are shewn rationals; Therefore D L and L G are rationals, commensurable in power
h) 37.10. onely: Therefore h the whole D G is a Binomial, and as in the 61 Proposition, we shall shew, that D L is the greatest name, and is more in power then L G the lesser, by the square of a line which shall be incommensurable thereto in length, and neither the one nor the other, D L nor L G, being shewn not to be commensurable in length to the Rational proposed, by the Definition, D G shall be a third Binomial: Therefore, &c. Which was to be demonstrated.

PROP. 64. THEOR. 46.

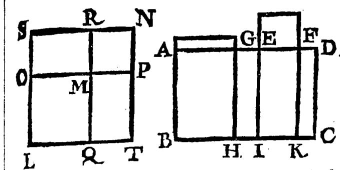
The Square of a Major line A B, *applyed to a Rational line* D E, *makes the breadth* D G *a fourth Binomial line.*

Demonstration FOr let the Construction be made as in the 61 Proposition:
a) 45. 10. Forasmuch as A C and C B, making the Major A B, a are incommensurable in power, and do make the Compound of their squares rational, and the Rectangle under them medial: D K equal to the Compound of those squares, shall be also rational: But L F the double of the Rectangle under A C and C B medial: Forasmuch then as D K is ap-
b) 21.10. plyed to the Rational D E: b D L shall be rational, commensurable in length to D E; in like manner, L F medial being applyed to the Rational
c) 22. 10. 13. 10. L K, c L G shall be rational, incommensurable in length to L K; that is to say, to D E: Therefore seeing that D L is commensurable in length to D E, and L G incommensurable: D L and L G shall be incommensurable in length: but they are shewn Rational: Therefore D L and L G are ratio-
d) 37. 10. nal, commensurable in power onely; therefore d D G is a Binomial.

Now, as is shewn in the 62 Proposition, the Rectangle under D I and I L is equal to the Square of L M: but forasmuch as the squares of A C and C B are incommensurable, (A C and C B being incommensurable in power,) their equal Rectangles D H and I K, shall be also incommensura-
e) 10.10. ble; and e therefore D I and I L being in the same reason, are incommensurable in length: But as is shewn in the 61 Proposition, D L is greater then L G, and to D L there hath been applyed a Rectangle conteined under D I and I L, equal to the square of L M: that is to say, to the quarter part of the square of L G, wanting a square figure, which divideth
f) 19.10. D L at I, in parts incommensurable in length; f D L shall be more in power then L G, by the square of a line which is incommensurable thereto in length: Wherefore D L (the greatest name of D G a Binomial,) being shewn commensurable in length to D E the rational proposed: D G by the Definition, shall be a fourth Binomial: Therefore, &c. Which was to be demonstrated.

PROP.

PROP. 65. THEOR. 47.



The Square of a line A B, *equal in power to a Rational* A C, *and a medial* C B, *applyed to a Rational line* D E, *makes the breadth* D G, *a fifth Binomial line.*

Demonstration FOrasmuch as A C and C B, which make A B equal in pow- 41.10.
er to a rational and a medial, are incommensurable in power, making the Compound of their squares medial, and the Rectangle under them rational, D K equal to the Compound of the squares of A C and C B, shall be medial: and L N equal to the Rectangle under A C and C B: and therefore the double thereof to wit L F, shall be rational: Therefore D K a medial, applyed to D E a rational, makes a D L the breadth rational, a) 23.10.
incommensurable in length to D E rational: but L F rational applyed to the Rational L K, b makes the breadth L G rational, commensurable in b) 21.10.
length to L K, that is to say, to D E.

Forasmuch c then as L G is commensurable in length to D E rational, c) 13.10.
and D L incommensurable in length, D L and L G shall be incommensurable in length; wherefore D L and L G are rational, commensurable in power onely; and therefore d D G is a Binomial. d) 37.10.

Now we shall shew as in the 61 Proposition, that the Rectangle under D I and I L is equal to the square of L M: and as in the precedent, D L is the greatest name, and is more in power then L G, by the square of a line which shall be incommensurable in length thereto; wherefore the least name L G being shewn commensurable in length to the rational exposed D E, by the Definition, D G shall be a fifth Binomial: Therefore, &c. Which was to be demonstrated.

PROP. 66. THEOR. 48.

The square of a line A B, *equal in power to two medials, applyed to a Rational line* D E, *makes the breadth* D G, *a sixth Binomial.*

Demonstration FOr having made the Construction as in the 61 Proposition: 42.10.
Forasmuch as A C and C B. making A B equal in power to two medials, are incommensurable in power, and make the Compound of their squares medial, and the Rectangle under them medial, and incommensurable to the Compound of their squares: as well D K equal to the same Compound, as L F equal to twice the Rectangle under A C and C B, is medial: and D K and L F medials, being applyed to the Rational

D E,

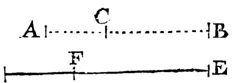
a) 23.10. D E, [a] their breadths D L and L G ſhall be rational, incommenſurable in length to D E rational.

b) 42.10. And [b] foraſmuch as the Compound of the ſquares of A C and C B is incommenſurable to the Rectangle under A C and C B, and twice the Rectangle under A C and C B, is commenſurable to the ſame Rectangle under A C and C B, being double thereto; c) 13.10. [c] the Compound of the ſaid ſquares, that is to ſay, D K, is commenſurable to twice the Rectangle under A C and C B; that is to ſay, to L F: Wherefore D L and L G, having d) 10.10. the ſame rate as D K and L F, [d] are incommenſurable in length; there- c) 37.10. fore D L to L G are rational, commenſurable in power onely [e]; Therefore D G is a Binomial.

And we ſhall demonſtrate as in the precedent, that D L is the greateſt name, and more in power then L G, by the ſquare of a line which is incommenſurable in length thereto; and being ſhewn that neither D L nor L G is commenſurable in length to D E the rational expoſed, by the Definition, D G ſhall be a ſixth Binomial: Therefore, &c. Which was to be demonſtrated.

PROP. 67. THEOR. 49.

The line D E *commenſurable in length to the line of two names* A B, *or Binomial, is alſo a line of two names, and of the ſame order.*



Demonſtration. For as the whole A B is to the whole D E; ſo [a] the part cut off A C, to the part cut off D F; therefore [b] the reſt C B ſhall be to the reſt F E, as the whole A B to the whole D E: And foraſmuch as D E is propoſed commenſurable in length to A B; [c] D F ſhall be alſo commenſurable in length to A C, and F E to C B: But [d] A C and C B names of A B Binomial, are rational; therefore D F and F E are alſo rational.

a) 12.6.
b) 19.5.
c) 10.10.
d) 37.10.

Again, ſeeing that as A C is to D F, ſo C B to F E; and alternately, as A C to C B, ſo D F to F E: but A C and C B are commenſurable in power onely: therefore [e] D F and F E are alſo commenſurable in power onely: Therefore D F and F E are rational, commenſurable in power onely; and [f] therefore D E is a Binomial: I ſay, that D E is alſo of the ſame order as A B, for A C is more in power then C B; by the ſquare of a line which is commenſurable or incommenſurable thereto; and if A C be more in power than C B, by the ſquare of a line that may be commenſurable in length thereto; [g] D F ſhall be more in power than E F, by the ſquare of a line which ſhall be commenſurable in length thereto.

e) 10.10.
f) 37.10.
g) 15.10.

And if A C be commenſurable in length to the Rational expoſed, in ſuch ſort as A B may be a firſt Binomial, [h] D F ſhall be alſo commenſurable in length to the ſame rational, both the one and the other, being rational; And D F being alſo commenſurable in length to A C: Wherefore D E according to the Definition, is a firſt Binomial; that is to ſay, of the ſame order.

h) 12.10.

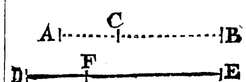
But if C B be commenſurable in length to the Rational expoſed: in ſuch ſort

ſort as A B is a ſecond Binomial: In like manner, F E ſhall be commenſurable in length to the Rational expoſed: wherefore D E by the Definition, is a ſecond Binomial.

Laſtly, if neither A C nor A B be commenſurable in length to the rational expoſed: in ſuch ſort, as A B may be a third Binomial; neither the one nor the other D F nor F E, is commenſurable in length to the Rational expoſed: Therefore D E by the Definition, is a third Binomial.

But if A C be more in power then C B, by the ſquare of a line which ſhall be incommenſurable thereto in length, D F ſhall be more in power than F E, by the ſquare of a line incommenſurable in length thereto: wherefore we ſhall ſhew (as before,) that D E is a fourth, fifth, or ſixth Binomial, according as A B is a fourth, fifth, or ſixth Binomial: Therefore, &c. Which was to be demonſtrated.

PROP. 68. THEOR. 50.



The right line D E *commenſurable to a Bimedial line* A B, *the ſame is alſo Bimedial, and of the ſame order.*

Demonſtration. For as [a] the whole A B is to the whole D E, ſo the part cut off A C, to the part cut off D F: Therefore [b] the reſt C B ſhall be to the reſt F E, as the whole A B to the whole D E; wherefore D E being propoſed commenſurable in length to A B, D F [c] ſhall be commenſurable in length to A C, and F E to C B: But A C and C B are medials; therefore [d] D F and F E commenſurable unto them, are alſo medials. a) 12.6. b) 19.5. c) 10.10. d) 24.10.

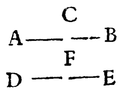
Again, A C being to D F as C B to F E; alternately, as A C is to C B, D F ſhall be to F E: but [e] A C and C B are commenſurable in power onely: Therefore [f] D F and F E ſhall be alſo commenſurable in power onely: and being ſhewn medials, the ſame D F and F E ſhall be medials, commenſurable in power onely: wherefore [g] D E ſhall be a Bimedial. e) 38.10. f) 10.10. g) 39.10.

I ſay, that it is of the ſame order as A B: for ſeeing that as A C to C B, ſo is D F to F E: but as A C to C B, ſo the ſquare of A C to the Rectangle under A C and C B, and as D F to F E, ſo the ſquare of D F to the Rectangle under D F and F E; as the ſquare of A C ſhall be to the Rectangle under A C and C B: ſo the ſquare of D F ſhall be to the Rectangle under D E and E F: and alternately, as the ſquare of A C to the ſquare of D F: ſo the Rectangle under A C and C B, to the Rectangle under D F and F E: but the ſquare of A C is commenſurable to the ſquare of D F: A C and D F being ſhewn commenſurable in length: [h] Therefore the Rectangle under A C and C B ſhall be commenſurable to the Rectangle under D F and F E: Wherefore if the Rectangle under A C and C B be rational; in ſuch ſort as A B may be a firſt Bimedial: the Rectangle under D F and F E, commenſurable thereto, ſhall be alſo rational; and [i] therefore D E ſhall be a firſt Bimedial. h) 10.10. i) 38.10.

But if the Rectangle under A C and C B were medial; in ſuch ſort as A B were a ſecond Bimedial, the Rectangle under D F and F E, commenſurable

k) 39.10. surable thereto, should [k] be also medial: Therefore D E should be a second Bimedial: Therefore, &c. Which was to be demonstrated.

PROP. 69. THEOR. 51.



The right line D E *commensurable to a Major line* A B, *is also a Major line.*

Demonstration LEt there be done as before, in such sort as A C and C B may bear the same rate to D F and F E, as the whole A B to the whole D E: Forasmuch then as A B and D E are commensurable either in length and power, or in power alone, as well [a] A C and D F, as as C B and F E shall be commensurable after the same manner. a) 10.10.

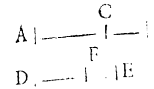
Again, forasmuch as A C is to D F, as C B is to F E; and alternately, as A C to C B, so D F to F E: as [b] the square of A C shall be to the square of C B, so the square of D F to the square of E F: And in compounding, as the Compound of the squares of A C and C B, to the square of C B, so the Compound of the squares of D F and F E, to the square of E F; and by conversion of reason, as the square of C B to the Compound of the squares of A C and C B, so the square of F E to the compound of the squares of D F and F E; and alternately, as the square of C B to the square of F E, so the compound of the squares of A C and C B is commensurable to the compound of the squares of D F and F E: but the square of C B is commensurable to the square of F E: C B and F E, being shewn commensurable, be it in length and power, or in power onely: Therefore [c] the compound of the squares of A C and C B, is commensurable to the compound of the squares of D F and F E; but the compound of the squares of [d] A C and C B is Rational: seeing that A C and C B compound the Major A B: therefore the compound of the squares of D F and F E is also Rational. b) 22.6. c) 10.10. d) 40.10. 9 Def.

Again, seeing that as A C is to C B so D F to F E; but as A C to C B, so the square of A C to the rectangle under A C and C B; and as D F to F E, so the square of D F to the rectangle under D F and F E, as the square of A C shall be to the Rectangle under A C and C B, so the square of D F shall be to the rectangle under D F & F E; and alternately, as the square of A C to the square of D F, so the rectangle under A C & C B, to the Rectangle under D F and F E.

But the square of A C is commensurable to the square of D F, A C and D F being shewn commensurable either in length and power, or in power alone; therefore [e] the Rectangle under A C and C B, is also commensurable to the Rectangle under D F and F E: but [f] the Rectangle under A C and C B is medial; therefore the Rectangle under D F and F E, commensurable thereto, is also medial: But A C being to C B as D F to F E; and [g] A C and C B, are incommensurable in power, (compounding the Major A B,) D F and F E shall be also incommensurable in power: Therefore D F and F E being incommensurable in power, and making the compound of their squares Rational, and the Rectangle under them Medial, as hath been shewn: [h] the whole D E shall be a Major line: Therefore, &c. Which was to be demonstrated. e) 10.10. f) C.24.10. g) 40.10. 10.10. h) 40.10.

PROP.

PROP. 70. THEOR. 52.



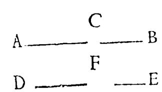
The line D E *commensurable to the line* A B, *equal in power to a Rational and a Medial, is also a line equal in power to a Rational and a Medial.*

Demonstration FOr having made the same Construction as before, we shall shew as in the precedent, that the compound of the squares of A C and C B is commensurable to the compound of the squares of D F and F E: but the compound of the squares of A C and C B is a Medial; seeing that A C and C B do compound A B equal in power to a Rational and a Medial; therefore the compound of the squares of D F and F E, commensurable thereto, is Medial.

In like manner, as in the precedent Proposition, the Rectangle under A C and C B, which [a] is rational, shall be commensurable to the Rectangle under D F and F E; and therefore the Rectangle under D F and F E shall be Rational. a) 41.10. 9. d.

Lastly, as in the precedent Proposition, D F and F E shall be incommensurable in power; therefore D F and F E being incommensurable in power, and making the compound of their squares Medial, and the Rectangle under them Rational, as is shewn, [b] the whole D E shall be a line equal in power to a Rational and a Medial: Therefore, &c. Which was to be demonstrated. b) 41.10.

PROP. 71. THEOR. 53.



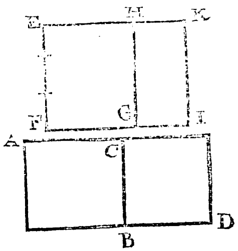
The line D E *commensurable to the line* A B, *equal in power to two medials, is also a line equal in power to two medials.*

Demonstration FOr, the Construction made, as before, it shall be shewn as in the 69 Proposition, that the compound of the squares of A C and C B is commensurable to the compound of the squares of D F and F E, and the Rectangle under A C and C B commensurable to the Rectangle under D F and F E: wherefore [a] as well the compound of the squares of A C and C B, or the Rectangle under A C and C B, being medial: also [b] as well the compound of the squares of D F and F E, as the Rectangle under D F and F E, shall be medial: But D F and F E are incommensurable in power, as before is said. a) 42.10. b) C.24.10.

Lastly, [c] the compound of the squares of A C and C B, and the Rectangle under A C and C B, being incommensurable, by Supposition, seeing that A B is equal in power to two medials: But the compound of the squares of D F and F E, is shewn commensurable to the compound of the squares of A C and C B, and the Rectangle under D F and F E shewn com- c) 42.10.

commensurable to the Rectangle under A C and C B, the compound of the squares of D F and F E, and the Rectangle under D F and F E shall be incommensurable: Therefore D F and F E being incommensurable in power, and making the compound of their squares medial, and the Rectangle under them medial, and incommensurable to the compound of their squares,
d) 42.10. [d] D E shall be a line equal in power to two medials: Therefore, &c. Which was to be demonstrated.

PROP. 72. THEOR. 54.



If a rational Superficies AB, *be compounded with a medial* CD, *there will be made four Irrationals, to wit, that which is called a Binomial, or a first Bimedial, or a line major, or a line equal in power to a rational and a Medial.*

Demonstration. For the Space A B, shall be greater then C D, or lesse, for it cannot be equal: For A B being rational, if C D were equal thereto, it should be also rational, contrary to the Supposition.

Let first of all A B be greater then C D, and to the rational proposed E F, let there be applyed the Rectangle E G equal to A B, and to H G let there be applyed H I equal to C D, insuch sort as the whole E I be equal to the whole A D, and A B being a rational, and C D medial; E G shall be also rational, and H I medial, which applyed to the rational E F: E H
a) 21.10. [a] shall be rational, commensurable in length to E F: [b] but H K rational,
b) 23.10. incommensurable in length to E F.

Again, E G and H I being incommensurable, as appeares by the Definition, E G being rational, and H I irrational, to wit, medial; [c] E H and H K,
c) 1.6. having the same rate, [d] shall be incommensurable in length, and there-
d) 10.10. fore are rational, commensurable in power only: [e] Therefore E K is a
e) 37.10. binomial, and A B being proposed greater then C D; that is to say, E G greater then H I: E H shall be also greater then H K, E H and H K having the same rate as E G and H I: Now E H the greatest name, shall be more in power then H K the lesser, by a square of a line commensurable in length or incommensurable thereto.

If E H be more in power then H K, by the square of a line commensurable in length thereto, E K shall be a first Binomial, by the Definition: E H being the greater name, shewn commensurable in length to the rational E F: Wherefore the line equal in power to the Space E I, conteined under the rational E F, and E K is a first Binomial; and therefore the Space A B
f) 55.10. compounded of the rational A B, and of the Medial C D [f] is irrational, called a Binomial.

But if E H be more in power then H K, by the square of a line which may be incommensurable in length thereto: E K shall be a fourth Binomial, by the Definition, E H being the greatest name, shewn commensurable in length

length to the rational E F: Therefore the right line equal in power to the square E I, conteined under E F rational, and E K a fourth binomial, and
g) 58.10. therefore also the square A D [g] is irrational, called Major.

Now let A B be the greatest irrational, and medial, and C D the least rational: having made the Construction as before, E H shall be incommensurable in length to E F, and H K commensurable, and therefore E H and H K rational, commensurable in power only, and the whole E K a Binomial; therefore E H the greatest name, shall be more in power then H K, by the square of a line which shall be commensurable thereto in length, or incommensurable: if it be commensurable, and E H is shewn incommensurable in length to the rational proposed, E K by the Definition, shall be a second Binomial: Therefore the line equal in power to the Superficies E I, a second Binomial, is irrational, [h] called a first Bimedial, if incom-
h) 56.10. mensurable, E K shall be a fifth Binomial; therefore the right line equal in power to the Superficies E I, (which is A B,) conteined under E F rational, and E K a fifth Binomial, [i] is irrational, called a line equal in power to a ratio-
i) 59.10. nal, and a medial: Therefore, &c. Which was to be demonstrated.

PROP. 73. THEOR. 55.

If two Superficies medials AB *and* C D, *incommensurable to one another, are compounded, two other irrational lines are made; to wit, either the second Bimedial, or the line equal in power to two medials.*

Demonstration. For A B shall be greater then C D, or lesse, and not equal; otherwise A B and C D should be commensurable, which is contrary to the Supposition.

Let it in the first place be supposed greater, and let it be done as in the precedent Proposition, and A B and C D being proposed medials, and incommensurable, their equals E G and H I, shall be also medials, and in-
a) 1.6. commensurable: Therefore [a] E H and H K having the same rate, shall [b] be
b) 10.10. incommensurable in length: (and E G and H I medials,) being applyed to
c) 23.10. the rational E F, both [c] E H and H K are rational, incommensurable in length to the rational E F: Therefore E H and H K are rational, commensurable in power onely, therefore the whole E K is a Binomial.

But seeing that A B is proposed greater then C D, as in the precedent Proposition, E H shall be the greatest name of E K: and therefore shall be more in power then H K, by the square of a line which is commensurable in length thereto, or incommensurable: If E H be more in power then H K by the square of a line which may be commensurable in length thereto, as well E H as H K being shewn incommensurable in length to E F rationals, by the Definition, E K shall be a third Binomial: Therefore the right line equal in power to the Superficies E I, conteined under E F a rational, and E K a third Binomial, and therefore the line equal in power to the Superficies A D is irrational, called a second Bimedial.

If E H be more in power then H K, by the square of a line incommensurable in length thereto, as well E H as H K, being incommensurable in length to E F rational, E K shall be a sixth Binomial, by the Definition,

Therefore the line equal in power to E I, or A D is irrational, called a line equal in power to two Medials: and if A B were lesse then C D, it may be demonstrated in like manner: Therefore, &c. Which was to be demonstrated.

COROLLARIE.

From all these things may be gathered easily that the line of two names, or Binomial, and the other irrational lines following it are different among themselves; and also from the medial line.

For the square of a medial line applyed to a rational line,
a) 23.10. *a makes the breadth rational, incommensurable in length to the said rational to which it is applyed.*

But the square of a line of two names applyed to a rational
b) 61.10. *line, b makes the breadth the first of two names, or first Binomial.*

And the square of a first Bimedial applyed to a Rational line
c) 62.10. *c makes the breadth a second Binomial.*

And the square of a second Bimedial, applyed to a rational
d) 63.10. *line, d makes the breadth a third Binomial.*

e) 64.10. *But the square of a Major line applyed to a Rational, e makes the breadth a fourth Binomial.*

And the square of a line equal in power to a Rational and a
f) 65.10. *Medial, applyed to a Rational line, f makes the breadth a fifth Binomial.*

Lastly, the square of a line equal in power to two Medials,
g) 66.10. *applyed to a Rational line, g makes the breadth a sixth Binomial.*

Now seeing that all these breadths do differ from the breadth of the Medial, and among themselves (to wit, from the breadth of the Medial, the one being rational, to wit, the medial, and the others irrational:) But forasmuch as they are not Binomials, of the same order they differ from one another; It is manifest that all the irrational lines whereof we have spoken hitherto, are different among themselves.

Here begins the Senaries of Irrational lines by intersection.

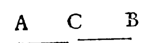
PROP. 74. THEOR. 56.

If from a Rational line A B *be cut off a Rational line* A C, *commensurable in power only to the whole* A B, *the remminder* B C *is irrational: and may be called an* Apotome, *or a* Residual.

Demon-

Demonstration. FOr as A B is to A C, so the square of A B to the Rectangle under A B & A C, and A B and A C, being incommensurable in length; the a square of A B shall be incommensurable to the Rectangle under A B and A C: (a) 10.10.) but the compound of the squares of A B and A C is commensurable to the square of A B, (for A B and A C being proposed commensurable in power, the squares of A B & A C shall be commensurable: b and therefore (b) 16.10.) their Compound commensurable to the square of A B:) but twice the Rectangle under A B and A C is commensurable to the Rectangle under A B and A C: therefore the Compound of the squares of A B and A C, and twice the Rectangle under A B and A C are incommensurable: Now c the Compound (c) 7.2.) of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C: therefore d the Compound of the (d) C.17.10.) squares of A B and A C is incommensurable to the remaining square of B C: wherefore the Compound of the squares of A B and A C being rational: (as being commensurable to the square of the rational A B;) the square of B C shall be irrational, and therefore B C irrational: And let it be called an *Apotome*, which others call a *Residual*: Therefore, &c. Which was to be demonstrated.

PROP. 75. THEOR. 57.



If from a medial line A B, *be cut off a medial line* A C, *commensurable only in power to the whole* A B, *which doth contein a Rational Rectangle with the whole* A B, *the rest* C B *is irrational: And let it be called a first Residual Medial.*

Demonstration FOr A B and A C being commensurable in power, the squares of A B and A C shall be commensurable; and a therefore (a) 16.10.) their compound shall be commensurable to the square of A C: but the square of A C medial is irrational, and medial: therefore b the compound (b) C.24.10.) of the squares of A B and A C, commensurable thereto, is irrational, and medial, and the Rectangle under A B and A C being proposed Rational, twice the Rectangle under A B and A C shall be Rational; wherefore the compound of the squares of A B and A C shall be incommensurable to twice the Rectangle of A B and A C: Therefore c seeing that the compound (c) 7.2.) of the squares of A B and A C, is equal to twice the Rectangle under A B and A C, and to the square of B C; twice the Rectangle under A B and A C, with the square of B C, shall be also incommensurable to twice the Rectangle under A B and A C: Wherefore d twice the Rectangle (d) 17.10.) under A B and A C, and the square of B C are incommensurable, and twice the Rectangle nnder A B and A C, being rational, the square of B C shall be irrational; and therefore B C irrational: And let it be called an *Apotome*, or first *Residual*: Therefore, &c. Which was to be demonstrated.

PROP.

PROP. 76. THEOR. 58.

A C B

A C B

If from a medial line A B, *be cut off a medial line* A C, *commensurable in power onely to the whole* A B, *which conteins with the whole* A B *a medial Rectangle, the remainder* B C *is irrational: And is called a Second* Apotome, *or* Residual *of a medial.*

Demonstration. FOrasmuch as the squares of A B and A C commensurable in power, are commensurable, their [a] compound shall be
a) 16. 10. commensurable to each of them: but each of those squares is medial; (A B
b) C. 24. 10. and A C being proposed medial:) therefore [b] their compound is medial, being commensurable to each of them.

Again, seeing that the Rectangle under A B and A C is proposed medial:
c) C. 24. 10. [c] the double thereof, to wit, twice the Rectangle under A B and A C, shall
d) 7. 2. be also medial: Therefore [d] seeing that the compound of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C, the compound of the squares of A B and A C which is medial, shall exceed twice the Rectangle nnder A B and A C, (which is
c) 27. 10. also medial,) by the square of B C: But [c] a medial doth not exceed a medial by a Rational: Therefore the square of B C shall not be rational: therefore irrational, and B C irrational: And is called a second *Apotome* of a medial line: Therefore, &c. Which was to be demonstrated.

PROP. 77. THEOR. 59.

A C B

If from a right line A B, *be cut off a right line* A C, *incommensurable in power to the whole* A B, *making with the whole* A B, *the Compound of their squares rational, and the Rectangle under them medial, the remainder* B C *is irrational: And is called a Minor.*

Demonstration. FOr the Compound of the squares of A B and A C, being rational: But the Rectangle under A B and A C: and [a] there-
a) C. 24. 10. fore the double thereof which is commensurable thereto; that is to say, twice the Rectangle under A B and A C medial, that is to say, irrational:
b) 10. 10. [b] the compound of the squares of A B and A C, shall be incommensurable to twice the Rectangle under A B and A C; But the compound of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C: therefore the compound of the squares of
c) C. 17. 10. A B and A C [c] shall be incommensurable to the remaining square of B C: but the compound of the squares of A B and A C is proposed rational: there-

d) 10. d. therefore [d] the square of B C incommensurable thereto, is irrational, and the line B C irrational: And is called Minor: Therefore, &c. Which was to be demonstrated.

PROP. 78. THEOR. 60.

A C B

If from a right line A B, *be cut off a right line* C D, *incommensurable in power to the Whole* A B, *making with the Whole* A B *the Compound of their squares medial: But the Rectangle conteined under them rational, the remainder* C B *is irrational: And is called a line making with a rational Space, a whole medial.*

Demonstration. FOr the compound of the squares of A B and A C being medial; that is to say irrational, and the Rectangle under A B and A C: and therefore the double thereof, to wit, twice the Rectangle under A B and A C rational: the [a] compound of the squares of A B and
a) 10. d. A C shall be incommensurable to twice the Rectangle under A B and A C:
b) 7. 2. But [b] the compound of the squares of A B and A C is equal to twice the Rectangle under A B and A C, and to the square of B C: therefore twice the Rectangle under A B and A C, with the square of B C, is incommensurable to twice the Rectangle under A B and A C; therefore [c] twice the re-
c) 17. 10. ctangle under A B and A C, and the square of B C are incommensurable: Therefore twice the rectangle under A B and A C being rational, the square of B C shall be irrational, and the line B C irrational: And is called a line making with a rational Space a whole medial: Forasmuch as its square added to twice the Rectangle under A B and A C, which is rational, makes the whole medial: Therefore, If, &c. Which was to be demonstrated.

PROP. 79. THEOR. 61.

A C B

If from a right line A B, *be cut off a right line* A C, *incommensurable in power to the whole* A B, *making with the whole* A B, *the Compound of their squares medial, and the Rectangle contained under them medial, and incommensurable to the compound of their squares: the remainder* C B *is irrational; and let it be called a line making a whole medial with a rectangle medial.*

Demonstration. FOrasmuch as the compound of the squares of A B and A C is equal to twice the Rectangle under [a] A B and A C, and to
a) 7. 2. the square of B C, the compound of the squares of A B and A C, proposed medial:

b) C.24.10. medial, shall exceed twice the rectangle under A B and A C, [b] which is medial, (being commensurable to the medial: conteined under A B and A C:) of the square of B C: But a medial doth not exceed a medial by a rational, wherefore the square of B C is not rational, therefore irrational, and B C irrational: And is called a line making a whole medial with a medial: Forasmuch as the square thereof with the Rectangle under A B and A C, which is medial, makes the whole compound of the squares of A B and A C, which is also medial: Therefore, If, &c. Which was to be demonstrated.

PROP. 80. THEOR. 62.

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     B   C
A —|—|—|D
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There belongs only to a Residual *or* Apotome A B, *one only right line* B C *rational, commensurable in power only to the whole* A D.

Demonstration For (if possible,) let another rational B D agree to it, commensurable in power only to A D, and B C being rational, A C commensurable in power only thereto, shall be rational: Therefore A C and B C are rational, commensurable in power only, In like manner A D aud B D shall be rationals, commensurable in power onely.

But there being the same excesse between the compound of the squares of A C and B C, and twice the Rectangle under A C and B C, as between the compounds of the squares of A D and B D, and twice the Rectangle under A D and C B; (for the excesse between the compound of the squares of A C and B C, and twice the Rectangle under A C and B C is the square of A B: the [a] compound of the squares of A C and B C, being equal to
a) 7.2. twice the Rectangle under A C and B C, and the square of A B.)

In like manner the square of A B is the excesse between the compound of the squares of A D and B D, and twice the Rectangle under A B and B D, (the compound of the squares of A D and B D, being equal to twice the Rectangle of A D and B D, and the square of A B;) also alternately, there will be the same excesse between the compound of the squares of A C and B C, and the compound of the squares of A D and B D, as between twice the Rectangle of A C and B C, and twice the Rectangle of A D and B D: but the excesse between those compounds is rational, the said compounds being rational: (for A C and C B being rational, commensurable in power onely, their squares shall be rational and commensurable: Therefore their compound shall be commensurable to each of them; and therefore Rational: In like manner, the compound of the squares of A D and B D shall be shewn rational;) therefore the excesse between twice the Rectangle under A C and B C, and twice the Rectangle under A D and B D is a space rational: And seeing that [b] the Rectangle under A C and B C rationals,
b) 22.10. c) C.24.10. commensurable in power onely, is medial, [c] the double thereof, to wit, twice the Rectangle under A C and B C, shall be medial: by the same reason, the

the Rectangle under A D and B D is medial; and [d] a medial exceeds not a medial by a rational; therefore the excesse between twice the Rectangle d) 27.10.
under A C and B C, and twice the Rectangle under A D and B D, is not a space rational: but it hath been shewn rational, which is absurd: Therefore any other rational agrees not with A B, commensurable in power onely to the whole, besides B C: Therefore, &c. Which was to be demonstrated.

PROP. 81. THEOR. 63.

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  B   C
A|—|—|—|D
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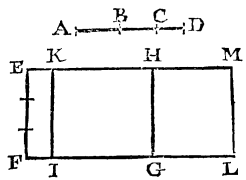
To a medial first Residual A B, *there agrees onely a medial line* B C, *commensurable in power only to the whole* A D, *conteining with the whole* A D *a rational Rectangle.*

Demonstration For, (if possible,) let another medial B D agree therewith, commensurable in power onely to the whole, and let the Rectangle under A D and B D, be rational, and B C being Medial, commensurable in power only to A C, [a] A C shall be medial; therefore A C and B C a) 24.10.
are medials commensurable in power only.

And seeing that there is the same excesse between the compound of the squares of A D and B D, as between twice the Rectangle under A C and B C, and twice the Rectangle under A D and B D, as is shewn in the precedent, but the excesse between those Rectangles is rational, the one and the other being rational: (for the Rectangle under A C and B C being proposed Rational, twice the Rectangle under A C and B C, commensurable thereto is rational, and by the same reason, twice the Rectangle under A D and B D is rational:) Therefore the excesse between the compounds of the squares of A C and B C; and that of the squares of A D and B D is rational, and A C and B C being medials, commensurable in power only, their squares shall be medials, and commensurable: Therefore [b] their com- b) 16.10.
pound is commensnrable to each of them, and [c] therefore medial; by the c) C.24.10.
same reason, the compound of the squares of A D and B D shall be medial: But a medial not exceeding a medial by a rational, the excesse between the compounds of the squares of A C and B C, and that of the squares of A D and B D is not rational: But it hath been shewn rational, which is absurd:. Therefore another medial agrees not to A B, besides B C, which may be commensurable in power onely to the whole, and contein a rational Rectangle with the whole: Therefore, &c. Which was to be demonstrated.

PROP. 82. THEOR. 64.



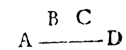
To the medial ſecond Reſidual AB, *there can agree but one only right line* BC *medial, commenſurable in power only to the whole* AC, *and conteining a medial Rectangle under* AC *and* BC, *with the whole.*

Demonſtration FOr, (if poſſible,) let BD medial agree therewith, commenſurable in power only to the whole AD; and let the Rectang'e under AD and BD be medial, and let EF the rational be expoſed, to
a) 45. 1. which [a] let there be applyed the Rectangle EG, equal to that which is compounded of the ſquares of AC and BC, and to EF let there be applyed the Rectangle EI, equal to the ſquare of AB; then the Rectangle EL equal to the Rectangle which is compounded of the ſquares of AD and BD: Foraſmuch
b) 7. 2. [b] then as the compound of the ſquares of AC and BC, is equal to twice the Rectangle under AC and BC, and to the ſquare of AB, KG ſhall be equal to twice the Rectangle under AC and BC: ſo KL ſhall be equal to twice the Rectangle under AD and BD, and AC and BC being medials, commenſurable in power only, (for BC being propoſed medial, AC
c) 16. 10. [c] commenſurable in power thereto, ſhall be medial alſo,) their ſquares
d) C. 24. 10. ſhall be medials, and commenſurable: Therefore [d] their compound is commenſurable to each of them, and therefore medial: Therefore EG equal to the ſame compound is alſo medial, and EG being applyed to EF rational, EH ſhall be rational, incommenſurable in length to EF.

Again, the Rectangle under AC and BC being medial, the double thereof, to wit, KG ſhall be medial, and being applyed to KI rational, KH ſhall be rational, incommenſurable in length to KI; that is to ſay, to EF: and ſeeing that AC and BC are incommenſurable in length, and as AC to BC, ſo the ſquare of AC to the Rectangle under AC and BC, the ſquare of AC ſhall be incommenſurable to the Rectangle under AC and BC: but the compound of the ſquares of AC and BC, that is to ſay, the Rectangle EG is commenſurable to the ſquare of AC; (for AC and BC being propoſed commenſurable in power, their ſquares ſhall be commenſurable to each of them; to wit, to the ſquare of AC:) and the double of the Rectangle under AC and BC, to wit KG, is commenſurable to the ſaid Rectangle under AC and BC: Therefore EG is incommenſurable to KG, therefore EH and KH bearing the ſame rate as EG and KG, are incommenſurable in length, and being ſhewn rational, they ſhall be rational, commenſurable in power only: Wherefore from EH rational, having cut off the rational KH, commenſurable in power only to EH,
e) 74. 10. [e] EK ſhall be reſidual, and KH agreeing therewith: So it may be ſhewn that EK is a Reſidual to KM, agreeing therewith: Therefore to the Reſidual there agrees only one rational line, commenſurable in power only to the whole, which is abſurd, as hath been demonſtrated: Therefore, to a medial Reſidual, &c. Which was to be demonſtrated.

PROP.

PROP. 83. THEOR. 65.

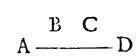


To a minor line AB, *there agrees only one right line* BC *incommenſurable in power to the Whole* AC, *and making with the Whole* AC *the Compound of their ſquares rational, and the Rectangle under them medial.*

Demonſtration FOr otherwiſe let another BD agree with AB incommenſurable in power to the whole AD, making the compound of the ſquares of AD and BD rational, and the Rectangle under AD and BD medial.

Foraſmuch then as hath been demonſtrated in the 80 Propoſition, the exceſſe of the compound of the ſquares of AC & BC, and the compound of the ſquares of AD & BD is the ſame as that between twice the rectangle under AC & BC, and of twice the rectangle under AD & BD: but the exceſſe between thoſe compounds is rational, both the one and the other, being propoſed rational: Therefore the exceſſe between twice the rectangle under AC and BC, and twice the rectangle under AD & BD is a rational Space: But it is alſo irrational, (for the rectangle under AC & BC being propoſed medial, twice the rectangle under AC and BC double thereto, [a] is alſo medial: a) C. 24. 10.
In like manner, twice the rectangle under AD and DB ſhall be medial; b) 27. 10.
therefore [b] the one ſhall not exceed the other by a rational, which is abſurd: Therefore another line agrees not to AB, beſides BC incommenſurable in power to the whole: Therefore, &c. Which was to be demonſtrated.

PROP. 84. THEOR. 66.



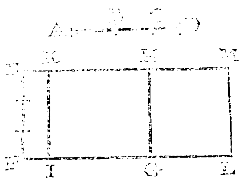
To a line AB, *making with a Space rational the whole medial, there agrees one only right line* BC, *incommenſurable in power to the whole* AC, *and making with the Whole the Compound of their ſquares medial: But the Rectangle conteined under them Rational.*

Demonſtration FOr (if poſſible) let another line BD agree therewith incommenſurable in power to the whole AD, and making the compound of the ſquares AD and BD medial, and the Rectangle under AD and BD rational: Foraſmuch as it is ſhewn in the 80th. Propoſition, that there is the ſame exceſſe between the compound of the ſquares of AC and BC, and the compound of the ſquares of AD and BD, as between twice the Rectangle under AC and BC, and twice the Rectangle under

AD

AD and BD: But the excesse between those Rectangles is a rational Space, as hath been shewn in the 81 Proposition; therefore the excesse between those compounds of the squares is also a rational Space: but it is not
a) 27.10. also rational, both the one and the other, being medial, [a] one medial exceeding not a medial by a rational, which is absurd: Therefore any other line then BC agrees not with AB, incommensurable in power to the whole: Therefore the line, &c. Which was to be demonstrated.

PROP. 85. THEOR. 67.



To a line AB, *making with a Space medial a whole medial, there agrees one only right line* BC, *incommensurable in power to the whole* AC, *and making with the whole* AC, *the Compound of their squares medial, and the Rectangle contained under them medial, and incommensurable to the compound of their squares.*

Demonstration. FOr (if possible, let BD agree thereto, also incommensurable in power to the whole AD, making what is before said; and let there be done as in the 82 Proposition: Forasmuch as the compound of the squares of AC and BC is medial, EG his equal is also me-
a) 23.10. dial; therefore [a] EH is rational, incommensurable in length to EF: Again forasmuch as the Rectangle under AC and BC is medial, the double
b) 23.10. thereof KG is also medial; therefore [b] KH is rational, incommensurable in length to EF; and forasmuch as KG is commensurable to the Rectangle under AC and BC, being double to the said Rectangle: But the Rectangle under AC and BC is proposed incommensurable to the compound of
c) 14.10. the squares of AC and BC: [c] KG shall be also incommensurable to the compound to the said Compound, that is to say to EG; and therefore EH
d) 1.10. and HK being in the same rate as EG and KG, are [d] incommensurable in length; but they are shewn rational; therefore are rational, commensurable in power only.

Taking then from the rational EH, the rational KH, commensurable in
e) 74.10. power only to EH, [e] EK shall be a Residual, and thereto agrees KH: in like manner we shall shew that EK is a Residual, to the which KM agrees. Therefore one only line doth not agree to a Residual, &c. Which is ab-
f) 80.10. surd, being [f] shewn the contrary: Therefore, &c. Which was to be demonstrated.

THIRD

THIRD DEFINITIONS.

A rational line being proposed and a Residual, when the whole compounded of the Residual and of the agreeing or added line, be more in power then the agreeing line, by the square of a line which is commensurable thereto in length.

1 *If the whole be commensurable in length to the rational proposed, let the residual be called a* First Residual.

2 *But if the agreeing or added line, be commensurable in length to the rational proposed, let it be called a* Second Residual.

3 *And if neither the whole nor the agreeing, be commensurable in length to the rational proposed, let it be called a* Third Residual.

Again, if the whole be more in power then the agreeing, by the square of a right line incommensurable in length thereto.

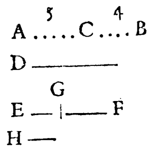
4 *If the whole be commensurable in length to the rational proposed, let it be called a* Fourth Residual.

5 *But if the agreeing be commensurable in length to the rational proposed, let it be called a* Fifth Residual.

6 *And if neither the whole, nor the agreeing, be commensurable in length to the rational proposed, let it be called a* Sixth Residual.

PROP.

PROP. 86. PROBL. 19.



To finde an Apotome, *or a* Firſt Reſidual.

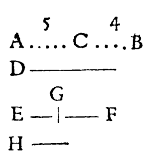
Conſtruction HAving found two ſquare numbers, A B and C B, whoſe exceſſe A C may be no ſquare; let the rational D be propoſed, and let E F be commenſurable in length thereto: E F ſhall
a) C.6.10. be alſo rational, and [a] as the number A B to A C: ſo let the ſquare of E F be made to the ſquare of G F: I ſay that E G is a firſt Reſidual.

Demonſtration FOraſmuch as the ſquares of E F and G F, being to one another as the number A B to the number A C are commenſurable, E F and G F ſhall be commenſurable at leaſt in power: Therefore ſeeing that E F is ſhewn rational, G F ſhall be rational alſo.

And ſeeing that A B and A C are not to one another as ſquare numbers, the ſquares of E F and G F ſhall not be to one another as a ſquare number to
b) 9. 10. a ſquare number; therefore [b] E F and G F are incommenſurable in length,
c) 74.10. they ſhall be then rational, commenſurable in power only: therefore [c] the remainder E G is a Reſidual: I ſay, it is a firſt Reſidual: For let E F the greateſt, be more in power then G F by the ſquare of H; and foraſmuch as the number A B is to the number A C, ſo the ſquare of E F to the ſquare of G F; by converſion, as A B to C B, ſo the ſquare of E F to the ſquare of H: but A B and C B are ſquare numbers; therefore the ſquare of E F ſhal be to the ſquare of H, as a ſquare number to a ſquare number: therefore E F and H are commenſurable in length.

Foraſmuch then as the whole E F is more in power then G F, agreeing with the ſquare of H, commenſurable in length thereto, and the whole E F is commenſurable in length to the rational expoſed D, by the Definition E G ſhall be a firſt Reſidual: Therefore, &c. Which was to be done.

PROP. 87. PROBL. 20.



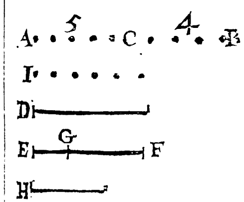
To finde a Second Reſidual.

Conſtruction HAving found two numbers A B and C B, as in the precedent, and propoſed the rational D, let G F be taken commenſurable in length thereto, G F ſhall be alſo rational, and as
a) Co.6.10. [a] the number A C to the number A B, ſo let the ſquare of GF be made to the ſquare of E F: I ſay, that EG is a Second Apotome.

Demonſtration FOr the ſquares of G F and E F being to one another as the numbers A C and A B are commenſurable, and G F and E F commenſurable at leaſt in power, and ſeeing that G F is ſhewn rational, E F ſhall be alſo rational: But foraſmuch as the numbers A C and A B, and by the ſame meanes the ſquares of G F and E F, are not to one another as a ſquare number to a ſquare number, [b] G F and E F ſhall be incommenſuarable in length, therefore G F and E F are rational, commenſurable in power only, and therefore [c] E G remainder, is a Reſidual: I ſay alſo, it is a ſecond Reſidual. For let E F be more in power then G F, by the ſquare of H. b) 9. 10. c) 74.10.

Foraſmuch then as A C is to A B, ſo the ſquare of G F is to the ſquare of E F, and by converſion of reaſon, as A B to A C, ſo the ſquare of E F to the ſquare of G F. Now we ſhall ſhew as in the preeedent, that the line H is commenſurable in length to E F: wherefore the whole E F being more in power then the agreeing G F, by the ſquare of H, commenſurable in length thereto; and that the ſame agreeing G F is commenſurable in length to the rational expoſed D, by the Definition, E G ſhall be an *Apotome*, or *Second Reſidual*: Therefore, &c. Which was to be done.

PROP. 88. PROBL. 21.



To finde an Apotome, *or* Third Reſidual.

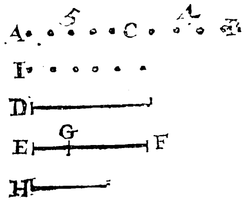
Conſtruction HAving found two numbers A B and C B, as in the 86 Propoſition; Let there be taken another number I, as is ſhewn in the 51 Propoſition, which may not be to either of them as a ſquare number to a ſquare number, and having propoſed D ratio-
a) C. 6. 10. nal, let it be made as I to A B: ſo [a] is the ſquare of D to the ſquare of E F, and thoſe ſquares of D and E F ſhall be to one another as the number I and A B, and therefore commenſurable: and therefore D and E F commenſurable in power, and D being rational, E F ſhall be alſo rational; and foraſmuch as the numbers I, and A B, and therefore the ſquares of D and E F are not to one another as a ſquare num-
b) 9. 10. ber to a ſquare number, [b] D and E F ſhall be incommenſurable in length.

c) C. 9. 10. Again, [c] as A B is to A C, ſo the ſquare of E F to the ſquare of G F: I
d) 6.10. ſay, that E G is a third Reſidual: For, [d] ſeeing that the ſquares of E F and G F being to one another as the numbers A B and A C, are commenſurable, E F and G F ſhall be commenſurable at leaſt in power, and E F being ſhewn rational, G F ſhall be alſo rational; and foraſmuch as A B and A C, and ſo likewiſe the ſquares of E F and G F, are not to one another as ſquare numbers, E F and G F ſhall be incommenſurable in length: Therefore E F and G F are rational, commenſurable in power only; and therefore G F being cut off from E F, commenſu able thereto in power only, E G ſhall be *Reſidual*, I ſay, that it is a *Third Reſidual*.

Demonſtration FOraſmnch as I is to A B, as the ſquare of D to the ſquare of E F, and as A B to A C, ſo the ſquare of E F to the ſquare of G F, in equal rate, as I to A C, ſo the ſquare of D to the ſquare of G F: Now

Now I and A C are not to one another as a square number to a square number: therefore (c) the squares of D and GF shall not be to one another as a square number to a square number: therefore D and G F are incommensurable in length: therefore neither the one nor the other, E F nor G F, is commensurable in length to the rational proposed D.

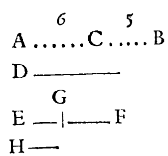
(c) 9. 10.



Now let E F be more in power then G F, by the square of H, we shall shew: as in the 86 Proposition, that H is commensurable in length to E F: Wherefore seeing that E F the whole is more in power then the agreeing G F, by the square of H, commensurable in length thereto, and that neither the one nor the other, E F nor G F, is commensurable in length to the rational proposed D, by the Definition, E G shall be a Third Residual: Therefore, &c. Which was to be done.

PROP. 89. PROBL. 22.

To finde a Fourth Residual.

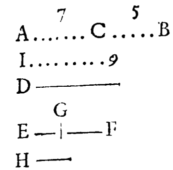


Construction HAving found two numbers A C and C B, so as A B compounded of them may not have to either of them such proportion as a square number to a square number: Let the rational D be exposed, to the which let E F be commensurable in length; E F shall be also rational, and having finished the rest as in the 86 Proposition, we shall shew (as there) that E G is a Residual; I say, that it is also a fourth Residual.

Demonstration FOr let E F be more in power then G F, by the square of H: and seeing that as A B to A C, so the square of E F to the square of G F; by conversion of reason, as A B shall be to C B, so the square of E F to the square of H. Therefore seeing that A B and C B are not to one another as square numbers, E F and H shall be incommensurable in length. Forasmuch then as the whole E F, is more in power then the agreeing G F, by the square of H, incommensurable in length thereto, and the same whole E F is commensurable in length to the rational D; E G by the Definition, shall be a fourth Residual: Therefore, &c. Which was to be done.

PROP. 90. PROBL. 23.

To finde a Fifth Residual.



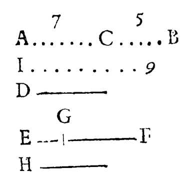
Construction HAving found two numbers A C and C B, as in the precedent Proposition: Let the same Construction be made as in the 87 Proposition; that is to say, let G F be taken commensurable in length to the rational D, &c. Therefore as in the 87 Proposition, we shall shew that E G is a Residual; I say also, that it is a Fifth Residual.

Demonstration FOr let E F be more in power then G F, by the square of H, and forasmuch as we shall shew as in the 86 Proposition, that by conversion of reason, as A B is to C B, so the square of E F is to the square of H.

Again, as in the precedent Proposition, E F and H, are incommensurable in length, and the agreeing G F is commensurable in length to the rational proposed; by the Definition, E G shall be a Fifth Residual: Therefore, &c. Which was to be done.

PROP. 91. PROBL. 24.

To finde a Sixth Residual.

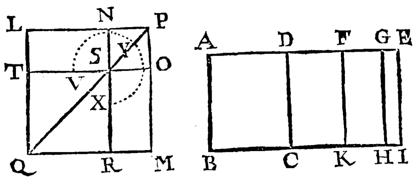


Construction HAving found three numbers A C, C B, and I, as in the 54 Proposition in such sort as A B be not to A C, nor to C B, and I be not to A B, nor to A C, as a square number to a square number: Let the rational D be exposed, and let the rest be done as in the 83 Proposition, we shall shew as there, that D and E F are incommensurable in length, and that E G is a Sixth Residual.

Demonstration FOr as in the 88 Proposition, D and G F, shall be also incommensurable in length; and therefore neither the one nor the other, E F nor G F, is commensurable in length to the rational proposed D. Now let E F be more in power then G F, by the square of H, which we shall shew to be incommensurable in length to E F, as in the 89 Proposition: Therefore seeing that the whole E F is more in power then the agreeing G F, by the square of H incommensurable in length thereto; and neither E F nor G F, is commensurable in length to the rational exposed D, by the Definition, E G shall be a Sixth Residual: Therefore, &c. Which was to be done.

PROP. 92. THEOR. 68.

If a Space A C, *be contained under a rational line* A B, *and a first Residual* A D, *the right line equal in power to the Space* A C, *is a Residual.*



Demonstration. FOr let D E be the agreeing line to A D; therefore A E and D E by the Definition of the first Residual, shall be rational, commensurable in power only: and the whole A E commensurable in length to the rational A B, and the whole A E shall be more in power then the agreeing D E, by the square of a line incommensurable in length thereto.

Let D E be divided in two equal parts in F, and let there be applied to A E a Rectangle, equal to the square of E F; that is to say, equal to the quarter of the square of D E, wanting by a square figure, to wit, under A G and G E; and forasmuch as A E is more in power then D E, by the
a) 18. 10. square of a line commensurable thereto in length, A G and G E [a] shall be
b) 16. 10. commensurable in length; wherefore [b] the one and the other shall be commensurable in length to the whole A E: but the whole A E is commen-
c) 12. 10. surable in length to the rational A B: therefore [c] as well A G as G E, is
d) 6. d. commensurable in length to A B, and therefore [d] rational: therefore having drawn G H and E I parallels to A B, meeting B C prolonged, in H
e) 20. 10. and I, [e] the one and the other Rectangle A H and G I contained under two rational lines commensurable in length, shall be rational.

Again, forasmuch as the one and the other, D F and F E, is commensurable in length to D E, and D E incommensurable in length to the rational A B, (for if it were commensurable in length, A E being also commensu-
f) 12. 10. rable in length to A B: [f] A E and D E, should be also commensurable in length, which is absurd, for they are proposed commensurable in power on-
g) 4. 10. ly,) [g] the one and the other of them, D E and F E, shall be also incommensurable in length to the rational A B.

And forasmuch as the one and the other D F and F E, is commensurable to the rational D E, they are rational; therefore as well A B and D F, as A B and F E, shall be rational, commensurable in power only: Therefore
h) 22. 10. having drawn F K parallel to A B, [h] the one and the other Rectangles D K and F I, contained under the rational, commensurable in power only, is Medial.

i) 14. 10. Now [i] let there be made the square L M equal to A H, and N O equal to G I, having one angle common with L M; to wit, L P M; therefore
k) 26. 6. [k] the squares L M and N O, shall be about one and the same Diameter; which let be P Q, and let the figure be finished as you see.

Forasmuch as the Rectangle under A G and G E is equal to the square of E F, by Construction, the three lines A G, F E, and G E, shall be proportional; and therefore A H, F I, and G I, having the same rate, F I shall be a mean proportional between A H and G I; that is to say, between the squares L M and N O, equal to those Rectangles: But L O is also a mean proportional between the same L M and N O: therefore the Rectangles F I and L O are equal: But [l] D K is equal to F I, and M N to L O; there- l) 36. 10.
fore the whole D I is equal to all the Gnomon R O, O N, N P, with the square N O: But the whole A I is equal to the squares L M and N O, by Construction, therefore the remainder A C shall be equal to the remaining square T R; therefore T S is equal in power to the Space A C: I say, that T S is a Residual.

For seeing that the spaces A H and G I are shewn rational, and to them are equal L M and N O, the same squares L M and N O shall be rational, and therefore T O and S O rational.

Again, seeing that F I is shewn medial, L O equal thereto shall be also medial: Therefore L O and N O, are incommensurable, the one being rational, and the other irrational; therefore [m] the lines T O and S O, ha- m) 1. 6.
ving the same rate as L O and N O, are [n] incommensurable in length; n) 10. 10.
therefore the remainder T S is a Residual: Therefore, &c. Which was to be demonstrated.

PROP. 93. THEOR. 69.

If a Space A C, *be contained under a rational line* A B, *and a second Residual* A D, *the right line equal in power to the same Space* A C, *is a Residual first Medial.*

Demonstration. FOr let D E be the agreeing line to A D; therefore A E and D E, by the Definition of the second Residual, shall be rational, commensurable in power only, and the agreeing D E commensurable in length to the rational A B; and lastly, A E shall be more in power then D B, by the square of a line commensurable in length thereto: Let D E be divided in two equal parts in F, and for the rest let it be done as in the precedent Proposition, [a] A G and G E shall be commensurable in a) 18. 10.
length; and therefore [b] each shall be commensurable to the whole A E; b) 15. 10.
but A E is incommensurable in length to the rational A B; (for if it were commensurable in length to A B, D E being proposed commensurable in length to the same A B: [c] A E and D E should be commensurable in length c) 12. 10.
to one another, which is absurd, being proposed commensurable in power only:) therefore [d] as well A G as G E is incommensurable in length to d) 14. 10.
A B: But A G and G E commensurable to the rational A E are rational: Therefore as well A B and A G, as A B and G E, are rational, commensurable in power onely: Therefore [e] as well the Rectangle A H as G I, con- e) 22. 10.
teined under the rational commensurable in power only, is Medial.

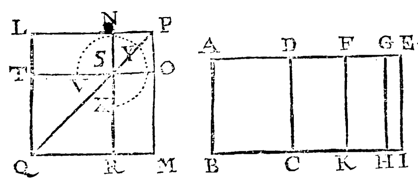
Again, as well D F as F E, being commensurable in length to D E, proposed commensurable in length to the rational A B, D F and F E, shall be commensurable in length to A B rational, as appears by the 12 Proposition, and therefore rational: therefore [f] as well the Rectangle D K as f) 20. 10.

 FI,

FI, conteined under the rational commenſurable in length is rational. And
it ſhall be ſhewn as in the 92 Propoſition, that TS is equal in power to the
Space AC. I ſay that TS is alſo a Reſidual firſt Medial.
g) 10. 10. For AG and GE, being commenſurable in length, [g] AH and GI ha-
ving the ſame rate as they ſhall be commenſurable, and therefore alſo
their equal ſquares LM and NO commenſurable: therefore the ſides TO
and SO, ſhall be at leaſt commenſurable in power. But TO and SO are
Medials, being the ſquares LM and NO equal to AH and GI (ſhewn
medials,) are Medials. And foraſmuch as FI, and therefore its equal LO,
h) 10.10. is rational, and therefore incommenſurable to the Medial NO; [h] TO
and SO having the ſame rate as they ſhall be incommenſurable in length,
and being ſhewn Medials, and commenſurable, the ſame TO and SO
ſhall be Medials, commenſurable in power only: and conteining the Re-
i) 73.10. ctangle LO, ſhewn rational, [i] TS ſhall be a firſt Reſidual: Therefore,&c.
Which was to be demonſtrated.

PROP. 94. THEOR. 70.

If a Space AC, *be conteined under a rational line* AB, *and a third Reſidual* AD, *the right line equal in power to the ſame Space is a Reſidual ſecond Medial.*



Demonſtration FOr let DE be the agreeing line to AD; therefore AE and
DE according to the Definition of the third Reſidual ſhall
be rational, commenſurable in power only; and neither AE nor DE
ſhall be comenſurable in length to the rational AB, and AE ſhall be more
in power then DE, by the ſquare of a line commenſurable in length there-
to: Let DE be divided in two equal parts in F, and let the reſt of the Con-
ſtruction be finiſhed, as in the 92 Propoſition.
a) 16.10. Again, as in the 92 Propoſition, AG and GE, ſhall be commenſurable
b) 14.10. in length to one another [a]: and therefore [b] as well AG as GE, commenſu-
rable in length to the whole AE: but AE is propoſed incommenſurable to
the rational AB: therefore as well AG as GE, ſhall be incommenſurable
in length to AB: Therefore as well AG as GE, commenſurable to AB
rational, being rational: as well AB and AG, as AE and GE, are ratio-
c) 22. 10. nal, commenſurable in power only: therefore [c] the one and the other Re-
ctangle AH and GI, ſhall be Medial.
Again, DE being propoſed incommenſurable in length to the rational
d) 14.10. AB, DF and EF commenſurable in length to DE, [d] ſhall be alſo incom-
menſurable in length to AB: therefore DF and FE, being commenſurable
to

to DE rational, as well AB and DF, as AB and FE ſhall be rational,
commenſurable in power onely; and therefore [e] as well the Rectangle DK c) 22. 10.
as FI, ſhall be Medial: and as in the 92 Propoſition, it ſhall be ſhewn
that TS is equal in power to the Space AC: I ſay, that TS is a Reſidual
ſecond Medial.
For AH and GI being ſhewn Medials, their equal ſquares LM & NO ſhall
be alſo Medial; and therefore TO and SO Medials; and [f] ſeeing that AH f) 10. 10.
and GI (having the ſame rate as AG and GE ſhewn commenſurable,) are
commenſurable, the ſquares LM and NO ſhall be alſo commenſurable;
therefore TO and SO are commenſurable at leaſt in power.
But foraſmuch as AE and DE are commenſurable in power only; that
is to ſay, incommenſurable in length; and GE is ſhewn commenſurable in
length to AE, and FE commenſurable in length to DE, GE and FE ſhall
be incommenſurable in length; and therefore [g] GI and FI, having the g) 10.10.
ſame rate as GE and FE; that is to ſay, their equals NO and LO are in-
commenſurable. Wherefore TO and SO having the ſame rate as LO and
NO are incommenſurable in length, and being ſhewn Medials, and com-
menſurable, they ſhall be Medials, commenſurable in power only; and
conteining LO a Medial: (for ſeeing that LO is equal to FI, as appears by
the 92 Propoſition, which is ſhewn medial, LO ſhall be alſo a medial;) TS
ſhall be a Reſidual ſecond Medial: Therefore, &c. Which was to be
demonſtrated.

PROP. 55. THEOR. 71.

If a Space AC *be conteined under a rational line* AB, *and a fourth Reſidual* AD, *the right line equal in power to the ſaid Space* AC, *is a line Minor.*

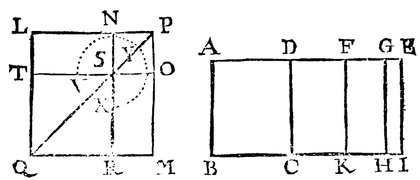
Demonſtration FOr let DE be the agreeing line to AD; therefore AE and
DE ſhall be rational, commenſurable in power only, by the
Definition of the fourth Reſidual: and AE commenſurable in length to
AB rational: and the ſame AE ſhall be more in power then DE, by the
ſquare of a line incommenſurable in length thereto: Let DE be divided in
two equal parts in F, and the reſt done as is before ſhewn: Therefore AG
and GE [a] ſhall be incommenſurable in length; ſeeing that AE is more in a) 19.10.
power then DE, by the ſquare of a line incommenſurable in length there-
to: And to AE is applyed a Rectangle under AG and GF, equal to a
quarter of the ſquare of DE, wanting a ſquare figure, and AE being ra-
tional, and commenſurable in length to the rational AB, [b] the Rectangle b) 20. 10.
AI ſhall be rational.
Again, DE being rational, and incommenſurable in length to AB ratio-
nal: DI [c] and therefore its half FI, ſhall be medial. And moreover AG c) 22. 10.
and GE being incommenſurable in length, AH and GL [d] having the d) 10.10.
ſame rate, ſhall be incommenſurable; and as in the 92 Propoſition, ſo here
it ſhall be ſhewn that TS is equal in power to the Space AC: I ſay, that
TS is a Minor.
For the compound of the ſquares LM and NO, deſcribed of TO and
SO, is equal to the Rectangle AI, by Conſtruction: but the ſame is
ſhewn

shewn rational; therefore the compound of the squares of T O and S O shall be rational: In like manner F I being shewn Medial, L O conteined under T O and S O, equal thereto, shall be Medial.

Lastly, A H and G I, being shewn incommensurable, their equal squares L M and N O, shall be incommensurable, and therefore T O and S O incommensurable in power: Therefore T O and S O being incommensurable in power, and the compound of their squares being rational, and the Rectangle under them medial, the remainder T S shall be a Minor: Therefore, &c. Which was to be demonstrated.

PROP. 96. THEOR. 72.

If a Space A C, *be conteined under a rational line* A B, *and a fifth Residual* A D, *the right line equal in power to the said Space* A C, *is a line making with a Space rational, a whole medial.*



Demonstration. Let D E agree to A D, by the Definition of the fifth Residual, A E and D E shall be rational, commensurable in power only, and D E commensurable in length to the rational A B, so A E shall be more in power then D E, by the square of a line incommensurable in length thereto.

Let D E be divided in two equal parts in F, and do the rest as is before shewn. Therefore as in the precedent Proposition, A G and G E shall be incommensurable in length; and forasmuch as A E is rational, incommensurable in length to A B rational, as is said in the 93 Proposition, A I [a] shall be Medial.

a) 22.10.

Also, seeing that D E is rational, commensurable in length to A B rational, [b] the Rectangle D I, and therefore its half F I, is rational.

b) 20.10.

Again, as in the precedent Proposition, A H and G I are incommensurable, and as is shewn in the 92 Proposition, T S equal in power to the Space A C, I say that T S is the line with which a Space rational makes a whole Medial.

Forasmuch as it hath been shewn that A I is a Medial, the compound of the squares L M and N O, of the lines T O and S O, equal thereto is medial, and F I being a rational, as we have shewn, the Rectangle L O conteined under T O and S O, equal thereto, is also rational, and T O and S O are incommensurable in power, as is shewn in the precedent: therefore seeing that the compound of their squares is medial, and the Rectangle under them rational, the remainder [c] T S is the line with which a rational makes a whole Medial: Therefore, &c. Which was to be demonstrated.

c) 78.10.

PROP.

PROP. 97. THEOR. 73.

If a Space A C *be conteined under a rational line* A B, *and a sixth Residual* A D, *the right line* T S, *equal in power to the said Space* A C, *is that which with a Rectangle Medial makes a whole Medial.*

Demonstration. For let D E agree to the same A D: therefore A E and D E by the Definition of the Sixth Residual, are rational, commensurable in power only, and neither one nor the other is commensurable in length to the rational A B; and finally, A E shall be more in power then D E, by the square of a line incommensurable in length thereto.

Let D F be divided in two equal parts in F, and let the rest be done as before is shewn: Therefore as in the 95 Proposition, A G and G E shall be incommensurable in length.

And seeing that as well A E as D E is rational, incommensurable in length to the rational A B: as well [a] A I as D I, and therefore F I the halfe of D I, shall be medial, and A H aud G I as in the 95 Proposition, shall be incommensurable. a) 22.10.

And forasmuch as A E and D E are commensurable in power only, [b] A I and D I which are in the same rate, are incommensurable; and D I and F I being commensurable, F I [c] shall be incommensurable to A I: Now we shall shew as in the 92 Proposition, that T S is equal in power to A C: and I say that T S is that, which with a Rectangle Medial makes a whole Medial. b) 10. 10. c) 14.10.

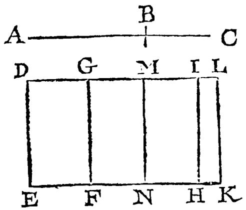
Forasmuch as A I is shewn medial, the compound of the squares L M and N O, of the right lines T O and S O, equal thereto, is Medial, also F I being shewn Medial, S O his equal conteined under T O and S O, is a Medial.

Again, F I being incommensurable to A I, as is shewn L O conteined under T O and S O, shall be incommensurable to the compound of the squares of T O and S O: seeing that L O is equal to F I, and the compound of the squares of T O and S O, equal to A I. Lastly, T O and S O are incommensurable in power, as is shewn in the 95 Proposition: Therefore seeing that the compound of their squares is medial, and the Rectangle under them Medial, and incommensurable to the compound of their squares, the [d] remainder T S is that which with a Medial makes a whole Medial: Therefore, &c. Which was to be demonstrated. d) 79.10.

PROP.

PROP. 98. THEOR. 74.

The ſquare of a Reſidual A B, *applyed to a rational line* D E, *makes the breadth* D G *a firſt Reſidual.*



Demonſtration. FOr to D E let there be applyed the Rectangle D H, equal to the ſquare of A C, and to H I let there be applyed I K, equal to the ſquare of B C, in ſuch ſort as the whole D K may be equal to the compound of the ſquares of A C and C B.

a) 7. 2. And [a] foraſmuch as the compound of the ſquares of A C and B C, is equal to twice the Rectangle under A C and B C, and to the ſquare of A B, if the ſquare of A B and the Rectangle D F be taken away, the Rectangle G K ſhall remaine equal to twice the Rectangle under A C and B C: therefore G L being divided into two equal parts in M, and M N being drawn parallel to D E; M K ſhall be equal to the Rectangle under A C and B C.

b) 16. 10. And foraſmuch as A C and B C are rationals; their ſquares ſhall be rational, and therefore commenſurable: Therefore [b] ſeeing that the compound of the ſquares of A C and B C is commenſurable to each of them, the ſaid compound, that is to ſay, his equal Rectangle D K ſhall be rational;

c) 21. 10. which being applyed to the rational D E, [c] D L ſhall be rational, commenſurable in length to D E.

d) 23. 10. Again, ſeeing that A C and B C are rational, commenſurable in power only, the Rectangle under them, and his double G K is Medial, which being applyed to the rational G F, G L [d] ſhall be rational, incommenſurable in length to C F; that is to ſay, to D E: And foraſmuch as D K rational, and G K Medial, that is to ſay irrational, are incommenſurable, D L

e) 10. 10. and G L [e] having the ſame rate as D K and G K, ſhall be incommenſurable in length: Therefore being ſhewn rational, they ſhall be rational,

f) 74. 10. commenſurable in power only, and therefore [f] the remainder D G ſhall be a Reſidual. I ſay it is a firſt Reſidual.

Foraſmuch as the Rectangle under A C and B C, that is to ſay M K, is a mean proportional between the ſquares of A C and B C, that is to ſay, between D K and I K: D H, M K, and I L, are continually proportional, and the lines D I, M L, and I L, continually proportional, being in the ſame

g) 17. 6. rate. Wherefore [g] the Rectangle under D I and I L, is equal to the ſquare of M L; that is to ſay, a quarter of the ſquare of G L.

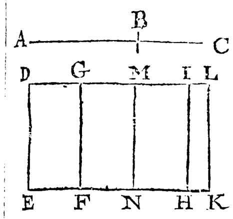
h) 16. 10. And foraſmuch as the ſquares of A C and B C, or their equals D H and I K are commenſurable: [h] D I and I L, being in the ſame rate, ſhall be commenſurable in length; therefore D L and G L being unequal, and to the greateſt there be applyed a Rectangle uuder D I and I L, equal to the quarter of the ſquare of G L the leaſt, wanting a ſquare figure, and D I being ſhewn commenſurable in length to I L, D L ſhall be more in power then G L, by the ſquare of a line commenſurable thereto in length: Wherefore

i) 18. 10. D G being ſhewn Reſidual, and that the whole D L [i] is more in power then G L, his agreeing line by the ſquare of a line commenſurable in length thereto;

thereto; and that the ſame whole D L is commenſurable in length to the rational D E, by the Definition, D G ſhall be a firſt Reſidual: Therefore the ſquare, &c. Which was to be demonſtrated.

PROP. 99. THEOR. 75.

The ſquare of a Reſidual firſt medial A B, *applyed to a rational line* D E, *makes the breadth* D G, *a ſecond Reſidual.*



Demonſtration FOr let it be done as in the precedent Propoſition, in ſuch ſort as D H and I K may be equal to the ſquares of A C and B C, and G K equal to twice the Rectangle under A C and B C, and therefore M K equal to once the ſame Rectangle; foraſmuch then as A C and B C are Medials, commenſurable in power only, and their ſquares, that is to ſay, their equals D H and I K medials: and commenſurable: therefore [a] the whole D K ſhall be commenſurable to each of them, [b] therefore medial; And D K being applyed to the rational D E, D L ſhall be rational incommenſurable in length to D E.

a) 16. 10.

b) C. 24. 10.

c) 22. 10.

Again, foraſmuch as the Rectangle under A C and B C is propoſed rational, the double thereof G K ſhall be alſo rational, which being applyed to the rational D E, [d] G L ſhall be rational, commenſurable in length to D E.

d) 21. 10.

But foraſmuch as D K Medial, that is to ſay irrational, and G K rational, are incommenſurable in length, and being ſhewn rational, [e] D L and L G ſhall be rational, commenſurable in power only; and therefore [f] the remainder D G a Reſidual: I ſay that it is a ſecond Reſidual: For (as in the precedent Propoſition) we ſhall ſhew that the whole D L is more in power then the agreeing line G L, by the ſquare of a line commenſurable thereto in length: therefore G L being ſhewn commenſurable in length to the rational D E, the ſame D G by the Definition, ſhall be a ſecond Reſidual: Therefore, &c. Which was to be demonſtrated.

e) 10. 10.

f) 74. 10.

PROP. 100. THEOR. 76.

The ſquare of a Reſidual ſecond medial A B, *applyed to a rational line* D E, *makes the breadth* D G *a third Reſidual.*

Demonſtration LEt there be done as is before ſhewn; and it ſhall be ſhewn as in the precedent Propoſition, that D K is a Medial, and therefore D L rational incommenſurable in length to D E: and foraſmuch as the Rectangle under A C and B C is medial, and therefore the double thereof, to wit G K, G L [a] ſhall be alſo rational, incommenſurable in length to D E.

a) 23. 10.

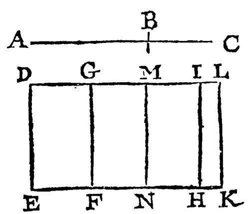
But

But foraſmuch as A C and B C are incommenſurable in length, and as A C to B C, ſo the ſquare of A C to the Rectangle under A C and B C;
b) 10. 10. [b] the ſquare of A C ſhall be incommenſurable to the Rectangle under A C and B C: But [c] the compound of the ſquares of A C and B C is com-
c) 16. 10. menſurable to the ſquare of A C, thoſe ſquares being commenſurable, deſcribed of lines commenſurable in power, and twice the Rectangle under A C and B C is commenſurable to the ſaid Rectangle under A C and B C: Therefore the compound of the ſquares of A C and B C, that is to ſay D K, is incommenſurable to twice the Rectangle under A C and B C, that is to
d) 10. 10. ſay G K: and therefore [d] D L and G L being in the ſame rate as D K and G K, are incommenſurable in length, and are ſhewn rational; therefore D L and G L are rational, commenſurable in power only; and therefore
e) 74. 10. [e] D G the remainder is a Reſidual: I ſay it is a third Reſidual.

For we ſhall ſhew as in the 98 Propoſition, that the whole D L is more in power then G L, its agreeing line, by the ſquare of a line commenſurable thereto in length: Therefore ſeeing that neither D L nor G L are commenſurable in length to the rational D E, as hath been ſhewn, D G ſhall be a third Reſidual, by the Definition: Therefore the ſquare, &c. Which was to be demonſtrated.

PROP. 101. THEOR. 77.

The ſquare of a line minor A B, *applyed to a rational line* D E, *makes the breadth* D G *a fourth Reſidual.*

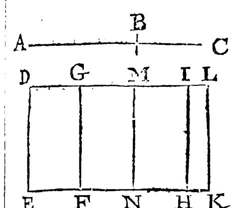


Demonſtration FOraſmuch as the Compound of the ſquares of A C and B C, that is to ſay D K, their equal, is rational; D L [a] ſhall be rational,
a) 21. 10. commenſurable in length to D E; and ſeeing that the Rectangle under A C and B C, and therefore the double
b) 23. 10. thereof G K is medial: G L [b] ſhall be rational, incommenſurable in length to D E.

Again, foraſmuch as D K rational, and G K irrational; to wit Medial, are incommenſurable; D L and G L [c] being in the ſame rate, are incom-
c) 10. 10. menſurable in length; and being ſhewn rational, D L and G L are rational, commenſurable in power only: therefore [d] D G the remainder is a Reſidu-
d) 74. 10. al: I ſay that it is a fourth Reſidual. For A C and B C being commenſurable in power, their ſquares or their equal Rectangles D H and I K, ſhall be
e) 10. 10. incommenſurable; and therefore [e] D I and I L incommenſurable in length: And ſeeing that the Rectangle under D I and I L is equal to the ſquare of M L, that is to ſay, to a quarter of the ſquare of G L, as is demonſtrated
f) 19. 10. in the 98 Propoſition, D L [f] ſhall be more in power then G L, by the ſquare of a line incommenſurable in length thereto, ſeeing that to D L the greateſt, there is applyed the Rectangle under D I and I L, equal to a quarter of the ſquare of G L the leaſt, wanting a ſquare figure, and dividing D L in parts incommenſurable in length D I and I L: Therefore ſeeing that D L the whole is more in power then the agreeing line G L, by the ſquare of a line incommenſurable in length thereto, and the whole D L is ſhewn commenſurable in length to the rational D E: by the Definition D G ſhall be a fourth Reſidual: Therefore, &c. Which was to be demonſtrated.

PROP. 102. THEOR. 78.

The ſquare of a line A B, *which with a Space rational* G K, *makes the whole Medial, applyed to a rational line* D E, *makes the breadth* D G, *a fifth Reſidual.*



Demonſtration LEt it be done as is before ſhewn; foraſmuch as the compound of the ſquares of A C and B C, or his equal D K is Medial: D L [a] ſhall be rational, incommenſurable in a) 23. 10.
length to D E; and foraſmuch as the Rectangle under A C and B C, or the double thereof G K is rational, G L [b] ſhall be rational, commenſurable b) 21. 10.
in length to D E.

And foraſmuch as D K Medial, or irrational, and G K rational, are incommenſurable; [c] D L and G L having the ſame rate, ſhall be incom- c) 10. 16.
menſurable in length: But they are ſhewn rational: Therefore D L and G L are rational, commenſurable in power only: therefore [d] the remainder d) 74. 10.
D G is a Reſidual: I ſay it is a fifth Reſidual.

For as in the precedent, it ſhall be ſhewn that D L the whole is more in power then G L, the agreeing line, by the ſquare of a line incommenſurable in length thereto: Therefore ſeeing that the agreeing line G L, is ſhewn commenſurable in length to the rational D E, by the Definition, D G ſhall be a fifth Reſidual: Therefore, &c. Which was to be demonſtrated.

PROP. 103. THEOR. 79.

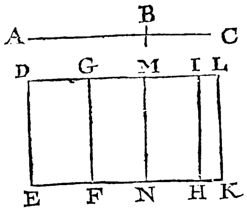
The ſquare of a line A B, *making with a Space medial, the whole Medial, applyed to a rational line* D E, *makes the breadth* D G, *a ſixth Reſidual.*

Demonſtration LEt the ſame be done as in the former; foraſmuch as, as well the compound of the ſquares of A C and B C, or D K his equal, as the Rectangle under A C and B C: and therefore the double thereof G K is medial, as wel [a] D L as G L, ſhall be rational, in- a) 23. 10.
commenſurable in length to D E, and the Rectangle under A C and B C, and

b) 14. 10. and the double thereof G K commensurable; G K [b] shall be incommensurable to the said compound, that is to say, to D K. And therefore [c] D L and G L having the same rate as D K and G K, shall be incommensurable in length. And they are shewn rational: Therefore D L and G L are rational, commensurable in power only; therefore [d] D G is a Residual: I say it is a sixth Residual.

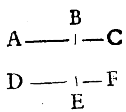
c) 10. 10.

d) 74. 10.



For as in the 101 Proposition, we shall shew that the whole D L is more in power then the agreeing line G L, by the square of a line incommensurable in length thereto: Therefore seeing that neither D L nor G L is commensurable in length to the rational D E; by the Definition, D G shall be a sixth Residual. Therefore, &c. Which was to be demonstrated.

PROP. 104. THEOR. 80.



The right line D E *commensurable in length to a Residual* A B, *is also a Residual, and of the same order as* A B.

Demonstration For let it be as A B to D E, [a] so B C to E F; therefore [b] the whole A C shall be to the whole D F, as A B to D E; or B C to E F: forasmuch then as A B and D E are proposed commensurable in length; [c] A C and D F, and B C and E F, shall be also commensurable in length; and forasmuch as A C and B C are rational, their commensurables D F and E F shall be rational: Again, forasmuch as A C is to D F, as B C is to E F; and alternately, as A C to B C, so D F to E F: and A C and B C are commensurable in power only: Therefore D F and E F shall be also commensurable in power only, and being shewn rational, the remainder D E is a Residual: I say that it is of the same order as A B. For first let A C be more in power then B C, by the square of a line commensurable in length thereto; that being [d] D F shall be more in power then E F, by the square of a line commensurable in length thereto: Therefore if A C be commensurable in length to the rational exposed, to the end that A B may be a first Residual, D F shall be also commensurable in length to the rational exposed; seeing that as well the rational exposed, as D F, is commensurable in length to the same A C; Therefore, by the Definition, D E shall be a first Residual, to wit, of the same order as A B. But if B C be commensurable in length to the rational, in the same manner E F shall be commensurable in length to the rational; therefore the one and the other A B and D E, by the Definition shall be a second Residual.

a) 12. 6.
b) 12. 5.
c) 10. 10.
d) 15. 10.

Lastly, if neither the one or the other A C nor B C, be commensurable in length to the rational exposed; [e] also neither the one nor the other D F nor E F shall be commensurable in length to the same rational: Therefore as

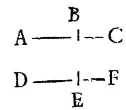
e) 14. 10.

as well the one as the other A B or D E, shall be a third Residual by the Definition.

Now let A C be more in power then B C, by the square of a line incommensurable in length thereto, that being [f] D F shall be more in power then E F, by the square of a line which shall be incommensurable in length thereto. Wherefore we shall shew as before, that D E is a fourth, fifth, or sixth Residual. Therefore, &c. Which was to be demonstrated.

f) 15. 10.

PROP. 105. THEOR. 81.



The right line D E, *commensurable to a Medial Residual* A B, *is also a Residual Medial, and of the same order as* A B.

Demonstration For as A B is to D E, [a] so let B C be to E F: Therefore the whole A C shall be to the whole D F, as A B to D E, and B C to E F: forasmuch then as A B and D E are proposed commensurable, [b] B C and E F, and A C and D F shall be also commensurable, and A C and B C being Medials D F and E F [c] commensurable to them, shall be also Medials.

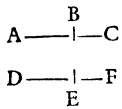
a) 12. 6.
b) 10. 10.
c) 24. 10.

Again, A C being to D F as B C to E F, and alternately, as A C to B C, so D F to E F; but A C and B C are commensurable in power only; Therefore [d] D F and E F shall be commensurable in power only: Wherefore D F and E F being shewn Medials, the remainder [e] D E shall be a Residual Medial: I say that it is of the same order as AB, for seeing that as A C to BC, so DF to E F: and as A C to B C, so the square of A C to the Rectangle under A C and B C, and as D F to E F, so the square of D F to the Rectangle under D F and E F: Therefore as the square of A C to the Rectangle under A C and B C, so the square of D F to the Rectangle under D F and E F: and alternately, as the square of A C to the square of D F: so the Rectangle under A C and B C, to the Rectangle under D F and E F: therefore the square of A C being commensurable to the square of D F, A C and D F being shewn commensurable, [f] the Rectangle under A C and BC shall be commensurable to the Rectangle under D F and E F: therefore if the Rectangle under A C and B C be rational, in such sort as A B may be a first Residual Medial, the Rectangle under D F and E F, commensurable thereto, shall be also rational; and therefore [g] D E shall be a Residual first Medial: but if the Rectangle under A C and B C be Medial, in such sort as A B may be a second Residual Medial: the Rectangle under D F and E F, commensurable thereto, [h] shall be also Medial: Therefore [i] D E shall be a second Residual Medial: Therefore, &c. Which was to be demonstrated.

d) 10. 10.
e) 75, 76. 10.
f) 10. 10.
g) 75. 10.
h) 24. 10.
i) 76. 10.

PROP.

PROP. 106. THEOR. 82.



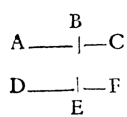
The right line D E, *commenſurable to a line minor* A B, *is alſo a line minor.*

Demonſtration. FOr let be done as is before ſhewn, in ſuch ſort as A B and B C, may have the ſame rate to D E and E F, as A C to D F, as in the former Propoſition, D F and E F ſhall be commenſurable to A C and B C, be it in length and power, or in power alone; and ſeeing that as A C is to D F, ſo B C is to E F; and alternately, as A C to B C, ſo D E to E F, [a] as the ſquare of A C to the ſquare of B C, ſo the ſquare of D F to the ſquare of E F; and in compounding, as the compound of the ſquares of A C and B C, to the ſquare of B C, ſo the compound of the ſquares of D F and E F, to the ſquare of E F: and by permutation, as the compound of the ſquares of A C and B C to the compound of the ſquares of D F and E F, ſo the ſquare of B C to the ſquare of E F: But the ſquare of B C is commenſurable to the ſquare of E F, B C and E F being ſhewn commenſurable: Therefore [b] the compound of the ſquares of A C and B C is commenſurable to the compound of the ſquares of D F and E F: But the compound of the ſquares of A C and B C is propoſed rational; Therefore [c] the compound of the ſquares of D F and E F is rational.

Again, as is ſhewn in the precedent Propoſition, the Rectangle under A C and B C is commenſurable to the Rectangle under D E and E F: but the Rectangle under A C and B C is propoſed Medial; therefore [d] the Rectangle under D F and E F, ſhall be alſo Medial; and ſeeing that as A C is to B C, ſo D F is to E F: and A C and B C are incommenſurable in power, D F and E F [e] ſhall be alſo incommenſurable in power: therefore D F and E F being incommenſurable in power, and making the compound of their ſquares rational, and the Rectangle under them Medial: [f] D E ſhall be a Minor: Therefore, &c. Which was to be demonſtrated.

a) 23. 6.
b) 10. 10.
c) 9. d.
d) C. 24. 10.
e) 10. 10.
f) 77. 10.

PROP. 107. THEOR. 83.



The right line D E, *commenſurable to a line* A B, *the which with a Space rational, makes the whole medial, the ſame is alſo a line making with a Space rational a whole medial.*

Demonſtration. FOr having made the Conſtruction as before, it ſhall be ſhewn as in the precedent Propoſition, that the compound of the ſquares of A C and B C is commenſurable to the compound of the ſquares of D F and E F, therefore the one being propoſed medial, [a] the other ſhall be alſo Medial.

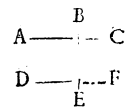
a) C. 24. 10.

Again,

Again, (as in the 105 Propoſition,) it ſhall be ſhewn that the Rectangle under A C and B C is commenſurable to the Rectangle under D F and E F; therefore the one being propoſed rational, [b] the other ſhall be alſo rational: And laſtly, as is ſhewn in the precedent Propoſition, D F and E F are incommenſurable in power; Wherefore D F and E F being incommenſurable in power, and making the compound of their ſquares medial, and the Rectangle under them rational: [c] D E ſhall make with a Space rational, a whole medial. Which was to be demonſtrated.

b) 9. d.
c) 78. 10.

PROP. 108. THEOR. 84.



The right line D E, *commenſurable to the line* A B, *the which with a Space medial, makes a whole medial, the ſame is alſo a line making with a Space medial a whole medial.*

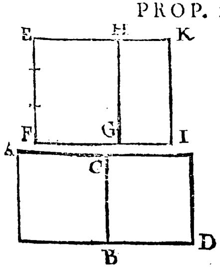
Demonſtration. FOr having made the Conſtruction as before, we ſhall ſhew as in the 106 Propoſition, that the compound of the ſquares of A C and B C, is commenſurable to the compound of the ſquares of D F and E F; therefore the compound of the ſquares of A C and B C being propoſed Medial; [a] that of the ſquares of D F and E F ſhall be alſo medial.

Again, (as in the 105 Propoſition,) it ſhall be ſhewn that the Rectangle under A C and B C is commenſurable to the Rectangle under D F and E F; therefore the one being Medial, the other ſhall be alſo Medial. And as in the 106 Propoſition, D F and E F ſhall be incommenſurable in power.

Laſtly, foraſmuch as the compound of the ſquares of D F and E F is commenſurable to the compound of the ſquares of A C and B C, as is ſaid, and the Rectangle under D F and E F is commenſurable to the Rectangle under A C and B C. But the compound of the ſquares of A C and B C, and the Rectangle under A C and B C are propoſed incommenſurable: Therefore the compound of the ſquares of D F and E F, and the Rectangle under D F and E F ſhall be incommenſurable: Wherefore D F and E F being incommenſurable in power, and the compound of their ſquares Medial, and incommenſurable to the compound of their ſquares; [b] D E makes with a Space Medial a whole Medial: Therefore, &c. Which was to be demonſtrated.

a) C. 24. 10.
b) 79. 10.

PROP. 109. THEOR. 85.



If a Space medial C D, *be cut from a rational* A D, *the right line which is equal in power to the reſt of the Space* A B, *is one of theſe two Irrationals; to wit, a Reſidual, or Minor.*

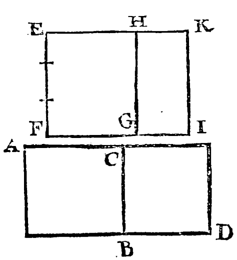
Demon-

a) 45. 1. *Demonstration.* FOr let the rational E F be exposed, [a] to which let there be applyed the Rectangle E G equal to A B, and to G H the Rectangle H I, equal to A B, and to G H the Rectangle H I, equal to C D, in such sort as the whole E I may be equal to the whole A D; forasmuch
b) 21. 10. then as E I equal to A D rational is rational, [b] E K shall be rational, commensurable in length to E F rational.

Again, forasmuch as H I equal to C D medial is medial: H K shall be rational, incommensurable in length to E F. Wherefore E K being commensurable in length to E F: But H K incommensurable in length to the
c) 23. 10. same E F, [c] E K and H K shall be incommensurable in length, and being rational, they shall be rational, commensurable in power only; and there-
d) 74. 10. fore [d] the remainder E H is a Residual, and H K an agreeing line thereto: Therefore E K is more in power then H K, by the square of a line which is thereto commensurable in length, or incommensurable.

If E K be more in power then H K, by the square of a line which is commensurable in length thereto, the whole E K being also shewn commensurable in length to E F rational: by the Definition, E H shall be a first Residual: Wherefore the line equal in power to the Space E G, conteined under E F rational, and the first Residual E H; that is to say, the Space A B, e-
e) 92. 10. qual to the same, [e] is a Residual: But if E K be more in power then H K, by the square of a line which shall be incommensurable thereto in length, The whole E K being shewn commensurable in length to E F rational, according to the Definition, E H shall be a fourth Residual; Wherefore the line equal in power to the Space E G, conteined under E F rational, and
f) 95. 10. E H a fourth Residual; that is to say, the Space A B equal thereto, [f] is a Minor: Therefore, &c. Which was to be demonstrated.

PROP. 110. THEOR. 86.

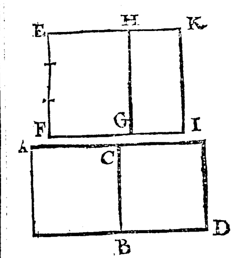


If a Superficie rational C D, *be cut from a medial* A D, *they make two other irrationals; to wit, either a Residual first medial, or a line making with a Space rational a whole medial.*

Demonstration. LEt be done the same Construction as before:
a) 23. 10. Forasmuch as E I equal to the Medial A D, is medial, E K [a] shall be rational, incommensurable in length to E F.
b) 21. 10. And forasmuch as H I equal to the rational C D is rational: H K [b] shall be rational, commensurable in length to E F: Therefore H K being commensurable in length to E F: but E K incommensurable in length to the
c) 13. 10. same E F, H K and E K [c] shall be incommensurable in length, and are rationals, they are then rational, commensurable in power only: Therefore
d) 74. 10. [d] the remainder E H is a Residual, and H K an agreeing line thereunto; there-

therefore E K is more in power then H K, by the square of a line commensurable thereto in length, or incommensurable, if commensurable, and the agreeing line H K being shewn commensurable in length to E F rational, by the Definition, E H shall be a second Residual: Therefore the line equal in power to the Space E G, conteined under E F rational, and E H, a second Residual, that is to say, A B equal thereto, [e] is a Residual first medial: But if E K be more in power then H K, by the square of a line c) 93. 10.
incommensurable in length thereto, the agreeing line H K being shewn commensurable in length to E F rational, E H shall be a fifth Residual, by the Definition: Therefore the line equal in power to the Space E G, conteined under E F rational, and the fifth Residual E H, that is to say A B, equal thereto, is a line [f] which with a rational makes a whole Medial: f) 96. 10.
Therefore, &c. Which was to be demonstrated.

PROP. 111. THEOR. 87.



If a superficies medial C D, *be cut from a superficies medial* A D, *incommensurable to the whole, there is made two other irrationals; to wit, a Residual second medial, or a line making with a superficies medial, a whole medial.*

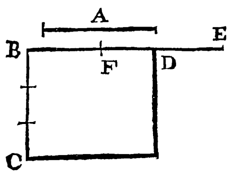
Demonstration. FOr having made the same Construction as before: Forasmuch as E I and H I, equal to the Medials A D and C D are Medials; E K and H K [a] shall be rational, incommensurable in length to a) 23. 10.
E F; and A D and C D, that is to say, E I and H I being proposed incommensurable, [b] E K and H K having the same rate, shall be incommensurable b) 10. 10.
in length, and are rational; therefore they are rational, commensurable in power only: therefore [c] E H remaining is a Residual, and H K an agreeing c) 74. 10.
line thereto; therefore E K is more in power then H K, by the square of a line commensurable thereto in length, or incommensurable.

If by the square of a line which may be commensurable thereto in length; both the one and the other E H and H K being shewn not commensurable in length to E F rational, according to Definition, E H shall be a third Residual. Wherefore the line equal in power to the Space E G, conteined under the rational E F, and the third Residual E H; that is to say A B equal thereto, is a Residual second Medial: But if E K be more in power then H K, by the square of a line incommensurable in length thereto, neither the one nor the other E K nor H K, being commensurable in length to E F rational, as is demonstrated by the Definition, E H shall be a sixth Residual: Therefore the line equal in power to E G, conteined under E F rati-

onal,

onal, and E H a sixth Residual; that is to say, A B equal thereto, is that which with a Space Medial makes a whole Medial: Therefore, If, &c. Which was to be demonstrated.

PROP. 112. THEOR. 88.



The line A, *called* Apotome *or* Residual, *is not the same as the line of two names, or* Binomial.

Demonstration. FOr let A be a Binomial (if possible) and having exposed the rational B C,
a)45. 1. a let there be applied to B C the Rectangle C D, equal to the square of A; Forasmuch then as A is a Residual, the breadth B D
b) 98.10. b shall be a first Residual: Let then D E be an agreeing line thereto; Therefore by the Definition of the first Residual, B E and D E shall be Rational, commensurable in power only, and B E shall be more in power then D E, by the square of a line which shall be commensurable in length thereto; and B E shall be commensurable in length to
c) 61.10. B C rational: Again A being also proposed to be a Binomial; c the same breadth B D shall be a first Binomial: Let B F be his greatest name: Therefore by the Definition of the first Binomial B F and F D shall be rational, commensurable in power only, and B F shall be more in power then F D, by the square of a line commensurable in length thereto, and B F shall be commensurable in length to the rational B C; there-
d) 12.10. fore as well B E, as B F being commensurable in length to B C; d B E and B F shall be also commensurable in length to one another: Wherefore the whole B E, being commensurable in length to his part B F, the same B E shall be also commensurable in length to his part B F, the same B E shall be also commmensurable in length to the other part F E; and therefore B E being Rational, F E shall be also Rational.

And forasmuch as the two B E and F E, are commensurable in length, B E is incommensurable in length to D E (B E and D E being Rational,
e) 14. 10. commensurable in power only,) e the remainder F E shall be incommensurable in length to the same D E: But as well F E as D E, is shewn Rational: Therefore F E and D E are Rational, commensura-
f) 74.10. ble in power only: Wherefore the remainder F D, f is a Residual, and therefore Irrational. But it is also shewn Rational, which is absurd: Therefore, &c. Which was to be demonstrated.

COROL-

COROLLARIE.

From these things demonstrated, may be gathered, that the Residual, and the other Irrationals following the same Residual, are neither Medials, nor the same to one another.

For the square of a Medial *applyed to a* Rational, *makes*
a) 23. 10. a *the breadth rational, incommensurable in length to the* Rational *proposed.*

But the square of the Residual *applyed to a rational line,*
b) 98.10. *makes* b *the breadth, a first* Residual.

And the square of the Residual *first* Medial, *applyed to a*
c) 99.10. *rational line,* c *makes the breadth a second* Residual.

But the square of the Residual *second* Medial, *applyed to*
d) 100. 10. *a rational line,* d *makes the breadth a third* Residual.

Furthermore the square of the Minor applyed to a rational
e) 101.10. *line,* e *makes the breadth a fourth* Residual.

But the square of a line, the which with a space rational, makes a whole Medial, *applyed to a rational line,* f *makes the*
f) 102.10. *breadth a fifth* Residual.

Lastly, the square of a line, which with a Space Medial *makes a whole* Medial, g *applyed to a rational line, makes*
g) 103.10. *the breadth a sixth* Residual.

Therefore these breadths being different from the breadth of the Medial, *and to one another; I say, from the breadth* Medial, *being the one is rational, and the others irrational: But to one another; forasmuch as they are not* Residuals *of the same order: it is manifest that the* Residual *and the other Irrationals following do differ among themselves, and from the* Medial.

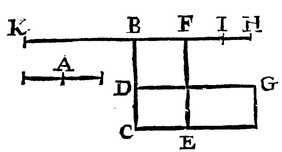
But forasmuch as it is shewn in this Theorem, that the Residual *is not the same as the* Binomal; *and that the squares of the* Residual, *and of the other five Irrationals following, applyed to a rational line, makes the breadths a First, Second, Third, Fourth, Fifth, and Sixth* Residual; *but the square of the* Binomial, *and the other five Irrationals following applyed to a rational line makes the breadth a First, Second, Third, Fourth, Fifth, and Sixth* Binomial. *It is manifest that the breadths of the* Residual, *and the other five Irrationals following, are not the same as the breadths of the* Binomials, *and the five other Irrationals following, forasmuch as not any*

Re-

Residual, *is the same as any* Binomial: *Therefore the* Residual *and the others following, do differ from the* Binomial, *and the others following: Wherefore as well those as the others, do differ from the* Medial, *in such sort as any whatsoever rational line being proposed, there will be* 13 *Irrational lines differing from one another, of which we have discoursed hitherto, and they are these following.*

1 The Medial line.
2 The Binomial line, or line of two names.
3 The first Bimedial.
4 The second Bimedial.
5 The line Major.
6 The line equal in power to a Rational and a Medial.
7 The line equal in power to two Medials.
8 The Residual, or Apotome.
9 The Residual first Medial.
10 The Residual second Medial.
11 The line Minor.
12 The line making with a Superficie Rational, a whole Medial.
13 The line making with a Superficie Medial, a whole Medial.

PROP. 113. THEOR. 89.



The square of a rational line A, *being applyed to a Binomial line* B C, *make the breadth* B F *Residual, of which the names are commensurable, and proportional to the names* B D *and* D C, *of the Binomial* B C, *and over and above the Residual* B F, *is of the same order as the Binomial* B C.

a) 45. 1. *Demonstration* For [a] to D C the lesser name, let there be applyed C G, equal to the square of A; and therefore to B E, making D G the breadth; and let B H be equal to D G and B E, C G being equal, as B C shall be to D C, so D G, or his equal B H, to B F, and in dividing, as

as B D to D C, so H F to F B: But B D is greater then D C; therefore H F shall be greater then F B. Let F I be equal to F B, and let it be as H I to I F, so F B to B K: Therefore in compounding H F shall be to I F, or to his equal F B, as F K to B K: But as H F to B F, so B D is shewn to be to D C: Therefore also as B D shall be to D C, so F K to B K: But B D and D C (names of the Binomial B C) are rational, commensurable in power only: Therefore [b] F K and B K are commensurable in power only. b) 10. 10.

Again, H F being to B F, as F K to B K, [c] the Antecedents H F and F K together, to wit, H K the whole, shall be to F B and B K consequents; that is to say, to F K the whole, as F K to B K: therefore F K is a mean proportional between H K and B K; therefore [d] as H K the first, to B K the third, so the square of H K the first, to the square of F K the second: But forasmuch as C G rational (being equal to the square of A rational, applyed to D C rational,) makes [e] the breadth D G rational, commensurable in length to D C; H B equal to D G, shall be also rational, commensurable in length to D C, and it being shewn that as B D is to D C, so F K is to B K: But as F K is to B K, so H K to F K; also as B D shall be to D C, so H K to F K: therefore [f] as the square of B D to the square of D C, so the square of H K to the square of F K. But the square of B D is commensurable to the square of D C: (B D & D C names of the Binomial B C rational, commensurable in power;) Therefore [g] the square of H K shall be also commensurable to the square of F K; but as the square of H K to the square of F K, so H K to B K: therefore H K is commensurable in length to B K: and therefore [i] to the rest B H: But B H is shewn rational, therefore H K commensurable thereto, is rational; and therefore B K commensurable to H K, is rational: Therefore F K being shewn commensurable in power only to B K, F K shall be also rational; therefore F K and B K being rationals, and shewn commensurable in power only; [k] the remainder B F shall be a Residual; and B K shall be an agreeing line thereto, which is first proposed. c) 12. 5. d) C. 20. 6. e) 21. 10. f) 22. 6. g) 10. 10. h) 10. 10. i) C. 16. 10. k) 74. 10.

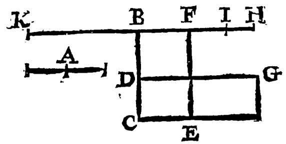
But forasmuch as the whole H K is shewn commensurable in length to his part B K, B K and B H [l] shall be commensurable in length: Therefore B H being shewn commensurable in length to D C, B K shall be also commensurable in length to D C, as appeares by the 12 Proposition of this Book. l) 16. 10.

And it being shewn that as B D is to D C, so F K to B K, and by permutation, as B D to F K, so D C to B K, but D C and B K are shewn commensurable in length: Therefore [m] B D and F K shall be also commensurable in length: Therefore F K being commensurable in length to B D, and B K to D C, F K and B K names of the Residual B F, shall be commensurable in length to B D and D C, names of the Binomial B C, which is in the second place proposed. m) 10. 10.

Also it being demonstrated, that as B D is to D C, so F K is to B K: F K and B K, names of the Residual, shall be in the same rate as B D and D C, names of the Binomial B C, which is in the third place proposed.

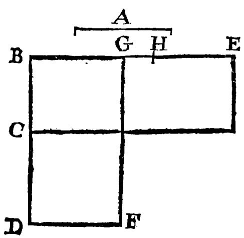
Lastly, B D is more in power then D C, by the square of a line commensurable in length thereto, or incommensurable; if it be more in power by the square of a line commensurable in length thereto; B D being to D C, as F K to B K, [n] F K shall be also more in power then B K, by the square of a line which shall be commensurable in length thereto, and if B D be more in power then D C, by the square of a line commensurable thereto in length, F K [o] shall be more in power then B K, by the square of a line n) 15. 10. o) 15. 10.

line incommenſurable thereto in length; and if B D be commenſurable in length to the rational expoſed F K, ſhewn commenſurable in length to BD, it ſhall be alſo commenſurable in length to the ſame rational, as appears by the 12 Propoſition of this Book. But if D C be commenſurable in length to the rational B K; by the ſame reaſon it ſhall be commenſurable in length to the ſame rational; and laſtly, if neither BD nor DC be commenſurable in length to the rational, neither F K [p] nor B K alſo ſhall be commenſurable in length to the rational propoſed: Therefore BF Reſidual is of the ſame order as B C Binomial, as appeares by the ſecond and third Definitions, which is in the fourth place propoſed: Therefore, &c. Which was to be demonſtrated.



p) 14. 10.

PROP. 114. THEOR. 90.



The ſquare of a rational line A, *being applyed to a Reſidual* B C, *makes the breadth* B E *a Binomial, whoſe names are commenſurable to the names* B D *and* C D, *of the Reſidual* B C, *and in the ſame rate: and over and above, the Binomial* B E *is of the ſame order as the Reſidual* B C.

Demonſtration. FOr let there be alſo applyed to the whole B D the Rectangle B F, equal to the ſquare of A; that is to ſay, to C E, making B G the breadth; foraſmuch as B F and C E are equal, [a] as B E, ſhall be to B G, ſo B D to B C: and by converſion of reaſon, as B E to G E, ſo B D ſhall be to C D.

a) 14, 16. 6.

Let E G alſo be divided in H, according to the rate of B E to G E, to the end that E A may be to H G as B E to G E: but as B E the whole is to G E the whole, ſo E H cut off from B E, to H G cut off from G E: B H the remainder of B E, ſhall be alſo to H E the remainder of G E, as the whole B E to the whole G E.

But as B E was to G E, ſo E H to H G; Therefore alſo as B H ſhall be to H E, ſo E H to G H; therefore H E is a mean proportional between B H and G H: Wherefore [b] as B H the firſt, to G H the third, ſo the ſquare of B H the firſt, to the ſquare of H E the ſecond: But as B D to C D, ſo B E to G E, that is to ſay, B H to H E, and B D to C D, names of the Reſidual B C, being rational, commenſurable in power only; [c] B H and H E are

b) C. 20. 6.

c) 10. 10.

are alſo commenſurable in power, and therefore the ſquares of B H and H E commenſurable: Therefore B H and G H being in the ſame rate as the ſquares of B H and H E, as hath been demonſtrated, are commenſurable in length, and the whole B H being commenſurable in length to its part G H; [d] it ſhall be alſo commenſurable in length to the other part B G.

d) C. 16. 10.

But B D being rational, to wit, the greateſt name of the Reſidual B C, and the Rectangle B F rational, being equal to the ſquare of A rational; [e] its breadth B G, ſhall be rational, commenſurable in length to B D: Therefore [f] B H is commenſurable in length to the ſame rational B D; and therefore is rational, B H and B G having been ſhewn commenſurable in length.

e) 21. 10.
f) 12. 10.

And ſeeing that B H and H E are ſhewn commenſurable in power only, and B E rational, H E commenſurable thereto, is alſo rational: Therefore B H and H E are rational, commenſurable in power only; Therefore [g] B E is a Binomial, which is firſt propoſed.

g) 37. 10.

But it being ſhewn that B H is to H E, as B D to C D, and alternately, B H to B D, as H E to C D; but B H being ſhewn commenſurable in length to B D, [h] H E ſhall be alſo commenſurable in length to C D: Wherefore B H and H E, names of the Binomial B E, are commenſurable in length to B D and C D, names of the Reſidual B C, which is in the ſecond place propoſed.

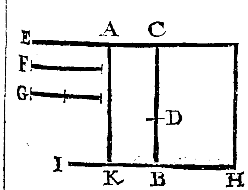
h) 10. 10.

They are alſo in the ſame rate. It being demonſtrated that B H is to H E, as B D to C D, which is in the third place propoſed.

Laſtly, either B D is more in power then C D, by the ſquare of a line commenſurable in length thereto, or is incommenſurable: if commenſurable, B H ſhall be alſo more in power then H E, by the ſquare of a line which ſhall be commenſurable in length thereto; and if incomenſurable, it ſhall be more in power, by the ſquare of a line which ſhall be incommenſurable thereto in length: and if B D be commenſurable in length to the rational expoſed, B H [i] ſhall be ſo alſo, being commenſurable in length to B D, commenſurable in length to the Rational. But if C D be commenſurable in length to the Rational, A E by the ſame reaſon, ſhall be ſo alſo. If neither the one nor the other, B D nor C D, be commenſurable in length to the Rational, alſo [k] neither B H nor H E, ſhall be commenſurable in length thereto: Therefore by the Second and the Third Definitions, B E is a Binomial of the ſame order as B C, the Reſidual which is in the fourth place propoſed: Therefore, The Square, &c. Which was to be demonſtrated.

i) 12. 10.
k) 14. 10.

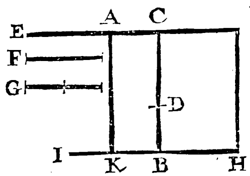
PROP. 115. THEOR. 91.



If a Space A B, *be contained under a Reſidual* A C, *and under a line called a Binomial* C B, *whoſe names* C D *and* D B, *are commenſurable to the names* C E *and* A E,

AE, of the Residual AC, *and in the same rate* (CD *to* DB, *as* CE *to* AE,) *the right line* F, *equal in power to the same Space* AB, *is a Rational.*

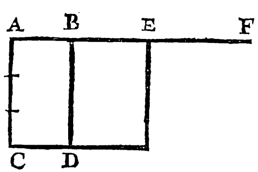
a) 45.1. *Demonstration* For having exposed the Rational G, let [a] there be applyed to the Binomial CB, the Rectangle CH, equal to the
b) 113.10. square of G: Therefore [b] the breadth BH, shall be a Residual: Therefore the names HI and BI are commensurable in length to the names CD and DB, and in the same reason; to wit, HI to BI, as CD to DB; and therefore as E to AE, and by permutation, as the whole HI to the whole CE: so the part cut off BI, to the part cut off AE: Therefore
c) 19.5. [c] the remainder BH shall be to the remainder AC, as the whole HI is to the whole CE: But HI is commensurable in length to CE, as well HI as CE being commensurable in length to CD: Therefore [d] BH is
d) 10.10. commensurable in length to AC, and therefore HC is commensurable to BA: HC being to BA, as HB to AC. But HC being equal to the square of the Rational G, is Rational: Therefore BA commensurable thereto, shall be Rational: Therefore F equal in power to the same, shall be Rational: Therefore, &c. Which was to be demonstrated.



COROLLARIE.

From this it is manifest, that a Space rational, may be conteined under two Irrational lines: For AB *conteined under* AC *and* CB *a Residual and a Binomial, which are Irrationals, is shewn to be rational.*

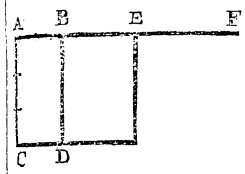
PROP. 116. THEOR. 92.



Of the Medial line AB, *there are made an infinite number of Irrationals, and not one of them the same with any one of the Antecedents.*

Demonstration For the Rational AC being proposed, let the Space AD be conteined under AB Medial, and AC rational; Therefore AD conteined under AD rational, and AB irrational, is irrational: Let

Let BE be equal in power to the Space AD, [a] BE shall be irrational; I say that BE is not any of the 13 [b] before spoken of. a) 11.d. b) c.112.10.
For seeing that the square of the Medial applyed to AC rational, makes the breadth [c] rational, incommensurable in length to AC, and the squares of the other 12 irrationals applyed to AC rational, makes the breadths either Binomials, or Residuals, as appears by the *61, 62, 63, 64, 65, 66, 98, 99,* 101, 102, and 103 Propositions of this Book. c) 23.10.
But the square of this irrational BE, applyed to the same rational AC, makes the breadth AB Medial: It is manifest that BE irrational, doth differ from all the 13; seeing that his square applyed to a rational line, makes the breadth different from the breadths made by the squares of those 13 lines applyed to the same rational: And if the Rectangle DE conteined under BD rational, and BE irrational be accomplished, the same shall be irrational: Let then EF be equal in power to the same, which shall be irrational.



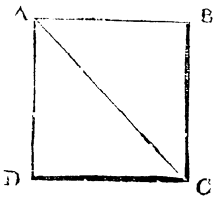
I say, [d] again that EF is not any of the 13 before mentioned lines, nor is also BE, which is manifest, seeing that the square of EF applyed to the rational, makes the breadth BE; but the squares of the same 13, and also the square of BE, applyed to the same rational, make the breadths differing to BE, as hath been demonstrated: In like manner, there might be found infinite other irrationals, differing from one another, and from the afore-mentioned 13: Therefore, &c. Which was to be demonstrated. d) 11.d.

PROP. 117. THEOR. 93.

Let it be proposed to us to shew that in square figures AB, *and* CD, *the Diameter* AC, *is incommensurable in length to the side* AB.

Demonstration For otherwise it should be commensurable in length to AB, and therefore [a] AC and AB should be to one another as a number to a number: Let AC be to AB, as the number EF is to the number G, and let EF and G be the least in their rate: Therefore AC being to AB as EF to G, the square of AC shall be also to the square of AB as the square number of EF to the square number of G: (for [b] the squares being in a double rate of their sides, and the sides having equal rates, the rates of the squares shall be also equal, being double the equal rates,) but the square of AC is double to the square of AB (AB and BC being equal;) therefore the square of the number EF, shall be double to the square of the number G; and therefore the square of EF having a half, and may be divided into two equal parts, shall be even, by the Definition: Therefore EF which produceth it, shall be also even; (for if it were odde, and multiplying its self, doth produce its square, the same square should be also odde, seeing that an odde number multiplying an odde number produceth an a) 5.10. b) 20.6.

an odde number, which is absurd, for it is shewn to be even:) But E F and G being the least in their rate, are primes to one another; and E F being shewn even, G shall be odde: (For if it were also even, the Binary should measure E F and G; and therefore should not be primes to one another, which is absurd:) Now let the even number E F be divided into two equal parts at H; forasmuch then as E F is double to E H, and the squares are in a double rate to their sides, the square of E F shall be quadruple to the square of E H, (for the proportion quadruple, is double to the proportion double, as appears by these numbers 4, 2, 1;) therefore the square of E F being double to the square of G, and quadruple to the square of E H, the square of E F is of 4 such equal parts, as the square of G is 2, and the square of E H 1. Therefore the square of G is double to the square of E H, being as 2 to 1; and therefore as hath been said of the number E F, the square of G having a half, shall be even, and the same G also even: But it is also shewn to be odde, which is absurd: Therefore the Diameter A C is not commensurable in length to the side A B: Therefore incommensurable in length.



E . . . H . . . F
G . . . . . .

Otherwise, (if possible,) let the Diameter A C be commensurable in length to the side A B, and that A C and A B may be to one another, as the numbers E F and G, which may be the least of their rate, and therefore primes to one another: Therefore G shall not be unity, for the square of A C being double to the square of A B, and as the square of A C to the square of A B, so the square of the number E F, to the square of the number G, as it is said in the first Demonstration, the square of E F shall be also double to the square of G; if therefore G be Unity, and therefore his square also Unity, the square of E F shall be a Binary, which is absurd.

And forasmuch as hath been already shewn, the square of E F being double to the square of G, the square of G shall measure the square of E F; and therefore G the side shall measure E F the side: Therefore G measuring also it self, E F and G shall be compounds to one another, having G for their common measure, but they are also primes to one another, which is absurd: Therefore the Diameter A C is not commensurable in length to the side A B: Therefore, &c. Which was to be demonstrated.

The End of the Tenth Element of EUCLIDE.

THE

THE ELEVENTH ELEMENT OF EUCLIDE.

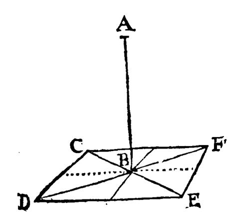
THE ARGUMENT.

IN this Eleventh Book is treated first of right lines in relation to Solid Bodies; *viz.* when they are in one Plain, when they are erect, or perpendicular to a Plain, when they are parallel, and how from a point given on high, perpendiculars may be drawn to a given Plain. And also of the intersections of Plains, and then of Solid Angles: Lastly, he speaketh of Solid Parallelepipedons, and something concerning Prismes.

DEFINITIONS.

1 *A Solid is that which hath length, breadth, and thicknesse.*

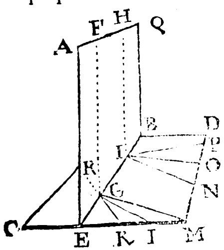
2 *But the Terms of a Solid are Superficies.*



3 *A right line as* A B, *is raised at right angles on a Plain, as* C D E F, *when it maketh right angles as* C B A, D B A, E B A, *and* F B A, *with all the right lines as* B C, B D, B E, and B F, *which do touch it on the Plain proposed.*

 I say

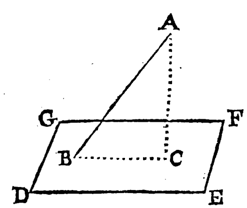
I Say, a line raiſed on thoſe conditions, is ſaid to be perpendicular to the propoſed Plain.



4 *A Plain as AB, is raiſed perpendicularly on a Plain CD, when all the lines GF, IH, &c. drawn on one of the ſaid Plains at right angles to the line of common Section BE, are at right angles to the other Plain CD.*

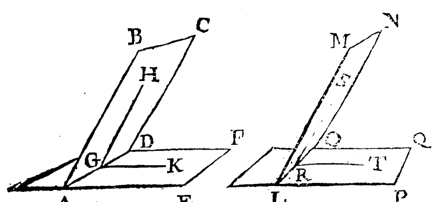
IF FG and HI are perpendicular to the other Plain CD; that is to ſay, if they are at right angles to GK, GL, and GM, IN, IO, and IP, &c. the Plain AB, ſhall be at right angles to the Plain CD, not being inclined toward the one part, or toward the other.

For KG being produced to R, and the angle FGK being put a right angle, FGR ſhall alſo be a right angle, and ſo of the reſt, &c. Wherefore the Plain AB, ſhall be at right angles to the Plain CD. And when a Plain is at right angles to another Plain, the perpendicular lines drawn upon one of them to the Common Section, ſhall be alſo perpendicular to the other Plain.



5 *The inclination of a right line as AB, to a Plain as CDEF, is the angle ABG, contained under the ſaid right line AB, and another right line as BG, drawn on the Plain, from the inclining line AB, by the point where there falleth a right line as AG, perpendicular, drawn from the top A of the inclining line AB, to the ſaid Plain.*

6 *The inclination of one Plain as ABCD, to another Plain, as EADF, is the acute angle HGK, conteined undr the right lines GH and GK, drawn on both the Plains at right angles to the line of Common Section AD, and from a point therein, as G, which are not perpendicular the one to the*



7 *A Plain as ABCD, is ſaid to be alike inclined to a Plain as EADF, and another Plain LMNO, to another Plain as PLOQ, when the angles of inclination HGK and SRT, are equal the one to the other.*

8 *Parallel Plains are ſuch as do not meet, being prolonged.*

9 *Like Solid figures are ſuch as are conteined under like plains, equal in number.*

10 *Equal and like Solid figures are ſuch are conteined of like plains, equal in number and magnitude.*

11 *A Solid angle, is the meeting of more than two right lines, touching one another in a certain point, and being not in one and the ſame ſuperficie, inclining to all the lines.*

12 *A Pyramyde is a Solid figure conteined of divers plains, which meet or terminate at one and the ſame point, being drawn from another plain, which is for the baſe of the Pyramyde.*

13 *Priſme is a Solid figure, conteined of plains, two of which (thoſe that are oppoſite to one another,) are equal, alike, and parallel, but the others are parallelograms.*

14 *Sphere is a figure conteined when a Semicircle is drawn about the diameter, remaining fixed, until it be again poſited there where it began to move.*

THat is to ſay, Sphere is a Solid figure conteined by the Superficies deſcribed by the revolution of a Semicircle drawn about the Diameter, fixed and unmoveable, until it be returned to the point where it began its motion.

15 *The Axis of a Sphere is a fixed right line, about which the Semicircle moves or turns.*

16 *But the Center of a Sphere is the ſame with that of the Semicircle.*

17 *The Diameter of a Sphere is a right line drawn by the Center and terminating on both parts in the Superficies of the Sphere.*

18 *A Cone is a figure which is conteined when one of the ſides*

which conteins the right angle of a rectangle triangle remaining fixed, the triangle is drawn about, until it return to the place where it began to move, and if the unmoveable right line be equal to the other side, which is moved about the right angle, the Cone shall be rectangle, if it be lesse, the Cone shall be an Ambligon, and if greater, an Oxigon.

THat is to say, a Cone is a Solid, conteined under the Superficies, described by a rectangle triangle, when the same triangle is drawn about one of the sides which conteineth the right angle, the same side remaining fixed until the same triangle be returned where it began its motion.

19 *The Axis of a Cone is the unmoveable right line about which the triangle turneth or moveth.*

20 *But the Base of a Cone is the circle described by the other side, drawn about.*

That is to say, by the other of the two, which conteins the right angle.

21 *A Cylinder is a figure which is conteined, when one of the sides of a rectangle parallelogram, of those that are about a right angle, remaining unmoveable, the parallelogram is drawn about, until it be again constituted, where it began to move.*

THat is to say, a Cylinder is a Solid conteined under the Superficie described by the revolution of a rectangled Parallelogram drawn about one of the sides which conteines one right angle, the same side remaining fixed, until it be returned where it began its motion.

22 *The Axis of a Cylinder is the unmoveable right line about the which the parallelogram is drawn.*

23 *But the Bases of a Cylinder are the circles described by the two opposite sides drawn about.*

24 *Like Cones, and Cylinders, are those whose Axes and Diameters of the Bases have the same proportion the one to the other.*

25 *A Cube is a Solid figure conteined under six equal squares.*

26 *A Tetrahedron is a Solid figure conteined under four triangles, equal and equilateral.*

27 *An Octahedron is a Solid figure conteined under eight triangles, equal and equilateral.*

28 The

28 *The Dodecahedron is a Solid figure conteined under twelve equal Pentagons, equilateral and equiangled.*

29 *An Icosahedron, is a solid figure conteine under twenty triangles equal, and equilateral.*

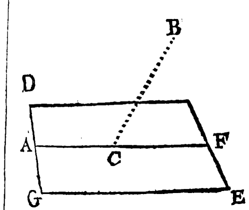
30 *A Parallelepipedon is a solid figure conteined under six quadrilateral figures, of which the opposites are parallels.*

31 *A solid figure is said to be inscribed in a solid figure, when all the angles of the figure inscribed are constituted either at the angles, or at the sides, or lastly, at the plains of the figure in which it is inscribed.*

32 *Reciprocally a solid figure is said to be circumscribed about another solid figure, when the angles or the sides, or lastly, the plains of the circumscribed figure do touch all the angles of the figure about which it is described.*

PROPOSITIONS, PROBLEMES, and THEOREMES.

PROPOSITION 1. THEOREM 1.



A part of a right line, as A C, *cannot be in a proposed plain* D E, *and another part in the aire.*

FOr let A B be the right line, a part whereof, to wit, A C, is on the Plain D E, and let the other part C B be (if possible,) in the aire, and so draw C F.

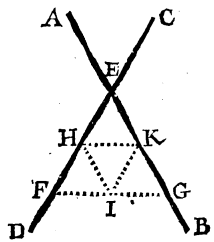
Demonstration. FOrasmuch then as A C part of A B a right line, is posited on the Plain D E, and the other part C B, remaines in the aire, that part of the Plain D E, to wit, from C to F, shall be lower or more depressed than the other part, to wit, from A to C; and therefore the Superficies D E shall not be conteined equally between his lines, which a is absurd; seeing that D E is proposed to be a plain Superficies. a) 7. def. 1.

Other-

Otherwise (if it be possible, let one part of the right line A B, that is to say, A C, be on the Plain D E, and the other part C B in the aire; then in the plain D E there shall be some right line posited directly to A C, as C F: Therefore the two right lines AB and AF have one common segment A C, which [b] is absurd: Therefore one part of a right line is not upon a Plain, and the other in the aire. Which was to be demonstrated.

b) 10. c. s. 1.

PROP. 2. THEOR. 2.



If two right lines, as A B *and* C D, *cut one the other, they are in one and the same plain, and every triangle as* E F G, *&c. is in one and the same plain.*

LEt A B and C D cut one another in E, then in the lines E B and E D, having assumed the points F and G, at pleasure, draw F G, to make the triangle E F G; I say the triangle E F G is in one and the same Plain, as also the right lines A B and C D.

Demonstration. FOr if one part of the triangle E F G, to wit, E H I G, be in the Plain, and another part thereof, to wit, the residue F H I, be in the aire; or contrariwise, E H and G I, parts of E F and G F, shall be also in a Plain; but the other parts H F and I F, shall be in the aire, which [a] is shewn to be impossible: And if E F I K, part of the same triangle, be said to be in the same Plain, but the other part G I K, in the aire; or contrariwise, by the same reason, E K and F I, parts of E G F G, shall be in a Plain; but the parts K G and I G shall be in the aire, or contrariwise.

a) 1. 11.

Lastly, if F H K G, part of the same triangle, be acknowledged to be in one and the same Plain, and the other part E H K in the aire, or contrariwise, F H and G K, parts of F E and G E, shall be also in one and the same Plain, and the other parts H E and K E, in the aire or contrariwise, which is absurd, as [b] hath been shewn; therefore the triangle E F G is in one and the same Plain, and so of every other triangle.

b) 1. 11.

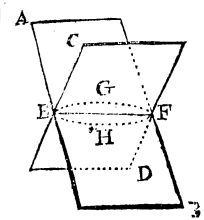
But seeing that E F and E G, sides of the triangle E F G, are in one and the same Plain therewith; but [c] C D and A B the whole, are in the same Plain with E F and E G, Therefore A B and C D shall be in one and the same Plain, to wit, that, in which we have demonstrated the triangle E F G to be: Therefore, If two right lines, &c. Which was to be demonstrated.

c) 11. 1.

PROP. 3. THEOR. 3.

If two plains, as A B *and* C D, *cut one another, their common section* E F, *is a right line.*

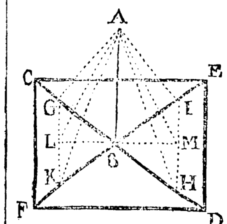
Demon-



Demonstration. FOr if you shall say it is not so, let there be drawn the right line E G F, on the Plain A B, and on the Plain C D, the right line E H F; therefore E G F and E H F being right lines, having the same terms E and F, do enclose a Superficies, which is absurd; therefore [a] the common section E F shall be a right line: Therefore, If two Plains, &c. Which ought to be demonstrated.

a) 12 Ax. or c. s. 1.

PROP. 4. THEOR. 4.



If two right lines, as C D *and* E F, *cut one another, and from the point of their common section be drawn another right line, as* A B, *at right angles, that right line* A B *shall also be at right angles to the Plain where the same right lines* C D *and* E F *are drawn.*

Demonstration. FOr seeing they cut one another at B, [a] they shall be in one and the same Plain, which let be C E D F: Let B G and B H be assumed, equal to one another, and also B I and B K, and draw G K and I H, and on the same Plain let there be drawn by the point B, L M, cutting G K and I H at L and M; and from the point A in the aire, let there be drawn to the same Plain, the right lines A G, A L, A K, A H, A M, and A I.

a) 2. 11.

Forasmuch as the sides BG and GK of the triangle BGK, are equal by Construction to BH and BI of the triangle BHI; [b] and the angles G B K and H B I, conteined of them, are equal at the top B, [c] the bases G K and H I shall be equal to one another, and the angles B K G & B I H also equal to one another.

b) 15. 1.
c) 4. 1.

Again, K B L and I B M being equal at the top B, the two angles K B L and B K L of the triangle B K L, shall be equal to the two I B M and B I M, of the triangle B I M, and the sides adjacent B K and B I, are also equal by Construction: Therefore the two other sides K L and L B, are equal to the two others I M and M B, each to his correspondent side.

d) 15. 1.

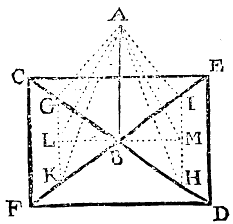
Moreover, the sides A B and B G of the triangle A B G, being equal to A B and B H, of the triangle A B H, by Construction; and the angles A B G and A B H, conteined of them, equal, (being proposed right angles) [e] the bases A G and A H shall be equal; and by the same A I and A K shall be equal.

e) 4. 1.

Furthermore, the sides A I and I H of the triangle A I H, being equal to A K and K G, of the triangle A K G, and the bases A H and A G shewn to be equal; [f] the angles A I H and A K G, conteined of those sides, shall be

f) 8. 1.

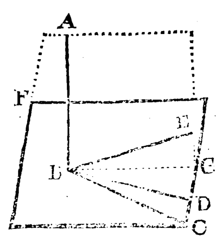
be equal; therefore the sides A K and K L being equal to A I and I M,
g) 4. 1. and the angles A K L and A I M conteined by them, being shewn equal, g the bases A L and A M shall be equal.



Lastly, the sides A B and B L of the triangles A B L being equal to A B and B M of the triangle A B M, and the bases A L and A M shewn to be equal, the angles
h) 8. 1. h A B L and A B M, conteined by those
i) 10. def. 1. sides, shall be also equal, and right angles i; Therefore A B makes right angles with
k) 3. def. 11. L M, k which doth touch it at B on the Plain proposed.

By the same reason it is demonstrated that A B makes right angles with all the right lines which do touch it in the point B on the same Plain, and in like sort may be taken on the said Plain as many lines as you shall please, which shall make right angles therewith, meeting in B, by the reasons above mentioned: Therefore A B shall be at right angles
l) 3. def. 11. l on the Plain C E D F, drawn by C D and E F: Therefore, If two lines, &c. Which ought to be demonstrated.

PROP. 5. THEOR. 5.



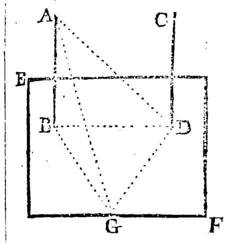
If to three right lines as B C, B D, *and* B E, *touching one another in* B, *another right line* A B *be constituted at right angles in the point of common section* B, *the said three lines are in one and the same Pain* F C.

Demonstration. For a seeing that any two of
a) 2. 11. them are in one and the same Plain, and being drawn forth, will cut one another at B: Let B C and B D be in one and the same Plain F C, and (if it be possible,) let not B E be in the same Plain, but raised up in the ayre: But b A B and
b) 2. 11. B E being in one and the same Plain, seeing they intersect at B, let both the two be in the Plain A E.

And forasmuch as the Plains F C and A E meet with one another at B, being produced, they must of necessity cut one another; let c therefore
c) 3. 11. the right line B G be their common section; seeing that A B is proposed
d) 4. 11. to be at right angles to B C and B D, d it shall also be at right angles to
e) 3. def. 11. the Plain F G, drawn by them; and e therefore shall be also at right angles to B G, which doth touch it at B: Wherefore the right angles A B E and A B G being in the Plain A G, drawn by A B and B E, shall be equal the one to the other, the part and the whole, which is absurd: Therefore the two lines B C and B D being on one and the same Plain F C; B E shall not be raised up in the aire, but shall be in the same Plain with them: Therefore, If to three right lines, &c. Which ought to be demonstrated.

PROP.

PROP. 6. THEOR. 9.

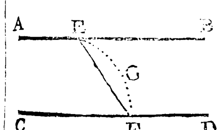


If two right lines as A B *and* C D, *are at right angles on one and the same Plain* E F, *the same right lines* A B *and* C D *are parallels.*

Demonstration. For having drawn B D on
a) 3. def. 11. the Plain E F, a the angles A B D and C D B shall be right angles; and on the same Plain E F, let D G be drawn perpendicular to B D, and let A B and D G be proposed equal; and let B G, G A, and A D, be joyned together: Then forasmuch as the sides A B and B D of the triangle B A D, are equal to G D and B D of the triangle G D B, by Construction; and the angles A B D and G D B, conteined by them, are also equal, being right angles, the bases A D and G B shall be equal.

Again, b the sides A B and B G of the triangle A B G, being equal to G D b) 4. 1.
and D A of the triangle G D A, and the base A G common; c the angles c) 8. 1.
A B G and G D A shall be equal: But d A B G is a right angle, therefore d) 3. def. 11.
G D A shall be a right angle, and the angle G D C being a right angle, G D also shall be at right angles to the three sides B D, D A, and D C:
Wherefore e B D, D A, and D C, shall be in one and the same Plain; that e) 5. 11.
is to say, C D shall be on the same Plain, drawn by D B and D A: But
f A B is on the same Plain with D B and D A: Therefore C D shall be on f) 2. 11.
the same Plain with A B; therefore the internal angles A B D and C D B being right angles, A B and C D shall be parallels: Therefore, If, &c. Which was to be demonstrated.

PROP. 7. THEOR. 7.

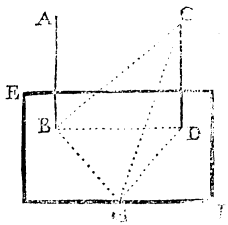


If there be two parallel right lines, as A B *and* C D, *in both of which be taken two points at pleasure, (suppose at* E *and* F,) *the right line* E F, *drawn from those points, is in one and the same Plain with the parallels.*

Demonstration. For if E F be not in the same Plain, let it be in another, and let that other cut the Plain of the parallels by E and F;
and let the common section of those Plains be E G F, a which shall be a a) 3. 11.
right line: Therefore the two right lines E F and E G F, having the same terms E and F, do enclose a space, which is absurd: Therefore the right line E F shall not be out of the Plain of A B and C D parallels: Therefore, If there be two, &c. Which was to be demonstrated.

 PROP.

PROP. 8. THEOR. 8.



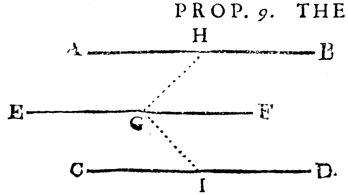
If there be two parallel right lines, as AB *and* CD, *one of which, as* AB, *is at right angles with the Plain* EF, *the other* CD, *shall be also at right angles to the same Plain* EF.

Demonstration. FOr having drawn BD on the Plain EF, the angle
a) 3. def. 11. CDB [a] shall be a right angle; but [b] the
b) 29. 1. two angles CDB and ABD are equal to two right angles; therefore ABD shall be a right angle: Now let BG be drawn on the Plain EF, perpendicular to BD; and let BG and CD be proposed equal, and let DG, G[illegible], and CD, be joyned.

Forasmuch as the sides CD and DB, of the triangle CDB, are equal to GB and BD, of the triangle GBD by Construction; and the angles CDB and GDB, conteined of the said sides are also equal, being
c) 4. 1. right angles, [c] the bases CB and GD shall be equal.

Again, the sides CD and DG of the triangle CDG, being equal to GB and BC of the triangle GBC, and the base CG common, the an-
d) 8. 1. gles [d] CDG and GBC, conteined of the said sides, shall be equal: But
e) 3. def. 11. CDG [e] is a right angle, therefore GBC shall be also a right angle; wherefore GB shall be perpendicular to BD, BC cutting the one and the other at B, being produced, (seeing they make an angle with B,)
f) 4. 11. therefore [f] shall be perpendiculrr to the Plain drawn by BD and BC:
g) 7. 11. but AB is also in the same Plain [g] with BD and BC, being in the same Plain with AB and CD parallels; the points B, C, and D being in the pa-
h) 3. def. 11. rallels AB and CD; therefore GB [h] shall be also perpendicular to BA: Wherefore AB being perpendicular to BC and BD, AB shall be also
4. 11. perpendicular to the Plain drawn by BG and BD; that is to say, to the Plain EF: Therefore, If there be two, &c. Which was to be demonstrated.

PROP. 9. THEOR. 9.



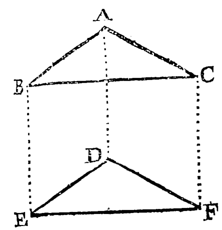
Right lines AB *and* CD, *parallel to one and the same right line* EF, *being not in the same Plain with them, those right lines are also parallel to one another.*

Demonstration. FOr from what point you please of EF (suppose from G,) let there be drawn two perpendiculars to EF, to wit, GH on the Plain of the parallels AB and EF, and GI on the Plain of the parallels CD and EF: Therefore EG being perpendicular to GH and GI,

GI, touching one another at G, shall be also perpendicular to the Plain 4. 11.
drawn by GH and GI; wherefore AB and EF being parallels, and EF perpendicular to the Plain drawn by GH and GI; AB shall be also per- 8. 11.
pendicular to the same Plain.

In like manner, CD and EF being parallels, and EF perpendicular to the Plain drawn by GH and GI, CD shall be also perpendicular to 8. 11.
the same Plain: Therefore AB and CD being perpendiculars to the Plain 8. 11.
drawn by GH and GI, shall be parallels the one to the other: Wherefore, Right lines, &c. Which was to be demonstrated.

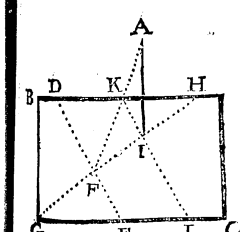
PROP. 10. THEOR. 10.



If two right lines as AB *and* AC, *touching one another, as at* A, *are parallel to two right lines, as* DE *and* DF, *touching one another, as at* D, *and being not in one and the same Plain, the same right lines shall contein equal angles as* BAC *and* EDF.

Demonstration. FOr let AB and DE be proposed equal to one another, and also AC and DF equal to one another, and let there be drawn BC, EF, BE, AD, and CF; therefore AB and DE being equal and parallel; [a] BE and AD shall be also be equal and parallel; by a) 33. 1.
the same reason CF and AD shall be parallels and equal: Wherefore BE and CF being equal and parallels to AD, they [b] shall be parallels b) 9. 11.
and equal to one another; and therefore [c] seeing BC and EF parallel c) 33. 1.
and equal to one another; therefore seeing the sides AB and AC, of the triangle BAC are equal to DE and DF of the triangle DEF, by Construction, each to his correspondent; and the base BC demonstrated equal to the base EF, [d] the angles BAC and EDF shall be equal: There- d) 8. 1.
fore, If two right lines, &c. Which was to be demonstrated.

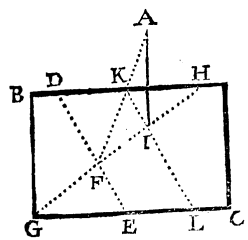
PROP. 11. PROBL. 1.



From a point given in the aire, as A, *to draw a right line perpendicular as* AI, *on the plain* BC, *which is under it.*

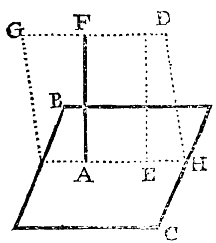
Construction. LEt the point A be in the aire, from which a perpendicular line ought to be drawn to the Plain BC; let DE be drawn to the said Plain BC at pleasure, [a] to which let the perpendicular a) 12. 1.
AF be drawn, and on the Plain BC, by F, let GH be drawn perpendicular

b) 11. 1. to D E, [b] to the which from A, let there be drawn the perpendicular A I; I say that A I is perpendicular to the said plain B C.



c) 31. 1. *Demonstration.* FOr [c] on the plain B C; let there be drawn K L, parallel to D E by I; seeing that D F is at right angles to F A, and to F H by Construction;
d) 4. 11. and therefore [d] perpendicular to the plain drawn by F A and F H; also [e] K I shall be
e) 8. 11. perpendicular to the plain drawn by F A
f) 2. 11. and F H: But [f] A I being in the same plain
& 3. def. 11. with F A and F H, and touching K I at I, the angle K I A shall be a right angle: Wherefore A I shall be at right angles to K I and I F; therefore A I shall be perpen-
g) 4. 11. dicular to the plain B C: Therefore [g] we have drawn a line perpendicular, &c. Which was to be done.

PROP. 12. PROBL. 2.

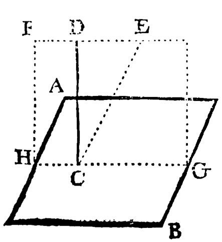


To a given plain as B C, *and from a given point therein as* A, *to draw a right line* A F, *at right angles.*

a) 11. 11. *Construction.* FRom [a] any point, (suppose D,) given above, let there be drawn D E, perpendicular to the plain B C, which if it fall in the point A, you have your desire; but if the point E fall
b) 31. 1. elsewhere, having drawn a right line by E and A, let [b] there be drawn A F parallel to D E, on the plain G H, drawn by D E and E A: I say that A F is perpendicular to the given plain B C.

c) 8. 1. *Demonstration.* FOr D E and A F being parallels, and D E perpendicular to the plain B C by Construction; [c] A F shall be also perpendicular to the same plain B C: Wherefore, To a plain, &c. Which was to be done.

PROP. 13. THEOR. 11.

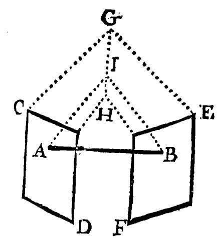


To a given plain, as A B, *from one and the same given point as* C, *cannot be drawn two right lines in the aire at right angles on the same part.*

Demonstration. FOr if it be possible, let there
a) 2. 11. drawn two perpendiculars C D and C E, [a] to the plain A B, and by C D

C D and C E which are in one and the same plain; let there be drawn the plain F G, cutting the plain A B by the right line G H; therefore E C and D C being perpendiculars to the plain A B, [b] the
b) 3. def. 11. angles E C G and D C G shall be right angles, and therefore equal, the part to the whole, which is absurd: Therefore to a given Plain, &c. Which was to be demonstrated.

PROP. 14. THEOR. 12.

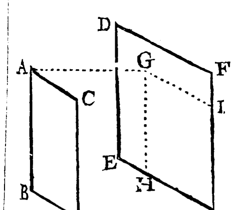


The plaines C D *and* E F, *to which one and the same right line* A B *is at right angles, those plains are parallel.*

Demonstration. FOr if it be not so, they being produced will meet with one another, which let be on the part of C E, and that G H be the line of their common section, in which [a] having taken
a) 3. 11. the point I at pleasure, let the right lines I A and I B be drawn on the plains G C D and G E F.

Forasmuch as A B is proposed to be at right angles to the plains G C D and G E F, [b] the two angles I A B and I B A shall be right angles in the
b) 3. def. triangle A B I: But [c] they are lesse than two right angles, which is absurd:
c) 17. 1. Therefore the plains C D and E F being produced, will never meet, they are therefore parallels: Wherefore, The plains, &c. Which was to be demonstrated.

PROP. 15. THEOR. 13.



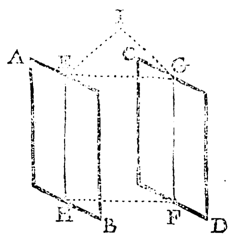
If two right lines A B *and* A C, *touching one another in* A, *are parallels to two other right lines* D E *and* D F; *in like manner touching one the other at* D, *neverthelesse being not in one and the same plain: the plains* B C *and* E F, *which are drawn by those lines are parallels.*

Demonstration. FOr from the point A, [a] to the plain E F, let there be drawn
a) 11. 11. the perpendicular A G, meeting with the plain E F in the point G; then [b] from the point G let there be drawn G H and G I, on the
b) 3. 11. plain E F, parallel to D E and D F; then A B and G H being parallels
c) 9. 11. to D E, [c] they shall be parallel to one another; and therefore [d] the an-
d) 29. 1. gles B A G and A G H are equal to two right angles: But [e] A G H is a
e) 3. def. right angle, therefore B A G shall also be a right angle. In like manner I con-

conclude that the angle C A G is a right angle: Wherefore G A being at right angles to the two lines f A B and A C, it ſhall be alſo at right angles to the plain B C, drawn by A B and A C: But A G is alſo at right angles to the plain E F by Conſtruction: Therefore g the plains B C and E F are parallels: Therefore, If two right lines, &c. Which was to be demonſtrated.

f) 4. 11.
g) 14. 11.

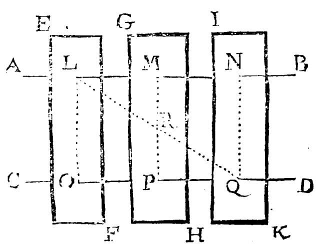
PROP. 16. THEOR. 14.



If two parallel plains A B *and* C D, *are cut by ſome other plain* E F, *the lines of their common ſections* E H *and* G F *ſhall be parallel.*

Demonſtration FOr if they be not parallels, being produced, they will meet with one another, being in the interſecting plain E F; let them meet than in the point I: Foraſmuch as the whole right line H E I, is in one and the ſame plain, to wit, in A B, produced; In like manner, the whole right line F G I is alſo in one and and the ſame plain, to wit, in C D, produced; the plains A B and C D alſo meet with one another in the point I, being produced; which is abſurd, being propoſed parallels; therefore E H and G F are parallels: Wherefore, If two plains, &c. Which was to be demonſtrated.

PROP. 17. THEOR. 15.



If two right lines A B *and* C D, *are cut by parallel plains* E F, G H, *and* I K, *thoſe right lines ſhall be cut proportionally.*

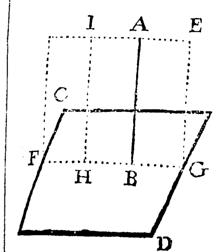
Demonſtration FOr let the right lines L O and and N Q be drawn on the plains E F and I K; and let L Q be joyned, meeting the plain G H in the point R, from which point R let there be drawn to the points M and P, the lines R M and R P, on the ſame plain G H; a the triangle L N Q ſhall be in one and the ſame plain: In like manner, alſo L O Q ſhall be in one and the ſame plain.

But foraſmuch as the parallel plains G H and I K are cut by the plain of the triangle L N Q; b their common ſections, to wit, the lines M R and N Q ſhall be parallel, by the ſame reaſon R P and L O ſhall be parallels: Wherefore c as L R is to R Q, ſo is L M to M N; in like manner, as L R to R Q, ſo is O P to P Q: Therefore as L M is to M N, ſo is O P to P Q: Therefore, If two right lines, &c. Which was to be demonſtrated.

a) 2. 11.
b) 16. 11.
c) 2. 6. 11. 5.

PROP.

PROP. 18. THEOR. 16.



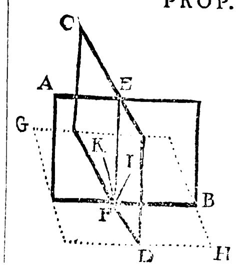
If a right line as A B, *be at right angles to a plain* C D, *all the plains drawn thereby ſhall be alſo at right angles to the ſame plain* C D.

Demonſtration FOr by A B let the plain E F be drawn, cuttng the plain C D in the right line F G; and having taken the point H at pleaſure in F G; a let there be drawn H I, to the plain E F, parallel to A B; therefore A B and I H being Parallels, and A B propoſed at right angles to the plain C D, b I H ſhall be alſo at right angles to the ſame plain C D; c therefore perpendicular to the common ſection F G, by the ſame reaſon, all the lines which are drawn parallel to A B, on the plain E F, ſhall be at right angles to the plain C D; and therefore perpendicular to the common ſection F G; therefore d the plain E F ſhall be at right angles to the plain C D.

a) 31. 1.
b) 8. 11.
c) 3. def.
d) 4. def.

By the ſame reaſons may be demonſtrated that all the other plains drawn by A B, are at right angles to the plain C D: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 19. THEOR. 17.

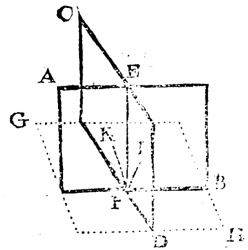


If two plains A B *and* C D, *interſecting one the other, are at right angles to a plain* G H, *their line of common ſection* E F, *ſhall be alſo at right angles to the ſame plain* G H.

Demonſtration FOr E F is either at right angles to B F and D F the common ſections of the plains A B and C D, with the plain G H, or is not, and if E F be at right angles to B F and D F, it a ſhall be alſo at right angles to the plain G H, drawn by them: But if E F be ſaid to be at right angles to the one or the other, either B F or D F, let it be only to B F; then foraſmuch as the plain A B is propoſed to be at right angles to the plain G H, b the line E F (which line E F is drawn on the plain A B, perpendicular to B F, as a ſection thereof with the plain G H,) ſhall be at right angles to the plain G H, we conclude the ſame, if E F be granted to be at right angles to D F; for then E F ſhall be perpendicular to the plain G H, being propoſed perpendicular to D F, the common ſection of the plain C D, with the plain G H, and drawn to the plain C D, which is at right angles to the plain G H.

a) 4. 11.
b) 4. def.
4. def.

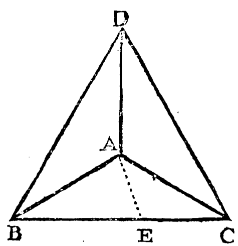
Finally,



11. 1. Finally, if E F be not at right angles to B F nor to D F, let there be drawn from the point F to the plain A B, the line F I, perpendicular to B F the common section with the plain G H; in like manner let there be drawn F K, to the plain C D, perpendicular to D F, as a section thereof, with the plain G H.

Forasmuch as the plain A B is proposed to be at right angles to the plain G H, [c] the
c) 4. def. line I F (which is drawn in the plain A B, perpendicular to B F, as a section thereof with the plain G H,) shall be at right angles to the same plain G H; by the same reason K F shall be at right angles to the same plain G H: Wherefore from the point F to the plain G H, there are
f) 13. 11. drawn two perpendiculars, which [f] is absurd: Wherefore E F shall be at right angles to the plain G H: Therefore, If two plains, &c. Which was to be demonstrated.

PROP. 20. THEOR. 18.



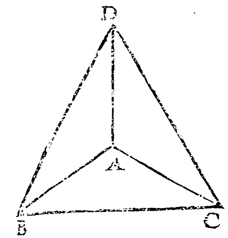
If a solid angle A, *be contained of three plain angles* B A C, C A D, *and* D A B, *two of them taken after what manner soever, are greater than the third.*

Demonstration FOr if it be possible, let one of them as B A C, be greater than the two others B A D and D A C, and on
a) 23. 1. the plain drawn by A B and A C, let [a] there be made the angle B A E, equal to the angle B A D, and the right line A E equal to A D; then from the same plain drawn by E, let there be drawn B C, touching A B and A C at B and C; and let there be drawn B D and D C: Forasmuch as the sides A D and A B of the triangle B A D, are equal to the sides A E and A B, of the triangle B A E, each to his correspondent side, and the angle con-
b) 4. 1. 20. 1. teined of them also equal, by Construction, [b] the bases B D and B E shall be equal.

But forasmuch as the sides D B and D C are greater than the side B C, if the equal right lines B D and B E be cut off, there will remain C D, greater than C E; seeing then the sides A D and A C, of the triangle D A C, are equal to the sides A E and A C, of the triangle E A C, each to his correspondent side; and the base C D greater than the base C E;
c) 25. 1. the angle [c] C A D shall be greater than the angle C A E; adding then the equal angles B A D and B A E, the two angles C A D and B A D shall be greater than the two C A E and B A E; that is to say, the whole angle B A C, which was proposed to be the greatest of all: Therefore two of them, which you please, are much greater than the other. Therefore, &c. Which ought to be demonstrated.

PROP.

PROP. 21. THEOR. 19.

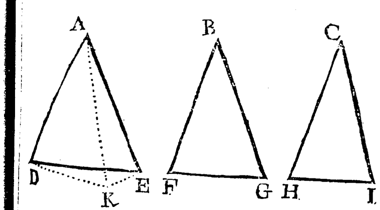


Every solid angle A, *is conteined under plain angles* B A C, B A D, *and* D A C, *which are lesse than four right plain angles.*

Demonstration FOr having drawn the right lines B C, C D and D B, the three solid angles B, C, and D, are made, each of which is conteined under three plain angles, *viz.* B, under C B A, A B D, and D B C, but C under B C A, A C D, and D C B, and D under C D A, A D B, and B D C: But [a] the two angles C B A and a) 20. 11.
A B D, being greater than the angle C B D; and in like manner, the two angles B C A and A C D are greater than B C D; and the two angles C D A and A D B, are greater than C D B, the six angles C B A, A B D, B C A, A C D, C D A and A D B, are greater than the three angles C B D, B C D, and C D B.

But [b] these three are equal to two right angles; therefore these six are b) 32. 1.
greater than two right angles; therefore seeing that these six together with the three at the point A, are equal to six right angles, because of the three angles B A C, C A D, and D A B; (for [c] the three angles of eve- c) 32. 1.
ry triangle are equal to two right angles,) if these six be taken away, which are greater than two right angles; the three which constitute the solid angle A, will remain lesse than four right angles: Therefore, Every, &c. Which ought to be demonstrated.

PROP. 22. THEOR. 20.



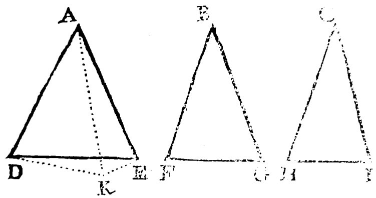
If there be three plain angles A, B, *and* C, *two whereof taken at pleasure, are greater than the other: But that they be conteined of equal right lines* A D, A E, B F, B G, C H, *and* G I, *it may be so ordered, that of the right lines* D E, F G, *and* H I, *joyning together the said equal lines, a triangle may be constituted.*

IF the three angles A, B, and C, be equal, [a] the bases D E, F G, a) 4. 1.
and H I, are equal: Therefore any two of them is greater than the third: But if two angles are only equal, and the third lesse; [b] two bases b) 4. 1.

shall

c) 24. 1. shall be only equal, and the base of the third [c] lesse than either of the other; therefore again, the two others (which you please,) are greater than the other; but if one of the said angles, as A, be greater, let it be so as the
d) 24. 1. two others B and C, be equal, or not equal; the base [d] D E, shall be also the greatest of all: Wherefore D E and F G shall be greater than H I; and D E and H I greater than F G. Now I say that F G and H I are greater than D E.

e) 23: 1. *Demonstration* For [e] let the angle D A K be made equal to the angle B, and let A K be proposed equal to A D, then the point K will fall under D E; forasmuch as the circumference of the circle described at the distance A D, on the center A, doth passe by the points D K E, because of the equality of the lines A D, A K, and A E; then let D K and K E be joyned together.



Forasmuch as the two angles B and C are proposed greater than the angle D A E, and B is equal to the angle D A K by Construction; the angle C shall be greater than the other angle K A E; and forasmuch as the sides A D and A K of the triangle A D K, are equal to the sides B F and B G of the triangle B F G, and the angles conteined of them D A K and
f) 4. 1. B are equal; the base [f] D K shall be equal to the base F G.

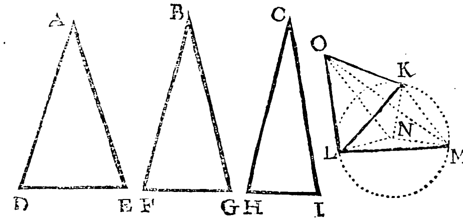
Again, seeing the sides C H and C I of the triangle C H I, are equal to the sides A K and A E of the triangle A K E; and the angle C demon-
g) 24. 1. strated greater than the angle K A E, [g] the base H I shall be greater than
20. 1. the base K E; therefore seeing that D K is demonstrated equal to F G; F G and H I shall be greater than D K and K E: But D K and K E are greater than D E: Therefore F G and H I shall be yet greater than D E. Which was proposed.

PROP. 23. PROBL. 3.

To constitute a solid angle of three plain angles, A, B, *and* C, *two of which taken at pleasure are greater than the other: But these three angles ought to be lesse than four right angles.*

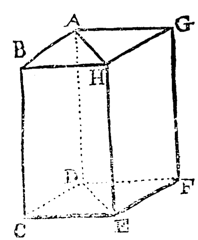
a) 22. 11. *Construction* Let [a] the six lines A D, A E, B F, B G, C H and C I, be proposed equal, conteining the said angles A, B, and C, subtended of the bases D E, F G, and H I, and making a triangle of D E, F G, and H I, let the triangle L M K be made, having the three sides
b) 22. 11. equal to D E, F G, and H I, about which [b] let there be described the circle K M L; and from the center N let N M, N L, and N K be drawn;
c) 5. 4. and [c] having drawn in the aire N O perpendicular, and [d] cut off, in such
d) 12. 11. sort as his square, with the square of N L, may be equal to the square of one of the sides of the said plain angles, and let L O, K O, and M O, be drawn, I say that L M O K is a solid angle, conteined of three plain angles, equal to A, B, and C.

Demon-



Demonstration For N L, N M, and N K being equal, and N O perpendicular to them; L O, M O, and K O, shall be equal; having [e] the like power the one as the other, and seeing they have the e) 47. 1. same power as one of the sides of the given plain angles, they shall be equal to the sides of the said plain angles; the bases L M, M K, and K L, being equal to the bases of the given angles: The three angles [f] at the point f) 8. 1. O shall be equal to the three given angles A, B, and C.

PROP. 24. THEOR. 21.



If a solid A B C D, *be conteined under parallel plains*, A C, C F, F H, H A, A F, *and* B E, *the plains opposite thereunto are parallelograms alike and equal.*

Demonstration For seeing the parallel plains B G and C F are cut by the plain A C; [a] their common a) 16. 11. sections A B and C D shall be parallel: In like manner, the parallel plains A F and B E, being cut by the plain A C, their common sections A D and B C shall be also parallel; therefore the quadrilateral figure A B C D is a parallelogram, and so it is demonstrated that the other figures quadrilateral, are parallelograms: Now I say that the opposite parallelograms are alike and equal.

For seeing that A B and B H are parallel to D C and C E, and are not in one and the same plain, but in opposite plains, [b] the angles A B H and b) 10. 11. D C E shall be equal; by the same reason the other angles of the parallelogram B G, shall be [c] equal to the other angles of the parallelogram c) 34. 1. C F.

But forasmuch as A B is equal to D C, in the parallelogram A C, and B H equal to C E, in the parallelogram B E; as A B is to B H, so D C is to C E, &c. and therefore as B H to H G, so C E to E F; and for the same reason, the sides of the parallelograms B G and C F, about the equal angles, shall be proportional: Therefore the parallelograms shall be alike.

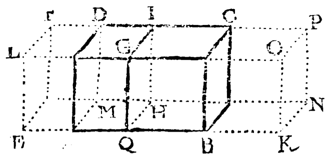
Now having drawn the diameters A H and D E; [d] seeing that the sides d) 34. 1.

A B

A B and B H of the triangle A B H, are equal to D C and C E, of the
e) 16. 11. triangle D C E; and [e] the angle A B H equal to D C E, as is shewn, the
f) 4. 1. triangles [f] A B H and D C E are equal to one another: Therefore [g] the
g) 34. 1. triangles A B H and D C E being the halves of the parallelograms B G and C F, they shall be equal to one another.

In like manner, we might shew that the opposite parallelograms A C and G E, and A F and B E are alike and equal to one another: Therefore, If a solid, &c. Which ought to be demonstrated.

PROP. 25. THEOR. 22.



If a solid Parallelepipedon A B C D, *be cut by a plain* I Q, *parallel to the opposite plains* A D *and* B C, *as the base* A H *shall be to the base* H B, *so the solid* A I *shall be to the solid* I B.

Demonstration. For A B being drawn forth towards E and K, let A E be proposed equal to A Q, and B K to B Q, and so finish the parallelograms E M and B N, and the solid A F and P B: Therefore A E and
a) 1. 6. A Q being equal, [a] the parallelograms E M and N Q shall be equal,
b) 24. 11. and [b] L D and D G opposite plains, equal and alike, and so A L and A G alike and equal to M F and M I, which are opposite plains, and so of the rest.

c) 9. def. 11. And the solid A F is equal to the solid A I, [c] and the solid B I to the solid B P: wherefore the solid E I is so multiplyed of the solid A I, as the base E H is of the base A H; and the solid Q P is so multiplyed of the solid C Q, as the base Q N is of the base B H: Therefore after the same manner that E H shall be greater, equal, or lesse than A H, so the solid E I shall be greater, equal, or lesse than the solid A I; also after the same manner that the base Q N shall be greater, equal, or lesse than H B, so the solid Q P shall be greater, equal, or lesse than the solid Q C.

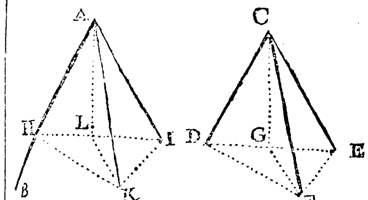
Let there be then four magnitudes A H and H B, bases, and H I and Q C solids, (whereof from the first and the third, *viz.* from A H base, and A I solid,) there were taken the equimultiplices E H base, and I E solid, and also from the second and fourth, (to wit, from the base B H, and the solid B Q,) the equimultiplices Q N and P Q, base and solid, the which are demonstrated greater, equal, or lesse, in what ever multiplication they be taken: Therefore [d] as the base A H is to the base H B, so
d) 6. def. 5. is the solid A I to the solid C Q: Therefore, &c. Which ought to be demonstrated.

COROLLARIE.

From these things it follows that if any Prisme be cut by a plain equidistant to the opposite plains, that the section is a figure equal and alike to the opposite plains; it having been shewn in the first Prisme that the triangle G H I *is equal and alike to the triangle* A B C; *and therefore also to the triangle* D E F; *and there is the same Demonstration in all, and the like is to be said of Parallelepipedons.*

PROP.

PROP. 26. PROBL. 4



To a given right line A B, *and to a point therein* A, *to constitute a solid angle equal to a solid angle given* C.

Construction. Let [a] there be drawn from F to the plain passing by C D and C E, the perpendicular F G, and let D F, D G, E F, F G, and C G be joyned; then let A H be cut off equal to C D; and [b] let the angle H A I be made equal to the angle D C E, and A I equal to C E. a) 11. 11.
b) 23. 1.

Again, on the plain drawn by A H and A I, let there be constituted the angle H A L, equal to D C G, which is on the plain drawn by C D, C E, and A L equal to C G; and [c] from L (to the plain where are A H, A L, and A I,) let there be drawn K L perpendicular, and put equal to F G, and joyn K A: I say the solid angle A, conteined under the three plain angles H A I, H A K, and K A I, is equal to C, the given solid angle. c) 12. 11.

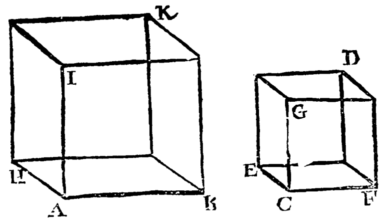
Demonstration. For having joyned H K, K L, I K, and I L; forasmuch as the sides A H and A L of the triangle A H L, are equal to the sides C D and C G of the triangle C D G, each to his correspondent side, and [d] the angles H A L and D C G equal by Construction, the d) 4. 1.
bases H L and D G shall be equal.

Again, having taken away the equal angles H A L and D C G, from the equal angles H A I and D C E, the remaining angles L A I and G C E are equal; then seeing that the sides A L and A I of the triangle A L I, are equal to C G and C E of the triangle C G E, each to his correspondent side by Construction; [e] L I and G E shall be equal: Foras- e) 4. 1.
much then as the sides L H and L K are equal to the sides G D and G F; and the angles H L K and D G F [f] are right angles, [g] the bases H K and f) 3. def.
D F are equal; therefore the sides A H and A K of the triangle A H K, g) 4. 1.
being equal to C D and C F of the triangle C D F, by Construction. (For seeing that A L and L K are equal to C G and G F by Construction, and do contein equal angles, to wit, [h] right angles; the bases A K h) 3. def.
and C F [i] are equal,) the angles [k] H A K and D C F shall be equal. i) 4. 1.

Lastly, L I and I K; being equal to G E and G F, and the angles I L K k) 8. 1.
and E G F right angles, [l] the bases I K and E F shall be equal. Therefore l) 3. def.
seeing the sides A I and A K of the triangle A I K, are equal to C E and C F of the triangle C E F, by Construction; the angles I A K and E C F shall be equal: Therefore the three plain angles H A I, H A K, and K A I, compounding the solid angle A, are equal to the three plain angles D C E, D C F, and F C E, compounding the solid angle C: Therefore the solid angle A is equal to the solid angle C: Wherefore, &c. Which ought to be done.

PROP.

PROP. 27. PROBL. 5.

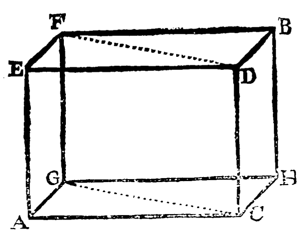


On a given right line A B, *to describe a solid parallelepipedon, alike, and alike posited to a given parallelepipedon* C D.

a) 26.11. *Construction* ON [a] the line A B, and at the point therein A, let there be made a solid angle, equal to the given solid angle C, in such manner as the three plain angles H A I, I A B, and B A H, may
b) 12.6. be equal to the three E C G, G C F, and F C E, and [b] as C F to C G; so let A B be made to A I, and as C G to C E, so A I to A H; and in reason of equality, as C F to C E, so A B to A H; and let the parallelepipedon A K be perfectly finished; to wit, the parallelograms B H, H I, and I B, being accomplished; and by the points I, B, and H, having drawn the plains I K, B K, and H K, which are parallels to the parallelograms B H, H I, and I B; I say the parallelepipedon A K is alike, and alike posited to the parallelogram C D.

Demonstration FOr seeing the angles B A H and F C E are equal, and the sides about them proportional; *viz.* as B A to A H, so F C to C E, by Construction; the parallelograms H B and E F, are alike, and alike posited: In like manner, H I and E G, and I B and G F; therefore the three plains B H, H I, and I B, of the solid A K are alike, and alike posited to the three F E, E G, and G F, of the solid C D: But
c) 24.11. [c] three of them, which you please, are equal, and alike to the three others opposite. Wherefore the six plains of the solid A K are alike, and
d) 9. def. alike posited to the six plains of the solid C D: Therefore [d] the solids A K and C D are alike and alike posited: Wherefore, &c. Which ought to be done.

PROP. 28. THEOR. 23.

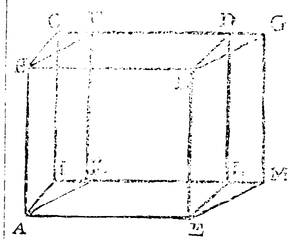


If a solid parallelepipedon A B, *be cut by a plain drawn by the diagonal lines of the opposite plains* A H *and* E B, *the solid shall be cut by that plain in two equal parts.*

a) 24.11. *Demonstration* FOr [a] seeing the Plains A H and E B are parallelograms, equal and alike, their halves, *viz.* the triangles A G C, G C H, E F D, and F D B, shall be equal the one to the other. But

But the sides about the equal angles G A C, C H G, F E D, and D B F, are proportional; therefore [b] the said triangles shall be also alike, and b) 6. 6.
the parallelogram A F being equal and alike to the parallelogram C B, and A D to G B, and C F common, the two triangles A G C and E F D, and [c] the parallelograms A F, A D, and C F, of the Prisme A C G F E D, c) 24. 11.
are equal and alike to the two triangles H C G and B D F, and to the parallelogram C B, B G and C F, of the prisme H G C D B F; wherefore [d] the said prismes shall be equal, which composing the parallelepipedon A B, the parallelepipedon A B shall be cut in two equal parts: d) 10. def.
Wherefore, &c. Which ought to be demonstrated.

PROP. 29. THEOR. 24.



The solids parallelepipedons A C D E *and* A F G E, *constituted on one and the same base* A B, *and of the same height, and from which the lines insisting, are placed in the same right lines, are equal to one another.*

Demonstration FOr [a] seeing the parallelograms A L and A M, made on the same base A E, and between the same parallels, are equal a) 35. 1.
to one another; taking away the common trapezium A L, the triangles A I K and E L M will remain equal.

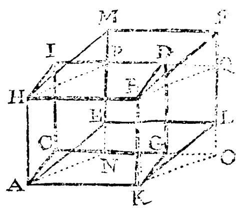
But [b] forasmuch as all the sides of the triangle A I K, are equal to all b) 34. 1.
sides of the triangle H C F, each to his correspondent side, [c] they shall c) Cor. 8. 1.
be equiangled and equal, and [d] shall have the sides about the equal angles d) 4. 6.
proportional, and therefore shall be alike to one another; and by the same reason the triangle E L M shall be alike and equal to the triangle B D G; Again [e] the parallelogram A C is equal and alike to the parallelo- e) 24. 11.
gram E D; and in like manner, the parallelogram A F to the parallelogram E G: But [f] I F is equal to L G the bases I K and L M being equal; (for f) 36. 1.
I L and K M being equal each to A E, are equal to one another; and taking away the common part K L, I K and L M will remain equal;) therefore [g] all the plains of the prisme A I K F C H are equal and alike to all the g) 34. 1.
plains of the prisme E L M G D B; therefore [h] these prismes shall be h) 10. def.
equal: Therefore if the common solid A H F K L D B E be added, the parallelepipedons shall be made equal, on the same base, &c. Which was to be demonstrated.

PROP. 30. THEOR. 25.

The solids parallelepipedons A I D K *and* A M F K, *made on the same base* A B, *and of the same height, and from which the lines insisting are not placed in the same right lines, are equal to one another.*

Demon-

Demonstration For seeing that the plains C D and E F, opposite to the base A B, are in one and the same plain, because of the same height of the parallelepipedons; let the lines C G and I D be prolonged on the same plain, of which let C G cut E M and L F, produced in the points N and O, and I D the same in the points P and Q; and let the right lines A N, K O, H P, and B Q, be joyned.



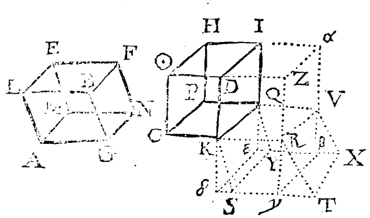
a) 34. 1. Forasmuch [a] as P Q and M F are equal, being opposite to the parallelogram F P, and M F is equal to H B, P Q and H B shall be equal: But they are parallelograms, H D B being a parallelogram;

b) 33. 1. therefore [b] H B and P Q are equal and parallel; and therefore H P Q B is a parallelogram; by the same reason H P N A, A N O K, and K O Q B, shall be parallelograms, and N O Q P is also a parallelogram; therefore A P Q R is a parallelepipedon, wherefore

c) 29. 1. the [c] parallelepipedon A I D K is equal to the parallelepipedon A P Q K, having the same base A B, and the lines insisting are placed in the same right lines C O and I Q; in like manner, the parallelepipedon A M F K shall be equal to the same A P Q K, having the same base A B, and the lines insisting being in the same right lines N M and O F: Wherefore the parallelepipedons A I D K, and A M F K, are equal to one another: Wherefore, &c. Which ought to be demonstrated.

PROP. 31. THEOR. 26.

The solids parallelepipedons A E F G, *and* C H I K, *made on equal bases* A B *and* C D, *and of the same height, are equal to one another.*



Demonstration For first of all, let the lines insisting A M, G N, L E, and B F, be perpendiculars to the base A B, and the insisting lines C P, K Q, O H, and D I, perpendiculars to the base C D, that being so, all the said perpendiculars shall be equal the one to the other, the parallelepipedons being of one and the same height. Let C K be produced directly, and let K R be put equal to L B, and the angle R K S (on the same plain produced, O K,) made equal to the angle B L A, and K S put equal to L A, and finish the parallelogram K T, on which constitute the parallelepipedons Q S T V, according to the height of the perpendicular R Q: Forasmuch as the sides K R and K S are equal to L B and L A, and the angles R K S and B L A equal, the parallelograms K T and L G shall be equal and alike.

Again, forasmuch as the sides K Q and K S are equal to L E and L A, and

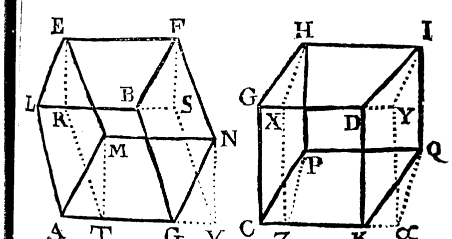
and [a] the angles Q K S and E L A are right angles, K Q and L E being propofed at right angles to the plains K T and L G, the parallelograms Q S and E A are equal and alike. a) 3. def. 11.

In like manner, the sides K R and C Q being equal to L B and L E; and the angles Q K R and E L B right angles, the parallelograms K V and L F are also equal and alike. Wherefore [b] seeing the three plains K T, b) 24. 11. Q S, and K V, of the parallelepipedon Q S T V are equal and alike to the three plains L G, E A, and L F of the parallellpipedon A E F G, as well those of the one, as those of the other, being equal to the three others opposite; the parallelepipedons [c] Q S T V and E A G F, are equal to one c) 24. 11. another.

Let D K and T S produced, meet in δ, and I Q, X Y in ε; and let the parallelepipedon $\mathcal{Q}\,\delta\,\gamma$ V be finished, and H I, β V produced to meet in ζ, and O D, γ R, in Z, and let the parallelepipedon I K R γ be finished: Forasmuch as the parallelepipedons $\mathcal{Q}$ S, T V, and $\mathcal{Q}\,\delta\,\gamma$ V have the same base K V, and the same height, to wit, between the same parallel plains K V, ad ζ X, their insisting lines K S, K δ, R T, R γ, $\mathcal{Q}$ Y, $\mathcal{Q}\,\varepsilon$, V X, and V ζ, are placed in the same right lines δ T and ε X, [d] they d) 29. 11. shall be equal to one another: But the parallelepipedon $\mathcal{Q}$ S T V is equal to the parallelepipedon E A G F; therefore $\mathcal{Q}\,\delta\,\gamma$ V shall be equal to the same E A G F.

But [e] forasmuch as K T and K γ, parallelograms, are equal to one an- e) 35. 1. other, and K T equal to L G and K γ, it shall be also equal to L G; that is to say, to C D, the base L G and C D being put equal; therefore [f] as f) 7. 5. C D is to D K, so is K γ to D R; but [g] as the base C D is to the base D R, g) 25. 11. so the solid C H I K to the solid K I α R; the parallelepipedon C H α R being cut by the plain I K, parallel to the opposite plains C H and α R: In like manner, as K γ, to D R: so the solid $\mathcal{Q}\,\delta\,\gamma$ V, to the solid I K R α; seeing that the parallelepipedon I $\delta\,\gamma\,\alpha$ is cut by the plain K V, parallel to the plains opposite D $\alpha\,\delta\,\zeta$. Therefore the parallelepipedons C H I K and $\mathcal{Q}\,\delta\,\gamma$ V, having the same proportion to the same solid I K R α; to wit, as the equal bases C D and K γ to the base D R; then seeing the parallelepipedon $\mathcal{Q}\,\delta\,\gamma$ V is demonstrated equal to the parallelepipedon A E F G, the parallelepipedons A E F G, and C H I K shall be equal: Which is proposed.

Now suppose that A M, G N, L E, and B F, and C P, K $\mathcal{Q}$, O H, and D I, be not perpendiculars to the bases A B and C D, and [h] from the h) 11. 11. points E, F, M, and N; let there be drawn the perpendiculars E R, F S, M T, and N V, to the plain of the base A B, and from the points H, I, P, and $\mathcal{Q}$, to the plain where is the base C D, let there be drawn the perpediculars H X I Y, P Z, $\mathcal{Q}\,\alpha$, they shall be



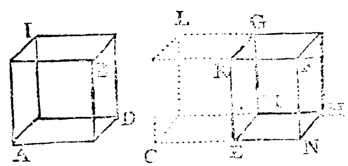
equal, the altitudes of the said parallelepipedons being equal; let there be

 drawn

drawn RS, SV, VT, TR, and XY; Zα, αI, and ZX, to make the paralellepipedons ETVF, and HZαI, the which being of the same height, and the lines insisting perpendicular, they shall be equal to one another, as is demonstrated.

i)29,30.11. But i ETVF is equal to AEFG, having the same base and height, and the paralellepipedon HZαI is equal by the same reason to CHIK; therefore AEFG and CHIK are equal to one another, the same demonstration may be made, if the insisting lines of one paralellepipedon are perpendiculars to the base, and of the other not perpendicular: Wherefore, &c. Which ought to be demonstrated.

PROP. 32. THEOR. 27.



The solid paralellepipedons ABCD *and* EFGH, *of the same height are to one another as their bases* AB *and* EF *are.*

a) 45. 1. *Demonstration.* FOr a let there be made on EK the parallelogram CK, equal to the parallellogram AB, having the angle CEK equal to the angle ENF; so the paralellograms EE and CK shall make the whole parallellogram, as is shewn in the 45. of the first.

Then if the other plains of the paralellepipedon EFGH are produced towards EG, and that the whole paralellepipedon CFLH be finished;
b) 31. 11. b the paralellepipedons ABID and CKLM shall be equal, being made on equal bases AB and CK, by construction, and of the same height by
c) 7. 5. supposition; therefore c as the solid CKLM is to the solid EFGH, so
d) 25. 11. is the solid ABID, to the same EFGH: But d CKLM is to EFGH as the base CK or the base AB his equal, is to the base EF; therefore the solid ABID shall be also to the solid EFGH, as the base AB is to the base EF: Therefore solid paralellepipedons of the same height, &c. Which ought to be demonstrated.

PROP. 33. THEOR. 28.

Like solid paralellepipedons ABCD *and* EFGH, *are the one to the other in a triple proportion to their homologal sides, or sides of the same proportion, as* AI *and* EK.

Demonstration. FOr, Let AI be produced to L, and let IL be equal to EK, or GR; and DI to M, and let IM be equal to HK or GO. and BI to N; and let IN be equal to KF or GS; then having accomplished the paralellograms LM, NL, and IT, let the paralellepipedon TXIV be finished.

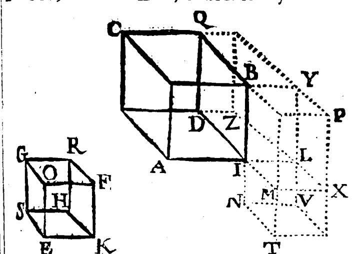
Forasmuch as the sides IL and IM are equal to the sides GR and
a) 15. 1. a GO, and the angles conteined of them also equal, the angle LIM being equal to AID, which because of the like paralellepiped. is equal to

EKH, or RGO, the parallelograms IX and GF shall be alike and equal, and in like manner LN, and RS, and IT and GE.

Therefore the three plains IX, LN, and IT, of the paralellepipedon TXIV are alike and equal to the three plains GF, RS and GE, of the paralellepipedon EFGH: But b three plains of each are alike and equal b) 24. 11.
to the three others opposite; therefore c the paralellepipedons TXIV c) 10. def.
and EFGH, are alike and equal.

Again, having accomplished the parallelograms MB, BL, and LM, finish the paralellepipedon MPBL, and the prralellograms IY, DL, and IQ, being accomplished, let the paralellepipedon IYQZ be also finished.

Forasmuch then as because of the like paralellepipedons ABCD and EFGH, as AI to EK; that is to say to IL, so DI to HK; that is to say to IM, and BI to FK, that is to say, to IN. But d as AI is to IL, so the paralellogram AD to DL; d) 1. 6.
and as DI to IM, so the parallelogram AD to LM; and as BI to IN, so the parallellogram BL to LN; then as AD to DL, so DL to LM; and BL to LN. But e as the base AD to the base DL, so the paralellepipedon ADCB e) 32. 11.



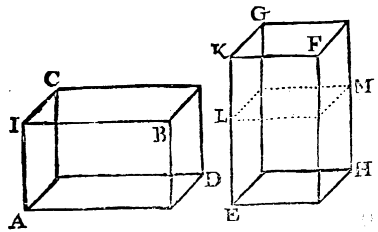
to the paralellepipedon DLYQ; and as the base DL to the base LM, so the paralellepipedon DLYQ, to the paralellepipedon LMBP, and as the base BL to the base LN, so the paralellepipedon LMBP to the parallelepipedon LNTX; therefore as ADCB to DLYQ, so DLYQ to LMBP, and LMBP to LNTX: Wherefore the four magnitudes ADCB, DLYQ, LMBP, and LNTX, are continually proportional; therefore f the first ADCB, is to the fourth LNTX; f) 10. def. 5.
that is to say, to EFGH in triple proportion to ADCB the first, to DLYQ the second.

But g as ADCB to DLYQ, so the base AD to the base DL; and g) 32. 11.
h as AD to DL, so the line AI to IL, that is to say, to EK; therefore h) 1. 6.
the paralellepipedon ADBC is in triple proportion to the paralellepipedon EFGH, of their homologal sides; to wit, of AI to AK: Wherefore, &c. Which ought to be demonstrated.

COROLLARIE.

From this it is manifest that if four right lines are continually proportional; as the first to the fourth, so the Paralellepipedon described on the first, shall be to the like Paralellepipedon, and alike described on the second: Forasmuch as it hath been shewn, that the Paralellepipedon is to the Paralellepipedon, as the first line to the fourth, in triple proportion of the first line to the second, to wit, of their Homologal sides.

PROP. 34. THEOR. 29.



The bases A D *and* E H, *and the altitudes of equal solid paralellepipedons* AD CB, *and* EHGF, *are reciprocal; and the solid paralellepipedons, whose bases and altitudes are reciprocal, are equal.*

Demonstration FOr if their altitudes be equal, the paralellepipedons being put equal, their bases shall be also equal; therefore as the base A D to the base E H, so the height E K, to the height A I; therefore the bases and the altitudes are reciprocal.

But if the heights A I and E K are unequal, let E K be the greater, from which let there be cut off E L equal to A I, and let the plain L M be understood to be parallel to the base E H, cutting the paralellepipedon.

Forasmuch as the solids A D C B and E H G F are equal, as
a) 7. 5. a A D C B to E H M L, so E H G F to E H M L; but b as the solid
b) 32. 11. A D C B to the solid E H M L; so the base A D to the base E H, the heights A I and E L being put equal; and as the solid EHGF to the solid E H M L; so by the same reason, the base K N to the base L N; since that, by that reason, the solids E H G F and E H M L have the same height K N and L N, being put for bases; for they shall be between the same parallel plains K N and G H; therefore as the base A D to the base
c) 1. 6. E H, so the base K N to the base L N: But c as K N to L N, so the line E K to the line E L; that is to say, to A I, equal to E L. Therefore as the base A D to the base E H, so the height E K to the height A I: Therefore the heights and the bases are reciprocal.

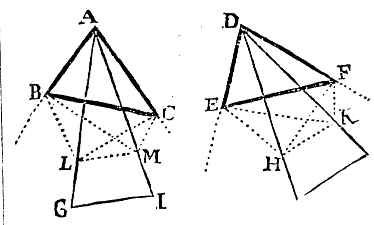
Now let the bases and heights be reciprocal, I say the paralellepipedons are equal; for if the heights E K and A I are equal, the base A D being put to the base E H, as the height E K to the height A I; the bases A D and
d) 31. 11. E H shall be equal; wherefore d the paralellepipedons A D C B and E H G F are equal to one another, having equal bases, and the same altitudes.

But if the height E K be greater, let E L be cut off equal to A I; and let the plain L M be understood to be paralel to E H.

Forasmuch then, as the base A D is to the base E H, so the height E K is to the height A I by supposition; that is to say, to E L, equal to A I:
e) 32. 11. But e as the base A D to the base E H, so the solid A D C B to the solid
f) 1. 6. E H M L, the heights A I and E L being put equal; and f as E K to E L,
g) 32. 11. so K N to L N; but g as the base K N to the base L N, so the solid E H G F to the solid E H M L, the solids E H G F, and E H M L having the same height, it K N and L N be put for bases: For so they shall be between parallel plains K N and G H, as the solid A D C B shall be to the solid E H M L,

E H M L, so the solid E H G F to the same solid E H M L: Therefore
h) 9. 5. h the solids A D C B and E H G F shall be equal, &c. Which ought to be demonstrated.

PROP. 35. THEOR. 30.



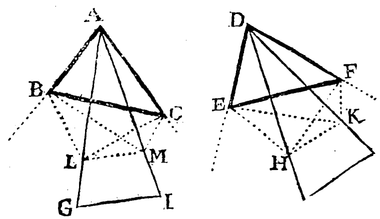
If there be two plain angles B A C *and* E D F *equal, at whose tops* A *and* D, *are raised in the aire two right lines* A G *and* D H, *conteining equal angles,* B A G *to* E D H, *and* C A G *to* F D H, *with the lines first proposed, each to his correspondent line; and in the lines raised in the aire, be taken any points at pleasure,* G *and* H, *and from them be drawn perpendiculars* G I *and* H K, *to the plains where are the angles first proposed,* B A C *and* E D F: *But at the points which are made on the plains, by the perpendiculars* G I *and* H K, *the right lines* I A *and* K D *are joyned, the same right lines shall contein equal angles with the lines drawn in the aire.*

Demonstration FOr if A G and D H be unequal, let there be taken A L from the greatest A G, equal to D H, and from L let there be drawn L M parallel to G I, to the plain of the triangle A G I; forasmuch as G I and L M are parallels, and G I is at right angles to the plain
a) 8. 11. of the angle B A C, L M a shall be also at right angles to the same plain: from the points M and K, let there be drawn the perpendiculars M B, M C, K E, and K F, to A B, A C, D E, and D F; and let B C, B L, L C, E F, E H, and H F, be joyned.

b) 3. def. And forasmuch as L M is at right angles to the plain B A C, it b will make a right angle with A M drawn to the same plain; wherefore
c) 47. 1. c the square of A L shall be equal to the squares of A M and M L; but
d) 47. 1. d the square of A M is equal to the squares of A C and C M, the angle A C M being a right angle by Construction; therefore the square of A L
e) 47. 1. is equal to the squares of A C, C M, and M L; but e the square of C L
f) 3. def. is equal to the squares of C M and M L, f the angle C M L being a right angle; therefore the square of A L is equal to the squares of A C and
g) 48. 1. C L; therefore g the angle A C L shall be a right angle.
h) 47. 1. Again, h seeing that the square of A L i is equal to the squares of A M
i) 47. 1. and M L: But the square of A M is equal to the squares of A B and B M, the

the angle A B M being a right angle by Conſtruction; the ſquare of A L is equal to the ſquares of A B, B M, and M L. But the ſquare of B L is
k) 3. def. equal to the ſquares of B M and M L, [k] the angle B M L being a right angle; therefore the ſquare of A L is equal to the ſquares of A B and B L;
l) 48. 1. therefore [l] the angle A B L ſhall be a right angle; by the ſame reaſon D F H and D E H ſhall be demonſtrated right angles.

Foraſmuch then as the angles A B L and L A B of the triangle L A B, are equal to D E H and H D E of the triangle D E H, and A L and D H
m) 26. 1. are equal; [m] the other ſides A B and B L ſhall be equal to the others D E and E H: In like manner, A C and C L ſhall be equal to D F and and F H; wherefore the ſides A B and A C, of the triangle A B C, are equal to D E and D F of the triangle D E F, and the angles B A C and E D F, conteined of them, are equal by



n) 4. 1. ſuppoſition: [n] the baſes B C and E F ſhall be equal, and A B C and A C B equal to D E F and D F E, each to his correſpondent angle. But the whole A B M and A C M are equal to the whole D E K and D F K, being all right angles; then the reſidues M B C and M C B ſhall be equal to the reſidues K E F and K F E: Wherefore B C and E F being demonſtrated equal, the ſides
o) 26. 1. [o] B M and C M ſhall be equal to the ſides E K and F K.

Foraſmuch as A C and C M of the triangle A C M are demonſtrated equal to D F and F K of the triangle D F K, and A C M and D F K right
p) 4. 1. angles, [p] the baſes A M and D K ſhall be equal to one another; and B L and E H being demonſtrated equal, their ſquares ſhall be equal: But foraſmuch as the ſquare of B L is equal to the ſquares of B M and M L,
q) 3. def. and the ſquare of E H equal to the ſquares of E K and K H; [q] B M L and E K H being right angles, the ſquares of B M and M L ſhall be equal to the ſquares of E K and K H.

Having then taken away the equal ſquares of B M and E K, demonſtrated equal; the remaining ſquares of L M and H K ſhall be equal; and therefore L M and H K equal: Wherefore ſeeing that the ſides A L and A M of the triangle A L M, are equal to D H and D K of the triangle D H K, and the baſe L M equal to the baſe H K, the angles L A M and H D K ſhall be equal: Therefore, If there be two plain angles, &c Which ought to be demonſtrated.

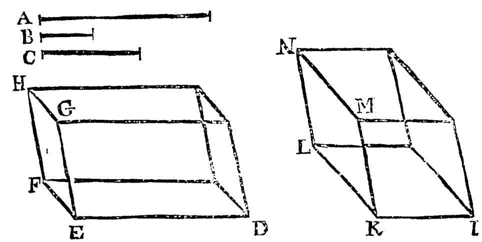
COROLLARIE.

If there be two equal plain angles, from the tops of which are drawn in the aire the equal right lines conteining equal angles with the lines firſt propoſed, each to his correſpondent angle, the perpendiculars drawn from the extremities of thoſe lines raiſed in the aire on the plain of the angles firſt propoſed, ſhall be equal to one another: For the plain angles B A C *and* E D F *being propoſed equal, and* A L *and* D H *raiſed on high, conſtituting the equal angles* L A B *and* H D E, *and* L A C *and* H D F, *it hath been ſhewn that* L M *and* H K *the perpendiculars, are equal to one another.*

PROP.

PROP. 36. THEOR. 31.

If there be three right lines proportional, A, B, *and* C, *the ſolid paralellepipedon* D H *made of them, is equal to the ſolid paralellepipedon* I N, *deſcribed of the mean* B, *provided it be equilateral, but equiangled to the aboveſaid.*



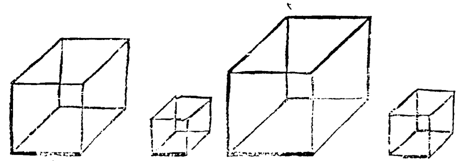
Conſtruction. LEt A, B, and C be continually proportional, and let there be made the ſolid angle E of any three plain angles D E F, D E G, and F E G, in ſuch manner as that D E be equal to A, and E F to B, and E G to C, and having accompliſhed the paralellograms D F, F G, and G D, let the paralellepipedon D H be finiſhed, which is ſaid to be conteined of the three lines A, B, and C, or made of them; [a] Then on the right line I K, and to a point therein K, let there be made the ſolid angle K, equal to the ſolid angle E, of the three plain angles I K L, I K M, and L K M, which let be equal to the three angles D E F, D E G, and F E C, in ſuch ſort as that I K, K L, and K M, may be equal each to the mean B, and having accompliſhed the paralellograms I L, L M, and M I; let the paralellepipedon I N be finiſhed, which is ſaid to be conteined under the line B, or deſcribed thereof: I ſay the ſolid D H is equal to the ſolid I N. a) 26. 11.

Demonſtration. FOr ſeeing that as D E to I K, ſo is K M to E G, (D E being taken equal to A, and I K and K M to B, and E G to C,) and the angles D E G and I K M equal to B, [b] the paralellograms D G and I M ſhall be equal, having the ſides about the equal angles reciprocal: But foraſmuch as the plain angles D E G and I K M are equal; from whoſe tops are drawn in the aire the lines E F and K L, conteining equal angles with the lines firſt propoſed, by conſtruction, each to his correſpondent angle, the perpendiculars drawn from F and L, to the plain of the baſes D G and I M, *viz.* the altitudes of the paralellepipedons D H and I N, (if the baſes D G and I M [c] be equal to one another:) Therefore [d] the paralellepipedons D H and I N, having the baſes D G and I M equal, and the altitudes alſo equal, they ſhall be equal to one another: Wherefore, If there be three right lines, &c. Which was to be demonſtrated.
b) 14. 6.
c) C. 35. 11.
d) 31. 11.

PROP.

PROP. 37. THEOR. 32.

If four right lines A, B, C, *and* D, *be proportional, the solid paralellepipedons* A, B, C, *and* D, *alike, and alike described of them, shall be also proportional; and if the solid paralellepipedons which are also alike and alike described are proportional, the right lines shall be also proportional.*



Construction. HAving constituted on the lines A and B, the two paralellepipedons A and B, alike and alike described; also on C and D describe the two others C and D, alike and alike posited, whether these be alike to the other or not, I say that as the solid A is to the solid B, so is the solid C to the solid D.

a) 33. 11.

Demonstration. FOr [a] seeing the solid A is to the solid B in triple proportion of the right line A, to the right line B, also the solid C to the solid D, in triple proportion of the right line C, to the right line D, the proportions of the solids A to B, and C to D shall be equal; forasmuch as they are triples of the equal proportions, to wit, of the proportion of the right line A to the right line B, and of the right line C to the right line D, which is first of all proposed.

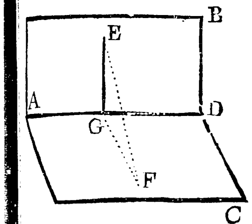
Secondly, let it be as the solid A is to the solid B, so the solid C to the solid D: I say that as the right line A is to the right line B, so the right line C is to the right line D.

b) 33. 11.

For [b] seeing that the solid A is to the solid B, in triple proportion of the line A to the line B, also the solid C is to the solid D in proportion triple of that of the line C to the line D, the proportions of the lines A to B, and of C to D are equal; forasmuch as their tripled proportions, to wit, of the solid A to the solid B, and of the solid C to the solid D, are put equal. Which was in the second place to be demonstrated.

PROP. 38. THEOR. 33.

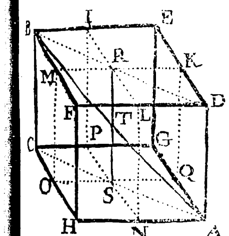
If a plain A B *be perpendicular to a plain* A C, *and from some point* E *of those that are in one of the plains* A B, *a line be drawn perpendicular to the other plain* A C, *the perpendicular drawn shall fall on the common section* A D, *of the plains* A B *and* A C.



a) 12. 1.
b) 4. def.
c) 3. def.
d) 17. 1.

Demonstration. FOr otherwise, let it fall elsewhere (if it be possible, as at F,) and [a] from some plain A C, let there be drawn F G perpendicular to A D, and let E G be joyned on the plain A B: Forasmuch as F G is perpendicular to the common section A D, [b] it shall be also perpendicular to the plain AB; and therefore [c] also perpendicular to G E: But E F is also perpendicular to F G: Therefore in the triangle E F G, the two angles E F G and E G F are right angles, which is absurd, being [d] they are lesse than two right: Therefore the perpendicular drawn from E on the plain A C, shall not fall out of the common section; therefore shall fall on it: Therefore, If a plain, &c. Which was to be demonstrated.

PROP. 39. THEOR. 34.



If the sides of the opposite plains A C *and* B D, *of a solid parallelepipedon* A B, *are cut in two equal parts, and that by the sections the plains be drawn, the common section of the plains* R S, *and the diameter of the solid paralellepipedon* A B *shall equally cut one another.*

a) 29. 1.
b) 4. 1.
c) 13. 1.
d) 14. 1.

Demonstration. FOr having joyned R B, R D, S A, and S C, let the two triangles A Q S and C O S be considered: Forasmuch as the sides A Q and Q S of the triangle A Q S, are equal to the sides C O and O S of the triangle C O S; (for A Q and C O are the halves of the equal sides A G and C H and [a] Q S and O S are equal to the two equal sides A N and H N; A S and H S being paralellograms) and [b] the angle A Q S equal to the alternate angle C O S; the bases A S and C S shall be equal, and the angles A S Q and C S O equal: But [c] the angles A S Q and A S O are equal to two right angles; therefore C S O and A S O are also equal to two right angles: Therefore [d] A S and C S shall make one only right line: In like manner, B R and D R shall be demonstrated equal, and shall make one only right line.

e) 9. 11.
f) 33. 11.

Again, forasmuch as A D as well as B C, is parallel and equal to F H, because of the paralellograms A F and F C, they [e] shall be also parallel to one another and equal: Wherefore [f] A C and B D which joyn them at the extreams, are also equal and parallel; therefore their halves A S and BR are equal.

g) 7. 11.

But forasmuch as A C and B D are parallels, [g] A B and R S shall be in one and the same plain with them, therefore they shall cut one another,

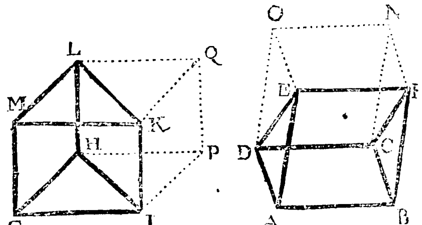
h) 15,29.1. to wit, in the point T. But [h] seeing that the two angles A S T and A T S, of the triangle A S T are equal to the two angles B R T and B T R of the
i) 26.1. triangle B R T, and the side A S to the side B R, [i] the other sides T A and T S shall be equal to the other sides T B and T R: Therefore A B and R S do cut one another in two equal parts in the point T. Wherefore, &c. Which was to be demonstrated.

CORLLARIE.

It follows from this Demonstration that in every paralellepipedon, all the diameters cut one another in two equal parts at one point, to wit T, *(as is here shewn by the line* R S,) *and every plain cutting the paralellepipedon in two equal parts, doth passe by the center thereof, as by the point* T.

PROP. 40. THEOR. 35.

If two Prismes ABCDEF, *and* GHIKLM, *are of the same height, of which the one hath a paralellogram* ABCD, *for the base, and the other a triangle* GHI, *and that the paralellogram be double the triangle, those prismes shall be equal.*



Demonstration FOr let the paralellepipedons A N and Q G be accomplished, prolonging the plains of the triangles, to make the paralellograms B N, A O, G P, and M Q: For having drawn N O and P Q, there shall be made two paralellepipedons A N and G Q, of the same height with the prismes, and to which the opposite plains are paralels, as is easie to be gathered from the 15th. Prop. of this Book.

a) 34.1. Forasmuch [a] as the paralellogram G P is double to the triangle G H I, and the paralellogram A C is put double to the same triangle G H I, the
b) 31.11. paralellograms A C and G P shall be equal; therefore [b] A N and G Q are of the same height, and on equal bases A C and G P, shall be equal to one another; therefore their halves, to wit, the prismes A B C D E F, and
c) 32.11. G H I K L M, (for [c] the paralellepipedons A N and G Q are cut each in two equal prismes, by the diameters of the opposite plains C F, D E, H I, and L K,) are alsoe equal to one another: Therefore, &c. Which was to be demonstrated.

The End of the Eleventh Element of EUCLIDE.

THE TWELFTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

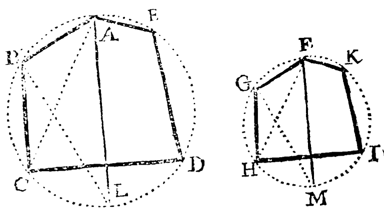
THis Twelfth Book setteth forth the Passions and Proprieties of Pyramids, Prismes, Cones, Cylinders and Spheres, and compareth Pyramids to Pyramids and Prismes, and likewise compareth Cones and Cylinders, and lastly Spheres one to another: But before he treateth of the afore-mentioned bodies, he proveth that like polygonal figures inscribed in Circles, and also the Circles themselves, are in such proportion the one to the other, as the squares of the Diameters of such Circles are, it being absolutely necessary that those things be first proved, for the better confirmation of the divers passions and proprieties of those bodies.

PROPOSITIONS, PROBLEMS, and THEOREMES.

PROPOSITION 1. THEOREM 1.

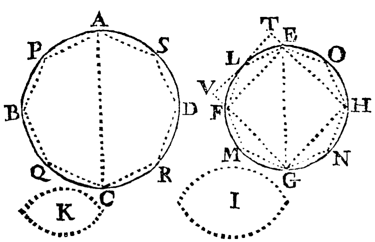
Like polygons ABCDE *and* FGHIK, *inscribed in circles, are in proportion the one to the other, as the squares described of the diameters* AL *and* FM, *of the circles.*

Demonstration FOr, let AC and FH be drawn, subtending the equal angles ABC and FGH; and let BL and GM be joyned: Then, by reason of the similitude of the polygons, AB being to BC, as
a) 6. 6. FG to GH, [a] the triangles ABC and FGH shall be equiangled, having the sides about the equal angles ABC & FGH,
b) 21. 3. proportional. But [b] the angle ALB is equal to the angle ACB, and the angle FMG equal to FHG; therefore ALB and FMG shall be equal; wherefore ABL and FGM being
c) 31. 3. also equal, [c] viz. right angles in the semicir-



d) 32. 1. cles [d] the remaining angles BAL and GFM shall be equal: Therefore
e) 4. 6. [e] as AL to AB, so FM to FG; and alternately, as AL to FM, so AB to FG: Therefore as the square of AL to the square of FM, so the polygon ABCDE described on the right line AB, to the polygon FGHIK described on FG; seeing that as well the squares as the polygons, are like figures, and alike describec: Therefore, Like polygons, &c. Which ought to be demonstrated.

PROP. 2. THEOR. 2.



Circles ABCD *and* EFGH, *are in proportion the one to the other, as the squares of their diameters* AC *and* EG.

Demonstration FOr if it be not so, I shall be lesse or greater; Suppose first of all I to be lesse, (if possible,) and that the circle
a) 10. 10. EFGH be greater than I, by the figure K: But [a] cutting off more than the half of the circle EFGH, and more than the half of the rest, so often
b) 6. 4. that at last there may remain a magnitude lesse than K: Let there be [b] inscribed in that circle the square EFGH, which shall be greater than the half the circle, if the four segments of the circle are lesse than K, you have what you require, if not, let the arches EF, FG, GH, and HE, be divided in two equal parts by L, M, N, and O; and let the Octogon be inscribed in the circle, drawing EL, LF, FM, MG, GN, NH, HO, and OE; it appeares that there will be four triangles equal in the four equal segments, and that each one, as FLE, shall exceed the
c) 41. 1. half of his section, the Isoscele triangle ELF [c] being the half of the Rectangle of the same height, described on the base EF, as EV, which is greater than the section FLE,

Now

Now let the eight remaining figures FM, MG, &c. be lesse than K, for if it were not so, it would be required to divide the last arches, and alwayes inscribe the polygons, whereof the last shall have the sides double in multitude to those of his precedent, and to take away still more than the half of each segment (to wit) his Isoscele triangle: It is evident that the last sections shall be in the end lesse than K.

Lastly, Let then the said eight sections remaining be lesse than K: It is manifest that the said Octogon shall be greater than I, I and K together, being equal to the circle EFGH. Let there be also described an Octogon alike, in the circle ABCD, dividing each of the semicircles ABC and ADC in two equal parts, in B and D, and again, each part in two equal parts in P, Q, R, and S; and having joyned the right lines to all the said points; it is manifest that the Octogon figure inscribed, shall be alike to the Octogon inscribed in the circle EFGH; [d] and the said Octogon of the circle ABCD, shall be to the Octogon of the circle EFGH, as the d) 1. 12.
square of the diameter AC, to the square of the diameter EG. But as the square of AC, to the square of EG, so the circle ABCD is to I: Therefore [e] as the polygon AQS to the polygon EMO: so the circle ABC shall be to I. But the polygon AQS is lesse than the circle ABC, e) 11. 5.
therefore [f] the polygon EMO shall be also lesse than I, which is absurd; f) 14. 5.
it being demonstrated to be greater: Therefore the figure I cannot be lesse than the circle EFGH.

Nor can it also be greater; For the circle ABC being to I, as the square of AC to the square of EG; alternately, I shall be to the circle ABC, as the square of EG to the square AC; but let it be understood as I is to the circle ABC, so the circle EFG is to another, as to K; therefore [g] as I is greater than the circle EFG, so the circle ABC shall be greater g) 14. 5.
than K, and [h] the circle ABC shall be to K, as the square of AC to the h) 11. 5.
square of EG, which is contrary to the first part of this Proposition; where it is shewn that one of the circles being to a certain figure in the proportion of the squares of the diameters, that that figure cannot be lesse than the other circle: Therefore I cannot be greater than the circle EFG, nor lesse, as by the first part hereof, therefore equal: Therefore, Circles, &c. Which ought to be demonstrated,

COROLLARIE.

From hence it follows, that as the circle is to the circle, so the polygon described in the one circle, to the like polygon described in the other circle; seeing that as well the circle is to the circle, and the polygon to the polygon, as the square of the diameter to the square of the diameter, as hath been demonstrated.

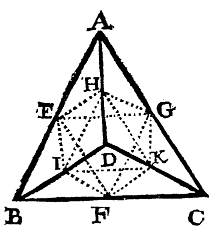
PROP. 3. THEOR. 3.

Every pyramid having the base ABC *triangular, may be divided into two pyramids* AEGH *and* HIKD, *and alike to one another, having the bases* AEG *and* HIK, *triangular, and alike to the whole, and in two equal prismes, which two prismes are greater than the half of the whole pyramid.*

Demon-

Demonstration. FOr the sides A D and D B, of the triangle A D B being divided in two equal parts, and therefore proportionally; a) 2.6. [a] H I and A B shall be parallels, and also I K, B C and H K, A C and E G, B C and E F, A C and F G, A B and E H, B D and E I, A D and I F, D C and H G, and D C, are parallels by the same reason. b) 9.11. But [b] F G and H I being parallels to A B, are parallel the one to the other, and so G H and F I being parallels to D C, are also parallel to one another; therefore A E I H, H E B I, I D H E, E B F G, G H K C, C K I F, and F G H I are parallellograms. But H E and H G being parallel to D B and D C, c) 10.11. [c] the angles E H G and B D C are equal; by the same reason H E G, D B C, and H G E and D C B shall be equal; d) 4.6. therefore [d] the sides of the triangle H E G are proportional to the sides of the triangle D B C, about the equal angles: Wherefore the triangle H E G is alike to the triangle D B C. e) Cor.4.6. But [e] the triangles H A E, H A G, and A E G, are also alike to the triangles D A B, D A C, and A B C; therefore f) 9.def.11. [f] the pyramid A E G H is alike to the pyramid A B C D.

Again, H I and H K being parallels to A B and A C, g) 10.11. [g] the angles I H K and B A C shall be equal; and in like manner H I K and A B C, and H K I and A C B equal; h) 4.6. [h] wherefore the sides of the triangle H I K are proportional to the sides of the triangle A B C, about the equal angles, and the triangle H I K is alike to the triangle A B C: But the triangles D H I, D I K, and D L H, i) Cor. 4.6. [i] are alike to the triangles D A B, D C B, and D C A; therefore the pyramid k) 9.def.11. [k] H I K D is alike to the pyramid A B C D. But the triangles A H E and H D I being alike to the triangle A D B, as is shewn, l) 21.6. [l] they shall be also alike to one another, and being made on equal lines A H and H D, they



shall be equal; by the same reason A H G and H D K shall be equal and alike, being proved alike to the triangle A D C, and made on equal lines A H and H D, and so the triangle A E G and H I K, shall be equal and alike, being proved alike to the triangle A B C, and posited on the equal lines A E and H I, being m) 34.1. [m] the opposite sides of the parallellogram A E H I.

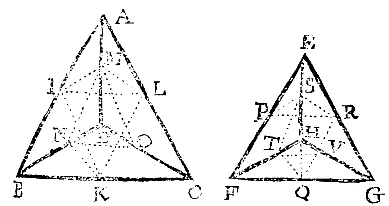
Also E H G and I D K shall be equal and alike, being proved alike to the triangle B D C, and having H E and D I equal n) 34.1. [n] to the parallellogram H E I D: Therefore the pyramids A E G H and H I K D o) 10.def.11. [o] are equal and alike, all the triangles of the one being proved equal and alike to all the triangles of the other.

Again, p) 34.1. [p] E H, H G, and G E, being equal and parallel to B I, I F, and F B, of the parallellograms E H I B, F G H I, and B F G E, the triangles E H G and B F I q) Cor.8.1. [q] shall be equiangled and equal to one another; and therefore alike; But they are also parallel to E H and H G, by which the plain E H G is drawn, being parallels to B I and I F, by which is drawn the plain B I F; therefore the solid B I F G H E, conteined of the two triangles E H G and B I F equal, alike, and parallel, and opposite one to the other, and by the three parallellograms E G F B, B E H I, and I F G H is a prisme by the definition. So the solid C F G H I K shall be shewn

shewn to be a prisme: For F C, C G and G F being equal and parallel, to I K, K H, and H I, of the parallelograms C F I K, C G H K, and F G H I, the triangles C F G and H I K shall be equal and equiangled to one another, and therefore alike: r) 15.11. [r] But they are parallels, C F and C G, by which is drawn the plain C F G, being parallel to K I and K H, by which passeth the plain K I H; therefore the solid C F G H I K, conteined of the two opposite triangles C F G and H I K, shall be equal, alike, and parallel and the three parallellograms C F I K, K H G C, and I F G H, is a prisme, and those prismes E B F G H I, and C F G H I K are of the same height, to wit, between the parallel plains B C G E and H I K, and the base quadrangular being double to the triangular, s) 40.11. [s] they shall be equal to one another.

Lastly, because the prisme E B F G H I is greater than the pyramid E B F I, the whole than its part: But the pyramid E B F I is equal and alike to the pyramid A E G H, and also to the pyramid H I K D, as is manifest by the equality and similitude of the triangles; the prismes E B F G H I and C F G H I K shall be greater than the pyramyds A E G H, and H I K D; therefore these prismes do exceed the half of the whole pyramid A B C D. For a whole being divided into two unequal parts, the greatest part exceeds the one half thereof, and the lesser part wants thereof: Therefore, Every pyramid having the base, &c. Which ought to be demonstrated.

PROP. 4. THEOR. 4.



If there be two pyramids ABCD, *and* EFGH, *of the same height, having the bases* ABC *and* EFG *triangular, and that each of them be divided in two pyramids, equal to one another,* AILM, MNOD, EPRS, *and* STVH, *and alike to the whole, and in two equal prismes* IBKLMN, CKLMNO, PFQRST, *and* GQRSTV, *and that in like manner both the one and the other of the pyramids produced by this first division, be divided, and that that be alwayes done after one manner; as the base* ABC *of one of the pyramids, shall be to the base* EFG *of the other, so also all the prismes which are in the one of the pyramids, shall be to all the prismes of the other equal in number.*

Demon-

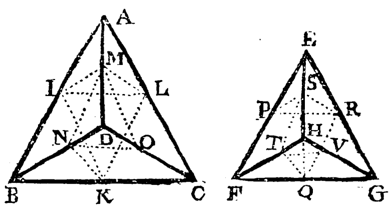
Demonstration. FOr seeing that as B C to C K, so F G to G Q, both the one and the other line being divided in two equal parts, and the triangles A B C and L K C being alike and alike posited.

a) Cor.4. 6.
b) 22. 5.

Also, [a] the triangles E F G and R Q G, then [b] as the triangle A B C shall be to the triangle L K G, so the triangle E F G to the triangle R Q G; and alternately, as A B C to E F G, so L K C to R Q G: But as L K C to R Q G; so the prisme C K L M N O to the prisme G Q R S T V, as shall be presently shewn; and therefore so the prisme I B K L M N to the prisme P F Q R S T V; these being equal to the others, and as one prisme alone, to wit, I B K L M N, to one prisme alone P F Q R S T; so

c) 12. 1.

[c] the two prismes, I B K L M N and C K L M N O together, are to the two prismes P F Q R S T and G Q R S T V together; therefore also as the base A B C to the base E F G, so the two prismes in the pyramid A B C D, to the two in the pyramid E F G H: In like manner may be shewn that the two prismes that are in the pyramids A I L M and M N O D, made in the pyramid A B C D, are to the two prismes which are in the pyramids E P R S and S T V H, made in the pyramid E F G H; as the bases A I L and M N O of those pyramids, are to the bases E P R and S T V of these, and so following, making alwayes the same division: But as these bases are to the others, so the base L K C, which is equal and alike to these, is to the base R Q G, which



is equal and alike to these, as is demonstrated by the precedent; that is to say, so the base A B C is to the base E F G: Therefore also as the base A B C to the base E F G, so the prismes of each pyramid made in the pyramid A B C D, shall be to the prismes of each pyramid made: Therefore also as the prismes of the pyramid A B C D, to the prismes of the pyramid E F G H, so the prismes as well of the pyramid A I L M, are to the prismes of the pyramid E P R S; as these of the pyramid M N O D, to these of S T V H, and so following.

d) 12. 5.

Wherefore seeing that as two prismes of the pyramid A B C D, are to two prismes of the pyramid E F G H, [d] so all the prismes which are in the pyramids A B C D, A I L M, and M N O D, &c. together, are to all the prismes together, which are in the pyramids E F G H, E P R S, and S T V H, &c. if those are equal in number to these: In like manner, as the base A B C to the base E F G, so all the prismes of the pyramid A B C D, to all the prismes of the pyramid E F G H: Therefore, If two pyramids, &c. Which ought to be demonstrated.

PROP. 5. THEOR. 5.

The pyramids A B C D *and* E F G H, *which are of the same height, and having the bases* A B C *and* E F G *triangular, are the one to the other as their bases.*

Demon-

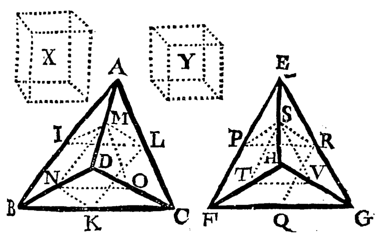
Demonstration. FOr if it be not so, let it be put as the base A B C to the base E F G, so the pyramid A B C D to the solid X, which shall be lesse or greater than the pyramid E F G H.

First, Let it be greater by the solid Y, and let the pyramid E F G H be divided in two equal pyramids, and in two equal prismes, according to Prop. 3. of the 12th. Again the two pyramids made in E F G H, let be in like sort divided in two equal pyramids, and two equal prismes, and so following.

Forasmuch as, if from the pyramid E F G H be taken more than the half, to wit, the two prismes P F Q R S T and G Q R S T V, [a] greater than the half of the pyramid E F G H; also from the remaining pyramids E P R S and S T V H, more than the halfe, to wit, their prismes, and so following; [b] there will remain in the end a magnitude lesse than Y, the excesse of E F G K, on the solid X. Now then let the remaining magnitude be lesse: But the pyramid E F G H being put equal to the solids X and Y, the prismes remaining in the pyramid E F G H, shall be greater than X.

a) 3. 12.
b) 1. 10.

Let the pyramid A B C D be divided into two equal pyramids and two equal prismes, and in like manner the pyramids A I L M and M N O D, in two equal pyramids, and two equal prismes; and let that be done so many times as in the pyramid E F G H.



c) 4. 12.

Forasmuch [c] as all the prismes in the pyramid A B C D, are to all those of the pyramid E F G H, equal in number; as the base A B C to the base E F G, that is to say, as the pyramid A B C D is to X. But all the prismes in the pyramid A B C D being lesse than the whole A B C D, also [d] all the prismes in the pyramid E F G H shall be lesse than the solid X. But they also are proved greater, which is absurd. Therefore X is not lesse than E F G H.

d) 14. 5.

Now let the solid X be greater than the pyramid E F G H; therefore the pyramid A B C D being put to X, as the base A B C to the base E F G; by conversion of proportion, X shall be to A B C D, as the base E F G to the base A B C, as X is to the pyramid A B C D, so let the pyramid E F G H be put to the solid Y, and X being put greater than E F G H; [e] the pyramid A B C D shall be greater than Y; therefore as the base E F G is to the base A B C, so the pyramid E F G H is to Y, which is lesse than the pyramid A B C D, which is absurd. It being already shewn that as the base is to the base, so the pyramid cannot be a solid lesse than the pyramid: Therefore the solid X is not greater than the pyramid E F G H, and is also shewn not to be lesse, therefore equal. Wherefore seeing that as the base A B C is to the base E F G, so the pyramid A B C D is put to X: But [f] the pyramid A B C D is to X, as the pyramid E F G H equal to the solid X; also as the base A B C to the base E F H, so the pyramid A B C D to the pyramid E F G H: Therefore, The pyramids, &c. Which ought to be demonstrated.

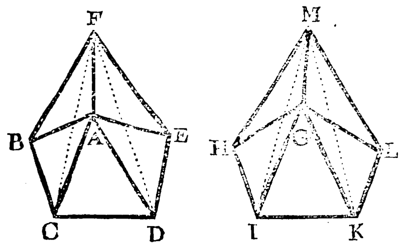
e) 14. 5.
f) 9. 5.

COROLLARIE.

Hence it comes that the pyramids of one height made on the same base, or equal triangular bases, are equal to one another, having the same proportion that their bases have, which are proposed equal, or one and the same.

It follows contrariwise that the equal triangular pyramids made on the same base, or equal bases are of the same height, and that equal triangular pyramids having the same height, have equal bases, or one and the same, the which two things is shewn by the first part of this Corollary; by the same discourse used in the Demonstration of the converse of the 30th. Prop. and 31th. of the Eleventh Book. If another pyramid be made, cut off as well in the height, as in the base.

PROP. 6. THEOR. 6.



Pyramids ABCDEF *and* GHIKLM, *which are of one height, and having the bases polygons,* ABCDE *and* GHIKL, *are the one to the other, as are their bases.*

Demonstration FOr having reduced the bases into triangles equal in number, each pyramid shall be divided into so many triangu-
a) 5. 12. lar pyramids: But [a] seeing that as the base A B C is to the base A C D, so the pyramid A B C F to the pyramid A C D F; and compoundedly, as the base A B C D to the base A C D, so the pyramid A B C D F to the py-
b) 5. 12. ramid A C D F: But [b] again, as the base A C D to the base A D E, so the pyramid A C D F to the pyramid A D E F: Therefore in proportion of equality, as the base ABCD to the base ADE, so the pyramid A B C D F to the pyramid A D E F, Therefore compoundedly, as the base A B C D E to the base A D E, so the pyramid A B C D E F shall be to the pyramid A D E F; by the same reason, as the base G H I K L shall be to the base G K L, so the pyramid G H I K L M to the pyramid G K L M; and by converse proportion, as the base G K L to the base G H I K L; so the pyramid G K L M to the pyramid G H I K L M.

c) 5. 12. Again, [c] forasmuch as the base A D E is to the base G K L; as the pyramid A D E F, to the pyramid G K L M, there will be four bases ABCDE, ADE, GKL, and GHIKL, in the same proportion as the four pyramids A B C D E F, A D E F, G K L M, and G H I K L M; therefore in proportion of equality, as the base A B C D E to the base G H I K L, so the pyramid A B C D E F to the pyramid G H I K L M; And therefore, Pyramids, &c. Which ought to be demonstrated.

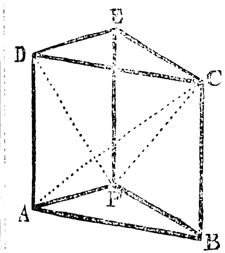
COROLLARIE.

It is manifest that pyramids of the same height made on bases equimultilateral, are

put in one and the same, are equal to one another, having the same proportion with their bases that are put equal to one and the same.

Again it followes contrariwise that the pyramids equimultilateral, and made on equal bases, equal to one and the same, have one height: And that the pyramids equimultilateral: having the same height, have the bases equal, if they be not on one and the same: These two things shall be demonstrated as we have said in the Corollary of the fifth Proposition of this Book.

PROP. 7. THEOR. 7.



Every prisme as ABCDEF, *having the base triangular, may be divided into three pyramids, equal to one another, having the bases triangular.*

Demonstration FOr in the three parallelograms let there be drawn three diameters, to wit, A C in the parallelogram A B C D, C F in the parallelogram B C E F, and F D in the parallelogram A D E F: Forasmuch [a] as the triangles A B C and A D C are equal, and [b] that as the base A B C is to the base A D C, so the pyramid A B C F to the pyramid A D C F, those pyramids having the same height, to wit, the perpendicular drawn from the top F, to the plain A B C D, the pyramids A B C F and A D C F, shall be equal to one another: In like manner the pyramids A D F G and E F D C, made on the equal bases A D F and E F D, and the same height, to wit, the perpendicular drawn from the top C, on the plain A D E F, shall be equal: But the pyramid A D C F is the same with the pyramid A D F C, that being conteined of four plains, to wit, of the base A D C, and the triangles A D F, A C F, and D C F, and this of the same four plains, to wit, of the base A D F, and the triangles A D C, A C F, and D C F: Therefore the three pyramids A B C F,) A D C F, and E F D C, or C D E F, (which is the same with E F C D, compounding the whole prisme, are equal to one another: Therefore, Every Prisme, &c. Which ought to be demonstrated.

a) 34. 1.
b) 5. 12.

COROLLARIE.

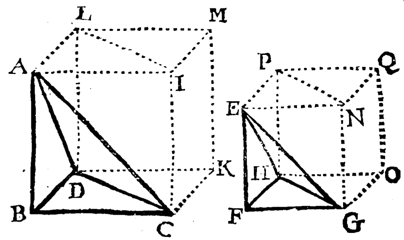
It follows from what hath been said, that every pyramid is the third part of the prisme that hath the same height and base, or equal thereunto, or else, every prisme is the triple of the pyramid which hath the same height and same base, or equal thereunto.

PROP. 8. THEOR. 8.

Like pyramids ABCD *and* EFGH, *which have the bases* ABC *and* FEG *triangular, are in a triple proportion of their homologal sides* BC *and* FG.

 Demon-

Demonstration. For let the parallelepipedons B M and F Q be finished, the pyramids being alike, the plain angles A B C and E F G shall be equal; and as A B to B C, so E F to F G, the parallelograms B I and F N shall be alike: In like manner, the angles A B D and E F H being equal, and as A B to B D, so E F to F H; also the angles D B C and H F G equal; and as D B to B C, so H F to F G; the parallelograms A L and B K shall be alike to the parallelograms F P and F O.
a) 14. 11. But [a] as well the three sides B I, B K, and B L of the parallelepipedon B M, are equal and alike to the three opposite sides D M, A M, and C M; as the three sides F N, F O, and F P, of the parallelepipedon F Q, are equal and alike to the three opposite sides H Q, E Q, and G Q; therefore the six plains conteining the solid B M, are equal to the six plains comprehending the solid F Q:



b) 9. def. 11. Wherefore [b] the parallelepipedon B M and F Q are alike: But forasmuch as the lines
c) 15. 5. L I P N being drawn, [c] the prismes D B C I L A and H F G N P E have the same proportion as the parallepipedons B M and F Q their double, and the pyramids A B C D and E F G H the same proportion as the said
d) 11. 5. prismes the triple of them, [d] the pyramids shall have also the same proportion as the parallelepipedons.
e) 33. 11. But [e] the parallelepipedons B M and F Q being in triple proportion of their homologal sides B C and F G, the pyramids A B C D and E F G H shall be also in triple proportion of B C to F G: Therefore, &c. Which was to be demonstrated.

COROLLARIE.

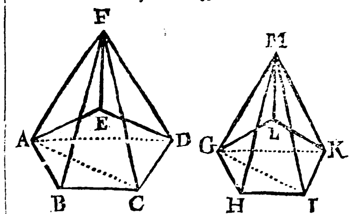
It is also manifest that the like pyramids whereof the bases have more than three sides, are in a triple proportion of their homologal sides.

For let A B C D E F *and* G H I K L M, *be like pyramids, having for their bases rectiline figures of several sides, I say that those pyramids are in triple reason of the homologal sides* A B *and* G H; *for from* A *and* G *having drawn the lines* A C, A D, G I, *and* G K, *to the opposite angles, the like bases shall be divided into an equal number of like triangles, to which* A B C, A C D, *and* A D E, *shall be alike to* G H I, G I K, *and* G K L; *therefore the pyramids being alike, the triangles* A F B *and* G M H *are alike, and the angle* F A B *equal to* M G H, *and as* F A *to* A B, *so* M G *to* G I; *the triangle* A B C *and* G H I *being alike, and therefore in equal reason as* F A *to* A C, *so* M G *to* G I.

Again, forasmuch as A C *is to* C B, *so is* G I *to* I H, A B C *and* G I H *being alike, and as* G B *to* C F, *so* I H *to* I M, *the triangles* B C F *and* H I M *being alike, by reason of the similitude of the pyramids, and in equal reason as* A C *to* C F, *so is* G I *to* I M: *Therefore being as* F A *to* A C, *so* M G *to* G I, *and as* A C *to* C F, *so* G I *to* I M; *also in equal reason, as* F A *to* F C, *so is* M G
5. 6. *to* M I: *Therefore* A F C *and* G M I *shall be equiangled, and therefore alike.*

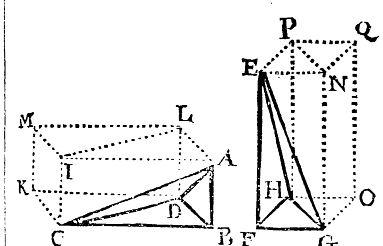
But A F B, B F C, *and* A B C *are also alike to the triangles* G M H, H M I, *and*

and G H I: *Therefore the pyramids* A B C F, *and* G H I M, *are alike, and in like manner also* A C D F, G H I K M, *and* A D E F *and* G K L M: *Therefore the pyramids* A B C F, A C D F, *and* A C D E F, *shall be to* G H I M, G I K M, *and* G K L M, *each to his correspondent; in triple reason of the homologal sides* A B, C D, D E, G H, I K, *and* K L, *each to his correspondent, there being therefore the* 8. def. 11. 8. 12.



same reason of A B, C D, *and* D E, *to* G H, I K, *and* K L, *the pyramids* A B C F, A C D F, *and* A D E F, *will have one and the same reason, to wit,*
triple to G H I M, *and* G K L M: *Therefore as* A B C F *alone, to* G H I M 12. 5.
alone, so all of them together, to wit, the whole A B C D E F, *to all of them together, to wit, the whole* G H I K L M. *Therefore the pyramid* A B E F *being to the*
pyramid G H I M *in triple reason of the homologal sides* A B *and* G H, *the pyra-* 8. 12.
mid A B C D E F *shall be also to the pyramid* G H I K L M, *in triple reason of* A B *to* G H. *Which was proposed.*

PROP. 9. THEOR. 9.



The bases and the altitudes of equal pyramids A B C D, *and* E F G H, *having the bases* A B C *and* E F G *triangular, are reciprocal; and pyramids having the bases triangular, whose bases and altitudes are reciprocal, are equal.*

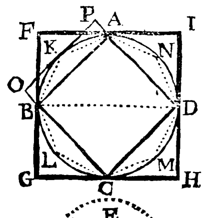
Demonstration. For having finished the parallelepipedons B M and F Q, of the same height as the pyramids, and having drawn L I and P N, the prismes D B C I L A and H F G N P E, (being triple to the pyramids supposed equal,) shall be equal to one another; and therefore the paralellepipedons B M and F Q also equal, being double to the prismes; and [a] therefore their bases and their heights are reciprocal, a) 34. 11.
that is to say, as the base B I to the base F N, so the height of the solid F Q to the height of the solid B M [b]. b) 15. 5.

But as the base B I to the base F N, so the triangle A B C to the triangle E F G; therefore also as the base A B C to the base E F G, so the height of the pyramid E F G H to the height of the pyramid A B C D; seeing the heights of the pyramids are the same as those of the paralellepipedons: Therefore the bases and the heights of equal pyramids are reciprocal.

Now

Now let the bases and heights be reciprocal; I say that the pyramids are equal. For having made the Construction as beforesaid: Forasmuch
c) 15. 5. c as A B C is to E F G as the parallelogram B I, to the parallelogram F N, and the heights of the parallelepiped. and he pyram. being the same, the bases of the paralelograms, and their heights are also reciprocal; There-
d) 34. 11. fore d the parallelepipedons B M and F Q shall be equal: Therefore the prismes D B C I L A and H F G N P E their halves, are also equal; and therefore also the pyramids the third parts of the prismes also equal: Therefore, The bases, &c. Which ought to be demonstrated.

PROP. 10. THEOR. 10.



Every Cone is the third part of the Cylinder, which hath the same base as the Cylinder, and the height equal thereto.

Demonstration FOr otherwise the cylinder shall be greater or lesse than the triple of the cone: Let it in the first place be greater (if possible) as by the magnitude of the solid E; that is to say, that from the cylinder (whose base is the circle A B C D,) E being cut off, the rest shall be the triple of the cone, where f the same circle A C is the base: In the same circle let there be inscribed the square A B C D, and divided in two triangles by the diameter B D, on which triangles let there be understood to be made two prismes of equal height with the cylinder, and the square being greater than the half of the circle, as it appears, it is evident that those two prismes shall exceed the half of the cylinder: But if (the two prismes taken away,) the residue of the cylinder are greater yet than E, on the bases of the said rests or segments, let there be made the four Isosceles triangles A K B, B L C, C M D, and D N A, on the which let there be imagined four prismes of the same height with the cone or cylinder, of which the circle A B C D is the base.

It is manifest that these Isosceles triangles are more than the halfe of the bases of the segments resting of the cylinder, being the halves of the rectangles of the same height as B P: Wherefore these four prismes shall be more than the half of the four segments: But if the eight little segments resting of the cylinder, are not lesse than E: After the same manner let there be still substracted more than the halfe of what remaineth;
a) 1. 10. and a in the end there will remaine a magnitude lesse than E; and in brief, let now these eight segments remaining of the cylinder, being taken together, be lesse than E.

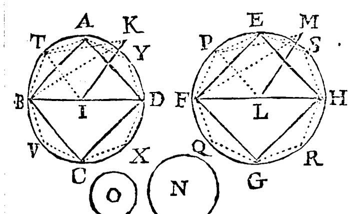
It is therefore evident as the column compounded of these six prismes, having for base the polygon inscribed in the circle A C, and of the same height as the given cylinder, shall be more than the triple of the cone, according to what hath been said before: But each prisme being the triple of his pyramid of the same height, and which hath the base equal,
b) 7. 12. b may be divided in three such equal pyramids: It followes that all these prismes making the column, which hath for base the polygons inscribed in the circle A C; that is to say, that column of the same height, as the cylinder

cylinder is the triple of these six pyramids which do make the pyramid, having the same polygon for base, and of the same height as the said column: Therefore the said pyramid shall be greater than the cone of the same height, having the circle A C for base, which is absurd, the pyramid being no other then part of the cone: Therefore the cylinder is not greater than the triple of the cone.

Let it be then lesser (if possible,) by the quantity of E, that is to say, that taking off E from the cone, that the rest may be the third of the cylinder: Now from the cone whereof the circle A C is the base, let there be taken more than the half, to wit, the pyramid of the same height, whereof the base is the square A B C D, and from the rest, to wit, the four segments K, L, M, and N, let there be taken more than the halfe, to wit, the pyramid of each segment of the same height as the said segment, and having for base the Isosceles triangle in the same segment; and let this cutting off be continued until that the remaining segments be lesser than E, which c will happen, and in five, let the eight small segments be lesse than E. c) 1. 10.

It is therefore manifest that the pyramid of the same height as the cone, having that octogon for base, is greater than the third of the given cylinder, being greater than the cone, having cut off from it E. But the cone thus cut off, is the third of the given cylinder, and the pyramid also the third of the column of the same height, having for base the same octogon, as hath been said above: Therefore that column shall be greater than the given cylinder, whereof it is a part, which is absurd: Therefore the given cylinder is neither greater nor lesse than the triple of the cone; therefore equal: Therefore, &c. Which was to be demonstrated.

PROP. 11. THEOR. 11.



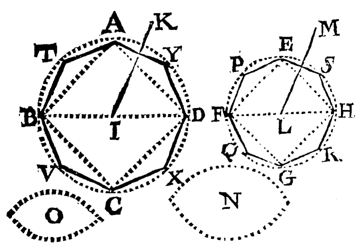
Cones and the cylinders which are of the same height, I K and L M, are the one to the other as their bases A B C *and* E F G.

Demonstration FOr if it be not so, it shall be greater or lesser, suppose it in the first place to be lesse, and that by the quantity of O, (if possible,) therefore N and O shall be equal to the cone F L M, and from the cone F L M, (as in the tenth Proposition) Let there be taken more than the halfe, to wit, a pyramid of the same height as the cone, whereof the base is the square E F G H, and from the rest also more than the halfe, to wit, four pyramids of the same height,) having for bases the four Isosceles triangles E P F, F Q G, G R H, and H S E, and so following, there will remain in the end a magnitude lesse than O, which let be the eight little segments: It follows that the pyramid F M of the same height as the cone F M, whereof the base is the octogon E P F Q G R H S, shall be greater than N.

Within the circle A B C D let there be inscribed a polygon alike to the po-

polygon inscribed in the circular base E F G H, and on the same let there
be understood to be a pyramid of the same height as the cone B K: But
a)Co.2.12. [a] as the circle A B C is to the circle E F G, so the polygon T V X Y is to
the polygon P Q R S; but as the circle A B C is to the circle E F G, so the
b) 6. 12. cone B K to N, and [b] as the polygon to the polygon, so the pyramid to the
c) 11. 5. pyramid of the same height as the cones; therefore [c] the pyramid A T B
V C X D Y K shall be to the pyramid E P F Q G K H S M, as the cone B K
to the solid N: But A T B V C X D Y G being lesse than the cone B K,
d) 14. 5. the part than the whole, ; [d] also the pyramid E P F Q G R H S M shall
be lesse than N; but it hath been shewn to be greater, which is absurd:
Therefore N is not lesse than the cone E F G H M.

Secondly, suppose N to be greater than E F G H M; therefore being
put as the circle A B C to the circle E F G, so the cone B K to N; by con-
version of proportion, as the circle E F G to the circle A B C, so N to the
cone B K, and as N to B K, so the cone E F G H M is put to O, and N be-
e) 14 5. ing put greater than the cone F M, [e] the cone D K shall be also greater than
O: Therefore seeing that
as the circle E F G is to the
circle A B C, so N to the
f) 11. 5. cone B K; [f] as the circle E F
G H is to the circle A B C, so
the cone F M to O, lesse than
the cone B K, which is ab-
surd; being already shewn
that the cone cannot be to
another magnitude lesse than
the other cone, as the base of
the one to the base of the o-



other: Therefore N is neither greater nor lesse than the cone F M;
therefore equal: Therefore, being put as the base A B C D to the
g) 7. 5. base E F G H, so the cone B K is to N; but [g] as the cone B K is to N, so
the same cone B K, to the cone F M; also as the base A B C D, to the base
E F G H, so the cone A B C D K, to the cone E F G H M.

h) 15. 5. But forasmuch as the cone A B C D K [h] is to the cone E F G H M; as
the cylinder A B C D K; (which is triple the cone A B C D K,) is to the cy-
linder E F G H M, (which is triple the cone E F G H M,) also as the base
A B C D shall be to the base E F G H, so the cylinder A B C D K shall be
to the cylinder E F G H M, which may also be demonstrated; so as of
the cones, if in lieu of cones and pyramids, be understood cylinders and
prismes: Therefore, Cones and cylinders, &c. Which was to be de-
monstrated.

CORLLARIE.

*Hence it followes that cones and cylinders made on the same base, or equal bases
and of the same height are equal, and contrariwise equal cones and cylinders, made
on the same base, or equal bases, are of the same height, and equal cones and cylin-
ders, and of the same height, are also on equal bases, or one and the same; which
shall be demonstrated as in the converse of the 31th. Prop. of the Eleventh Book.*

PROP. 12. THEOR. 12.

*Like cones and cylinders are the one to the other in triple
pro-*

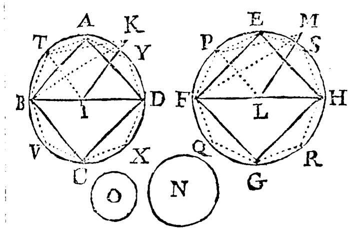
proportion of the Diameters BD *and* FH, *of their bases*
ABC *and* EFG.

Demonstration. If it be not so, let the cone A B C D K be greater or lesser
than the cone E F G H M, by another magnitude N, in triple
proportion of the diameter B D to the Diameter F H.

And let N in the first place be lesse than the cone E F G H M, by O,
and let there be made the same construction as in the precedent Propositi-
on, so as that the pyramid E P F Q G R H M be again shewn greater
than N; and let the right lines K B, K T, M F, and M P, be drawn, to
have two triangles B K T and E M P, of the pyramids A T B V C X D Y K,
and E P F Q G R H S M; and let T I and P L be joyned.

Forasmuch as the cones A B C D K and E F G H M are put alike, [a] the a)24.def.11
axis I K shall be to the axis L M, as the diameter B D to the diameter
F H: Therefore as the Semidiameter B I to the Semidiameter F L, so
the axis I K to the axis L M; and alternately, as B I to I K, so F L to
L M.

Then seeing that the angles B I K and F L M [b] are right angles, (the b)3.def.11.
cones being put right,) therefore the axes being perpendicular to their
bases, [c] the triangles B I K and F L M shall be equiangled; therefore as c) 6. 6.
K B to B I, so M F to F L. But as B I to B T, so F L to F P; because of the
similitude of the triangle
B I T to F L P, (for seeing
B I T and F L P, insisting on
like arches, are equal; and
as B I is to I T, so F L to
L P, as well by the equality
of the lines B I and I T, as
F L and L P, [d] the trian- d) 6. 6.
gles B I T and F L P are
alike,) and in proportion of
equality, as K B to B T, so
M F to F P.



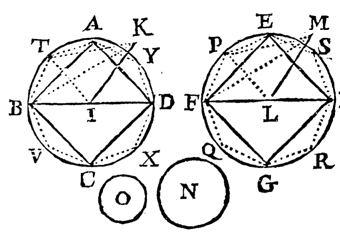
Again, forasmuch as the sides K I and I B of the triangle K I B, are e-
qual to K I and I T of the triangle K I T, and the angles conteined of them
[e] right angles, the axis I K being put perpendicular to the circle A B C, e)3.def.11.
[f] the bases K B and K T shall be equal; and in like manner M F and M P f) 4. 1.
equal; therefore K B and K T shall be proportional to M F and M P, there
being proportion of equality on both parts; but seeing that as K B to B T,
so K T to the same B T; also as M F to F P, so M P to the same F P; but as
K B was to B T, so M F to F P: In like manner, as K T shall be to B T,
so M P to F P; and by conversion, as B T to T K, so F P to P M; there-
fore seeing that as T K is to K B, so P M to M F, and as K B to B T,
so M F to F P, and as B T to T K, so F P to P M, as is shewn; the trian-
gles B K T and F M P shall have the sides proportional; therefore [g] are g) 6. 6.
equiangled and alike; and [h] in like manner, the other triangles about h)1. def. 6.
the pyramids A T B V C X D Y and E P F Q G R H S M, shall be alike the
one to the other, which being equal in number, [i] the same pyramids i) 9. def. 11.
shall be alike, and [k] shall be in proportion triple of their homologal sides k)Co.8.12.
B T and F P: But as B T to F P, so B I to F L; by the similitude of the
triangles B I T and F L P; and [l] as B I to F L, so B D to F H: Therefore l) 11. 5.

the

the pyramid shall be to the pyramid in triple proportion of the diameters B D and FH. But the cone A B C D K was put to N in triple proportion of the same diameters: Therefore as the pyramid A T B V C X D Y K to the pyramid E P F Q G R H S M, so the cone A B C D K to N: Wherefore seeing that the pyramid A T B V C X D Y K is lesse than the cone m) 14. 5. A B C D K the part than the whole, m the pyramid A P F Q G R H S M shall be lesse than N. But it is shewn to be greater, which is absurd: Therefore N is not greater than the cone E F G H M.

Secondly, suppose N to be greater than the cone E F G H M: Therefore the cone A B C D K being put to N in proportion triple of the diameter B D to F H, and the pyramid A T B V C X D Y K being to the pyramid E P F Q G R H S M in triple proportion of the same diameters, as is shewn, as the cone A B C D K shall be to N, so the pyramid A T B V C X D Y K, to the pyramid E P F Q G R H S M; and alternately, as N to the cone A B C D K, so the pyramid E P F Q G R H S M to the pyramid A T B V C X D Y K.

n) 20. 12. Therefore n the pyramid E P F Q G R H S M being to the pyramid A T B V C X D Y K in triple proportion of the homologal sides P F and T B, that is to say, of the diameter F H to the diameter B D, N shall be also to the cone A B C D K in triple proportion of the diameter F H to the diameter B D, as N is to the cone A B C D K, so the cone E F G H M is put to O. Therefore the cone E F G H M shall be also to O in triple proportion of the diameter F H, to the diameter B D, and N being put greater than the cone E F G H K, o the cone A B C D K shall be also greater than O; therefore the cone



o) 14. 5. E F G H M shall be to O, lesse than the cone A B C D K in triple proportion of the diameter F H to the diameter B D, which is absurd: For it is shewn that the cone cannot be in triple proportion of the diameters of the bases to another magnitude lesse than the other cone; therefore N is not greater than the cone E F G H M, and is shewn not to be lesse, therefore equal: p) 7. 5. Wherefore p the cone A B C D K hath the same proportion to the cone E F G H M, as to N.

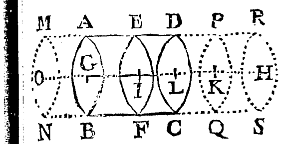
Therefore seeing that the cone A C B D K is to N in triple proportion of the diameters B D and F H, the cone A B C D K shall be also to the cone E F G H M, in triple proportion of the same diameters.

q) 15. 1. But q forasmuch as the cylinders are in the same proportion as the cones whereof they are triples, the cylinder shall be to the cylinder in the same triple proportion of the diameters of the bases; which shall be demonstrated; as of the cones, if in lieu of cones and pyramids, you take cylinders and prismes: Therefore, Like cones and cylinders, &c. Which was to be demonstrated.

PROP. 13. THEOR. 13.

If a cylinder A B C D, *be cut by a plain* E F, *parallel to the opposite plains* A B *and* C D, *as the cylinder* A F *shall be to the cylinder* E C, *so the axis* G I *shall be to the axis* I L.

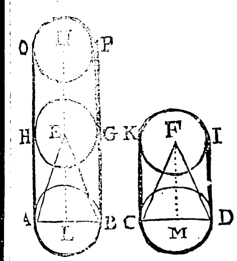
Demonstration. FOr having prolonged the axis G L on both parts, let G O be put equal to G I, and L K and K H each equal to I L, and let it be understood that the cylinder is prolonged from both parts to O and H, that being, it is manifest that the cylinders A N, A F, E C, C P, and Q R, are all on equal bases; a and therefore A N and A F having the same height, that is to say, having the axes equal, I G and G O are equal to one another: In like manner, F D, D Q, and Q R, being of the same height, are also equal to one another; wherefore the axis I O is so multiplex of the axis G I, as the cylinder E N, of the cylinder A F, and the axis I H so multiplex of the axis I L, as the cylinder F R of the cylinder F D, as is manifest. a) C. 11. 12.



Therefore if the axis O I, multiplex of G I first magnitude, be equal, greater, or lesse than I H, multiplex of I L second magnitude; also the cylinder E N multiplex of the cylinder E B third magnitude, shall be greater, equal, or lesse than the cylinder F R, multiplex of the cylinder F D fourth magnitude, in any multiplication whatever; wherefore b as the axis to the axis, so is the cylinder to the cylinder: Therefore, If a cylinder, &c. Which was to be demonstrated. b) 6. def.

PROP. 14. THEOR. 14.

Cones A B E *and* C D F, *and cylinders* A B G H *and* C D I K, *made on equal bases* A B *and* C D, *are to one another as the heights* L E *and* M F.



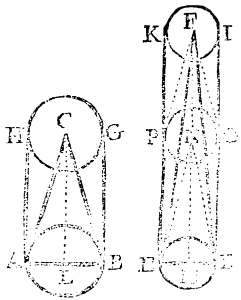
Demonstration. FOr let the cylinder A G be prolonged on the part of G H, with his axis L E, and the rectangle A G; and let the axis E N be taken equal to the axis M F, and about the center N let there be understood to be a circle O P, equal and parallel to G H, lastly, let the cylinder G O be made of the same height as the cylinder C I.

Forasmuch as the cylinders H P and C I, having the bases and heights equal, a are equal; the cylinder A G shall have the same proportion to them. But b the cylinder A G is to the cylinder H P, as the axis or height L E, to the axis or height E N; that is to say, to the height M F his equal: Therefore the cylinder A G shall be also to the cylinder C I, as the height L E to the height M F. a) C. 11. 12. b) 13. 12.

But c forasmuch as the cones A B E and C D F are the third parts of the cylinders A G and C I; d they shall be in the same proportion as the cy- c) 10. 12. d) 15. 5.

cylinders: Therefore the cone ABE shall be to the cone CDF, as the
height LE, to the height MF: Therefore, Cones, &c. Which was to
be demonstrated.

PROP. 15. THEOR. 15.



The bases AB *and* DE, *and the heights* LM *and* MF, *of equal cones* ABC *and* DEF, *and cylinders* AG *and* DI, *are reciprocal, and cones and cylinders, whose bases and heights are reciprocal, are equal.*

Demonstration For if the heights LC and
MF are equal, the cylinders
a) Co. 11. 3. being put equal, [a] the bases shall be equal;
therefore as AB to DE, so MF to LC; therefore the bases and the heights are reciprocal.

And if the one be greater, as MF; let there be cut off MN, equal to
LC, and let the plain PO be understood to passe by N, and parallel to
DE, cutting the cylinder DI in two; therefore the two cylinders AG
b) 7. 5. and DI, being put equal; [b] as AG shall be to DO, so DI to DO; but
c) 11. 12. [c] as the cylinder AG to the cylinder DO, so the base AB to the base
d) 14. 12. DE, the heights CL and MN being equal [d].

Also as DI to DO, so the height MF to MN, being the bases are
equal; to wit, one and the same DE; therefore also as the base AB to
the base DE, so the height MF to the height MN; that is to say, to his
equal LG; therefore the bases and the heights are reciprocal.

And if the cones ABC and DEF are equal, the cylinders AG and
e) 10. 12. DI shall be equal, [e] the cones being the thirds of the cylinders: Therefore
from the equality of the cylinders, it followes that the bases and the
heights are reciprocal, as is shewn; and therefore from the equality of
the cones it will also follow that the bases and the heights are reciprocal;
which may also be demonstrated by the discourse used about the cylinders,
if there be constituted two cones, as appears by the figure.

Now let the bases and heights be reciprocal; I say that the cones and
the cylinders are equal: If the heights be equal, being to one another
f) Co. 11. 12. as the bases, their bases shall be also equal; therefore [f] the cylinders
are equal.

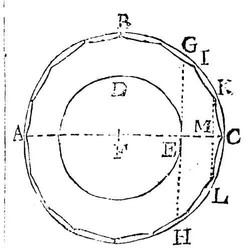
If unequal, let the same construction be made as before; therefore
the height MF being put to the height LC; that is to say, to MN his
g) 11. 12. equal, as the base AB to the base DE; but [g] as AB to DE, so the cy-
h) 14. 12. linder AG to the cylinder DO, of the same height [h].

Also as the height MF to the height MN, so the cylinder DI, to the
cylinder DO, the bases being equal, as the cylinder AG to the cylinder
i) 9. 5. DO, so DI to the same cylinder DO; therefore [i] the cylinder AG shall
be equal to the cylinder DI.

And if the bases and heights of the cones ABC and DEF are reci-
procal, the bases and the heights of the cylinders AG and DI, being the

the same bases and heights of the cones, shall be reciprocal: Therefore
as hath been shewn, the cylinders, and therefore the cones, the third
parts of them, are equal: Neverthelesse, the same demonstration may
be made of the cones that hath been made of the cylinders: Therefore,
The bases, &c. Which was to be demonstrated.

PROP. 16. PROBL. 1.



Two circles ABC *and* DE, *being about one and the same center* F, *to inscribe in the greatest of them* ABC, *an equilateral polygon, and of an even number of sides, and which shall not touch the lesser circle* DE.

Construction Let AC be drawn by the center F, cutting the circle DE
a) Co. 16. 3. in E, and by E let GH be drawn perpendicular to AC, [a] which will
touch the circle at E: Forasmuch then as the arch AGC is greater than
the arch GC, if from AGC the half be taken AB, and from the rest
BC, the half BI; and of the rest IC, the half IK, and so following;
b) 1. 10. [b] there will remain at last an arch lesse than CG, which let be the sub-
tense CK; I say that CK is one side of the polygon which ought to
be inscribed.

Demonstration For if the arch BI be divided into equal parts in number
and magnitude, in the parts of the arch CI, and the
quarter of the circle AB; let be divided into so many equal as there
are equal parts in the arch BC: and also the semicircle AHG: Then let
c) 29. 3. there be drawn the right lines subtending all the arches, which [c] shall be
equal to the right line CK; forasmuch as they subtend arches equal to
the arch CK, and so you shall have described an equilateral polygon in
the circle ABC, and of an even number of sides which shall not touch
the lesser circle DE.

For from K to AC, let there be drawn the perpendicular KL, cutting
AC in M; forasmuch as the angles GEM and KME are right angles,
d) 29. 1. [d] HG and KL shall be parallels; therefore GH touching the circle DE
in the point E, the right line KL shall be wholly without the said circle,
and shall never touch it, never meeting GH: Therefore the right line
GK which is farther distant from the circle DE, shall lesse touch the
circle DE than KL; and therefore also neither shall the other sides of the
inscribed polygon, being equal to the side CK; and therefore shall be
so farre distant from the center F as CK, and shall not touch the circle DE:
e) 14. 3. [e] Therefore, Two circles, &c. Which was to be demonstrated.

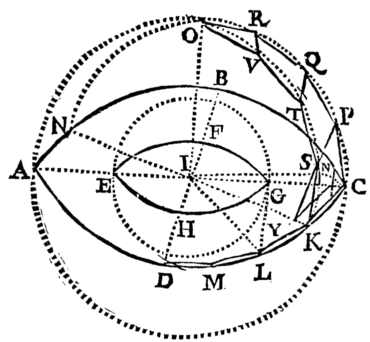
COROLLARIE.

It is manifest from this, that if from the extremity of the side of the polygon inscribed, which doth meet with the diameter, be drawn a line perpendicular to the diameter, that line cannot in any wise touch the lesser circle, but shall fall wholy with-

without it. For such is the line K L, *which drawn from the extremity* K *of the side* C K, *meeting with the diameter* A C, *perpendicular to the diameter* A C, *is demonstrated not to touch the circle* D E.

PROP. 17. PROBL. 2.

Two Spheres A B C D *and* E F G H, *being about one and the same center* I, *to inscribe in the greatest Sphere* A B C D, *a polyhedron which shall not touch the Superficies of the lesser Sphere* E F G H.



Construction LEt the two Spheres be cut by some plain drawn by the center I, and let the common sections be made in the two Spheres, the plains A B C D and E F G H, (which shall be circles, by the definition of the Sphere,) having the same center I with the Spheres; for the semicircles, by the revolution of which the spheres are described, will agree with the sections A B C D and E F G H: wherefore the said sections shall be circles: Or, seeing that all the right lines drawn from the center I, to the circumferences of the sections are equal; and in them are drawn the diameters A C and B D, cutting one another at right angles in the center I;
a) 16. 12. [a] then in the greatest circle A B C D, let there be inscribed a polygon, that may not touch the lesser circle E F G H; and let C K, K L, L M, and M D, be sides of the quarter C D; and let K N
b) 12. 11. be drawn by the center I; and [b] from I let there be drawn I O, perpendicular to the plain of the circles A B C D and E F G H, meeting in O the superficies of the greatest Sphere, and by A C, I O, and N K, I O, let there be drawn the plains A O C and N O K, they shall be semicircles, the greatest amongst those of the sphere, having the same center, as is said, and I O the line of common section of the plains of the semicircles N O K and A O C, being perpendicular to the plain of the circle
c) 18. 11. A B C D; [c] those semicircles shall be also perpendicular to the plain of the circle A B C D, and the three semicircles A D C, A O C, and N O K, being equal, their diameters being equal, their halves, to wit, the quarters O C,

O C, O K, and D C, shall be equal: Wherefore O C and O K shall contein each as many sides of the polygon as D C.

Let those sides be C P, P Q, Q R, R O, K S, S T, T V, and V O, and let there be drawn S P, T Q, and V R, and from P and S, the perpendiculars P X and X Y; [d] they shall fall on I C and I K, common se- d) 38. 11.
ctions of the squares I O C and I O K, and the plain I C D, raised perpendicularly on the same plain, [e] they shall be also parallel, and let e) 6. 11.
X Y be also drawn.

Now the arches P C and S K being equal, and [f] the right lines subtending f) 29. 3.
(P C and S K) also equal; the angles P X C and S Y K right angles, and [g] the angles P C X and S K Y equal, insisting on equal circumferences g) 27. 3.
A O P and N O S, the two angles P C X and P X C of the triangle P X C, are equal to the two angles S K Y and S Y K, and the sides P C and S K equal, [h] the two other sides P X and X C shall be equal to the two other h) 26. 5.
sides S Y and Y K each to his correspondent side. Therefore the diameters A C and N K being equal, and the segments C X and K Y also equal, A X and N Y shall be also equal, and I X and I Y equal. Wherefore as I X to X C, so I Y to Y K; therefore [i] X Y is parallel to K C, and P X i) 2. 6.
and S Y being shewn equal and parallel, S P and Y X [k] shall be also equal k) 33. 1.
and parallel, and [l] S P and K C shall be parallel, and [m] S K and P C in l) 9. 11.
one and the same plain with them. m) 7. 11.

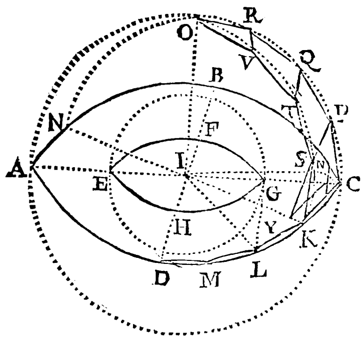
Therefore the whole quadrilateral figure S K C P is in one and the same plain, so it's shewn that each of the quadrilateral figures, as T P and V Q hath all his parts in one and the same plain, and the triangle O R V in like manner; and if from the points P, Q, R, S, T, and V, be drawn lines to the center I, there may be imagined a solid figure, composed of four pyramids, between O C and O K, whose tops are in the center I, and their bases are the quadrilateral figures C S, P T, and Q V, and the triangle O V R; and doing the same on the other sides K L, L M, and M D, as on C K, and also on the three other quarters B C, B A, and A D, there will be inscribed a polyhedron in the proposed Sphere: I say it touches not the lesser Sphere.

For having drawn the line I Z, from the center I, perpendicular to the plain C S, it shall be greater than I G, the semidiameter of the lesser Sphere: For to inscribe the polygon, (having drawn G L perpendicular to A C from the point G, which being greater than C K, the side of the inscribed polygon, as is manifest;) let I L be drawn, and also C Z and K Z, [n] the angles I Z C and I Z K being right angles, the square n) 2. 11.
of I C [o] shall be equal to the two squares of I Z and C Z, and the o) 47. 1.
square of I K equal to the squares of I Z and Z K: Wherefore the squares of the diameters being equal, the squares of I Z and Z K shall be equal to the squares of I Z and C Z: Therefore if the common square of I Z be taken away, the squares of C Z and K Z will remain equal; and therefore K Z and C Z equal; and having drawn the lines from Z to S and P, it will be demonstrated by the same reasons that they shall be equal, as well to one another, as to C Z and Z K; therefore if from Z, taken for the center, there be described a circle, at the distance of one of them, it will passe by all the points C, K, S, and P; and therefore shall be described about the said quadrilateral figure, whose three sides C P, C K, and K S, are equal, [p] subtending equal arches of equal circles, and the fourth p) 29. 3.
S P lesser.

For seeing that the triangles I C K and I X Y are alike, and the side

I C

q) 14. 5. I C greater than the side I X, q C K shall be also greater than Y X, that is to say, than S P, shewn to be equal to Y X, the angle C Z K shall be greater than a right angle; (for the four lines drawn from the point Z to the points K, C, P, and S, being demonstrated equal, the four triangles
r) 4. 1. which are made, whereof the tops are at the point Z, r shall have the four angles at Z equal to four right angles, and the bases of the three triangles S Z K, K Z C, and C Z P, being shewn equal, the three angles opposed to them shall be equal; therefore each one shall be greater than a right angle, seeing the base C K is shewn to be greater than S P, and the angle C Z K greater than the angle S Z P, therefore greater than a right angle, and so the greatest of the triangle C Z K; therefore the side C K shall be greater than C Z, and G L being greater than C K, shall be also greater than C Z. Wherefore the square of G L shall be greater than the square of C Z, and the squares of I G and G L being equal to the squares of I Z and C Z, (as well those two, as the other two, being equal to



the square of the semidiameter, seeing that the triangles I G L and I C Z are rectangles,) and the square of G L being greater than the square of C Z, the square of I Z shall be also greater than the square of I G; and therefore I Z greater than I G; therefore the plain C K S P touches not the lesser Sphere E F G H; and so it may be demonstrated that all the other plains of the polyhedron in the greatest Sphere, touches not the superficies of the lesser Sphere E F G H: Therefore, Two Spheres, &c. Which was to be demonstrated.

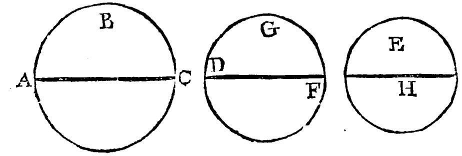
COROLLARIE.

From this Demonstration follows, that if in another Sphere there be described a solid polyhedron, alike to the abovesaid, they shall be in triple proportion of the diameters of the Spheres in which they are inscribed; for being alike by the definition of like solids, they will contein in their convexitie as many like plains the one as the other. Wherefore they may be divided into as many like pyramids the one as the other, having all of them the semidiameter of their Spheres, for one of their sides: There-
s) Co.8.12. *fore being taken one and one, shall be in triple proportion of their s homologal sides,*
t) 12. 5. *which are the semidiameters of their Spheres, and t the whole to the whole, also in*
v) 15. 5. *triple proportion of their semidiameters; v and therefore also in triple proportion of the diameters intire.*

PROP.

PROP. 18. THEOR. 16.

Spheres A B C *and* D G F, *are the one to the other in triple proportion of their diameters* A C *and* D F.



Demonstration FOr if it be not so, it shall be greater or lesser; First then, Suppose E be lesse than D E F, then E may be inclosed in D G F, putting both the two about one and the same center: Where-
a) 17. 12. fore a in D G F may be inscribed a solid polyhedron, which shall not touch the lesser circle E, and by the same reason may be described
c) C.17.12. a solid polyhedron in the other, which b shall be to that which is described in D G F in triple proportion of the diameter A C to the diameter D F; but the Sphere A B C is put in such proportion to the Sphere
c) 11. 5. E: Therefore c as the Sphere A B C to the Sphere E, so the polyhedron of the Sphere A B C shall be to that of D G F; but the Sphere A B C is
d) 14. 5. greater than his polyhedron, the whole than his part; d therefore E shall be greater than the polyhedron of the Sphere D G F, the part than the whole, which is absurd: Wherefore A B C is not to another Sphere lesse than D G F in triple proportion of the diameter A C to the diameter D F.

Now suppose E to be greater than D G F, if possible, and it being put as the Sphere A B C is to the Sphere E, so the triple proportion of the diameter A C to the diameter D F; alternately, E shall be to A B C in triple proportion of D F to A C; in like manner it may be understood that
e) 14. 5. as E is to A B C, so D G F is to a fourth proportional, which e shall be lesse than A B C, E being put greater than D G F; wherefore this fourth
f) 11. 5. may be inscribed in ABC, and f D G F shall be to the said fourth proportional inscribed in A B C, in triple proportion of D F to A C, which is shewn to be absurd: Therefore E is neither greater nor lesse than D G F, therefore equal: Therefore, Spheres, &c. Which was to be demonstrated.

COROLLARIE.

It is manifest from this Demonstration that one Sphere is to another Sphere, as the polyhedron inscribed in the one, is to a like polyhedron and alike inscribed in the other, being shewn that as well the Spheres as the polyhedrons inscribed in them, are in triple proportion of their diameters.

The End of the Twelfth Element of EUCLIDE.

THE THIRTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

THe chief matter conteined in this Thirteenth Book is concerning the Passions of a line divided by extream and mean proportion, a thing of wonderful use in Geometry, as by this and the following Books will evidently appear: This Book also treateth of the composition of the five Regular Bodies, *viz.* the Tetrahedron, Cube, Octohedron, Dodecahedron, and Icosahedron, and how to inscribe them in a Sphere given. It also setteth forth certain similitudes and comparisons of the said Bodies one with another, and also with the Sphere within which they are inscribed, with divers other matters to the said Bodies relating.

PROPOSITIONS, PROBLEMES, and THEOREMES.

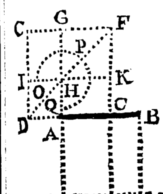
PROPOSITION 1. THEOREM 1.

If a right line AB, *be cut according to extream and mean proportion,* (*as in* C,) *the greatest segment* AC, *comprised with*

with the half AD, *of the whole* AB, *is in power quintuple of the square which is described of the half of the whole.*

Construction THat is to say, that the square of the greatest segment and of the halfe of the whole, as of one only line, is quintuple of the square of the half of the whole.

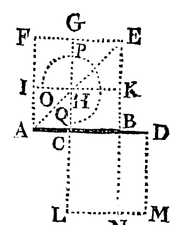
Demonstration FOr on CD let there be described the square DF, and having drawn the diameter DF, let there be drawn AG and IK, cutting it in H, parallel to DC and CF, and prolonging GA, let the square AM be made, and prolonging FC to N, [a] AI and KG shall be the squares of AD and AC; and [b] forasmuch as AB to AC, as AC to CB, [c] the rectangle CM conteined under AB and CB, shall be equal to KG the square of AC, and AB being put the double of AD, and AL equal to AB, and AH to AD, AL shall be also double to AH. But [d] as AL to AH, so the rectangle AN to the rectangle AK: Therefore AN is double to AK; but [e] forasmuch as AK is equal to IG, AN shall be equal to the two AK and IG.



a) Cor. 4. 2.
b) 3. def. 6.
c) 17. 6.
d) 1. 6.
e) 43. 1.

Adding then the equal figures CM and KG, the square AM shall be equal to the Gnomon OPQ; therefore the square AM being the quadruple of the square AI (seeing that DA is put half of AB) the Gnomon OPQ shall be quadruple to the same AI: Wherefore if to OPQ be added AI, the square FD shall be made quintuple to the square AI: Therefore, If a right line, &c. Which was to be demonstrated.

PROP. 2. THEOR. 2.



If a right line AB *be in power quintuple of his segment* AC, *the line* CD *double, the said segment being divided according to extream and mean proportion, the other part is the greatest segment of the line first cut.*

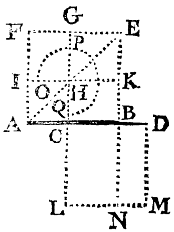
Construction THat is to say, if the square of a right line be quintuple of the square of one part thereof, the double of that part being divided by extream and mean proportion, the greatest segment is the other part of the said line.

Demonstration FOr [a] having finished the Construction as in the precedent, CI and KG shall be the squares of AC and CB, and forasmuch as the square BF is put quintuple to the square CI, if the square CI be taken away, there will remain the Gnomon OPQ, quadruple to the same square CI; but the square CM is quadruple to the square CI; seeing that CD is double to AC; therefore the Gnomon OPQ shall be equal to CM.

a) Cor. 4. 2.

 Again,

Again, forasmuch as C D is put double to A C, and L C is equal to C D, and C H to A C, A C shall be double to C H; therefore seeing that as L C is to C H, so the rectangle L B to the rectangle C K, that shall be also double to this: But [b] C K is equal to H F; therefore L B is equal to C K and H F; and therefore the square resting K G, to the rectangle resting B M.

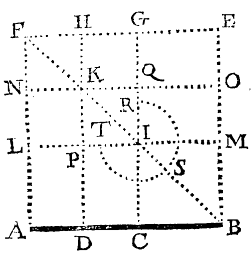


b) 13.6.

Forasmuch then as the rectangle B M conteined under C D and B D, is equal to K G the square of C B, and [c] as C D shall be to C B, so C B to B D: Therefore C D is divided in B by extream and mean proportion, and C B is the greatest segment, &c. Which was to be demonstrated.

c) 17.6.

PROP. 3. THEOR. 3.



If a right line A B, *be divided in* C, *according to extream and mean proportion, the line made of the lesser segment, and of the half of the greatest, is in power the quintuple of the square described of the half of the greatest segment.*

Construction. That is to say, the square of the lesser segment, with the half the greatest, as of one only line, is the quintuple of the square of the half of the greatest.

Demonstration. For on A B let there be described the square A E, whose diameter is B F, by C and D let there be drawn C G and D H parallel to A F and B E, and cutting B F by I and K, by which let there be drawn L M and N O, parallel to A B and E F, cutting C G and D H, at P and Q, [a] L G, P Q, N H, and D O, are the squares of A C, C D, and B D.

a) Cor.4.2.

Forasmuch then as A C is the double of C D, the square L G shall be quadruple to the square P Q, but the rectangle C E is equal to the square L G; (for seeing that as A B to A C, so A C to C B, the [b] rectangle under A B and C B, to wit, C E, is equal to the square of A C, to wit, to L G:) Therefore C E shall be also quadruple to P Q.

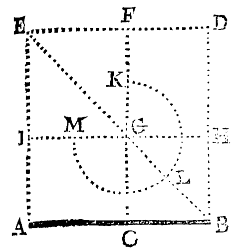
b) 17.6.

But forasmuch as the squares N H and P Q are equal, A D and C D being equal, their sides H K and I Q shall be equal; and [c] therefore E O and O M opposite to them equal; therefore [d] the rectangles I O and Q E shall be equal: But [e] I O is equal to I D; therefore Q E shall be equal to the same I D, adding therefore the common rectangle C O, all the Gnomon R S T shall be equal to the rectangle C E, and C E being shewn quadruple to the square P Q, the Gnomon R S T shall be also quadruple to the same

c) 34.1.
d) 36.1.
e) 12.1.

same square P Q, adding the common square P Q, the square D O described of B D, shall be quintuple to the square P Q, described of C D: Therefore, If a right line, &c. Which was to be demonstrated.

PROP. 4. THEOR. 4.

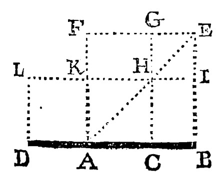


If a right line A B, *be divided in* C, *according to extream and mean proportion, the one and the other square, to wit, that of the whole* A B, *and that of the lesser segment* C B *together are triple to the square described of the greatest segment* A C.

Demonstration. For on A B let there be described the square A D, and having drawn B E the diameter, and by G drawn C F, parallel to B D, and H I to A B, cutting the diameter at G, [a] C H and I F shall be the squares of B C and A C; and forasmuch then, as A B is to A C, as A C is to C B, [b] the rectangle A H conteined under A B and B C, is equal to I F the square of A C: Therefore A H being equal to C D, the Gnomon K L M with the square C H, shall be double to the square I F: Wherefore adding I F and A D the square of A B, with C H square of B C, shall be triple to I F square of A C: Therefore, If a right line, &c. Which ought to be demonstrated.

a) Cor.4.2.
b) 17.6.

PROP. 5. THEOR. 5.



If a right line A B, *be divided in* C, *according to extream and mean proportion, and that to it there be added a line* A D, *equal to the greatest segment* A C, *the whole right line* B D *is cut in extream and mean proportion, and the greatest segment* A B, *is the right line which was at the beginning.*

Demonstration. For upon A B, let there be described the square B F, and let A E the diameter, be drawn, and C G parallel to B E, cutting A E in the point H, by which let there be drawn I K, parallel to B D, and by D, D L, parallel to A F, meeting I K, prolonged at L, and [a] C K and I G shall be the squares of A C and B C.

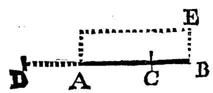
a) Cor.4.2.

Forasmuch then, as A B is to A C, as A C is to C B, [b] the rectangle I F, conteined under A B and B C, shall be equal to C K, the square of A C: But [c] C K is equal to A L, therefore I F is also equal to A L; adding then the common part B K, the rectangle B L, conteined under B D and D A, shall be equal to B F, the square of A B: Therefore [d] as B D shall

b) 17.6.
c) 36.1.
d) 17.6.

ſhall be to A B, ſo A B to A D: Wherefore B D is divided at A in extream and mean proportion, and A B is the greateſt ſegment: Which was to be demonſtrated.

PROP. 6. THEOR. 6.



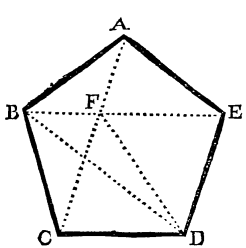
If a rational right line A B, *be divided in* C, *according to extream and mean proportion, the one and the other of the ſegments* A C *and* C B, *is an irrational line, the which is called an Apotome or a Reſidual.*

Demonſtration. FOr let there be added to the greateſt ſegment A C, the right line A D, equal to the half of the whole A B; foraſ-
a) 1. 13. much [a] then as the ſquare of C D is quintuple to the ſquare of A D, the ſquare of C D ſhall be to the ſquare of A D, as a number to a number:
b) 6. 10. Wherefore [b] the ſquares of C D and A D ſhall be commenſurable, and therefore the lines C D and A D commenſurable at leaſt in power. But A D is rational, being the half of A B rational, therefore C D ſhall be
c) Cor. 24. 8. alſo rational [c].

But foraſmuch as the ſquares of C D and A D are the one to the other, as a ſquare number to a ſquare number (being as 5 to 1, ſo 25 to 5,) C D
d) 9. 10. and A D [d] ſhall be incommenſurable in length; and therefore only rational, commenſurable in power: Wherefore if from C D rational,
e) 74. 10. be taken A D rational, commenſurable in power only, [e] the remain A C ſhall be irrational, which is called a Reſidual.

Again, the rectangle A E being applyed to A B, conteined under A B
f) 17. 6. and C B, ſeeing that as A B to A C, ſo A C to C B, [f] the rectangle A E ſhall be equal to the ſquare of A C: Wherefore the ſquare of the Reſidual A C, that is to ſay, the rectangle A E, applyed according to the ra-
g) 98. 10. tional A B, makes [g] the other ſide B E, that is to ſay, C B, equal to the ſame, firſt Reſidual: Therefore, If a right line, &c. Which was to be demonſtrated.

PROP. 7. THEOR. 7.



If three angles A, B, *and* C, *of an equilateral Pentagon* A B C D E, (*whether taken in order, or not in order*) *are equal, the ſame Pentagon ſhall be equiangled.*

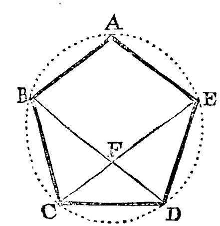
Demonſtration. FOr to thoſe equal angles let B E, A C, and C D, be ſubtendents, and from F, where B E and A C interſect one another, let F D be drawn: Foraſmuch as the ſides A B and A E, of the triangle A B E, are equal to C B and C D, of the

the triangle C D B, and the angles conteined of them put equal, [a] the a) 4. 1.
baſes B E and B D; and the angles A E B and C D B are equal: But [b] the b) 5. 1.
angles B E D and B D E are alſo equal, the ſides B E and B D, being ſhewn equal: Therefore the whole A E D and C D E ſhall be equal.

Again, the ſides A B and A E of the triangle A B E, being equal to B A and B C of the triangle B A C, and the angles conteined of them equal,
by ſuppoſition, [c] the baſe B E ſhall be equal to A C, and the angles A B E c) 4. 1.
and A E B equal to B A C and B C A, each to his correſpondent angle: Therefore ſeeing the angles A B E and B A C of the triangle A B F are
equal, [d] the ſides B F and A F are alſo equal; therefore if they be taken d) 6. 1.
from the equal lines B E and A C, the reſt F E and F C ſhall be equal: Therefore the ſides F E and E D of the triangle F E D, being equal to
F C and C D of the triangle F C D, and F D common, [e] F E D and e) 8. 1.
F C D, conteined of thoſe ſides ſhall be equal: But A E B and B C A are ſhewn to be equal, therefore the whole A E D and B C D are equal; therefore C D E being ſhewn equal to A E D, and the angles A B C, B A E, and B C D, being put equal, the ſame Pentagon ſhall be equiangled.

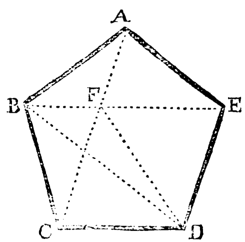
Secondly, Let the three angles A, C, and D, (not taken in order,) be equal: Foraſmuch then, as the ſides A B and A E of the triangle A B E, are equal to the ſides C B and C D of the triangle C B D, and the angles
conteined of them put equal, [f] the baſes B E and B D, and the angles f) 4. 1.
A E B and C D B ſhall be equal: But [g] the angles B E D and B D E are g) 5. 1.
alſo equal, the ſides B E and B D being ſhewn equal; therefore the whole A E D and C D E are equal: Therefore B A E and C D E being put equal, the three angles taken in order A, E, and D, ſhall be equal: Therefore, If three angles, &c. Which was to be demonſtrated.

PROP. 8. THEOR. 8.



If two right lines D B *and* C E, *ſubtend two angles* C *and* D, *of a Pentagon equiangled and equilateral* A B C D E, *the which are in order, they ſhall divide one another in extream and mean proportion, and their greateſt ſegments* F B *and* F E, *are equal to the ſide of the Pentagon.*

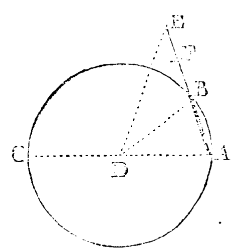
Demonſtration. FOr [a] having deſcribed a circle about the Pentagon, [b] the a) 14. 4.
five arches A B, B C, C D, D E, and E A, ſhall be equal; b) 28. 3.
and foraſmuch as the ſides C D and C B of the triangle C D B, are equal to the ſides D C and C E of the triangle D C E, and the angles conteined of them put equal; the baſes D B and C E, and the angles C D B and
D C E are equal. Wherefore [c] in the triangle C D F, the two angles c) 4. 1.
F C D and F D C being equal, and [d] B F C the external, equal to them, d) 32. 1.
B F C ſhall be the double of D C E; but [e] the angle B C E is alſo double e) 33. 6.
to the ſame D C E: Foraſmuch as the arch B A E is double to the arch D E:



DE: Therefore the angles B F C and
f) 6. 1. B C F are equal; and f therefore B F equal to the side of the Pentagon B C.
g) 17. 5. But g forasmuch as the angles B D C and E C D, insisting on the equal arches, are equal, the two angles D B C and C D B of the triangle B C D, are equal to the two D C F and F D C of the tri-
h) 32. 1. angle C F D; therefore h the triangles B C D and C F D are equiangled; there-
i) 4. 6. fore i as B D to D C, that is to say, to B F, equal to D C, so C D, that is to say, B F, his equal, is to F D; therefore B D is cut in F in extream and mean proportion, and the greatest segment B F is shewn equal to the side of the Pentagon.

In like manner may be shewn that C E is cut at F, according to extream and mean proportion, and that his greatest segment E F is equal to the side of the Pentagon: Therefore, If two right lines, &c. Which was to be demonstrated.

PROP. 9. THEOR. 9.



If the side E B *of the Hexagon, and the side* A B *of the Decagon inscribed in one and the same circle* A B C, *are compounded, the whole right line* E A *is divided in* B, *in extream and mean proportion, and the greatest segment* E B *thereof, is the side of the Hexagon.*

Demonstration FOr from A let there be drawn the diameter A C, and from the center D, let there be joyned D B and D E: Forasmuch as the arch A B, is the tenth part of the whole circumference of the circle; the arch of the semicircle will contein the arch A B five times; therefore the arch B C shall be the quadruple of the arch A B:
a) 33. 6. Wherefore a the angle B D C shall be also quadruple of the angle A D B.

Again, forasmuch as the sides B D and B E are equal, to wit, sides of
b) Co. 15. 4. the Hexagon, (for b B E side of the Hexagon, is equal to the semidiame-
c) 5. 1. ter B D,) c the angles B D E and B E D shall be equal, d to which the
d) 32. 1. external angle A B D being equal, A B D shall be the double of B E D:
e) 5. 1. But e B A D is equal to A B D, the sides D A and D B being equal; therefore B A D shall be also double of B E D; therefore the two angles D A B and D B A together, shall be the quadruple of B E D.

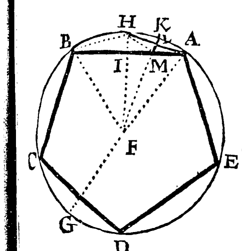
f) 32. 1. Therefore f seeing the external angle B D C is equal to the two D A B and D B A, the external angle B D C shall be the quadruple of the same B E D, but the same angle B D C hath been shewn the quadruple of A D B;

A D B; therefore the angles A E D and A D B are equal. Therefore the two angles A E D and D A E of the triangle A B E being equal, the angles A E D and A D B are equal. Therefore those two equal to the two
g) 32. 11. others A D B and B A D, of the triangle A B D; g the triangles A D E and A B D are equiangled: Therefore as E A shall be to A D, or to E B, equal to A D, so A D, that is to say, E B, his equal, to B A; therefore E A is divided in B in extream and mean proportion, and E B is the side of the Hexagon: Therefore, If the side, &c. Which was to be demonstrated.

COROLLARIE.

From this Demonstration it is manifest that if the side af the Hexagon be divided in extream and mean proportion, his greatest segment shall be the side of the Decagon described in the same circle. For from the side of the Hexagon B E *being taken* B F, *a part equal to* A B, *as the whole* A E *shall be to the whole* B F, *a part equal to* A B, *as the whole* A E *shall be to the whole* B E, *so the part cut off* B E, *to*
b) 19. 5. *the part cut off* B F; *therefore* h *the rest* A B, *that is to say,* B F, *shall be to the rest* F E, *as the whole* A E *to the whole* B E, *or* B E *the part cut off from* A E *to* B F,
i) 3. def. 6. *the part cut off from* B F, *Therefore* i B E *shall be cut as* A E *in extream and mean proportion in* F, *and* B F *the greatest segment is equal to* A B, *the side of the Decagon.*

PROP. 10. THEOR. 10.



If an equilateral Pentagon A B C D E, *be inscribed in a circle* A B C D, *the side of the Pentagon shall be equal in power to the side of the Hexagon and the side of the Decagon, inscribed in the same circle.*

THat is to say, That the square of the side of the equilateral Pentagon inscribed in the circle, is equal to the two squares together of the side of the Hexagon and Decagon inscribed in the same circle.

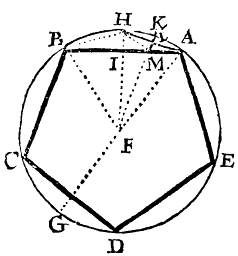
Demonstration FOr let the diameter A F G be drawn, and joyn F B, then having divided the arch A B in two equal parts at H, let the right lines A H, B H, and F H, be joyned, dividing the right line A B in I, the right line A H shall be the side of the Decagon, and B F the side of the Hexagon.

Again, having divided the arch A H in two equal parts in K, let F K be joyned, cutting the right line A H in the point L, and the right line A B in the point M, to which let there be drawn H M: Forasmuch then,
a) 27. 3. as the arches A H and B H are equal, a the angles A F H and B F H insisting on them are equal: Wherefore the sides A F and F I, of the triangle A F I, being equal to B F and F I of the triangle B F I, and the argles
b) 4. 1. conteined of them also equal, b the bases A I and B I shall be equal, and the angles A I F and B I F equal, and therefore right angles; by the same reason, the right lines A L and H L shall be equal, and the angles A L F and H L F right angles.

Then if from the equal semicircles A B C G and A E D G, be taken
the equal arches A B C and A E D, there will remain the arches C G
and D G equal to one another; therefore the arch C G shall be the half
of the arch C D. But the arch A H is the half of the arch A B; there-
fore the arches A B and C D being equal, their halves, which are the
arches C G and A H, shall be equal; therefore seeing that the arch A H
is double to the arch H K, the arch C G shall be also the double of the
arch H K.

In like manner, forasmuch as the arches A B and B C are equal, and
the arch A B the double of the arch B H, the arch B C shall be also the
double of the same B H: Wherefore the arches C G and B C are equi-
c) 1. 5. multiplices, to wit, the doubles of the arches H K and B H; and c there-
fore the whole arch B G shall be also the double of the whole arch B K,
d) 33. 6. and d therefore the angle B F G the double of the angle B F K: But e foras-
e) 20. 3. much as the same angle B F G at the center, is also the double of the angle
F A B, the angles B F M and F A B are equal; and therefore the triangles
A B F and F B M having the angles F A B and M F B equal, and the angle
f) 4. 6. A B F common, shall be equiangled; therefore f as A B to B F, so B F
g) 17. 6. to B M: Therefore g the rectangle conteined under A B and B M shall be
equal to the square of the right line B F.

Again, forasmuch as the sides A L and L M of the triangle A L M, are
equal to H L and L M of the triangle H L M, and the angles conteined
of them equal, to wit, right angles; the
h) 4. 1. bases h A M and H M, and the angles



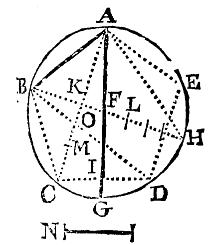
i) 5. 1. L A M and L H M shall be equal; but
i the angle L A M is equal to the angle
H B A: Forasmuch as the sides H A and
H B are equal, therefore the angle L H M
shall be also equal to the same H B A;
k) 32. 1. therefore k the triangles A B H and
A H M, having the angles A B H and
A H M equal, and H A M common, they
l) 4. 6. shall be equiangled: Wherefore l as A B
m) 17. 6. to A H, so A H to A M. Therefore m the
rectangle under A B and A M shall be
equal to the square of A H. But the rectangle under A B and B M is shewn
equal to the square of B F; therefore the rectangles under A B and B M,
and under A B and A M together, are equal to the squares of B F and A H.
n) 2. 2. But n the rectangle under A B and B M, and under A B and A M, are
equal to the square of A B; therefore the square of A B, that is to say,
the side of the pentagon is equal to the squares of B F, and A H the side
of the Hexagon, and the side of the Decagon: Therefore, If in a
circle, &c. Which was to be demonstrated.

COROLLARIE.

It is manifest from this Demonstration that the right line drawn from the center which doth divide one arch in two equal parts, doth also divide the right line subtending it, at right angles in two equal parts it being shewn that F H *dividing the arch* A B *in two equal, divideth also the right line* A B *in two equal parts, and at right angles to* I, *and the same Demonstration serveth for all the others.*

It is manifest also that the Diameter of the circle drawn from the angle of the Pentagon, doth divide also the arch subtending the side opposite to the same angle in two equal parts, and also the said side in two equal parts and at right angles, it being shewn that the diameter A G *divideth in two equal parts the arch* C D, *subtend of the side* C D. *Therefore by what hath been demonstrated, it divideth also the said side* C D *in two equal parts and at right angles, the same Demonstration may be made of every equilateral Polygon inscribed in a circle, the number of the sides being odde.*

PROP. II. THEOR. II.



If in a circle A B C D, *whose diameter is rational, there be inscribed an equilateral Pentagon* A B C D E, *the side of the Pentagon* A B, *is an irrational line, called Minor.*

Demonstration. FOr let the diameters A G
and B H be drawn, inter-
secting at the center F, and let A G cut C D at I, and let A C and A H
be joyned; and A C cut B H at K, and let there be taken F L, the quarter
part of F H, and C M the quarter part of A C: Forasmuch as B H is put
rational, F L and B F the aliquot parts thereof, shall be rational, it being
commensurable; and therefore the whole B L, compounded of them,
a being commensurable to each of them, F L and B F shall be rational. a) 16. 10.
Again, F B drawn from the center, cutting the arch A B C in two equal
parts at B, b it will cut the right line A C also in two equal parts at K, b) C. 10. 12.
and at right angles: But c A G cuts C D at I, in two equal parts, and at c) C. 10. 13.
right angles, therefore the triangles A C I and A F K, having the angles
C I A and A K F right, and C A I common, d shall be equiangled; there- d) 32. 1.
fore e as C I to C A, so F K to F A; and alternately, as C I to F K, so e) 3. 6.
C A to F A, or to F H, equal to F A. But f as C A to F H, so C M the f) 15. 5.
quarter part of C A, to F L, quarter part of F H: Therefore as C I to F K,
so C M to F L, and alternately; as C I to C M, that is to say, as C D the
double of C I, to C K the double of C M, so F K to F L; and in com-
pounding, as C D and C K together, to C K, so F K and F L together, to
wit, K L, to F L; and g therefore as the square of the compounded of g) 22. 6.
C D and C K, to the square of C K, so the square of K L to the square
of F L.

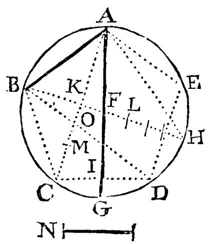
But forasmuch as, if A C be cut in extream and mean proportion (B D
being drawn) h his greatest segment O A, shall be equal to the side of the h) 8. 13.
Pentagon, to wit, to C D, i the square of the compounded of C D or i) 1. 13.
O A the greatest segment, and of C K the half of the whole, shall be the
quintuple of the square of C K, the half of the whole; wherefore the
square of K L shall be also the quintuple of the square of F L. Therefore
the square of K L shall be commensurable to the square of F L, and
therefore K L and F L also commensurable at least in power, &c. But F L
is shewn rational, therefore K L shall be also rational.

Now forasmuch as B F is 4 such parts as F L is 1, and such 5 parts as is
B L; F L is 1 thereof, the square of B L shall be 25 such parts as the
square of F L 1, (as appeares by these numbers 1, 5, 25. For k the k) 20. 6.
squares are in a double proportion of their sides.

But the ſquare of K L is ſhewn to be 5 ſuch parts as the ſquare of F L is 1; therefore the ſquare of B L ſhall be 25 ſuch parts as the ſquare of K L is 5: Therefore the ſquare of B L ſhall be the quintuple of the
l) Co. 24. 8. ſquare of K L l.

Foraſmuch then, as the ſquares of B L and K L are not the one to the other as a ſquare number to a ſquare number, (being as 25 to 5, or 5 to 1,)
m) 9. 10. m B L and K L ſhall be incommenſurable in length, and being ſhewn rational, they ſhall be rational, commenſurable in power only; therefore if from the rational B L, be taken the rational K L, commenſurable in
n) 74. 10. power only, n the remaining B K ſhall be irrational, named Reſidual, and his agreeing line ſhall be K L.

But now let B L be greater in power than K L, by the ſquare of N; foraſmuch as the ſquare of B L is equal to the ſquares of K L and N.



But the ſquare of B L was 5 ſuch parts as the ſquare of K L 1, the remaining ſquare of N ſhall be four ſuch parts as the ſquare of B L is 5: Therefore the ſquares of B L and N ſhall be to one another as a number
o) 6. 10. to a number: Therefore o commenſurable, and B L and N commenſurable at leaſt in power. But B L is ſhewn rational; there-
p) Co. 24. 8. fore N ſhall be ſo alſo, and p the ſquares of B L and N being not the one to the other as a ſquare number to a ſquare number (being as 5 to 4,) B L and N ſhall be incommenſurable in length, therefore rational, commenſurable in power only.

Wherefore the whole B L being rational, commenſurable in length to the rational B H, (for B L is 5 ſuch parts as B H is 8,) and being more in power than his agreeing line K L, by the ſquare of N ſhewn incommenſurable in length thereto, B K ſhall be the fourth Reſidual by the Definition.

q) Co. 8. 6. Laſtly, Foraſmuch q as A B is a mean proportional between B H and B K, (the triangle A B H having the angle B A H right, from which is
r) 31. 3. drawn A K, perpendicular to the baſe,) r the ſquare of A B ſhall be equal to the rectangle under B H and B K; therefore A B being in power as the ſuperficies conteined under the rational B H, and the fourth Reſidual
s) 17. 6. B K; s A B ſhall be a Line Minor: Therefore, If in a Circle, &c. Which was to be demonſtrated.

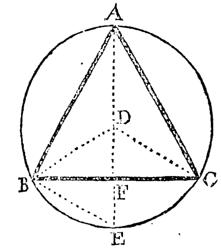
PROP. 12. THEOR. 12.

If a equilateral triangle A B C *be inſcribed in a circle* A B C; *the ſide* A B *of the triangle is triple in power to the line drawn from the center of the circle to the circumference.*

Conſtruction THat is to ſay, that the ſquare of the ſide of the equilateral triangle, inſcribed in the circle, is triple the ſquare of his ſemidiameter.

Demonſtration FOr having drawn the diameter A E, cutting the arch B C
a) C. 10. 13. in two equal parts at E, a and the right line B C alſo in two

two equal parts and at right angles at F, the arch B C being the third part



of the circumference, the arch B E ſhall be the ſixth part, and the right line B E being drawn, ſhall be the ſide of an Hexagon, and b equal to the ſemidiameter: Foraſ- b) Co. 15. 4.
much then as the ſquare of A E is equal to the ſquares of A B and B E, c the angle c) 47. 1.
A B E being a right angle in the ſemicircle: But d the ſquare of A E is the quadruple d) 31. 3.
of the ſquare of B E, ſeeing that A E is the double of E B, the ſquares of A B and B E together, ſhall be in like manner the quadruples of the ſame ſquare of B E; and therefore ſuch four parts as are the ſquares A B and B E, the ſquare of B E ſhall be one, and ſo the ſquare of A B ſhall be three ſuch parts: Therefore the ſquare of A B is the triple of the ſquare of B E, the which is equal to the ſemidiameter: Therefore, If a triangle, &c. Which was to be demonſtrated.

COROLLARIE.

From theſe things it followes, that the diameter of the circle is ſeſquitertia in power to the ſide of the equilateral triangle inſcribed in the ſame circle; that is to ſay, the ſquare of the diameter is in proportion ſeſquitertia, or as 4 *to* 3, *to the ſquare of the ſide of the equilateral triangle inſcribed in the ſame circle; for the ſquare of* A B *being ſhewn triple the ſquare of the ſemidiameter* A D, *the ſquare of* A B *being put* 3, *the ſquare of* A D *ſhall be* 1, *and therefore the ſquare of* A E *quadruple the ſquare of* A D, *ſhall be* 4. *Wherefore the ſquare of* A E *is ſeſquitertia of the ſquare of* A B, *that is to ſay, as* 4 *to* 3; *and* e *foraſmuch then as* A F c) Cor. 8. 6.
is to A B *as* A B *to* A F *perpendicular, drawn from one angle on the ſide. The ſquare of* A B *ſhall be alſo to the ſquare of* A F *as* 4 *to* 3.

Secondly, it may be gathered that D E *the ſemidiameter is cut in two equal parts at* F, *by the ſide* B C *of the triangle* A B C, *for the ſquare of* A B *being triple the ſquare of* B E, *if the ſaid ſquare of* A B *be put* 12, *the ſquare of* B E *ſhall be* 4.
But f *the ſquare of* B E *is equal to the ſquares of* B F *and* F E, *therefore the ſquares* f) 47. 1.
of B F *and* F E *ſhall be* 4. *But the ſquare of* B F *is* 3, *(the ſquare of* A B *being quadruple the ſquare of* B F) *for* A B *is the cube of* B F; *therefore the ſquare of* F E *ſhall be* 1; *therefore the ſquare of* B E *quadruple the ſquare of* F E; *therefore* D E *ſhall be double to* F E. *Wherefore* D E *is cut in two equal parts at* F.

PROP. 13. PROBL. 1.

To conſtitute a pyramid, and encompaſſe it with a given ſphere, and demonſtrate that the diameter A B *of the ſphere is ſeſquialtera in power to the ſide of the pyramid.*

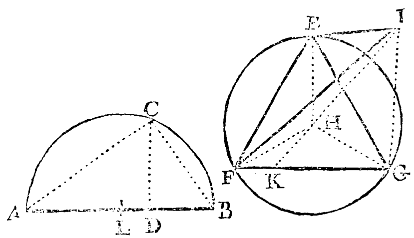
Conſtruction LEt A B be the diameter of the given ſphere, about which let there be deſcribed the ſemicircle A C B, and let there be taken B D, the third part of A B, in ſuch manner as that A B be double to D B, and A B triple to the ſame D B, and ſeſquialtera of A D; and having drawn C D perpendicular to A B, let A C and C D be joyned, and let the circle F E G be deſcribed at the ſpace H E, put equal to D C, on

on the center H, in which let the equilateral triangle E F G be inscribed, and H F and H G joyned, each of which shall be equal to D C, being equal to H E, and from H let there be raised H I, at right angles to the plain of the circle, and put equal to A D; and from I let there be drawn I E, I F, and I G. I say that the solid conteined of the four triangles F E G, I E F, I F G, and I E G, is a Pyramid or Tetrahedron.

Demonstration FOrasmuch as the sides D A and D C of the triangle A D C, are equal to H I and H E of the triangle I H E, and [a] the angles conteined of them put right angles, [b] the base A C shall be equal to the base I E: In like manner, A C shall be shewn equal to I F and I G.

a) 3. def. 11.
b) 4. 1.

Again, Forasmuch as the three lines A D, D C, and D B, are proportional, as A D to D B, so the square of A D to the square of D C; and in composing, as A B to D B, so the squares of A D and D C together, to the square of D C. But the squares of A D and D C are equal to the square of A C; therefore also as A B to D B, so the square of A C to the square of D C; therefore A B being the triple of D B, also the square of A C shall be the triple of the square H E, H E being equal to D C; then the square of E F is also the triple of the same square of H E; therefore the square of A C



shall be equal to the square of E F; and therefore A C equal to E F: Wherefore A C being shewn also equal to I E, I F, and I G, the four triangles E F G, E F I, F G I, and G E I, shall be equilateral and equiangled to one another, therefore there is made a Pyramid or Tetrahedron, whose base is E F G, and his top I: I say that the said pyramid is comprehended in the given sphere, whose diameter is A B, and the diameter of the sphere A B is sesquialtra in power to the side F E or A C.

Let I H be prolonged, perpendicular to K, in such sort as that I K be equal to D B, and the whole I K to the whole A B; forasmuch then as the three lines A D, D C, and D B, are proportional, to which I H, H E, and H K, are equal; I H, H E, and H K, shall be also proportional: Wherefore H E being perpendicular to I K, and mean proportional between I H and H K, the semicircle described about I K, in the plain of I K and H F, will passe by E, as shall be by and by shewn.

In like manner, as well the semicircle described about I K, in the plain of I K and H F, will passe by F, as the semicircle described about I K, in the plain of I K and H G by G; therefore each of these semicircles drawn about the diameter remaining fixed, shall describe a sphere, which shall comprehend the Tetrahedron constituted, seeing that it passeth by all the angles thereof E, F, G, and I; and that sphere being equal to the sphere given, the diameter I K being put equal to the diameter A B, the given sphere shall comprehend the same Tetrahedron.

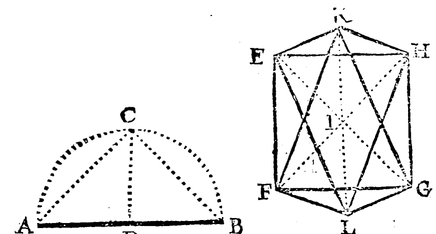
But

But forasmuch [c] as A B, A C, and A D, are proportional, as A B to A D, [d] so the square of A B shall be to the square of A C. But A B is put sesquialtra to the square of A D. Therefore the square of A B shall be sesquialtra to the square of A C; and therefore seeing that A B is the diameter of the sphere, and A C equal to E F the side of the pyramid. It is manifest that the diameter of the sphere is sesquialtra in power to the side of the Tetrahedron or pyramid: Therefore we have constituted, &c. Which was to be done.

c) Co. 8. 6.
d) Co. 20. 6.

PROP. 14. PROBL. 2.

To constitute an Octohedron, and to encompasse it with one and the same Sphere as the Pyramid, and shew that the diameter A B *of the Sphere, is double in power to the side of the said Octohedron.*



Construction LEt A B be the diameter of the sphere comprehending the constituted pyramid, about which let there be described the semicircle A C B, and from the center D let there be drawn D C, perpendicular to A B, and let A C and B C be joyned, which [a] shall be equal to one another, and taking E F equal to A C or B C, let there be described thereon the square E F G H, to which let there be drawn the diameters E G and F H, cutting one another in I, and I E, I F, I G, and I H, shall be equal, [b] being semidiameters of the circle described about the square.

a) 4. 1.
b) 9. 4.

Then from I let there be drawn K L, on both parts, perpendicular to the plain of the square, and I K and I L put equal to E, and from K to L let there be drawn K E, K F, K G, K H, L F, L E, L H, and L G: I say that the solid conteined of the eight triangles K E F, K F G, K G H, K F H, L E F, L E H, L H G, and L F G, is the Octohedron required.

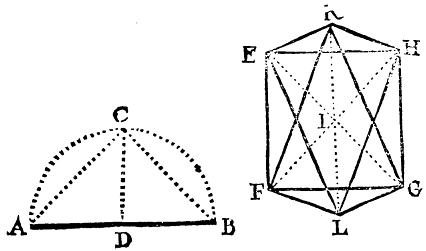
Demonstration FOr seeing the sides E I and I K of the triangle F I K, are equal to E I and I H of the triangle E I H, and the angles conteined of them right angles, (to wit, E I K [c] right, and E I G [d] being equal to G I H by consequence, by reason of the equality of the sides E I and I H, and G I and I H, and of the bases E H and G H, [e] the base E K shall be equal to the base E H.

c) 3. def. 11.
d) 8. 1.
e) 4. 1.

In like manner, K H shall be equal to E H; therefore K E H shall be an equi-

equilateral triangle, and so we conclude that the other triangles shall be equilateral, and for that cause equal to the triangle K E H, being their sides are equal to the sides of the square E F G H, each to each: Therefore the Octohedron is constituted of eight triangles equilateral and equal, the which I say is conteined of the sphere, whose diameter is A B, and of the which the pyramid constituted is conteined; and the diameter of the sphere A B, is double in power to the side E F or A C.

f) 3. def. 11. Now forasmuch [f] as I E is perpendicular to K L, and the mean proportional betweeen the segments K I and I L, (K I, I E, and I L, being equal, and therefore in the same proportion;) the semicircle described about K L on the plain of K L and I F, shall passe by E; In like manner, the semicircle described about K L, on the plain of K L and I F, shall passe by F, and the same of the points G and H: Therefore each of these semicircles drawn about the fixed diameter K L, will describe a sphere, the which shall



comprehend the constituted Octohedron; seeing that it passeth by all the angles thereof: Therefore that sphere being equal to the given sphere, whose diameter is A B; (For A C and E F being equal, and therefore their squares equal; and as well the square of A C donble of the square of A D, as the square of F E of the square of E I, A D and E I, that is to say, K I, semidiameters are equal; therefore the whole A B and K L diameters, shall be equal, and therefore their spheres equal;) the same Octohedron shall be conteined of the given sphere.

But forasmuch as the square of A B is equal to the equal squares of A C and B C, the square of A B shall be double the square of A C. Wherefore A B being the diameter of the sphere, and A C equal to the side of the Octohedron E F; it is manifest that the diameter of the sphere is double in power to the side of the Octohedron: Therefore we have constituted, &c. Which ought to be done.

COROLLARIE.

Hence it is manifest that in the Octohedron the three diameters cut one another at right angles in the center of the sphere, as K L, E G, *and* F G, *at* I, *all the angles at* I *being shewn right angles; therefore* K I *being perpendicular to* E G, F H *shall be so also to the plain made by* E G *and* F H; *therefore the plains* K F L H *and* E L G K, *drawn by* K I, *shall be at right angles to* E F G H; *and by the same reason will cut one the other at right angles, the which are also squares, having for their sides those of the Octohedron, which are equal; and the angles right angles: Forasmuch as (the square of the diameter of the sphere being double the square of the side of the Octohedron, as is shewn,) the square of the diameter* H F *is equal to the squares*

squares of the sides of the Octohedron FK *and* KH, [g] *the angle* F K H *shall be a right angle, and so of the others.* g) 48. 1.

So then it is evident, that the Octohedron may be divided into two Pyramids alike and equal, having for base the common square E F G H, *and four equal triangles and alike, as well on one part as on the other.*

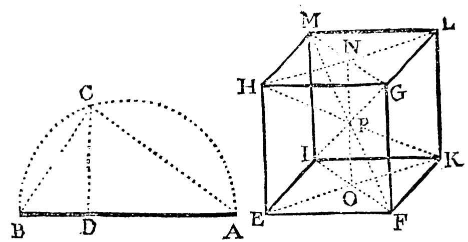
It followes also that if the Tetrahedron and the Octohedron, are described in one and the same sphere, the side of the Tetrahedron shall be sesquialtera in power to the side of the Octohedron, for supposing that square of the diameter of the sphere be divided into six parts; [h] *then the diameter of the sphere being sesquialtera in power to the side of the Tetrahedron, the square of the side of the Tetrahedron is four such parts as the square of the diameter of the sphere is six.* h) 13. 13.

Again, [i] *the diameter of the sphere being double in power to the side of the Octohedron, the square of the side of the Octohedron shall be three such parts as the square of the diameter of the sphere is six; therefore the square of the side of the Tetrahedron being four such parts as the square of the side of the Octohedron is three; it is manifest that the square of the side of the Tetrahedron is sesquialtera to the square of the side of the Octohedron.* i) 14. 13.

Lastly, seeing that in the square E F G H, E H *is parallel to* F G, *and in the square* E L G K, E K *is parallel to* G L; [k] *the plain* E H K *drawn by* E H *and* E K, *shall be parallel to the plain* F G L, *drawn by* F G *and* G L, *there being the same reason given of the other bases of the Octohedron, it followes that the opposite sides of the Octohedron are parallel to one another.* k) 15. 11.

PROP. 15. PROBL. 3.

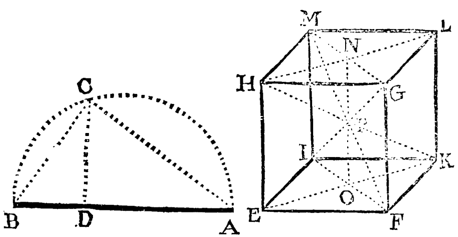
To constitute a Cube, and enclose it in one and the same sphere, as the foregoing figures, and to demonstrate that the diameter A B *of the sphere, is triple in power to the side of the said Cube.*



Construction. Let A B be the diameter of the sphere, which doth contein the precedent figures, and about the which there be described the semicircle A B C, and from A B let there be taken D B the third part, then having drawn D C perpendicular to A B, let A C and B C be joyned, and let E F and G H the square of E F, be described, which let be taken equal to B C, on which let there be drawn at right angles E I, F K, G L, and H M, also equal to B C, at whose extreams let there be joyned I K, K L, L M, and M I: Forasmuch then, as E I and F K are at right angles to the plain E F G H, [a] E I and F K shall be parallel; but they are equal, each being put equal to B C, that is to say, to E F; there- a) 6. 11.

b) 33. 1. fore [b] E F and I K are parallel and equal; therefore the parallelogram E F K I, in which E I, I K, and K F, being each equal to E F, all the four
c) 34. 1. are equal: But [c] all the four angles are right angles, E I K and F K I being equal to the two opposite angles K F E and I E F; therefore E F K I is a square; by the same reason F K L G, G L M H, and H M I E, are squares,
d) 24. 11. and so I K L M shall be a square, [d] being alike and equal to the opposite
e) 15. 11. square E F G H; for E L [e] is a solid parallelepipedon, having the opposite plains parallel, to wit, drawn by parallel lines touching one another: Wherefore E L shall be a cube, the which I say is comprehended of the sphere, whose diameter is A B; for let E K, F I, G M, and H L, be the diameters of the opposite plains E F K I and G H M L, by which let there be drawn the plains E K L H and F I M G.

f) 28. 11. *Demonstration* FOrasmuch as, [f] as well the plain E K L H as F I M G, cuts the cube in two equal parts, both the one and the other will passe by the center of the cube, to wit, by P, in which point also all
g) C. 39. 11. the diameters of the cube [g] are divided into two equal parts, therefore the common section of the plains, to wit, N O, shall passe by the same point P. But forasmuch as the plains E K L H and F I M G are rectangles, (for H E being perpendicular to E I and E F, by reason of the squares E M
h) 4. 11. and E K, [h] it shall be also perpendicular to the plain E K; and therefore
i) 3. def. 11. [i] to the line E K, therefore the angle H E K is a right angle, by the same reason the other angles on the plain E K L H, shall be right angles, and



also on the plain F I M G. Therefore the plains E K L H and F I M G are rectangles,) and equal, the sides of the one being equal to the sides of the other, and their diameters E L, H K, F M, and G I, are equal, as is manifest from the 34th. of the first: Therefore P E, P L, P H, P K, P F, P M, P G, and P I, shall be equal: Wherefore the semicircle about E L, described from the center P, and drawn about the diameter E L remaining fixed, will describe a sphere passing by all the angles of the cube, which shall be shewn equal to the sphere, whose diameter is A B.

k) 47. 1. For seeing that the square of E K is equal [k] to the equal squares of E F and F K; and therefore double the square of E F, that is to say, the square of K L; the squares of E K and K L shall be triple square of K L; but
l) 47. 1. [l] the square of E L is equal to the squares of E K and K L, the angle E K L being a right angle to the rectangle E K L H: Therefore the square of E L shall be triple the square of K L, that is to say, of the square of B C. But the
m) Co. 8. 6. square of A B is triple the same square of B C, [m] (for A B, B C, and B D,
n) 20. c. 6. are proportional,) therefore [n] as A B to B D, so the square of A B to the square of B C.

Then seeing that A B is triple to B D, the square of A B shall be also triple

triple to the square of B C; therefore the squares of E L and A B shall be equal: Wherefore E L and A B the diameters of the spheres, shall be equal, and therefore the said spheres also equal.

But forasmuch as the square of the diameter E L is shewn triple to the square of the side of the cube K L, it is manifest that the diameter of the sphere is triple in power to the side of the cube: Therefore we have constituted, &c. Which was to be done.

COROLLARIE. I.

From this it is manifest, that all the diameters of the cube are equal to one another and doe cut one another in two equal parts in the center of the sphere, and by the same reason, the right lines joyning the centers of the opposite squares, divide themselves in two equal parts in the same center.

For it is demonstrated that the diameters of the cube E L, H K, F M, *and* G I, *are equal, and do cut one another in two equal parts in the center* P.

Let N O *joyning the centers* N *and* O *of the opposite squares be also cut equally in two parts in the same center, we shall shew that the two angles* O F P *and* O P F *of the triangle* O P F, *are equal to* N M P *and* N P M *of the triangle* N M P, *(for* O F P [o] *is equal to* N M P *the alternate angle between* F I *and* G M [p] *parallels,* o) 29. 1.
being common sections of the parallel plains E K, G M, *and* [q] O P F *equal to* p) 16. 11.
N P M, *opposite to the top.) But the sides adjacent* P F *and* P M *are equal, being* q) 15. 1.
semidiameters, therefore P O *and* P N *shall be also equal; therefore* N O *shall be cut in two equal parts at* P, *and so of the other lines conjoyning the centers of the opposite squares.*

COROLLARIE. II.

Again, the power of the diameter of the sphere or of the cube, is equal to the powers of the sides of the Tetrahedron and cube together; For [r] *the square of the diameter* r) 13. 13.
is nine such parts as the square of the side of the Tetrahedron is six. But [s] *the said* s) 15. 13.
square of the diameter of is of nine such parts as the square of the side of the cube is three. Therefore the square of the side of the Tetrahedron and the square of the side of the cube together are of nine parts even as the square of the diameter is: Therefore the square of the diameter is equal to the squares of the said sides; that is also manifest of the semicircle described about the diameter of the sphere where the side of the Tetrahedron is A C, *and of the cube* B C; *But it appeares that the diameter of the sphere* A B *hath power as the line* A C *and* B C; *seeing that* A C B *is a right angle.*

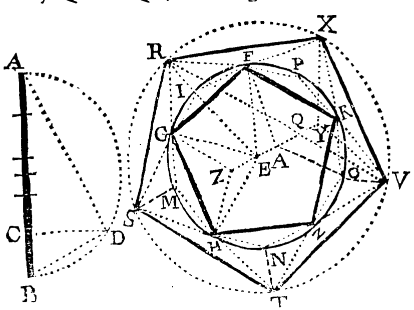
PROP. 27. PROBL. 4.

To constitute an Icosahedron and enclose it with one and the same sphere, as the figures beforesaid, and shew that the side of the Icosahedron is a line irrational, the which is called Minor.

Construction LEt A B be the diameter of the sphere, conteining the precedent figures, about which let there be described the semicircle A D B, and let B C be taken, the fifth part of A B, to the end that A B may be quintuple to B C, and to A C sesquiquarta; but A C quadruple of B C. Then having drawn D C perpendicular to A B, let A D and B D be drawn, and at the space of E F, which let be equal to B D, let there be described a circle from the center E, in which let there be described an equilateral pentagon F G H I K.

Afterwards having divided the arches F G, G H, H I, I K, and K F,
in two equal parts, by L, M, N, O, and P, let the right lines (F L, L G,
G M, and M H, &c. be joyned,) to wit, the sides of the decagon: Then
from the center E, and from the points L, M, N, O, and P, let there be
erected the perpendiculars to the plain of the circle F G H I K, to wit,
E Q, L R, M S, N T, O V, and P X, the which let be put equal to the
semidiameter E F, or to C D, and they shall be all equal to one another:
a) 6. 11. But [a] they are parallel, therefore the right lines which joyn them, to wit,
E L, Q R, E M, Q S, E N, Q T, E O, Q V, E P, and Q X, (the which
b) 33. 1. notwithstanding we have not drawn all to avoid confusion,) [b] shall be
equal to one another, two to two; and therefore E L and E M being
equal to the semidiameter E F; Q R, Q S, Q T, Q V, and Q X, shall
be also equal, both to one another, and to the semidiameter E F, or
c) 15. 1. to C D [c].

But forasmuch as the plain drawn by Q R and Q S, is parallel to the
plain F G H I K, drawn by E L and E M; by the same reason the plain
drawn by Q S and Q T, is parallel to the same plain F G H I K, drawn
by E M and E N. But the plain drawn by Q R and Q S, agrees with the
plain by Q S and Q T, to the right line Q S; therefore these two plains
will make together one only plain, as shall be presently demonstrated. In like manner, the plain by Q T and and Q V, and this plain will make one only plain, and also the plain by Q V and Q X, and also the plain by Q X & Q R. There are then five equal lines in one and the same



plain Q R, Q S, Q T, Q V, and Q X; therefore if from Q, at the
space Q R, there be described a circle in the same plain, it will passe
by the other points R, S, T, V, and X, and shall be equal to the
circle F G H I K; let the points R, S, T, V, and X, be joyned by the
d) 6. 11. lines R S, S T, T V, V X, and X R; forasmuch [d] as L R and P X are equal
and parallel; if there be understood a right line drawn from L to P,
e) 33. 1. [e] L P and R X shall be also equal and parallel: Therefore [f] in equal cir-
f) 28. 3. cles, they shall take away equal arches: But L P takes the fifth part of
the circle F G H I K, to wit, $\frac{2}{10}$, F L and L P: Therefore R X shall also
take the fifth part of the circle R S T V X.

Even so we shall conclude that the other right lines R S, S T, T V, and
V X, taking the fifth parts: Wherefore R S T V X is an equilateral pen-
tagon, having all the sides equal to the pentagon F G H I K. For from the
angles of the pentagon R S T V X, let there be drawn to the angles of the
pentagon F G H I K, the right lines R F, R G, S G, S H, T H, T I, V I,
V K, X K, and and X F: Forasmuch then as L K perpendicular, is put e-
qual to the semidiameter E F, that is to say, to the side of the Hexagon of
g) 10. 13. the circle F G H I K, and L F being the side of the decagon, [g] the square
of

of the side of the pentagon of the same circle, shall be equal to the squares
of L R and L F. But [h] the square of F R is equal to the same squares, h) 47. 1.
[i] the angle F L R being a right angle; therefore the square of F R is equal i) 3. def. 11.
to the square of the side of the pentagon: Therefore F R shall be equal to
the side of the pentagon L P or F G, that is to say, to K X.

And by the same reason, the other lines R G, S G, and S H, &c. shall
be equal to the other sides of the one and the other pentagon; and there-
fore the ten triangles R F X, R F G, R G S, S G H, S H T, T H I, T I V,
V I K, V K X, and X K F, shall be equilateral and equal to one another;
their sides being equal to these of the equilateral pentagon; then after, let
the perpendicular E Q be prolonged on both parts, in such manner as
Q Y and E Z may be equal each to the side of the decagon of the circle
F G H I K, or R S T V X; and let V Q, V Y, X Q, X Y, G E, G Z,
H E, and H Z, be joyned: Forasmuch as Q X is the semidiameter of the
circle R S T V X, that is to say, the side of the hexagon, and Q Y is the
side of the decagon of the same circle, [k] the square of the side of the k) 10. 13.
pentagon X V is equal to the squares of Q X and Q Y; but [l] the square l) 47. 1.
of X Y, is equal to the same squares; the angle X Q Y being [m] a right m) 3. de. 11.
angle, (for the plains of the circles F G H I K and R S T V X, being shewn
parallel, and [n] E Q being perpendicular to the plain of that circle, by n) 14. 11.
construction, E Q shall be also perpendicular to the plain of the circle,
as it appeares, and therefore perpendicular to Q X in the same plain:)
Therefore the square of X Y is equal to the square of X V, therefore X Y
shall be equal to X V: so X V shall be shewn equal to V Y; therefore the
triangle V Y X is equilateral.

Even so it will be demonstrated, if the right lines R Y, S Y, and T Y,
be drawn (the which notwithstanding have not been drawn for other reason
than to avoid confusion) that the four triangles R Y X, R Y S, S Y T, and
T Y V, are equilateral and equal to the triangle V Y X, that is to say, to
the ten first, all their sides being equal to the side of the pentagon: In
like manner, G Z H shall be equilateral, (seeing that E G is the side
of the hexagon, to wit, the semidiameter, and E Z the side of the
decagon, and the angle G E Z a right angle,) and [o] also the four tri- o) 3. def. 11.
angles H Z I, I Z K, K Z F, and F Z G, (from which we have drawn the
lines to avoid confusion) all of them shall be equal to the fifteen first, each
to his correspondent, for the same cause: Therefore those twenty triangles
being equilateral and equal, and each joyned to other, by the right lines, to
wit, their sides; there shall be constituted of them an Icosahedron, the which
I say is comprehended of the sphere, whereof A B is the diameter.

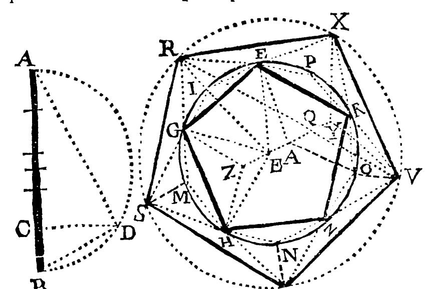
For let E Q be divided in two equal parts by A, and let A F, A Y, and
A V, be drawn. Forasmuch as the sides A Q and Q V, of the triangle
A Q V, are equal to A Q and Q X, of the triangle A Q X; seeing that
Q V and Q X are semidiameters of the circle A S T V X; but the angles
A Q V and A Q X are [p] equal, to wit, right angles, [q] the bases A V and p) 3. def. 11.
A X shall be equal. q) 4. 1.

In like manner, if the right lines be drawn from A and Q, to R, S, and
T, the right lines A R, A S, and A T, shall be shewn equal both to one
another, and to A V and A X. Again, forasmuch as the sides A Q and Q X,
of the triangle A Q X, are equal to A E and E F of the triangle A E F:
Seeing that Q X and E F are semidiameters of equal circles, and that
E Q is divided in two equal parts at A. But the angles A Q X and A E F
are equal, to wit, [r] right angles, the bases A X and A F shall be [s] equal: r) 3. def. 11.
In s) 4. 1.

In like manner, if from A E there be drawn the right lines to G, H, I, and K, it will be shewn that A G, A H, A I, and A K, will be equal to A X; and therefore the ten lines drawn to the ten angles F, G, H, I, K, R, S, T, V, and X, are equal: But forasmuch as Q E is the semidiameter, that is to say, the side of the hexagon of the circle F G H I K, and E Z
t) 9.13. the side of the decagon of the same circle, t Q Z shall be divided in E by extream and mean proportion, and the greatest segment shall be Q E: Wherefore the lesser segment Z E; taking E A the half of the greatest segment, that is to say, A Z, is in power as the quintuple of the square described of E A. But A F is in power also as the quintuple of the same square of E A, (for the square of E F being quadruple of the square of A E, E F being the double of E A, the square of E F and E A together, shall be the quintuples of the square of E A. And v the square of A F being equal to the
v) 47.1. squares of E F and E A, the square of A F shall be also the quintuple of the square of E A;) therefore the squares of A Z and A F shall be equal; and therefore A Z and A F equal; then seeing that Z Y is cut in two equal parts at A, (forasmuch as, if to the equal lines A E and A Q, there be added the equal lines E Z and Q Y, the whole Z A and Y A are made equal;) all the right lines drawn from A, to all the angles of the Icosahedron shall be equal: Wherefore the sphere described about the diameter Z Y, from the center A, shall passe by all the angles of the constituted Icosahedron: I say it is equal to the sphere of the diameter A B.

w) 15.5.
x) 22.6.
Demonstration FOr seeing w that as A Z to A E, so Y Z the double of A Z, to Q E the double of A E, and x so as the square of A Z to the square of A E, so the square of Y Z to the square of Q E, and the the square of A Z being shewn quintuple of the square of A E; also the square of Y Z shall be quintuple of the square of Q E, that is to say, of the square of B C; Q E being put equal to the semidiameter E F, that is to say, to B C: But y the square of A B is also quintuple of the same square of B C: (For A B being to B C, as B C to B D, as z A B shall be to B D, so the square of A B to the square of B C; and therefore A B

y) Co.20.6.
z) Cor. 8.6.



being quintuple of the square of B D, the square of A B shall be also quintuple of the square of B C:) therefore the squares of Y Z and A B are equal: Wherefore X Z and A B also equal, and therefore the spheres described about the same shall be equal.

Lastly, forasmuch as the diameter of the sphere A B is put rational, (for in comparing of it, it is shewn that the side of the Icosahedron is a line irrational, called Minor,) and is in power as the quintuple of the square of B C, or of E F his equal, E F the semidiameter of the circle
a) 6.10. F G H I K shall be also rational; (for a the squares of A B and E F being to

to one another as a number to a number, to wit, as 5 to 1, or 10 to 2, shall be commensurable: Therefore A B and E F commensurable at least in power: Therefore seeing that A B is rational, E F shall be so also;) therefore the whole diameter of the circle F G H I K shall be rational: Wherefore F G the side of the pentagon, that is to say, of the Icosahedron, 11.13.
shall be a line irrational called Minor: Therefore we have constituted, &c. Which was to be done.

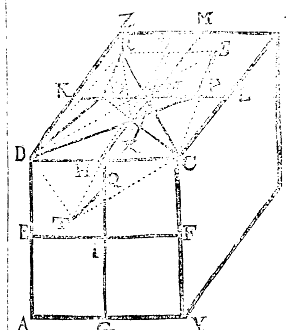
COROLLARIE.

From this it may be concluded that the diameter of the sphere is quintuple in power to the semidiameter of the circle which doth encompasse the five sides of the Icosahedron, to wit, of that whereof the Icosahedron is constituted, and which passeth by the five angles of the Icosahedron. For it is shewn that the square of the diameter A B *is quintuple to the square of* B C, *that is to say, to the semidiameter* E F.

Secondly, In like manner it is manifest that the diameter of the sphere is compounded of the side of the hexagon, that is to say, of the semidiameter, and the two sides of the decagon of one and the same circle; for Y Z *diameter of the sphere is compounded of* E Q *side of the Hexagon, and of* Q Y *and* E Z, *sides of the decagon* F G H I K.

Thirdly, It appeares lastly, that the opposite sides of the Icosahedron, as R X *and* H I, *are parallel, for* R X *is shewn parallel to the right line which shall be drawn from* L *to* P: *Therefore* H I *being also parallel to the same* L P, *(the alternate angles* H I L *and* I L P *made of the right line drawn from* L *to* P, b *being equal; subtending equal arches, to wit, each* $\frac{3}{10}$ *of the circumference,) it is manifest that* R X *and* H I *are parallels, and so of the other opposite sides.* b) 27.3.

PROP. 17. PROBL. 5.

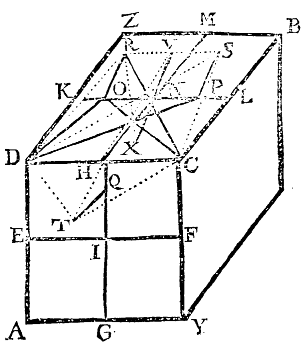


To constitute a Dodecahedron, and enclose it with one and the same sphere, as the figures aforesaid, and demonstrate that the side of the Dodecahedron is an irrational line, the which is called Residual.

Construction LEt there be proposed two plains of the cube aforesaid at right angles to one another, A C and B D, whose sides may be cut each in two equal parts, by E, G, F, H, K, L, and M; and let the lines E F, G H, K L, and H M, be drawn, to cut one another in the points I and N; then I H, K N, and
N L, a are cut by extream and mean proportion, by O, P, and Q, whose a) 30.6.
greatest segments are I Q, P N, and N O, and b at the exteriour parts b) 12.11.
of the plains B D and C A, let there be drawn O R, P S, and Q T, perpendiculars, equal to P N, N O, and I Q, and let D T, D R, S R, C T, and

and C S, be joyned: I say that D R S C T is an equilateral and equiangled pentagon, and is on one and the same plain.

Demonstration. Suppose it otherwise, Let D P, D O, and D S be joyned in the plain D B and D S; seeing that N K is cut by extream and mean proportion at O, and N O is the greatest segment, the squares [c] of N K and of K O, shall be triple of the square of N O, N K,
c) 4. 13. and K D, being equal, and O N and O R also equal by construction, the squares of D K and K O shall be also the triple of the square of N O, that is to say, of the square of O R; but the square of D O is equal to the
d) 47. 1. two squares of D K and K O, ([d] the angle O K D being a right angle;) therefore the square of D O is also the triple of the square of O R: Wherefore adding the said square of O R, the two squares of D O and O R, shall be the quadruples of the square of O R: But the square of D R
e) 47. 1. is equal to the two squares of D O and O R: ([e] the angle D O R being a right angle,) it shall be then also quadruple of the square of O R; and R S being the double of R O, his square shall be also the quadruple of the square of R O: Wherefore D R and R S shall be equal. By the same reason, the three other sides D T, T C, and C S, shall be shewn equal to one another, and also to D R and R S: Therefore the pentagon is equilateral.



It is also in one and the same plain: For let there be drawn N V parallel to O R, and let there be also drawn V H and T H, they will meet with one another directly, and V H T shall be one only right line: For H I being cut according
f) 3. def. 6. to extream and mean proportion at Q, [f] I H shall be to I Q as I Q to Q H, and I H and H N are equal, and N V and I Q, and Q T and Q I, also equal: Wherefore as H N to N V, so Q T to Q H. Therefore the triangles V N H and H Q T, having two sides proportional to two sides, and being conjoyned at the angle H, in such sort as the sides of the same
g) 6. 11. proportion, N H, and Q T, and N V and Q H [g] are parallels, being
h) 32. 6. perpendicular to the plains A C and D B, [h] the line V H T shall be one only right line, the which being in one and the same plain, it followes that the whole pentagon D T C S V R drawn by the same, is in one and the same plain.

It is also equiangled, For K N being cut in extream and mean proportion, and O N is the greatest segment, if K N be added to N P, equal to the greatest segment N O, the whole K P shall be divided by extream and mean proportion at N, and K N shall be the greatest segment; there-
i) 4. 13. fore [i] the squares of K P and N P, or of P S equal to N P, are the triple of the square of K N, or of the square of K D, equal to K N.

And adding the square of K D, with the two squares of K P and P S, these three squares shall be quadruple of the square of K D; but the
k) 47. 1. square of P D is [k] equal to the two squares of P K and K D; therefore the

the two squares of D P and P S shall be quadruples of the square of K D;
but [l] the square of D S is equal to the two squares of D P and P S: There- l) 47. 1.
fore the square of D S shall be also quadruple of the square of K D, and the square of D C is also quadruple of the square of D K, (D C being the double of D K:) Wherefore D C and D S shall be equal.

And all the sides of the Pentagon being equal, the two triangles D R S and D T C shall have the sides D R and R S equal to D T and T C, each to his correspondent side, and the bases D S and D C equal: Therefore
[m] the angle D R S shall be equal to D T C, and so it shall be shewn that m) 8. 1.
the angle R S C is equal to the angle D T C. Wherefore three angles of the Pentagon being equal, [n] it shall be equiangled, and being equilateral n) 7. 13.
on D C, one of the twelve equal sides of the cube.

And if on each side of the eleven remaining sides, there be made a Pentagon, there will be a solid figure conteined of twelve Pentagons, equilateral and equiangled, which shall be a Dodecahedron by the Definition.

It may also be inscribed in the proposed sphere, for V N being perpendicular to the plain B D, drawn from N, the center of that plain being
prolonged, [o] shall cut the diagonal of the cube in two equal parts at X, o) 39. 11.
which [p] shall be the center of the sphere inclosing the cube, and X N is p) Co. 15. 3.
equal to the half of the side of the cube.

Let X R be drawn, the squares of K P and N P being [q] shewn triples, q) 4. 13.
the square of K N and V X being equal to K P, and R V to N P, (N X being equal to K N, and V N to N P:) Wherefore the squares of R V and V X
are triple the square of K N: But the square of R X [r] is equal to the two r) 47. 1.
squares of R V and V X: Wherefore the square of R X shall be also triple the square of K N. But [s] the square of the diameter of the sphere de- s) 15. 13.
scribed about the cube, is triple the square of the side of the cube, and the parts being in the same proportion as their equimultiplices, [t] the t) 15. 5.
square of the semidiameter shall be triple the square of the half side. But K N is the half of the side of the cube, therefore R X shall be the semidiameter of the sphere described about the cube A B, whose center is X.

In like manner is shewn, that all the right lines drawn from X, to the other angles of the Dodecahedron, are equal to the semidiameters of the sphere described about the cube: Therefore one and the same sphere may be circumscribed about a cube and a Dodecahedron.

Lastly, the side of the Dodecahedron is an irrational line, called a Residual, for the two half sides of the cube K N and N L, being divided by
extream and mean proportion, [v] the whole K L shall be to the two great- v) 15. 5.
est segments together, to wit, to O P, as O P, which is the two same and the two greatest segments, to the two lesser, to wit, to the line compounded of P L and K O: Therefore if K L the side of the cube be divided by extream and mean proportion, O P shall be the greatest segment: But K L being cut after this manner, is rational, (his square being commensurable to the square of the diameter of the sphere proposed ratio-
nal, [w] being the third part thereof:) Wherefore O P the greatest w) 15. 13.
segment, or R S the side of the Dodecahedron his equal, shall be an irrational line, called Residual: Therefore, &c. Which was to be done.

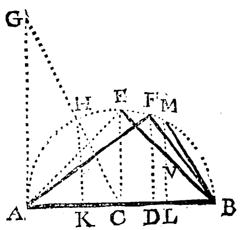
COROLLARIE.

It followes from this Demonstration that the side of the cube being divided in ex-

extream and mean proportion, the greatest segment is the side of the dodecahedron inscribed in the same sphere.

And the side of the cube is equal to the right line subtending the angle of the pentagon of the dodecahedron. But forasmuch as the same is divided in extream and mean proportion, x the greatest segment is the side of the pentagon, y the right
x) 8. 13. y) 5. 13. *line compounded of the said subtence of the angle of the pentagon, that is to say, of the side of the cube, and of the greatest segment, to wit, of the side of the dodecahedron, is divided in the like sort, and the lesser segment is the side of the dodecahedron, and the greatest segment is the side of the cube: It followes that a right line being divided by extream and mean proportion, the lesser segment shall be the side of the dodecahedron, and the greatest segment shall be the side of the cube inscribed in the same sphere.*

PROP. 18. PROBL. 6.



To expound the sides of the five aforegoing figures, and to compare them to one another.

THat is to say; To find the sides of the five figures inscribed in one and the same sphere, and their proportion, the diameter of the sphere being given.

Construction. LEt the diameter of the sphere be A B, and let it be divided in two equal parts at C; and let it be divided at D, so as that A D may be the double of D B, and let the semicircle A E B be described, and let C E and D F be drawn at right angles to A B, from C and D, and let A F, F B, and E B, be joyned; then forasmuch as A D is the double of D B, A B shall be the triple of B D; and by conversion of proportion, B A sesquialtra of A D; but as B A to A D, so the square of B A to the
a) 8. 6. square of A F: For a the triangle A F B is equiangled to the triangle A F D, therefore the square of B A is sesquialtra to the square of A F. But the diameter of the sphere is also sesquialtra in power to the side of the pyramid, and A B is the diameter of the sphere; therefore A F is equal to the side of the pyramid.

Again, Forasmuch as A D is the double of D B, A B shall be the triple of B D: But as A B to B D, so the square of A B to the square of F B:
b) 15. 13. Therefore the square of A B is the triple of the square of B F: But b the diameter of the sphere is triple in power to the side of the cube, and A B is the diameter of the sphere: Therefore B F is the side of the cube, and A C being equal to C B, A B shall be the double of B C; but as A B to B C, so the square of A B to the square of B E: Therefore the square of
c) 14. 13. A B is the double of the square of B E, and c the diameter of the sphere is double in power to the side of the Octohedron, and A B is the diameter of the given sphere: Wherefore B E is the side of the Octohedron.

From A let there be drawn A G at right angles to A B, and put equal to the same A B, and having joyned G C, cutting the circumference at H, from H let there be drawn H K, perpendicular to A B.

De-

Demonstration. FOrasmuch as A G is the double of A C (for G A is equal to A B) but d as G A to A C, so H K to K C, H K shall be double
d) 4. 6. to K C; therefore the square of H K is the quadruple of the square of K C; therefore the squares of H K and K C, that is to say, the square of H C is quintuple of the square of K C; but H C is equal to C B: Therefore the square of B C is the quintuple of the square of C K.

And forasmuch as A B is the double of B C, of which A D is the double of D B, the remaining B D shall be the double of C D: Wherefore B C is the triple of C D; therefore the square of B C is the noncuple of the square of C D; but the square of B C is the quintuple of the square of C K: Therefore the square of C K is greater than the square of C D, and K C greater than C D. Let C L be put equal to K C, and from L let there be drawn L M, at right angles to A B, and let M B be joyned.

And forasmuch as the square of B C is the quintuple of the square of K C, and B A the double of C B, and K L the double of C K, e the square
e) 15. 5. f) C. 16. 13. of A B shall be quintuple of the square of K L; but f the diameter of the sphere is quintuple in power of the semidiameter of the circle of which the Icosahedron is described, and A B is the diameter of the sphere: Therefore K L is the side of the hexagon of the same circle.

Moreover, Forasmuch as the diameter of the sphere is composed of the side of the Hexagon, and of two sides of the decagon described in the same circle, and A B is the diameter of the sphere, K L the side of the Hexagon, and A K equal to L B, both the one and the other, A K and L B, shall be the side of the Decagon, described in the same circle of which the Icosahedron is described; and forasmuch as the side of the Decagon is L B, and that of the Hexagon M L, for it is equal to K L, it being also equal to H K, for that they are equally distant from the center, and both the one and the other H K and K L is the double of K C; M B shall be the side of the pentagon; for that which is of the pentagon, is the same as that of the Icosahedron: Therefore M B is the side of the Icosahedron.

And forasmuch as F B is the side of the cube, let it be divided in extream and mean proportion at N, and let B N be the greatest segment, N B shall be the side of the Dodecahedron, and the diameter of the sphere being shewn sesquialtra in power to the side of the pyramid A F, and double in power to B E, the side of the Octohedron. and triple in power to F B, the side of the cube, of which parts the diameter of the sphere is 6 in power of the same parts; the side of the pyramid shall be 4, of the Octohedron 3, and of the cube 2: Therefore the side of the pyramid is sesquitertia in power to the side of the Octohedron, and to the side of the cube double in power, and the side of the Octohedron is sesquialtra in power to the side of the cube; therefore the abovesaid sides of the three Figures, to wit, of the Pyramid, of the Octohedron, and of the Cube, are the one to the other in rational proportion: But the two others, to wit, of the Icosahedron, and of the Dodecahedron, are not in rational proportion, neither to one another, nor to the abovesaid, to wit, the Minor and Residual.

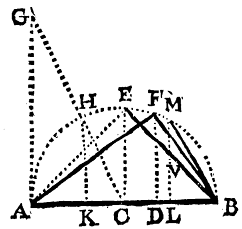
But we shall demonstrate it so as that the side B M of the Icosahedron, is more than the side B N, of the Dodecahedron.

For seeing the triangle F D B is equiangled to F A B, as D B shall be to B F, so F B to B A, and having three lines proportional, as the first shall be to the third, so the square of the first, to the square of the second.

 There-

Therefore as D B to B A, so the square of D B to the square of B F; and by conversion of proportion, as A B to B D, so the square of F B to the square of B D. But A B is the triple of B D; therefore the square of F B is the triple of the square of B D: But the square of A D is the quadruple of the square of D B, for A D is the double of D B: Therefore the square of A D is greater than F B; and therefore A L much greater than F B; and A L being divided by extream and mean proportion, the greatest portion is L K: Forasmuch as L K is the side of the Hexagon, and K A of the Decagon, and F B being divided by extream and mean proportion, the greatest portion is B N; therefore K L is greater than B N: but K L is equal to L M, therfore L M is greater than B N; but B M is greater than M L: therefore M B the side of the Icosahedron, shall be greater than B N the side of the Dodecahedron.

Otherwise, Forasmuch as A D is the double of D B, A B shall be the triple of B D; but as A B to B D, so the square of A B to the square of B F: Forasmuch as the triangle F A B is equiangled to the triangle F B D; therefore the square of A B is triple to the square of B F. But the square of A B is shewn quintuple to the square of K L; therefore five squares of K L are equal to three of B F; but three of B F are greater than six of those which are made of B N; therefore also five of K L are greater than six of B N; therefore one of K L is greater than one of B N; therefore one of K L is greater than one of B N; but K L is equal to L M; therefore L M is greater than B N. Wherefore M B is much greater than B N. Which was to be demonstrated.



But we shall demonstrate it so as that three of F B, are greater than six of those of B N.

Forasmuch as B N is greater than N F, the rectangle under F B and B N shall be greater than the rectangle under B F and F N; therefore that which is conteined of F B and B N, with that of B F and F N, is greater than the double of that which is conteined under B F and F N; but that which is conteined of F B and B N, with that conteined under B F and F N, is the square of F B; but that which is conteined under B F and F N, is equal to the square of B N; for F B is divided by extream and mean proportion at N, and the rectangle conteined under the extreams is equal to the square of the mean: Therefore the square of F B is greater than the double of the square of B N: Wherefore one square of F B is greater than two of B N, and therefore three of F B are greater than six of those which are made of B N. Which was to be demonstrated.

The End of the Thirteenth Element of EUCLIDE.

THE

THE FOURTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

IN this Book which is a continuation of the former subject, is intreated of the Dodecahedron and Icosahedron described in the same sphere: And although this be commonly accounted the Fourteenth Book in order of *EUCLIDES* Elements, yet by others it is supposed to be the First Book of *Hypsicles* of *Alexandria*, concerning the five regular Bodies, as it may appear by the following Preface of the said *Hypsicles*.

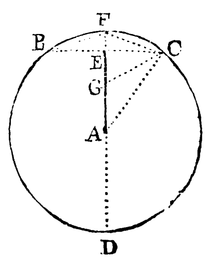
The Preface of *Hypsicles* before the Fourteenth Book.

FRiend Protarchus, *when that* Basilides of Tire *came into* Alexandria, *having familiar friendship with my father, by reason of his knowledge in the Mathematical Sciences, he remained with him a long time, yea even all the time of the Pestilence. And sometime reasoning between themselves of that which* Apollonius *had written touching the comparison of a Dodecahedron and of an Icosahedron inscribed in one and the*

the self same sphere, what proportion such bodies have the one to the other, they judged that Apollonius *had somewhat erred therein. Wherefore they (as my father declared unto me) diligently weighing it, wrote it perfectly. Howbeit afterward I happened to find an other Book written of* Apollonius, *which conteined in it the right demonstration of that which they sought for: which when they saw, they much rejoyced. As for that which* Apollonius *wrote, may be seen of all men, for it is in every mans hand. And that which was of us more diligently afterward written again. I thought good to send and dedicate unto you, as to one whom I thought worthy commendation, both for that deep knowledge which I know you have in all kinds of learning, and chiefly in Geometry, so that you are able readily to judge of those things which are spoken, and also for the great love and good will which you bear towards my father and me. Wherefore vouchsafe gently to accept this, which I send unto you. But now I think it is time to end our Preface, and to begin the matter.*

PROPOSITIONS, and THEOREMES.

PROPOSITION 1. THEOREM 1.



The perpendicular right line AE, *drawn from the center* A, *to the side* BC, *of the Pentagon inscribed in the circle* BDC, *is the half of the one and the other side, to wit, of that of the Hexagon and of the Decagon together, inscribed in the same circle.*

Demonstration. For having continued AE on both parts, to finish the diameter DF, and having taken EG equal to EF, let AC, CG, CF, and BF, be joyned: Forasmuch [a] as AE cutteth BC in two equal parts, the two sides BE and EF, of the triangle BEF, shall be equal to CE and EF of the triangle CEF, and the angles conteined of them being equal, to wit, right angles, [b] the bases CF and BF shall be equal, and [c] the arches CF and BF shall be equal; therefore BFC being the fifth part of the whole circumference CF, shall be the tenth part; therefore the right line CF is the side of the Decagon.

a) 3. 5.
b) 4. 1.
c) 28. 3.

Again, the sides CE and EF of the triangle CEF, being equal to CE and EG, and the angles conteined of them right angles, [d] the bases CF and CG shall be equal, and the angles CFG and CGF equal, and the arch CF being the fifth part of the half circumference DCF, being the tenth part of the whole, the arch DC shall be the quadruple of CF: Wherefore [e] the angle DAC shall be also the quadruple of CAF [f].

d) 4. 1.
e) 33. 6.
f) 20. 3.

But the angle CFD at the circumference, is the half of DAC at the center, therefore CFD; and therefore CGF shewn equal thereto, shall be the double of CAF; but [g] CGF the external angle, is equal to the two internal angles GAC and GCA, of the triangle GAC, and GAC being shewn the half of the same CGF; GCA shall be the half of the same CGF; and therefore GAC and GCA equal, and GA and GC equal.

g) 32. 1.

But CG is shewn equal to CF, therefore AG shall be also equal to CF; therefore adding the equal lines GE and EF, AE shall be equal to GF and FE together; therefore the three lines AE, FE, and FC, together, that is to say, the two together AF and FC, shall be the double of AE; and contrariwise, AE perpendicular to the side of the Pentagon, shall be the half of the two together AF and FC, to wit, of the side of the Hexagon and the Decagon: Therefore, The perpendicular right line, &c. Which was to be demonstrated.

COROLLARIE.

From whence it follows that the perpendicular drawn from the center to the right line, fitted in the circle, doth cut the arch subtending the said line in two equal parts, being demonstrated that AE *drawn from the center* A, *perpendicular to* CB, *divides the arch* BC *in two equal parts at* F.

Secondly, That the perpendicular from the center to the side of the said Pentagon, is equal to the perpendicular from the same center drawn on the side of the equilateral triangle, and to the half of the side of the Decagon inscribed in the same circle.

For as it is here demonstrated, the perpendicular drawn from the center to the side of the Pentagon, is equal to the half of the side of the Hexagon and of the Decagon together: But the perpendicular drawn from the center on the side of the equilateral triangle, is [h] equal to the half of the semidiameter, or side of the Hexagon. Therefore it is manifest that the same perpendicular drawn from the center to the side of the Pentagon, is equal to the perpendicular drawn to the side of the equilateral triangle, and to the half of the side of the Decagon together.

h) C. 12. 13.

PROP. 2. THEOR. 2.

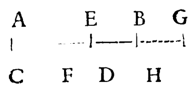
If two right lines AB *and* CD, *be cut in* E *and* F, *in extream and mean proportion, they shall be alike cut, to wit, in the same proportion.*

Demonstration. For seeing that as AB is to AE, so AE to EB, and as CD to CF, so CF to FD, [a] the rectangle conteined under AB and EB, shall be equal to the square of AE, and the rectangle under CD and FD equal to the square of CF. There-

a) 17. 6.

Therefore as the rectangle under AB and EB ſhall be to the ſquare of AE, ſo the rectangle under CD and FD to the ſquare of CF (there being proportion of equality on the one part and the other:) Wherefore as the quadruple of the rectangle under AB and EB, to the ſquare of AE, ſo the quadruple of the rectangle under CD and FD, to the ſquare of CF, and by compounding, as the quadruple of the rectangle under AB and EB, and the ſquare of AE to the ſame ſquare of AE, ſo the quadruple of the rectangle under CD and FD, and the ſquare of CF to the ſquare of CF: But [b] four times the rectangle under AB and EB, with
b) 8. 2. the ſquare of AE, is equal to the ſquare of AG, compounded of AB and EB.

And four times the rectangle under CD and FD, with the ſquare of CF, is equal to the ſquare of CH, compounded of CD and FD; therefore as the ſquare of AG compounded of AB and EB, to the ſquare of AE, ſo the ſquare of CH compounded of CD and FD, to the ſquare
c) 22. 6. of CF: Therefore [c] as AG to AE, ſo CH to CF, and in compounding, as AG and AE, that is to ſay, the double of AB, is to AE, ſo CH and CF, that is to ſay, the double of CD, is to CF; and by permutation, as the double of AB to the double of CD, ſo



d) 15. 5. AE to CF: But [d] as the double of AB to the double of CD, ſo AB to CD; therefore as AB ſhall be to CD, ſo AE to CF; therefore ſeeing that as the whole AB is to the whole CD, ſo the part cut off AE, to
e) 19. 5. the part cut off CF; alſo [e] the remainder EB ſhall be to the remainder
f) 11. 5. FD, as the whole to the whole; therefore [f] as AE to CF, ſo EB to FD; and alternately, as AE to EB, ſo CF to FD, and ſo according to all or any other manner of arguing in proportion, it will be ſhewn, that the whole lines and their parts, are proportional to one another: Therefore, If two right lines, &c. Which was to be demonſtrated.

PROP. 3. THEOR. 3.

One and the ſame circle conteines the Pentagon of the Dodecahedron, and the triangle of the Icoſahedron, inſcribed in one and the ſame ſphere.

Before I ſhall ſhew this Propoſition, I ſhall ſhew that the ſquare of the ſide of the Pentagon, with the ſquare of the line ſubtending one angle thereof, is the quintuple of the ſquare of the ſemidiameter of the circle where the ſaid Pentagon is inſcribed.

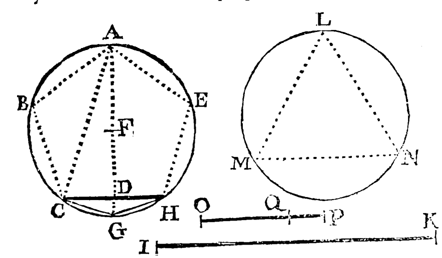
Let CH be the ſide of the Pentagon inſcribed in the circle, and let CA be the line ſubtending one of the angles of the Pentagon, and FG
a) C.10.13. the ſemidiameter, [a] cutting CH, and the arch CGH, in two equal parts: I ſay that the ſquares of CA and CH together, are the quintuple of the ſquare of FG.

Demonſtration. For the right line CG being drawn, ſhall be the ſide of the Decagon, and the ſquare of AG ſhall be the quadruple of the ſquare of FG the ſemidiameter: Wherefore the two ſquares of

of AC and CG, [b] equal to the ſaid ſquare of AG, ſhall be alſo the qua- b) 47. 1.
druple of the ſquare of FG: Therefore the three ſquares of AC, CG, and GF, together, ſhall be the quintuple of the ſquare of GF; but the ſquares of CG and of GF together, ſides of the Hexagon and of the
Decagon, are [c] equal to the ſquare of CH, the ſide of the Pentagon; c) 10. 13.
therefore the ſquares of AC and of CH, are quintuples of the ſquare of GF. Which was propoſed. Now

Let IK be the diameter of the ſphere conteining the Dodecahedron and the Icoſahedron; and let ABCHE be one Pentagon of the ſame Dodecahedron, and LMN a triangle of the Icoſahedron: I ſay that one and the ſame circle conteineth the Pentagon ABCHE, and the triangle LMN; that is to ſay, that the circles which circumſcribe ABCHE and LMN, are equal, and that it is no other than one and the ſame; for
having drawn AC ſubtending the angle B of the Pentagon, [d] AC ſhall be d) C.17.13
the ſide of the cube inſcribed in the ſame ſphere.

Let there be taken OP, ſo as that the ſquare of IK the diameter of the ſphere, may be quintuple the ſquare of OP, and OP equal to the ſemidiameter of the circle where the Icoſahedron is inſcribed: And let
OP [e] be cut in extream and mean proportion at Q; OQ the greateſt ſeg- e) C.16.13.
ment ſhall be the ſide of the Decagon inſcribed in the ſame circle,
whereof OP [f] is the ſemidiameter or ſide of the Hexagon. But [g] CA be- f) C. 9. 13.
ing divided by extream and mean proportion, his greateſt ſegment is g) 7. 13.



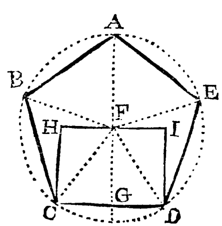
AB, the ſide of the Pentagon: Therefore [h] as AC the whole, to OP h) 2. 14.
the whole, ſo the greateſt ſegment AB, to the greateſt ſegment OQ:
Therefore [i] as the ſquare of AC to the ſquare of OP, ſo the ſquare of AB i) 22. 6.
to the ſquare of OQ, and [k] as the triple of the ſquare of AC to the quintuple k) 4. 5.
of the ſquare of OP, ſo the triple of the ſquare of AB, to the quintuple of the ſquare of OQ. But the triple of the ſquare of AC is equal to the quintu-
ple of the ſquare of OP ([l] the ſquare of IK the diameter of the ſphere l) 15. 13.
being put equal, as well to the triple of the ſquare of AC the ſide of the cube, as to the quintuple of the ſquare of OP:) Wherefore the triple of the ſquare of AB ſhall be alſo equal to the quintuple of the ſquare of
OQ: But [m] ML the ſide of the triangle of the Icoſahedron is equal to m) C.16.13
the ſide of the Pentagon inſcribed in the circle, whereof OP is the ſemi-
diameter: Therefore the ſquare of the ſaid ſide being [n] equal to the n) 10. 13.
ſquares of OP and OQ, the ſides of the Decagon and of the Hexagon, five times the ſquare of ML ſhall be equal to five times the ſquare of OP, and to five times that of OQ, or to three times the ſquare of BA, and to three times the ſquare of AC; therefore the ſquares of BA and AC

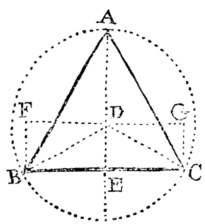
being quintuples of the square of the semidiameter F A (as is demonstrated) three times the square of AB and three times the square of AC shall be equal to fifteen times the square of F A the semidiameter: But five times the square of ML the side of the equilateral triangle, is equal to fifteen times the square of the semidiameter of the circle L M N (each square [o] of ML being triple the square of the semidiameter.) Wherefore the three squares of B A and A C being equal to five of M L, fifteen of F A the semidiameter, shall be equal to fifteen of the semidiameter of the circle L M N: Wherefore each square of them shall be equal to each square, and the semidiameter to the semidiameter: Therefore the circles A B C and L M N shall be equal: Which was to be demonstrated.

o) 12. 13.

PROP. 4. THEOR. 4.

If from the center F *of the circle, circumscribing the Pentagon of the Dodecahedron* A B C D E, *be drawn a right line* F G, *perpendicular to one side* C D, *of the same pentagon, thirty times the rectangle conteined under the said side and the the perpendicular, shall be equal to the Superficies of the Dodecahedron.*





Demonstration FOr from the center F, having drawn the right lines F A, F B, F C, F D, and F E, to all the angles of the Pentagon, the Pentagon shall be divided into five equal triangles, as [a] it appeares, all the sides being equal, and all the bases; let there be made the rectangle C I, conteined under C D and F G: Forasmuch [b] as the rectangle C I is the double of the triangle F C D, and by consequents the double of each of the four remaining triangles, equal to F C D; the rectangle C I taken five times, shall be equal to ten such triangles; that is to say, to two Pentagons of the Dodecahedron (five triangles being equal to one Pentagon;) therefore the sextuple of the rectangle C I (that is to say, six rectangles, such as is C I,) taken five times, to wit, thirty times the rectangle C I shall be alsо equal to the sextuple of the two Pentagons (that is to say, to twelve Pentagons) to wit, to the whole superficies of the Dodecahedron: Therefore, If from the center, &c. Which was to be demonstrated.

a) Cor. 8. 1.
b) 4. 1 1.

In like manner may be shewn, that if from the center of the circle circumscribing the triangle of the Icosahedron A B C, be drawn a perpendicular

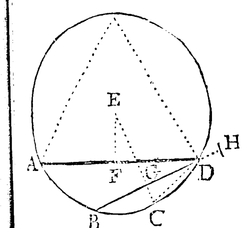
dicular D E on one side, to wit, on B C; the rectangle B G conteined under B C and D E taken thirty times, shall be equal to the superficies of the Icosahedron.

For the rectangle B G being the double of the triangle D B C, that is to say, equal to the said triangle taken two times, the rectangle B G taken thirty times shall be equal to the triangle D B C taken sixty times; but sixty such triangles as D B C make twenty triangles of the Icosahedron, seeing that three of them make one, as it appeares in the triangle A B C, where the sides D A, D B, and D C, are equal, and the three bases A B, B C, and C A, equal by supposition: Therefore B G taken thirty times, shall be equal to the whole superficies of the Icosahedron.

COROLLARIE.

Whence it followes, that as the rectangle C I *is to the rectangle* B G, *so the whole superficies of the Dodecahedron, to the superficies of the Icosahedron.*

PROP. 5. THEOR. 5.



As the Superficies of the Dodecahedron is to the Superficies of the Icosahedron, so the side of the cube is to the side of the Icosahedron inscribed in one and the same sphere.

Construction IN the circle A B C D, [a] enclosing the Pentagon of the Dodecahedron, and the triangle of the Icosahedron; let B D be the side of the Pentagon, and A D the side of the triangle; and from the center E let there be drawn E F and E G, perpendicular to A D and B D, and let F G be prolonged to the circumference in the point C, and let C D be drawn, E C shall cut the arch B C D in two equal parts, and C D shall be the side of the Decagon, as [b] we have shewn.

a) 13. 14.
b) 1. 14.

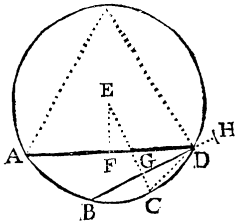
And let B D be prolonged to H, in such manner as that B H be equal to the side of the cube inscribed in the same sphere.

I say now that the superficies of the Dodecahedron is to the superfices of the Icosahedron, as B H the side of the cube, is to A D the side of the Icosahedron.

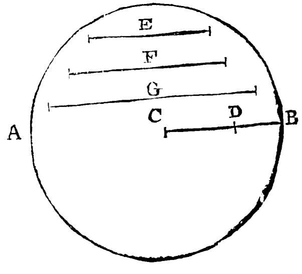
Demonstration FOr E C being the side of the Hexagon, and D C the side of the Decagon, inscribed in one and the same circle, E C and C D, taken as one only line, [c] shall be divided by extream and mean proportion, and E C the greatest segment, and [d] E G is the half of the whole compounded of E C and C D, and E F is the half of the greatest segment E C ([e] the semidiameter being cut in two equal parts by A D) and E C being divided by extream and mean proportion, the half E F shall be the greatest segment, as is manifest; for the half of the greatest segment and the half of the lesser, makes the half of the whole; [f] their whole being to one another as their parts: But B H the side of the cube, being divided by extream and mean proportion, [g] B D the side of the Dode-

c) 9. 13.
d) 1. 14.
e) C. 12. 13.
f) 15. 5.
g) C. 17. 13.

Dodecahedron ſhall be his greateſt ſegment; therefore [h] as the whole
h) 2. 14. B H to B D his greateſt ſegment, ſo the whole E G ſhall be to E F his greateſt ſegment; therefore [i] the rectangle
i) 16. 6. under B H and E F extreams, ſhall be equal to the rectangle under B D and E G means, and the rectangles under B H and E F [k] being
k) 1. 6. ing to the rectangle under A D and E F, as B H is to A D; and as B H ſhall be to A D, ſo the rectangle under B D and E G (ſhewn equal to the rectangle under B H and E F) ſhall be to the rectangle under A D and E F; therefore [l] the rectangle under B D
l) Cor. 4. 14. and E G being to the rectangle under A D and E F, as the ſuperficies of the Dodecahedron to that of the Icoſahedron, alſo as B H the ſide of the cube, ſhall be to A D the ſide of the Icoſahedron, ſo the Superficies of the Dodecahedron ſhall be to the Superficies of the Icoſahedron. Which was to be demonſtrated.



PROP. 6. THEOR. 6.



If a right line C D, *be divided by extream and mean proportion at* D, *as the ſquare of the whole* C B, *and the ſquare of the greateſt ſegment* C D, *are to the ſquares of the whole* C B, *and the leaſt ſegment* D B, *ſo the ſquare of the ſide of the cube* G, *is to the ſquare of the ſide of the Icoſahedron* F, *inſcribed in one and the ſame ſphere with the cube.*

Demonſtration FOr E being the ſide of the Pentagon, and F the ſide of the triangle inſcribed in the circle propoſed: If G the ſide of
a) C. 17. 13. the cube be divided by extream and mean proportion, [a] E ſhall be his greateſt ſegment: Wherefore F being the ſide of the triangle inſcribed in
b) 12. 13. the given circle, [b] his ſquare ſhall be triple the ſquare of the ſemidiame-
c) 4. 13. ter C B, and [c] the ſquares of C B and D B are triple the ſquare of C D: Therefore the ſquare of F is to the ſquare of C B, as the two ſquares of C B and D B are to the ſquare of C D; and by permutation, the ſquare of F ſhall be to the two ſquares of C B and D B, as the ſquare of C B to the ſquare of C D: But as the ſquare of C B is to the ſquare of C D, ſo
d) 2. 14. the ſquare of G to the ſquare of E (ſeeing that as [d] C B the whole is to his greateſt ſegment C D, ſo G the whole, is to E his greateſt ſegment: There-

Therefore [e] as the ſquare of C B ſhall be to the ſquare of C D, ſo the ſquare e) 22. 6.
G to the ſquare of E;) therefore [f] as the ſquare of F is to the ſquares of C B f) 11. 5.
and D B, ſo the ſquare of G to the ſquare of E; and alternately, as the ſquare of F to the ſquare of G, ſo the ſquares of C B and D B are to the ſquare of
E, which is the ſide of the Pentagon, [g] equal to the ſquares of C B and C D, g) 10. 13.
the ſides of the Hexagon and of the Decagon which are inſcribed in one and the ſame circle: Therefore the ſquare of G the ſide of the cube, is to the ſquare of F the ſide of the Icoſahedron, as the ſquares of C B and C D are to the ſquares of C B and D B: Wherefore, If a right line, &c. Which was to be demonſtrated.

PROP. 7. THEOR. 7.

As the ſide of the Cube is to the ſide of the Icoſahedron, ſo the ſolid of the Dodecahedron is to the ſolid of the Icoſahedron inſcribed in one and the ſame ſphere.

Demonſtration FOraſmuch as the Pentagon of the Dodecahedron, and the triangle of the Icoſahedron inſcribed in one and the ſame
ſphere, are [a] inſcribed in one and the ſame circle or equal circles; and a) 3. 14.
that in the ſphere the equal circles are equally diſtant from the center; for the perpendiculars drawn from the center of the ſphere, to the plain of circles, are equal, and fall in the centers of the ſame circles: Therefore the perpendiculars drawn from the center of the ſphere to the center of the circle conteining the Pentagon of the Dodecahedron, and the triangle of the Icoſahedron, are equal: Wherefore the pyramids having for baſes the Pentagons of the Dodecahedron, and the triangles of the Icoſahe-
dron, are of the ſame height: But [b] the pyramids of the ſame height are b) 5. 6. 12.
to one another as their baſes: Therefore as the Pentagon to the triangle, ſo the pyramid which hath for baſe the Pentagon, to the Dodecahedron, and the center of the ſphere for its top; is to the pyramid, whereof the baſe is the triangle of the Icoſahedron, and the center of the ſphere the top.

Therefore alſo as 12 Pentagons are to 20 triangles, ſo 12 Pyramids having the baſes Pentagonal, are to 20 Pyramids having the baſes triangular: But 12 Pentagons are the ſuperficies of the Dodecahedron, and 20 triangles are the ſuperficies of the Icoſahedron: Therefore as the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, ſo 12 Pyramids having the baſes pentagonal, to 20 Pyramids, having the baſes triangular, and 12 Pyramids having the baſes Pentagonal, are the Solid of the Dodecahedron, and 20 pyramids having the baſes triangular, are the ſolid of the Icoſahedron.

Therefore as the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, ſo the Solid of the Dodecahedron to the Solid of the Icoſahedron: But as the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, ſo is ſhewn the ſide of the cube to the ſide of the Icoſahedron; ſo the Solid of the Dodecahedron to the Solid of the Icoſahedron. Which was to be demonſtrated.

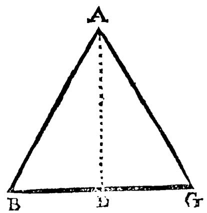
COROLLARIE.

The Solid of a Dodecahedron is to the Solid of an Icoſahedron, as the ſuperficies of

of the one are to the superficies of the other, being described in one and the selfe same sphere.

Namely, as the side of the cube is to the side of the Icosahedron, as was before manifest, for they are resolved into Pyramids of one and the selfe same altitude.

PROP. 8. THEOR. 8.



If the side AB *of an equilater triangle* ABG, *be rational, the superficies* ABG *shall be irrational, called Medial.*

Demonstration FOrasmuch as the line AB is in power sesquitertia to the
a) C. 12. 13. line AD, [a] of what parts the line AB conteineth in power 12 of the same parts, the line AD conteineth in power 9: Wherefore the residue BD conteineth in power of the same parts 3. (For the line
b) 47. 1. AB conteineth in power the lines AD and BD [b]: Wherefore the lines
c) 6. 10. AD and DB are rational and commensurable to the rational line AB [c]: But forasmuch as the power of the line AD is to the power of the line DB in proportion as 9 a square number, is to 3 a number not square:
d) C. 25. 8. Therefore they are not in the proportion of square numbers [d], and there-
e) 9. 10. fore they are not commensurable in length [e]. Wherefore that which is conteined under the lines AD and DB, which are rational lines commensurable in power only, is Medial [f]: But that which is conteined under
f) 22. 10. g) 41. 1. the lines AD and DB is double to the triangle ABD [g]. Wherefore that which is conteined under the lines AD and DB, is equal to the whole tri-
h) 1. 6. angle ABG (which is double to the triangle ABD [h].) Wherefore the the triangle ABG is Medial. If therefore the side, &c. Which was required to be proved.

COROLLARIE.

If an Octohedron and a Tetrahedron be inscribed in a Sphere, whose Diameter is Rational, their Superficies shall be Medial.

For those Superficies consisting of equilater triangles, whose sides are
i) 13, 14. 13. commensurable to the diameter which is rational [i]; and therefore they are rational. But they are commensurable in power onely to the perpendicular line, and therefore they contein a Medial triangle as was before manifested.

PROP. 9. THEOR. 9.

If a Tetrahedron and an Octohedron be inscribed in one and the selfe same sphere, the base of the Tetrahedron shall be sesquitertia to the base of the Octohedron, and the Superficies of the Octohedron shall be sesquialtera to the Superficies of the Tetrahedron.

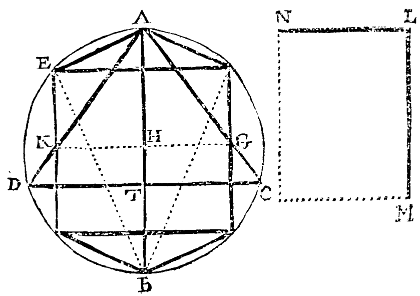
Demonstration FOrasmuch as the diameter of the sphere is in power sesquialtera to the side of the Tetrahedron [a], and the same a) 13. 13.
diameter is in power double to the side of the Octohedron [b]: Therefore of b) 14. 13.
what parts the diameter conteineth in power 6, of the same the side of the Tetrahedron conteineth in power 4, and of the same the side of the Octohedron conteineth in power 3. Wherefore the power of the side of the Tetrahedron is to the power of the side of the Octohedron in the same proportion that 4 is to 3, which is sesquitertia. And like triangles (which are the bases of the Solids) described of those sides, shall have the one to the other the same proportion that the squares made of those sides shall have. For both the triangles are the one to the other, and also the squares are the one to the other in double proportion of that in which the sides
are [c]: Wherefore of what parts one base of the Tetrahedron was 4, of c) 20. 6.
the same are four bases of the Tetrahedron 16. Likewise of what parts of the same one base of the Octohedron was 3, of the same are eight bases of the Octohedron 24. Wherefore the bases of the Octohedron are to the bases of the Tetrahedron, in that proportion that 24 is to 16, which is sesquialtera. If therefore a Tetrahedron and an Octohedron be inscribed in one and the self same sphere, the base of the Tetrahedron shall be sesquitertia to the base of the Octohedron, and the superficies of the Octohedron shall be sesquialtera to the superficies of the Tetrahedron: Which was required to be proved.

PROP. 10. THEOR. 10.

A Tetrahedron ADC, *is to an Octohedron* AEKBG, *inscribed in one and the self same sphere* ADBC, *in proportion as the rectangle Parallelogram conteined under the line* NL, *which conteineth in power* $\frac{27}{64}$ *parts of the side of the Tetrahedron* AC, *and under the line* ML, *which is subsesquioctava to the same side of the Tetrahedron, is to the square of the diameter of the sphere* AB.

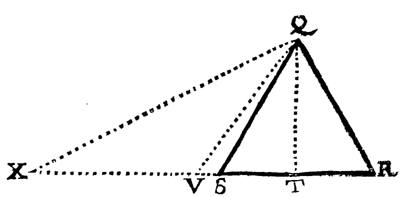
Demonstration 1. FOrasmuch as the line drawn from the angle A, by the center H, perpendicular upon the base of the Tetrahedron, falleth upon the center I of the circle which conteineth that base, and maketh the right line HT the sixth part of the diameter AB [a]; there- a) 13. 13.
fore the line HA (which is drawn from the center to the circumference) is triple to the line HT; and therefore the whole line AT is to the line AH as 4 is to 3. Let the Tetrahedron ADC be cut by a plain GHK, passing by the center H, and being parallel to the base DTC [b]. Now b) C. 15. 11.
then the triangle ADC of the Tetrahedron shall be cut by the right line KG, which is parallel to the line DC [c]: Wherefore as the line AT is c) 16. 11.
to the line AH, so is the line AC to the line AG [d]: Wherefore the line d) 2. 6.
AC is to the line AG sesquitertia, that is, as 4 to 3. And forasmuch as the triangles ADC, AKG, and the rest which are cut by the plain KHG, are like the one to the other [e]: The pyramids AD and AKG shall be e) 5. 6.
like

f) 7. def. 11. like the one to the other f: Wherefore they are in triple proportion of that
g) 8. 12. in which the sides of A C and A G are g. But the proportion of the sides
h) 2. 8. A C to A G is as the proportion of 4 to 3. Now then if h you find out four of the least numbers in continual proportion, and in that proportion that
i) 15. def. 1. 4 is to 3, which shall be 64, 48, 36, and 27, it is manifest i that the extreams 64 to 27 are in triple proportion of that in which the proportion given 4 to 3 is: Or the quantity of the proportion of 4 to 3, which is 1 and $\frac{1}{3}$) being twice multiplyed into it self, there shall be produced the proportion



of 64 to 27: Wherefore the Pyramis or Tetrahedron A D C is to the Pyramis A K G as 64 is to 27, which is triple to the proportion of 4 to 3. And forasmuch as the line A C is to the line A G in length sesquitertia of what parts the line A C conteineth in power 64, of the same parts doth the line
k) 2. 6. A G contein in power 36. For k the proportion of the powers or squares, is duple to the proportion of the sides which are as 64 to 48.

Demonstration 2. Now then upon the line R S, which let be equal to the line A G, let there be described an equilateral triangle
l) 1. 1. Q R S l, and from the angle Q drawn to the base R S, a perpendicular line
m) C. 6. 10. Q T, and extend the line R S to the point X. And as 27 is to 64, so m let the line R S be to the line R X, and divide the line R V into two equal parts in the point X, and draw the line Q V. And forasmuch as the line R S is equal to the line A G, of what parts the line A C conteineth in power 64, of the same parts the line R S conteineth in power 36, for it is proved that the line A G conteineth in power 36 of those parts: And of what parts the line R S conteineth in power 36, of the same parts the line Q T conteineth in power



n) C. 12. 13. 27 n. Wherefore of what parts the line A C conteineth in power 64, of the same parts the line Q T conteineth in power 27. Wherefore the right line Q T shall be equal to the right line L N by supposition. Again, forasmuch as the line R S is put equal to the line A G, and of

and of what parts the line R S conteineth in length 27, of the same parts is the line R X put to contein in length 64, and of what parts the line R X conteineth in length 64, of the same the line A C (which is in length sesquitertia to the line A G or R S) conteineth 36. Wherefore the line R V (which is the half of the line R X) conteineth in length of the same parts 32, of which the line A C conteined in length 36. Wherefore the line R V is to the line A C subsesquioctava, and therefore the line R V is equal to the line L M, which is also subsesquioctava to the same line A C. And forasmuch as the line N L is equal to the line Q T, and the line L M to the line R V (as before hath been proved) the rectangle Parallelogram conteined under the lines Q T and K V, shall be equal to the rectangle conteined under the line N L, which is in power $\frac{27}{64}$ to the side A C, and under the line L M, which is in length subsesquioctava to the same side A C. But that which is conteined under the lines Q T and R V is double to the triangle Q V R o, and to the same triangle Q V R is the triangle Q X R double p. Wherefore the whole triangle Q X R is equal to that which is conteined under the lines Q T and R V, and therefore is equal to the Parallelogram M N. And forasmuch as the line R X by supposition conteineth in length 64 of those parts of which the line R S conteineth 27, and the triangles Q R X and Q R S are q in the proportion of their bases; that is as 64 to 27: But as 64 is to 27, so is the Pyramis or Tetrahedron A D C to the Pyramis A K G: Wherefore as the Parallelogram N M or the triangle Q R X is to the triangle Q R S, so is the Pyramis A D C to the Pyramis A K G: And forasmuch as the semidiameter A H is the altitude of the Pyramis A K G, and also of the two equal and like pyramids of the Octohedron which have their common base in the square of the Octohedron r: Therefore as the base of the Pyramis A K G (which is the triangle Q R S) is to two squares of the Octohedron, that is, to the square of the diameter A B, which is equal to those squares s; so is the Pyramis A K G to the Octohedron A E B t. And forasmuch as the Parallelogram M N is to the base Q R S, as the Pyramis A D C is to the Pyramis A K G, and the base Q R S is to the square of the line B E, as the Pyramis A K G is to the Octohedron A E B: Therefore by proportion of equality, taking away the meanes v, as the Parallelogram N M is to the square of the line B E, so is the Pyramis A D C to the Octohedron A E B, inscribed in one and the same sphere. But the parallelogram N M is conteined under the line N L, which by supposition is in power $\frac{27}{64}$ to L M, which is also by supposition in length subsesquioctava to the same line A C: Wherefore, &c. Which was required to be proved.

o) 41. 1.
p) 1. 6.
q) 1. 6.
r) C. 14. 13.
s) 47. 1.
t) 6. 12.
v) 22. 5.

PROP. 11. THEOR. 11.

If a Cube be conteined in a sphere, the square of the diameter doubled, is equal to all the Superficies of the Cube taken together. And a perpendicular line drawn from the center of the Sphere to any base of the Cube, is equal to half the side of the Cube.

Demonstration Forasmuch a as the diameter of the sphere is in power triple to the side of the Cube: Therefore the square of the diameter doubled is sextuple to the base of the same Cube. But the sextuple of the
a) 15. 13.

the power of one of the sides conteineth the whole Superficies of the Cube,
for the Cube is composed of six square Superficies,[b] whose sides therefore
b) 1.def.11. are equal: Wherefore the square of the diameter doubled is equal to the whole Superficies of the cube. And forasmuch as the diameter of the Cube, and the line which falleth perpendicularly upon the opposite bases of the Cube do cut the one the other into two equal parts, in the center of the
c) 2C.15.13 Sphere which conteineth the Cube [c], and the whole right line which coupleth the centers of the opposite bases, is equal to the side of the Cube [d]; for
d) 33. 1. it coupleth the equal and parallel semidiameters of the bases: Therefore the
e) 15. 5. half thereof shall be equal to the half of the side of the Cube [e]: If therefore a Cube, &c. Which was required to be proved.

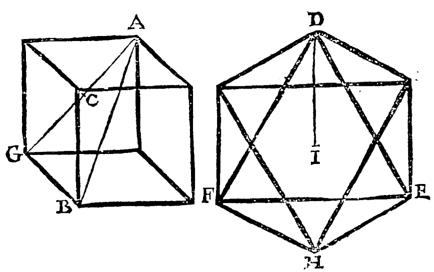
COROLLARIE.

If two thirds of the power of the diameter of the sphere be multiplyed into the perpendicular line equal to half the side of the Cube, there shall be produced a Solid equal to the Solid of the Cube.

For it is before manifest that two third parts of the power of the diameter of the sphere are equal to two bases of the Cube: If therefore unto each of those two thiads be applyed half the altitude of the Cube,
f) 31. 11. they shall make each of those Solids equal to half of the Cube [f]; for they have equal bases: Wherefore two of those Solids are equal to the whole Cube.

PROP. 12. THEOR. 12.

One and the self same circle conteineth both the square of a Cube ABG, *and the triangle of an Octohedron described in one and the self same Sphere.*



Construction LEt the diameter of the Sphere be A B or D H, and let the lines drawn from the Centers (that is, the semidiameters of the circles which contein the bases of those Solids) be CA and ID. Then I say that the lines C A and I D are equal.

Demonstration FOrasmuch as A B the diameter of the Sphere which conteineth the Cube, is in power triple to B G the side of the
a) 15. 13. Cube [a], unto which side A G the diameter of the base of the Cube is in power

power double [b], conteineth the base [c]. Therefore A B the diameter of the b) 47. 1.
the sphere is in power sesquialter to the line A G: namely, of what parts c) 9. 4.
the line A B conteineth in power 12 of the same line A G, shall contein in power 8. And therefore the right line A C which is drawn from the center of the Circle to the circumference, conteineth in power of the same parts 2. Wherefore the diameter of the Sphere is in power sextuple to the line which is drawn from the center to the circumference of the circle which conteineth the square of the Cube. But the diameter of the same Sphere which conteineth the Octohedron is one and the same with the diameter of the Cube, namely, D H is equal to A B, and the same diameter is also the diameter of the square, which is made of the sides of the Octohedron: Wherefore the said diameter is in power double to the side
of the same Octohedron [d]. But the side D F is in power triple to the line d) 14. 13.
drawn from the center to the circumference of the circle which conteineth
the triangle of the Octohedron (namely, to the line I D) [e]. Wherefore e) 12. 13.
the same diameter A B or D H, which was in power sextuple to the line drawn from the center to the circumference of the circle which conteineth the square of the Cube, is also sextuple to the line I D, drawn from the center to the circumference of the circle which conteineth the triangle of the Octohedron. Wherefore the lines drawn from the centers of the Circles to the circumferences which contein the bases of the Cube and of the
Octohedron are equal, and therefore the Circles are equal [f]. Wherefore f) 1. def. 3.
one and the self same Circle, &c. Which was required to be proved.

COROLLARIE.

Hereby it is manifest, that Perpendiculars coupling together in a Spere, the Centers of the Circles which contein the opposite bases of the Cube and of the Octohedron, are equal.

For the Circles are equal, and the lines which passing by the center of the Sphere, do couple together the centers of the bases, are also equal. Wherefore the Perpendicular which coupleth together the opposite bases of the Octohedron, is equal to the side of the Cube, for either of them is the altitude erected.

PROP. 13. THEOR. 13.

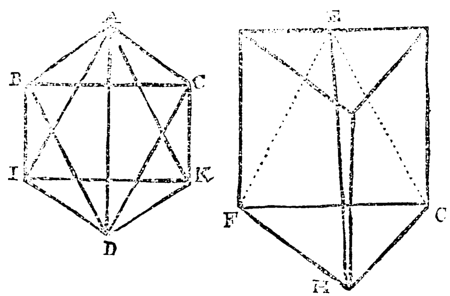
An Octohedron ABCD, *is to the triple of a Tetrahedron* EFGH, *conteined in one and the same Sphere, in that proportion as their sides are.*

Construction UPon the base F G H erect a Prisme, which is done by erecting from the angles of the base perpendicular lines, equal to the altitude of the Tetrahedron, which Prisme shall be triple to the Te-
trahedron E F G H [a]. Then I say that the Octohedron is to the Prisme a) 1 C.7.12.
which is triple to the Tetrahedron, as the side B C is to the side F G.

Demonstration FOr, forasmuch as the sides of the opposite bases of the Octohedron are right lines touching one another, and are parallels to other right lines which touch one another, for the sides of the squares which are composed of the sides of the Octohedron, are opposite:

 Where-

Wherefore the opposite plain triangles, namely, A B C and K I D, shall
b) 15. 11. be parallels, and so the rest [b]. Let the diameter of the Octohedron be the line A D. Now then the whole Octohedron is cut into four equal and like Pyramids, set upon the bases of the Octohedron, and having the same altitude with it, and being about the diameter A D, namely the Pyramis set upon the base B I D, and having his top the point A, and also the Pyramis set upon the base B C D, having his top the same point A. Likewise the Pyramis set upon the base I K D, and having his top the same point A: and moreover the Pyramis set upon the base C K D, and
c) 8.def.11. having his top the former point A, which Pyramids shall be equal [c] (for they each consist of two bases of the Octohedron, and of two triangles conteined under the diameter A D, and two sides of the Octohedron.) Wherefore the Prisme which is set upon the base of the Octohedron, and having the same altitude with it, namely, the altitude of the parallel bases, as it is manifest by the former, is equal to three of those Pyramids
d) C. 7. 12. of the Octohedron [d]. Wherefore that prisme shall have to the other Prisme under the same altitude composed of the four Pyramids of the
e) Co. 7. 12. whole Octohedron, the proportion of the triangular bases [e]: And forasmuch as 4 Pyramids are unto 3 Pyramids in sesquitertia proportion to the



base of the Prisme which conteineth three Pyramids of the same Octohedron and are set upon the base of the Octohedron; and under the altitude thereof, that is in subsesquitertia proportion to the base of the Octohedron. But the base of the same Octohedron is in sesquitertia proportion
f) 10. 14. to the base of the pyramis [f]: Wherefore the triangular bases, namely, of the Prisme which conteineth four pyramids of the Octohedron, and is under the altitude thereof, are equal to the triangular bases of the Prisme, which conteineth three Pyramids under the altitude of the Pyramis E F G H. But the Prisme of the Octohedron is equal to the Octohedron, and the Prisme of the Pyramis E F G H is proved triple to the same Pyramis E F G H. Now then the Prismes set upon equal bases, are the one to
g) C. 25. 11. the other as their altitudes are [g]: namely, as are the Parallelepipedons
h) 31. 11. their doubles [h]. But the altitude of the Octohedron is equal to the side of
i) C. 13. 14. the Cube conteined in the same sphere [i], and the side of the Cube is in power to the altitude of the Tetrahedron in that proportion that 12 is to
k) 18. 13. 16 [k]. And the side of the Octohedron is to the side of the Pyramis in that
l) 18. 13. proportion that 18 is to 24 [l], which proportion is one and the same with the proportion of 12 to 16. Wherefore that Prisme which is equal to the Octo-

Octohedron, is to the Prisme which is triple to the Tetrahedron in that proportion that the altitudes, or that the sides are. Wherefore an Octohedron is to the triple of a Tetrahedron, &c. Which was required to be demonstrated.

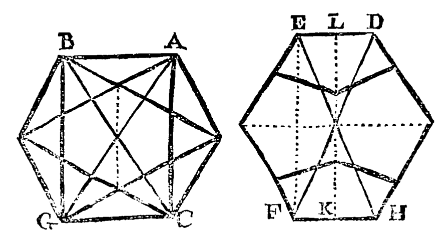
COROLLARIE.

The sides of a Tetrahedron and of an Octohedron are proportional with their altitudes.

For the sides and altitudes were in power sesquitertia. Moreover the diameter of the Sphere is to the side of the Tetrahedron as the side of the Octohedron is to the side of the Cube, namely, the powers of each is in sesquialter proportion [m]. m) 18. 13.

PROP. 14. THEOR. 14.

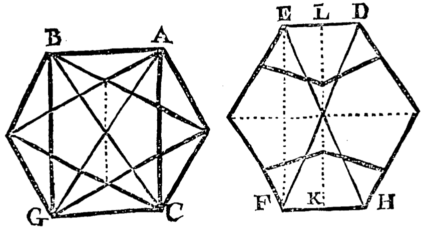
If a rational line A G *or* D F, *conteining in power two lines* A B *and* B G, *make the whole and the greater segment, and again conteining in power two lines* E F *and* E D, *make the whole and the lesser segment, the greater segment* A B *shall be the side of the Icosahedron* A B G C, *and the lesser segment* E D *shall be the side of the Dodecahedron* D E F H, *conteined in one and the same Sphere.*



Construction Suppose that A G be the diameter of the sphere which conteineth the Icosahedron A B G C; and let B G subtend the sides of the Pentagon described of the sides of the Icosahedron [a]. Moreover, upon a) 16. 13. the same diameter A G, or D F equal unto it, let there be described a Dodecahedron [b], whose opposite sides E D and F H, let be cut into two equal b) 17. 13. parts in the points I and K, and draw a line from I to K. And let the line E F couple two of the opposite angles of the bases which are joyned together: Then I say that A B the side of the Icosahedron, is the greater segment which the diameter A G conteineth in power together with the whole line, and the line E D is the lesser segment which the same diameter A G or D F conteineth in power together with the whole.

Demonstration For seeing that the opposite sides A B and G C of the Icosahedron being coupled by the diameters A G and B C, are equal and parallels, [c] the right lines B G and A C which couple c) 2 C. 16. 13. them

d) 33. 1. them together are equal and parallel [d]: Moreover, the angles B A C and
e) 8. 1. A B G being ſubtended of equal diameters, ſhall [e] be equal, and [f] they
f) 29. 1. ſhall be right angles. Wherefore the right line A G conteineth in power
g) 47. 1. the two lines A B and B G [g]. And foraſmuch as the line B G ſubtendeth
the angle of the Pentagon compoſed of the ſides of the Icoſahedron, the
h) 8. 13. greater ſegment of the right line B G ſhall be the right line A B [h], which
line A B together with the whole line B G, the line A G conteineth in
power. And foraſmuch as the line I K coupling the oppoſite and parallel



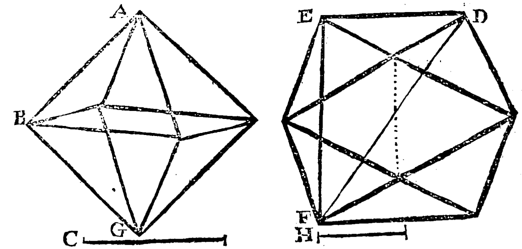
ſides E D and F H of the Dodecahedron, maketh at thoſe points right
i) C. 17. 13. angles [i], the right line E F which coupleth together equal and parallel
k) 33. 1. lines E I and F K, ſhall be equal to the ſame line I K [k]. Wherefore the
l) 29. 1. angle D E F ſhall be a right angle [l]: Wherefore the diameter D F conteineth in power the two lines E D and E F. But the leſſer ſegment of
m) C. 17. 13 the line I K is E D the ſide of the Dodecahedron [m]. Wherefore the ſame
line E D is alſo the leſſer ſegment of the line E F (which is equal to the
line I K:) Wherefore the diameter D F conteining in power the two lines
n) 47. 1. E D and E F [n], conteineth in power ED the ſide of the Dodecahedron the
leſſer ſegment, together with the whole: If therefore, &c. Which
was required to be demonſtrated.

PROP. 15. THEOR. 15.

If the power of the ſide A B *of an Octohedron* A B G, *be expreſſed by two right lines* C *and* H *joyned together by an extream and mean proportion, the ſide* D E *of the Icoſahedron* D E F, *conteined in the ſame Sphere, ſhall be duple to the leſſer ſegment* H.

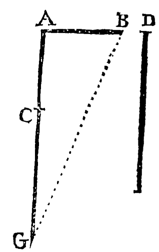
a) 15. 14. *Demonſtration* Foraſmuch as [a] it was manifeſt that E D the ſide of the Icoſahedron is the greater ſegment of the line E F, and that
the diameter D F conteineth in power the two lines E D and E F, namely, the whole and the greater ſegment; but by ſuppoſition, the ſide A B
conteineth in power the two lines C and H joyned together in the ſame
proportion. Wherefore the line E F is to the line E D, as the line C is to
b) 2. 14. to the line H [b]. And alternately, [c] the line E F is to the line C, as the line
c) 16. 5. E D is to the line H; and foraſmuch as the line D F conteineth in power
the two lines E D and E F, and the line A B conteineth in power the two
lines

lines C and H. Therefore the ſquares of the lines E F and E D are to the
ſquare of the line D F, as the ſquares of the lines C and H to the ſquare



of the line A B. And alternately, the ſquares of the lines E F and E D
are to the ſquares of the lines C and H, as the ſquare of the line D F is to
the ſquare of the line A B. But D F the diameter is [d] in power double to d) 14. 13.
A B the ſide of the Octohedron inſcribed (by ſuppoſition) in the ſame
Sphere. Wherefore the ſquares of the lines E F and E D are double to
the ſquares of the lines C and H. And therefore one ſquare of the line
E D is double to one ſquare of the line H [e]. Wherefore E D the ſide of the e) 12. 5.
Icoſahedron, is in power dupla to the line H, which is the leſſer ſegment.
Therefore, &c. Which was required to be demonſtrated.

PROP. 16. THEOR. 16.



If the ſide A B *of a Dodecahedron, and the right line of which the ſaid ſide is the leſſer ſegment* A G *be ſo ſet, that they make a right angle (as in the point* A) *the right line* D *which conteineth in power half the line* B G, *ſubtending the angle, is the ſide of an Octohedron conteined in the ſame Sphere.*

Demonſtration Foraſmuch as the line A G maketh the greater ſegment
G C the ſide of the Cube conteined in the ſame ſphere [a], a) C. 17. 13.
and the ſquares of the whole line A G, and of the leſſer ſegment A B,
are triple to the ſquare of the greater ſegment G C [b]: Moreover, the dia- b) 4. 13.
meter of the Sphere is in power triple to the ſame line G C, the ſide of the
Cube [c]. Wherefore the line B G is equal to the diameter, for it contein- c) 15. 13.
eth in power the two lines A B and A G [d]; and therefore it conteineth in d) 47. 1.
power the triple of the line G C. But the ſide of the Octohedron conteined in the ſame Sphere, is in power triple to half the diameter of the
Sphere [e]. And by ſuppoſition, the line D conteineth in power the half of e) 14. 13.
the

the line B G. Wherefore the line D (conteining in power the half of the same diameter) is the side of an Octohedron. If therefore, &c. Which was required to be proved.

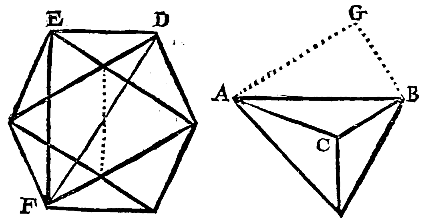
COROLLARIE.

Unto what right line the side of the Octohedron is in power sesquialter, unto the same line the side of the Dodecahedron inscribed in the same Sphere, is the greater segment.

For the side of the Dodecahedron is the greater segment of the segment C G, unto which D the side of the Octohedron is in power sesquialtera, that is, is half of the power of the line E G, which was triple to the line C G.

PROP. 17. THEOR. 17.

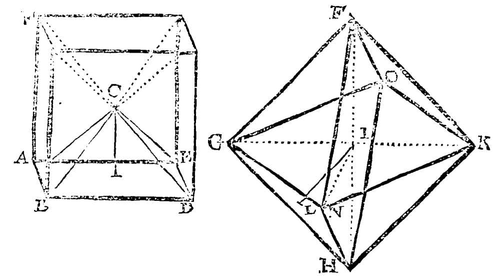
If the side A B *of a Tetrahedron* A B C, *contein in power two right lines* A G *and* G B, *joyned together by an extream and mean proportion, the side* E D *of a Icosahedron* E D F, *described in the same Sphere, is in power sesquialter to the lesser right line* G B.



Demonstration Forasmuch as [a] the side E D is the greater segment of the line E F, which subtendeth the angle of the Pentagon.
a) 15. 14. But as the whole line E F is to the greater segment E D, so is the same greater segment to the lesser [b]; and by supposition A G was the whole
b) 30. 6. line, and G B the greater segment: Wherefore as E F is to E D, so is A G to G B [c]. And alternately, the line E F is to the line A G, as the
c) 2. 14. line E D is to the line G B. And forasmuch as (by supposition) the line A B conteineth in power the two lines A G and G B, therefore [d] the angle
d) 48. 1. A G B is a right angle: But the angle D E F is a right angle [e]. Where-
e) 15. 14. fore the triangles A G B and F E D are equiangled [f]: Wherefore their
f) 6. 6. sides are proportional, namely, as the line E D is to the line G B, so is the line F D to the line A B [g]. But by what hath been demonstrated, F D
g) 4. 6. is the diameter of the Sphere which conteineth the Icosahedron, which diameter is in power sesquialter to A B the side of the Tetrahedron inscribed in the same Sphere [h]. Wherefore the line E D the side of the Ico-
h) 13. 13. sahedron, is in power sesquialter to G B the greater segment, or lesser line. If therefore, &c. Which was required to be done.

PROP. 18. THEOR. 18.

The Superficies of a Cube A B C D, *is to the Superficies of an Octohedron* F G H K, *inscribed in one and the same Sphere, in that proportion that the Solids are.*

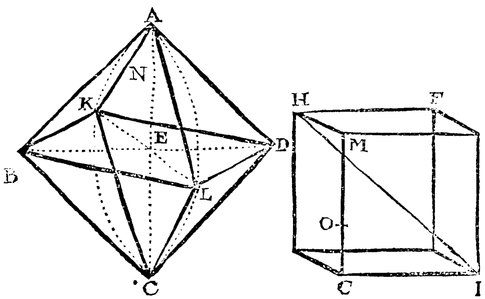


Construction The four diameters of the Cube are A C, B C, D C, and E C, produced on each side, and the three diameters of the Octohedron are F H, G K, and O N. Draw from the center of the Cube to the base A B E D, a perpendicular line C R, and from the center of the Octohedron draw to the base G N H, a perpendicular line I L.

Demonstration And forasmuch as the three diameters of the Cube do passe by the center C. Therefore [a] there shall be made of a) 2 Cor. of 15. 13.
the Cube six Pyramids, as the Pyramis A B D E C, equal to the whole Cube; for there are in the Cube six bases, upon which fall equal perpendiculars from the center, for the bases are conteined in equal circles of the Sphere. But in the Octohedron the three diameters do make upon the ten bases eight Pyramids, having their tops in the center [b]. Now the ba- b) 3 Cor. of 14. 13.
ses of the Cube and of the Octohedron are conteined in equal circles of the Sphere [c]. Wherefore they shall be equally distant from the center, c) 13. 14.
and the perpendicular lines C R and I L shall be equal. Wherefore the Pyramids of the Cube shall be under one and the same altitude with the Pyramids of the Octohedron, namely, under the perpendicular line, drawn from the center to the bases. Wherefore six Pyramids of the Cube are to eight Pyramids of the Octohedron, being under one and the same altitude, in that proportion that their bases are [d]; that is, one Py- d) 6. 12.
ramis set upon six bases of the cube, and having to his altitude the perpendicular line, which Pyramis is equal to the six Pyramids, [e] is to one e) 6. 12.
Pyramis set upon the eight bases of the Octohedron, being equal to the Octohedron, and also under one and the same altitude, in that proportion that six bases of the Cube, which contein the whole Superficies of the Cube are to eight bases of the Octohedron, which contein the whole Superficies of the Octohedron, For the Solids of those Pyramids are in proportion to one another as their bases are [f]. Wherefore, &c. Which f) 6. 12.
was required to be proved.

PROP. 19. THEOR. 19.

If a Cube FGHIM, *and an Octohedron* AECDB, *be contained in one and the same Sphere* ABCD, *they shall be in proportion to one another as the side* MG *of the Cube, is to the semidiameter* AE *of the Sphere.*



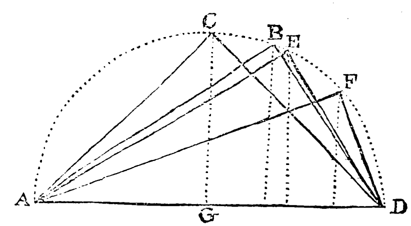
Demonstration. FOrasmuch as the diameter A C is in power double to B K
a) 14. 13. the side of the Octohedron a, and is in power triple to
b) 15. 13. M G the side of the Cube b; therefore the square B K D L shall be sesquialter to F M the square of the Cube. From the line A E cut off a third
c) 9. 6. part A N, and from the line M G cut off likewise a third part G O c. Now then the line E N shall be two third parts of the line A E, and so also shall the line M O be of the line M G. Wherefore the Parallelepipedon set upon the same base B K D L, and having his altitude the line E A, is triple to the Parallelepipedon set upon the same base, and having his al-
d) C. 31. 11. titude the line A N d. But it is also triple to the Pyramis A B K D L, which
e) 2 C. 7. 12. is set upon the same base, and is under the same altitude e. Wherefore the Pyramis A B K D L is equal to the Parallelepipedon which is set upon the base B K D L, and hath to his altitude the line A N. But unto that Parallelepipedon is double the Parallelepidedon which is set upon the
f) C. 31. 1. same base B K D L, and hath to his altitude a line double to the line A N f;
g) 2 Cor. of 14. 13. and unto the Pyramis is double the Octohedron A B K L D C g. Wherefore the Octohedron A B K D L C is equal to the Parallelepipedon set
h) 15. 5. upon the same base B K L D, and having his altitude the line E N h. But the Parallelepipedon set upon the base B K D L, which is sesquialter to the base F M, and having to his altitude the line M O, which is two third
i) 2 part of 34. 11. parts of the side of the Cube M G, is equal to the Cube F G i. (For it was before proved that the base B K D L is sesquialter to the base F M.) Now then these two Parallelepipedons, namely, the Parallelepipedon which is set upon the base B K D L (which is sesquialter to the base of the Cube) and hath to his altitude the line M O (which is two third parts of M G the side of the Cube) which Parallelepipedon is proved equal to the Cube and the Parallelepipedon set upon the same base B K D L, and having his alti-

altitude the line E N, which Parallelepipedon is proved equal to the Octohedron) these two parallelepipedons (I say) are to one another as the altitude M O is to the altitude E N k. Wherefore also as the altitude M O k) C. 31. 11.
is to the altitude E N, so is the Cube F G H I M to the Octohedron A B K D L C l. But as the line M O is to the line E N, so is the whole line l) 7. 5.
M G to the whole line E A m. Wherefore as M G the side of the Cube is m) 18. 5.
to E A the semidiameter, so is the Cube F G H I M to the Octohedron A B K D L C inscribed in one and the same Sphere. If therefore, &c. Which was required to be demonstrated.

COROLLARIE.

Distinctly to notifie the powers of the sides of the five Solids by the power of the Diameter of the Sphere.

The sides of the Tetrahedron and of the Cube do cut the power of the Diameter of the Sphere into two squares, which are in proportion double to one another. The Octohedron cutteth the power of the Diameter into two equal squares. The Icosahedron into two squares, whose proportion is double to the proportion of a line divided by an extream and mean proportion, whose lesser segment is the side of the Icosahedron. And the Dodecahedron into two squares, whose proportion is quadruple to the proportion of a line divided by an extream and mean proportion, whose lesser segment is the side of the Dodecahedron. For A D the Diameter of the Sphere conteineth in power A B the side of the Tetrahedron, and B D the side of the Cube, which B D is in power half of the side A B. The Diameter also of the Sphere conteineth also in power A C and C D, two equal sides of the Octohedron. But the Diameter conteineth in power



the whole line A E, and the greater segment thereof E D, which is the side of the Icosahedron n. Wherefore their powers being in dupla pro- n) 15. 14.
portion of that in which the sides are o, have their proportion dupla to o) 1 C. 20. 6.
the proportion of an extream and mean proportion. Again the Diameter conteineth in power the whole line A F, and his lesser segment F D, which is the side of the Dodecahedron p. Wherefore the whole having to the p) 15. 14.
lesser a double proportion of that which the extream hath to the mean, namely, of the whole to the greater segment q, it followes that the pro- q) 10. de. 5.
portion of the power is double to the doubled proportion of the sides r; r) 1 C. 20. 6.
that is, is quadruple to the proportion of the extream and of the mean s. s) def. 6.

By this meanes therefore the Diameter of a Sphere being given, there shall be given the side of every one of the bodies inscribed. And forasmuch as three of those bodies have their sides commensurable in power only, and not in length unto the Diameter given (for their powers are in the proportion of a square number to a number not square. Wherefore they have not the proportion of a square number to a square number t. t) C. 25. 8.

 Where-

v) 9. 10. Wherefore also their sides are incommensurable in length [v]: Therefore it is sufficient to compare the powers and not the lengths of those sides to one another; which powers are conteined in the power of the Diameter, namely, from the power of the Diameter let there be taken away the power of the Cube, and there shall remain the power of the Tetrahedron, and taking away the power of the Tetrahedron, there remaineth the power of the Cube; and taking away from the power of the Diameter half the power thereof, there shall be left the power of the side of the Octohedron. But forasmuch as the sides of the Dodedecahedron and of the Icosahedron are proved to be irrational, for the side of the Icosahedron is a lesser line [w], and the side of the Dodecahedron is a Residual line [x]; therefore those sides are unto the Diameter which is a Rational line, incommensurable both in length and power. Wherefore their comparison cannot be defined or described by any proportion expressed by numbers [y], neither can they be compared to one another; for irrational lines of divers kinds are incommensurable to one another; for if they should be commensurable, they should be of one and the same kind [z], which is impossible. Wherefore we seeking to compare them to the power of the diameter, though they could not be more aptly expressed then by such proportions, which cut that rational power of the Diameter according to their sides, dividing the power of the Diameter by lines in that proportion, as the greater to the lesser segment, to put the lesser segment to be the side of the Icosahedron, and dividing the said power of the Diameter by lines in proportion as the whole to the lesser segment, to expresse the side of the Dodecahedron by the lesser segment, which may well be done between Magnitudes incommensurable.

w) 16. 13.
x) 17. 13.
y) 8. 10.
z) 103, 105. 10.

The End of the Fourteenth Element of EUCLIDE.

THE

THE FIFTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

THis Fifteenth and last Book of *EUCLIDE*, or rather the Second Book of *Appollonius* or *Hypsicles*, teacheth the inscription and circumscription of the five regular bodies one within & about another, a thing undoubtedly pleasant and delectable in mind to contemplate, and also profitable and necessary to practice. For without practice in act it is very hard to see and conceive the constructions and Demonstrations of the Propositions of this Book, unlesse a man have a very deep, sharp, and fine imagination. Wherefore I would wish the diligent student in this Book (to make the study thereof more pleasant unto him) to have presently before his eyes, the bodies formed and framed of pasted paper: And then to draw and describe the Lines and Divisions, and Superficies, according to the constructions of the Propositions. In which Descriptions if he be careful and diligent, he shall find all things in these Solid matters, as cleer and manifest to the eye, as were things before taught only in plain or superficial figures.

DE-

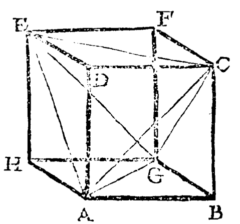
DEFINITIONS.

1 *A solid figure is then said to be inscribed in a solid figure, when the angles of the figure inscribed touch together at one time, either the angles of the figure circumscribed, or the superficies, or the sides.*

2 *A solid figure is then said to be circumscribed about a solid figure, when together at one time either the angles, or the superficies, or the sides of the figure circumscribed, touch the angles of the figure inscribed.*

PROPOSITIONS, and PROBLEMES.

PROPOSITION 1. PROBLEM 1.



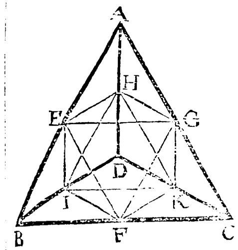
In a given Cube ABCDE FGH, *to inscribe a Pyramid.*

Construction Let the given Cube be A B C D E F G H, to inscribe therein a Pyramid or Tetrahedron; from one angle thereof E, let there be drawn to the bases constituting it three diameters E A, E G, and E C, from the extremities of which A, G, and C, let there be also drawn the diameters A G, G C, and C A, to the other three bases which joyne the extremities of the three first Diameters.

Demonstration Forasmuch as the diameters of the equal squares are equal to one another, being double in power [a] to the sides of the
a) 47. 1.
equal squares: It is manifest that the four triangles A C E, G A C, G A E, and G C E compounded of the said diameters, are equilateral and equal to one another; therefore do constitute a Pyramid; and all the angles of
b) 3.def.11. the Pyramid being compared to the angles of the Cube, [b] it is manifest that it is inscribed in the Cube. Therefore in a given Cube we have inscribed a Pyramid. Which was to be done.

PROP.

PROP. 2. PROBL. 2.



In a given Pyramid to inscribe an Octohedron.

Construction Let the given Pyramid be A B C D, in which an Octohedron ought to be inscribed. Having divided all the sides in two equal parts at E, F, G, H, I, and K, and drawn E F, F G, G E, H I, I K, K H, E I, I F, F K, K G, G H, and H E, there shall be constituted eight triangles, four of which E H I, I H K, K H G, and H G E, are on the plain E I K G: But the four others I F E, E F G, G F K, and K F I, below the same plain.

Demonstration But forasmuch as the sides A E and A H of the triangle A E H, are equal to A G and A H of the triangle A G H, being the halves of the equal lines A B, A D, and A C, and the angles conteined of them E A H and G A H equal, being angles of the equilateral triangles A B D and A C D, [a] the bases E H and G H shall be equal, a) 4. 1.
and by the same reason E H and I H and the others, in like manner: Wherefore these eight triangles are equilateral and equal to one another; and therefore do compound the Octohedron E I G H F, [b] the b) 31.def.11.
which is inscribed in the Pyramid, seeing that all the angles do touch all the sides of the Pyramids, by the Construction. Therefore in a Pyramid we have inscribed an Octohedron. Which was to be done.

COROLLARIE.

Then forasmuch as the three plains E I K G, G H I F, *and* F K H E, [c] *are equal squares, cuting one another at right angles each of which cutteth the Pyramid* A B C D *in two equal parts (for the square* E K *divideth it in the Pyramid* c) Cor. 1. of 14. 13.
H I K D, *and in the Prisme* H I K G A E, *and also in the Pyramid* E B F I, *and in the Prisme* E F C G K I: *But it appeares that the Pyramid is equal to the Pyramid, and the Prisme to the Prisme: In like manner the other squares* [d] G I *and* d) 8. 12.
F H *do divide the same Pyramid* A B C D *in two equal parts:) It is manifest that if in the Tetrahedron there be inscribed an Octohedron, that the Tetrahedron shall be cut in two equal parts, by three equal squares, the which do cut one another at right angles, and the Tetrahedron in two equal parts.*

PROP. 3. PROBL. 3.

In a given Cube to inscribe an Octohedron.

Construction Let the given Cube be A H, in which an Octohedron ought to be inscribed. Let the sides of the base A D and C D, be divided in two equal parts by I, K, L, and M, which let be joyned by I L and K M, cutting one another in N, which is the center of the square B D, as appeares by the demonstration of the 8th. of the 4th. and so the centers of the other bases, O, P, Q, R, and S, shall be found. Therefore all the right lines drawn from the said centers to the middle of the bases,

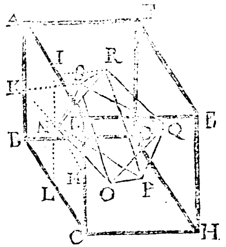
bases, as are N I, R I, N K, S K, &c. are equal to the halves of the sides of the Cube, or the squares, as appeares by the said Demonstration of the eighth of the fourth.

Lastly, if the said centers be joyned by N O, O P, P Q, Q R, R S, S N, N P, P R, R N, S O, O Q, and Q S, there will be constituted eight triangles, four of which N S R, R S Q, Q S O, and O S N, are on the plain N O Q R, and the other four O P Q, Q P R, R P N, and N P O, below the same Plain.

Demonstration BUt forasmuch as the sides I N and N R, of the triangle I N R, are equal to K N and R S of the triangle K N S, being all halves of the equal sides of the Cube as is said, also the angles conteined of them are equal; for N I and I R being parallel to B A and A F, a the angle N I R is equal to the right angle K A F of the square A G; by the same reason N R S being equal to the right angle I A F of the square A E, b the bases N R and N S shall be equal; so we shall demonstrate all the other lines equal both to one another, and to these two: If from the centers be drawn right lines to the middle of the sides, in such manner as that each two be drawn to the middle of the side common to two squares of the Cube, from which these two points, from whence come the right lines, are the centers; so as N I and R I which are drawn to I, halfe of the side A D, common to the squares A C and A E, whose centers are N and R: In like manner N K and S K are drawn to K, the middle of A B, common to the squares of A C and A G, whose centers are N and S, &c. Therefore we have constituted eight equilateral triangles, and equiangled to one another, which do constitute the Octohedron N O Q R S P, which c is inscribed in the Cube, seeing that all his six angles touch all the six bases of the Cube at their centers. Therefore in the given Cube we have inscribed an Octohedron, &c. Which was to be done.

a) 10.11.

b) 4.1.



c) d. 31.11.

Otherwise, d in the Cube let there be inscribed a Pyramid, and e in that Pyramid let there be inscribed an Octohedron, and what was proposed is done. For seeing that according to the Demonstration of the second Proposition of this Book, the angles of the Octohedron do touch the sides of the Pyramid; but the sides of the Pyramid (by the Demonstration of the first Proposition of this Book) are in the plains of the bases of the Cube, being the diameters of the same bases: It is manifest that the angles of the Octohedron do touch in like manner the bases of the Cube; and therefore that the Octohedron is inscribed in the Cube.

d) 1.15.
e) 2.15.

COROLLARIE.

Then forasmuch as the diameters of the Octohedron inscribed in the Cube, N Q *and* R O *(if they were drawn)* f *cut one another at right angles, and joyn the centers of the opposite bases of the Cube. It is manifest that the right lines which joyn together the centers of the opposite bases of the Cube divide themselves not only in two equal parts,* g *as is shewn; but also at right angles, as we have here shewn.*

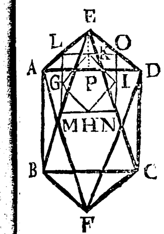
f) 1 C.14.13.

g) 1 C.15.13.

PROP.

PROP. 4. PROBL. 4.

In a given Octohedron ABCDEF, *to inscribe a Cube.*



Demonstration FForasmuch as six Pyramids quadrangular are conteined in the Octohedron whose tops are the six angles of the Octohedron, and each two constitute an Octohedron: Let one of them, to wit, the Pyramid A B C D E, whose base is the square A B C D, and the equilateral triangles A B E, E B C, C E D, and D E A, whose centers are G, H, I, and K, by the which let there be drawn L M, M N, N O, and O L parallel to A B, B C, C D, and D A, the triangles L M E, E M N, N E O, and O E L, shall be equilateral, a being alike to the triangles equilateral abovesaid, and therefore equal to one another, having the sides common, and the bases L M, M N, and N O, by consequence equal, and if from E by the center K, be drawn E P, it will divide the angle at E in two equal parts; therefore L K shall be equal to K O; that is to say, that L is divided into two equal parts by K the center; and in like manner L M, M N, and N O, shall be cut each in two equal parts by the centers G, H, and L; therefore the four triangles G L K, K O I, G M H, and H N I, are Isosceles triangles (b the angles G L K, K O I, I N H, and H M G, being equal to the right angles of the square G K I H) therefore the bases G K, K I, I H, and H G, shall be equal, and shall contein right angles. Wherefore G H I K is a square, and if in the same manner, in the other five Pyramids of the Octohedron, the centers of the triangles are joyned by right lines, there shall be described in like manner five squares, which shall be equal to one another, having the sides common. Wherefore six such squares shall compound a Cube, c which shall be described in the Octohedron, seeing that the eight angles thereof do touch the eight bases of the Octohedron at their centers. Therefore in the Octohedron given we have inscribed a Cube. Which was to be done.

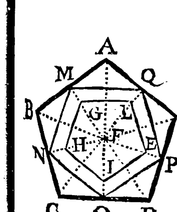
a) Cor. 4.6.

b) 10.11.

c) 31.de.11.

PROP. 5. PROBL. 5.

In a given Icosahedron to inscribe a Dodecahedron.

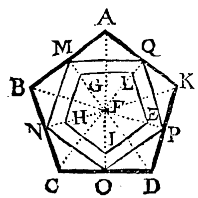


Construction LEt one of the twelve Pyramids of the Icosahedron, whose bases are Pentagons, A B C D E F, and the base the Pentagon A B C D E, and the equilateral triangles A B F, B C F, C D F, D E F, and E A F, and their centers G, H, I, K, and L, joyned, by G H, H I, I K, K L, and L G. Again, from the top F, by the centers of the triangles let there be drawn F M, F N, F O, F P, and F Q, cutting a A B, B C, C D, D E, and E A, at M, N, O, P, and Q, in such sort as M A, M B, N B, N C, O C, O D, P D, P E, Q E, and Q A, may be equal and comprehend equal angles, to wit, of the

a) C.10.13.

b) 4. 1. the Pentagon [b] MN, NO, OP, PQ, and QM, joyning the points M, N,
c) 8. 1. O, P, and Q, are also equal; therefore [c] the angles MFN, NFO, OFP, PFQ, and QFM, are equal, the sides FM, FN, FO, FP, and FQ, being equal, to wit, perpendiculars of the angles of equilateral triangles drawn to the opposite bases.

Demonstration. FOr FB and BM of the triangle FBM, being equal to FB and BN of the triangle FBN, and conteining equal
d) 4. 1. angles, to wit, of the equilateral triangles, the bases [d] FN and FM shall be equal, and so of the others. But forasmuch as FG, FH, FI, FK, and FL, semidiameters of the equal circles circumscribing the equilateral
e) 4. 1. triangles are equal, aud contein equal angles as is demonstrated; [e] GH, HI, IK, KL, and LG, shall be equal; and seeing that as well AMQ, QMN, MNB, as BNM, NMO, and ONC, are equal to two right angles. But AMQ and NMB are equal to BNM and ONC, the remainders QMN and MNO shall be equal. By the same reason the other angles NOP, OPQ, and PQM, shall be equal both to one another and to these: Wherefore the Pentagon MNOPQ shall be equilateral and equiangled to the plain of the Pentagon ABCDE. Lastly, FM, FN, FO, FP, and FQ, being cut proportionally at G, H, I, K, and L: Forasmuch also as

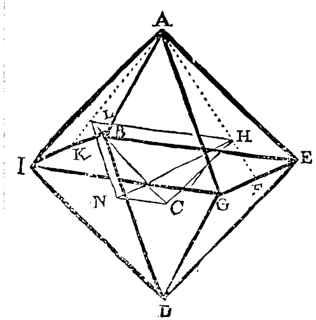


FG, FH, FI, FK, and FL, are equal lines; drawn from the center;
f) 2. 6. [f] GH, HI, IK, KL, and LG, shall be parallel to MN, NO, OP, PQ,
g) 10. 11. and QM; and [g] therefore the angles IGH, GHI, HIK, IKL, and LKG, are equal to the equal angles QMN, MNO, NOP, OPQ,
h) 15. 11. FQM, therefore also equal to one another, and [h] the plains drawn by GH, HI, and HI, IK, being parallels to the plain of the Pentagon MNOPQ, drawn by MN, NO, and OP, and meeting one another at HI, shall make one only plain. In like manner may be shewn that the plains IK, KL, and KL, LG, and LG, GE, will make one and the same plain with the pl ain drawn by GH, HI, and IK. Wherefore GHIKL is an equilateral Pentagon and equiangled, having the angles and the sides equal; and if in the same manner to the other eleven Pyramids of the Icosahedron, there be joyned right lines to the centers of the triangles, there will in like manner be described the equilateral triangles equiangled, the which having the sides common, shall be equal to one another: Wherefore twelve such Pentagons shall constitute a Dodecahedron, the
i) 31. d. 11. which [i] shall be inscribed in the Icosahedron, the twenty angles of the Dodecahedron being constituted at the centers of the twenty bases of the Icosahedron. Therefore in the Icosahedron given we have inscribed a Dodecahedron. Which was to be done.

PROP. 6. PROBL. 6.

In an Octohedron given ABGDEI, *to inscribe a trilateral equilateral Pyramis.*

Construction. TAke four bases of the Octohedron, that is, three which close in the lowest triangle EGD, namely, AEG, BED, IGD, and let the fourth be AIB, which is opposite to the lowest triangle before put, namely, to EGD. And take the centers of those four bases, which let be the points H, C, N, and L, and upon the triangle HCN erect a Pyramis HCNL. Now forasmuch as these two bases of the Octohedron, to wit, AGE and ABI, are set upon the right lines EG and BI, which are opposite to one another in the square GEBI of the Octohedron; from the point A draw by the centers of the bases, to wit, by the centers H and L, perpendicular lines AHF and ALK, cutting the lines EG and BI into two equal parts in the points F and K [a]: a) C. 12. 13.
Wherefore a right line drawn from the point F to the point K, shall be a parallel, and equal to the sides of the Octohedron, to wit, to EB and GI [b]. And the right line HL, which cutteth the equal sides AF and AK proportionally (for AH and AL are drawn from the centers of equal circles, to the circumferences) is a parallel to b) 33. 1.
the right line FK [c], and also to c) 2. 6.
the sides of the Octohedron, to wit, to EB and IG [d]. Where- d) 9. 11.
fore as the line AF is to the line AH, so is the line FK to the line HL [e]. e) 4. 6.



Demonstration. FOr the triangles AFK and AHL are like [f]: But the line AF is f) Cor. 2. 6.
in sesquialter proportion to the line AH: (for the side EG maketh HF the half of the right line AH [g]:) Wherefore FK g) C. 12. 13.
or GI the side of the Octohedron is sesquialter to the right line HL. And by the same reason may we prove that the sides of the Octohedron are sesquialter to the rest of the right lines which make the pyramis HNCL, to wit, to the right lines HN, NC, CL, LN, and CH: Wherefore those right lines are equal, and therefore the triangles which are described of them, to wit, the triangles HCN, HNL, NCL, and CHL, which make the Pyramis HNCL, are equal and equilateral. And forasmuch as the angles of the same Pyramis, to wit, the angles H, C, N, and L, do end in the centers of the bases of the Octohedron: Therefore it is inscribed in the same Octohedron [h]. Wherefore, &c. h) 1 def. 15.
Which was required to be done.

COROLLARIE I.

The bases of a Pyramis inscribed in an Octohedron, are parallels to the bases of the Octohedron.

For seeing the sides of the bases of the Pyramis touching one another, are parallels to the sides of the Octohedron, which also touch one another: As for example, HL was proved to be a parallel to GI, and LC to DI. Therefore [i] the plain Superficies which is drawn by the lines HL i) 15. 11.
and LC, is a parallel to the plain Superficies drawn by the lines GI and DI, and so also of the rest.

COROLLARIE II.

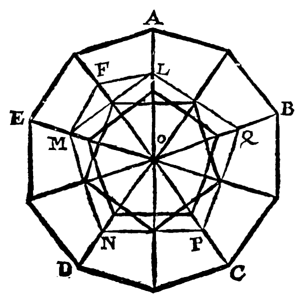
A right line joyning together the centers of the opposite bases of the Octohedron, is sesquialter to the perpendicular line drawn from the angle of the inscribed Pyramis, to the base thereof.

For seeing the Pyramis and the Cube which conteineth it, do in the same points end their angles [k]: Therefore they shall both be inclosed in one and the same Octohedron [l]. But the diameter of the Cube joyneth together the centers of the opposite bases of the Octohedron, and therefore is the diameter of the sphere which conteineth the Cube and the Pyramis inscribed in the Cube [m]; which diameter is sesquialter to the perpendicular which is doawn from the angle of the Pyramis to the base thereof; for the line which is drawn from the center of the sphere to the base of the Pyramis is the sixth part of the diameter [n].

k) 1. 15.
l) 4. 15.
m) 13, 14. 13
n) 2 C. 13. 13

PROP. 7. PROBL. 7.

In a Dodecahedron given ABCDE, *to inscribe an Icosahedron.*



Construction LEt the centers of the Circles, which contein six bases of the Dodecahedron, be the points L, M, N, P, Q, and O. And draw those right lines O L, O M, O P, and OQ, as also the right lines L M, M N, N P, P Q, and Q L.

Demonstration ANd now forasmuch as equal and equilateral Pentagons are conteined in equal Circles: Therefore perpendicular lines drawn from their centers to the sides, shall be equal [a], and shall divide the sides of the Dodecahedron into two equal parts [b]: Wherefore the foresaid perpendicular shall concurre in the point of the Section, wherein the sides are divided into two equal parts, as L F and M F do. And they also contein equal angles, to wit, the inclination of the bases of the Dodecahedron [c]: Wherefore the right lines L M, M N, N P, P Q, and Q L, and the rest of the right lines which joyn together two centers of the bases, and which subtend the equal angles conteined under the said equal perpendicular lines, are equal to one another [d]: Wherefore the triangles O L M, O M N, O N P, O P Q, O Q L, and the rest of the triangles which are set at the centers of the Pentagons are equilateral and equal. Now forasmuch as the 12 Pentagons of a Dodecahedron contein 60 plain superficial angles, of which 60 every 3 make one solid angle of the Dodecahedron: It followes that a Dodecahedron hath 20 solid angles, but each of these solid angles is subtended of each of the triangles of the Icosahedron, to wit, of each of those triangles which joyn together the centers of the Pentagons which make the solid angle, as hath been proved. Wherefore 20

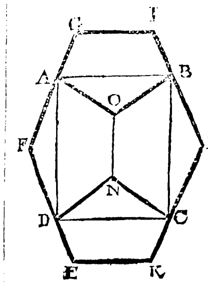
a) 14. 3.
b) 3. 3.
c) 3 C. 18. 11
d) 4. 1.

20 equal and equilateral triangles, which subtend the 20 solid angles of the Dodecahedron, and have their sides which are drawn from the centers of the Pentagons common, do make an Icosahedron [e]; and it is inscribed in the Dodecahedron given [f], for that the angles thereof do all at one time touch the bases of the Dodecahedron. Wherefore, &c. Which was required to be done.

e) 25. de. 11.
f) 1. def. 15.

PROP. 8. PROBL. 8.

In a Dodecahedron given, to include a Cube.



Construction DEscribe [a] a Dodecahedron: And take the 12 sides of the Cube, each of which subtend one angle of each of the 12 bases of the Dodecahedron; for the side of the Cube subtendeth the angle of the Pentagon of the Dodecahedron [b]: If therefore in the Dodecahedron described we draw the twelve right lines subtended under the foresaid 12 angles, and ending in 8 angles of the Dodecahedron, and concurring together in such sort that they be in like sort situate (as was plainly proved [c],) then shall it be manifest that the right lines drawn in this Dodecahedron from the foresaid 8 angles thereof, do make the foresaid Cube, which therefore is included in the Dodecahedron, for that the sides of the Cube are drawn in the sides of the Dodecahedron.

a) 17. 13.
b) 2 Cor. of 17. 13.
c) 2 Cor. of 17. 13.

Demonstration AS for Example, take four Pentagons of a Dodecahedron, to wit, AGIBO, BHCNO, CKEDN, and DFAON; and draw these right lines A B, B C, C D, and D A, which four right lines make a square, for that each of those right lines do subtend equal angles of equal Pentagons, and the angles which those four right lines contein are right angles: Wherefore the six bases being squares, do make a Cube [d]; and for that the eight angles of the said Cube are set in eight angles of the Dodecahedron: Therefore is the said Cube inscribed in the Dodecahedron [e]: Wherefore, &c. Which was required to be done.

d) 21. d. 11.
e) 1. def. 15.

PROP. 9. PROBL. 9.

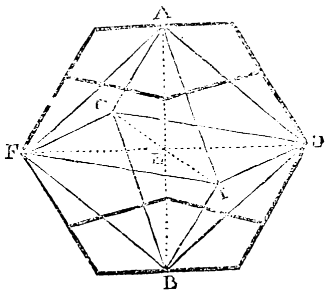
In a Dodecahedron given ABGD, *to include an Octohedron* AGBDCI.

Construction TAke [a] the 6 sides which are opposite to one another, those 6 sides I say whose sections wherein they are divided into two equal parts, are coupled by three right lines within the center of the sphere wherein the Dodecahedron is conteined, do cut one another perpendicularly. And let the points wherein the foresaid sides are cut into two equal parts be A, B, G, D, C, and I. And let the foresaid three right lines joyning togther the said sections, be A B, G D, and C I, and let the center of the sphere be E.

a) 3 C. 17. 13

Demon-

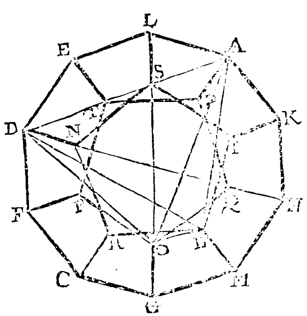
c) Cor.3.of 17. 13. *Demonstration* NOw forasmuch b as those three right lines are equal, it followeth c that the right lines subtending the right angles which they make at the center of the sphere, which right angles are
c) 4. 1.
conteined under the halves of the said three right lines, are equal to one another, that is, the right lines A G, G B, B D, D A, C A, C G, C B, C D, and I A, I G, I B, and I D, are equal to one another. Wherefore also the eight triangles C A G, C G B, C B D, C D A, I A G, I G B, I B D, and I D A, are equal and equilateral. And therefore A G B D C I is an Octohedron d. And the said Octohedron is included in the Dodecahedron e; for that all the angles thereof do at one time touch the sides of the Dodecahedron. Wherefore, In the, &c. Which was required to be done.



d) 23.de.11
e) 1.def.15.

PROP. 10. PROBL. 10.

In a Dodecahedron given ABCD, *to inscribe an equilateral trilateral Pyramis.*



Construction TAke three bases of the Dodecahedron, meeting at the point S, to wit, the bases A L S I K, D N S L E, and S I B R N, and of those three bases take the three angles at the points A, B, and D, and draw these right lines A B, B D, and D A; and let the diameter of the sphere conteining the Dodecahedron be S O, and there draw these right lines A O, B O, and D O.

a) 17. 13. *Demonstration* NOw forasmuch as a the angles of the Dodecahedron are set in the Superficies of the Sphere described about the Dodecahedron: Therefore if upon the diameter S O, and by the point A, be described a semicircle, it shall make the angles S B O and S D O right angles. Wherefore the diameter S O conteineth in power both the right lines S A and A O, or the lines S B and B O, or else S D and D O; but the lines S A, S D, and S B, are equal to one another; for they each subtend one of the angles of equal Pentagons: Wherefore the other lines remaining, to wit, A O, B O, and D O, are equal to one another, and by the same reason may be proved that the diameter H D, which subtendeth the two right lines H A and A D, conteineth in power both the said two right lines, and also conteineth in power both the right lines H B and B D, which two right lines it also subtendeth. And moreover, by the same reason, the diameter A C, which subtendeth the right lines C B and B A, conteineth in power both the said right lines C B and B A. But the right lines H A, H B, and C B, are equal to one another, for that each of them also subtendeth one of the angles of equal Pentagons: Wherefore the right lines remaining, to wit, A D, B D, and B A, are equal to one another. And by the same reason may be proved that each of these right lines A D, B D, and B A, is equal to each of the right lines A O, B O, and D O: Wherefore the six right lines A B, B D, D A, A O, B O, and D O, are equal to one another: And therefore the triangles which are made of them, to wit, the triangles A B D, A O B, A O D, and B O D, are equal and equilateral; which triangles therefore do make a Pyramis A B D O, whose base is A B D, and tops the point O, each of the angles of which Pyramis, to wit, the angles at the points A, B, D, and O, do in the same points touch the angles of the Dodecahedron. Wherefore the said Pyramis is inscribed in the Dodecahedron b. Wherefore, In a Dodecahedron, &c. Which was required to be done. b) 1.def.15.

PROP. 11. PROBL. 11.

In an Icosahedron given, to inscribe a Cube.

IT was manifest a that the angles of a Dodecahedron are set in the centers of the bases of the Icosahedron. And b it was proved that the angles of a Cube are set in the angles of a Dodecahedron. Wherefore the same angles of the Cube shall of necessity be set in the centers of the bases of the Icosahedron. Wherefore the Cube shall be inscribed in the Icosahedron c. Wherfore, &c. Which was required to be done.

a) 7. 15.
b) 8. 15.
c) 1.def.15.

PROP. 12. PROBL. 12.

An Icosahedron given, to inscribe a trilateral equilateral Pyramis.

IT was manifested a that the angles of a Cube are set in the centers of the bases of the Icosahedron: And b it was plain that the four angles of a Pyramis are set in four angles of a Cube. Wherefore it is evident c that a Pyramis described of right lines joyning together these four centers of the bases of the Icosahedron, shall be inscribed in the same Icosahedron. Wherefore, In an Icosahedron, &c. Which was required to be done.

a) 11. 15.
b) 1. 15.
c) 1.def.15.

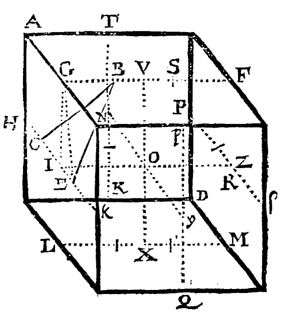
PROP. 13. PROBL. 13.

In a Cube A D F L, *to inscribe a Dodecahedron.*

Construction 1. DIvide each of the sides of the Cube into two equal parts in the points T, H, K, P, G, L, M, F, and p k Q f. And draw these right lines T K, G F, p Q, H k, P f, and L M, which lines again

again divide into two equal parts in the points N, V, Y, I, Z, and X, and draw the lines N Y, V X, and I Z. Now the three lines N Y, V X, and I Z, together with the diameter of the Cube, shall cut one another into
a) 39. 11. two equall parts in the center of the Cube [a]: Let that center be the point O. And not to stand long about the Demonstration, understand all these right lines to be equal and parallels to the sides of the Cube, and to cut
b) 29. 1. one another at right angles [b]. Let their halves, to wit, F V, G V, H I and K I, and the rest such like, be divided by an extream and mean pro-
c) 30. 6. portion [c], whose greater segments let be the lines F S, G B, H C, and *k* E, &c. and draw the lines G I, G E, B C, and B E.

Demonstration NOw forasmuch as the line G I is equal to the whole line C V, which is the half of the side of the Cube; and the line I E is equal to the line B V, that is, to the lesser segment: Therefore the squares of the lines C I and I E, are triple to the square of the line
d) 4. 13. G B [d]. But unto the squares of the lines G I and I E, the square
e) 47. 1. of the line G E is equal [e]; for the angle G I E is a right angle: Wherefore the square of the line G E is triple to the square of the line G B; and forasmuch as the line F G is erected perpendicularly to the plain
f) 4. 11. A G K L [f], for it is erected perpendicularly to the two lines A G and G I; therefore the angle B G E is a right angle, for the line G E is drawn in the plain A G *k* L. Wherefore the line B E conteining in power



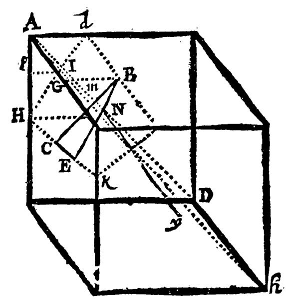
g) 47. 1. the two lines B G and G E [g], is in power quadruple to the line G B (for the line G E was proved to be in power triple to the same line G B:)
h) 20. 6. Wherefore the line B F is in length double to the B G [h]. But (by Construction) the line C E is double to the line I E: Wherefore the halves G B and I E are in proportion the one to the other as their doubles B E and C E:
i) 15. 5. Wherefore [i] the line C E is the greater segment of the line B E, divided by an extream and mean proportion. And forasmuch as the same thing may be proved touching the line B C: Therefore the lines B E and B C are equal, making an Isosceles triangle.

Construction 2. NOw let us prove that three angles of the Pentagon of the Dodecahedron are set at the points B, C, and E, and the other two angles are set between the lines B C and B E.

Forasmuch as the circle which conteineth the triangle B C E circum-
k) 11. 4. scribeth the Pentagon, whose side is the line C E [k]: Extend the plain of the triangle B C E, by the parallel lines *d* B and H E, cutting the line A D, to wit, the diameter A D, the base of the Cube in the point I; and let it cut the line A H the diameter of the Cube in the point M. And by the point I, draw in the base A D a parallel to the line A *d*, which let be I *l*.

Demon-

Demonstration FOrasmuch as from the triangle A H N, there is by the parallel line *l* I, taken away the triangle A *l* I, like unto the
l) Cor. 2. 6. whole triangle A H N [l], the lines A I and *l* I shall be equal. But as the
m) 2. 6. line H A is to the line A *d*, so [m] is the line H *l* to the line *l* I, or to the line *l* A, which is equal to the line *l* I; and the greater segment of the line H A, (which is half the side of the Cube) is as before hath been proved, the
n) 33. 1. line A *d*, that is, the line G B, which is equal to the line A *d* [n]: Wherefore the greater segment of the line H *l* is the line *l* A, and as the whole line H *l* is to the greater segment, so shall the same greater segment H *l*, be to the lesser seg-
o) 5. 13. ment *l* A [o]. Wherefore the line H A is divided by an extream and mean proportion in the point *l*. But in the triangle A H N, the line N A which is drawn from the center of the base A D, is in the point I, cut like unto the line A H, by the
p) 2. 6. parallel line *l* I [p]; for the lines H N and I *l* are parallels by construction: Wherefore the line N A is in the point I divided by an extream and mean proporti-



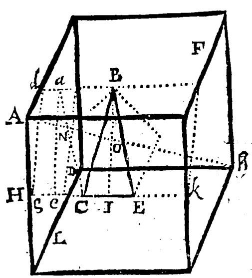
on by the Superficies *d* B E H. And forasmuch as the line Y O N, which coupleth the centers of the opposite bases, is a parallel to the line H E; the plain Superficies extended by the line Y O N, parallel to the plain *d* B E H, the two plains shall cut the lines A O and A N (the semidiameter of the Cube, and the semidiameter of the base A D) into the same
q) 17. 11. proportions in the points *m* and I [q]. But the line A N is in the point I divided by an extream and mean proportion: Wherefore the semidiameter of the Cube is in the point *m* divided by an extream and mean proportion by the plain of the triangle B C E. And forasmuch as the rest of the triangles described in the Cube after the like manner, may by the same reasons be proved to be in a plain, which cutteth the semidiameter of the Cube by an extream and mean proportion, it is manifest that three plains of the Dodecahedron shall under every angle of the Cube concurre in one and the same point of the semidiameter, being cut by an extream and mean proportion.

Construction 3. NOw resteth to prove that the right lines which couple that point of the semidiameter, with the angles of the triangle B E C are equal; whereby may be proved that the Pentagons are equilateral and equiangled.

Take the two bases of the Cube, whereon are set the triangle B C E, to wit, the bases A F and A *k*, take also the same diameter of the Cube, that was before, to wit, A *h*, and let the side set at the point *n* of the section of the diameter by an extream and mean proportion, be the line C *n* or B *n*, and let the center of the Cube be as before the point O, and extend the line C *n* to the line B *d*, and let it concurre with it in the point *a*.

Demon-

Demonstration. FOrasmuch as the plain which passeth by the line H C E, and the center O (cutting the Cube into two equal parts) is parallel to A F the base of the Cube by Construction: Imagine that by the point *n* be extended a plain Superficies parallel to the former parallel plains which shall cut the semidiameter O A, and the line C *a* proportionally in the point *n* r: For those lines do touch the extream parallel plains extended by the lines H E and E O, and by the lines A *d* and *d* B; but it is proved that the line O A is divided by an extream and mean proportion in the point *n*: Wherefore the line C *a* is divided also by an extream and mean proportion in the point *n*. Again, forasmuch as B C E is an Isosceles triangle, and it is proved that the line B I cutteth the base C E into two equal parts in the point I, the angles B I C and B I E shall be right angles. Imagine by the line B I and the center O, a plain to passe (cutting the Cube into two equal parts) parallel to the base A D: And unto those plains let there be imagined another parallel plain passing by the point *n*, which let be *n e*, which shall cut the semidiameter A O, and the half side of the Cube, to wit, the line I H, like, in the points *n* and *e*. Wherefore the line I H is in the point *e* divided by an extream and mean proportion: Wherefore the line H *e* is equal to the line C I or I E; to wit, each are lesser segments. And forasmuch as the line I *e* is to the line I C (which is equal to the line *e* H) as the whole is to the greater segment, take away from the whole line I *e* the greater segment I C; there shall remain the lesser segment C *e* s: Wherefore the line I *e* is divided by an extream and mean proportion in the point C. Again, unto the same plaines imagine another plain to passe by the point *a*, parallel to them, and let the same be *a g*. Now then t the lines C *a* and C *g* are in like sort cut in the points *n* and *e*. But the line C *a* was in the point *n* cut by an extream and mean proportion. But the line I C is to the line C *e* as the greater segment is to the lesser: Wherefore the line C *e* is to the line *e g*, as the greater segment is to the lesser; and therefore their proportion is as the whole line I C to the greater segment C *e*; and as the greater segment C *e* is to the lesser segment *e g*: Wherefore the whole line C *e g* which maketh the greater segment and the lesser, is equal to the whole line I C or I E. And forasmuch as two parallel plain superficies (to wit, that which is extended by I O B, and that which is extended by the line *a g*) are cut by the plain of the triangle B C E, which passeth by the lines *a g* and I B, their common sections *a g* and I B shall be parallels v. But the angle B I E or B I C is a right angle: Wherefore the angle *a g* C is also a right angle w, and those right angles are conteined under equal sides, to wit, the line *g* C is equal to the line C I, and the line *a g* to the line B I x: Wherefore the bases C *a* and C B are equal y: But of the line C B the line C E was proved to be the greater segment; wherefore the same line C E is also the greater segment of the line C *a*: But *c n* was also the greater segment of the same line C *a*: Wherefore unto the line C E



r) 17. 11.
17. 11.
s) 5. 13.
t) 17. 11.
v) 16. 11.
w) 29. 1.
x) 35. 1.
y) 4. 1.

C E the line *c n*, which is the side of the Dodecahedron, and is set at the Diameter, may be proved equal to lines equal to the line C E. Wherefore the Pentagon inscribed in the circle, wherein is conteined the triangle B C E, z is equiangled and equilateral. And forasmuch as two Pentagons set upon every one of the bases of the Cube, do make a Dodecahedron, and six bases of the Cube do receive twelve angles of the Dodecahedron, and the eight semidiameters do in the points where they are cut by an extream and mean proportion, receive the rest: Therefore the twelve Pentagon bases conteining 20 solid angles do inscribe the Dodecahedron in the Cube a. Wherefore, In a Cube, &c. Which was required to be done.

z) 11. 4.
a) 1. def. 15.

COROLLARIE I.

The Diameter of the Sphere which conteineth the Dodecahedron, conteineth in power these two sides, to wit, the side of the Dodecahedron, and the side of the Cube wherein the Dodecahedron is inscribed.

For in the first figure, a line drawn from the center O to the point B the angle of the Dodecahedron, to wit, the line O B, conteineth in power the two lines O V the half side of the Cube, and V B the half side of the Dodecahedron b. Wherefore c the double of the line O B, which is the Diameter of the Sphere conteining the Dodecahedron, conteineth in power the double of the other lines O V and V B, which are the sides of the Cube and of the Dodecahedron.

b) 47. 1.
c) 15. 5.

COROLLARIE II.

The side of a Cube divided by an extream and mean proportion, maketh the lesser segment the side of the Dodecahedron inscribed in it; and the greater segment the side of the Cube inscribed in the same Dodecahedron.

For it was before proved that the side of the Dodecahedron is the greater segment of B E the side of the triangle B E C; but the side B E (which is equal to the lines G B and S F) is the greater segment of G F, the side of the Cube; which line B E (subtending the angle of the Pentagon) was d the side of the Cube inscribed in the Dodecahedron.

d) 8. 15.

COROLLARIE III.

The side of a Cube is equal to the sides of a Dodecahedron inscribed in it, and circumscribed about it.

For it was manifest by this Proposition that the side of a Cube maketh the lesser segment the side of the Dodecahedron inscribed in it, to wit, as in the first figure, the line B S the side of the Dodecahedron, is the lesser segment of the line G F the side of the Cube, and it was proved e that the same side of the Cube subtendeth the angle of the Pentagon of the Dodecahedron circumscribed; and therefore it maketh the greater segment the side of the Dodecahedron, or of the Pentagon f. Wherefore it is equal to both those segments.

e) 17. 13.
f) 1 Cor. of 17. 13.

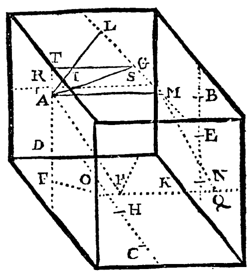
PROP. 14. PROBL. 14.

In a Cube given A B C, *to inscribe an Icosahedron.*

Construction. LEt the centers of the bases of the given Cube be the points D, E, G, H, I, and K, by which points, draw in the bases

unto the other sides parallels not touching one another: And divide the lines drawn from the centers, as the line D T, &c. by an extream and mean proportion in the points A, F, L, M, N, B, P, Q, R, S, C, and O,
a) 30. 6. [a] and let the greater segments be about the centers: And draw the lines A L, A G, A M and T G.

Demonstration. Forasmuch as the lines cut are parallels to the sides of the Cube, they shall make right angles to one another [b]:
b) 29. 1. And forasmuch as they are equal, their sections shall be equal, for that
c) 2. 14. the sections are like [c]: Wherefore the line T G is equal to the line D T, for they are each half sides of the Cube. Wherefore the square of the whole line T G, and of the lesser segment T A, is triple to the square
d) 4. 13. of the line A D the greater segment [d]: But the line A G conteineth in power the lines A T and T G, for the angle A T G is a right angle. Wherefore the square of the line A G is triple to the square of the line A D: And forasmuch as the line M G L is erected perpendicularly to the plain passing by the lines A T and T G, which is parallel
e) C. 14. 11. to the bases of the Cube [e]; therefore the angle A G L is a right angle. But the line L G is equal to the line A D, for they are the greater segments of equal lines: Wherefore the line A G (which is in power triple to the line A D) is in power triple to the line L G: Wherefore adding to the same square of the line A G the square of the line L G, the square of the



f) 47. 1. line A L, which [f] conteineth in power the two lines A G and G L, shall be quadruple to the line A D or L G. Wherefore the line A L is
g) 20. 6. double to the line A D [g]; and therefore is equal to the line A F, or to the line L M. And by the same reason may we prove that every one of the other lines which couple the next sections of the lines cut, as the lines A M, P F, P M, M Q, and the rest, are equal: Wherefore the triangles A L M, A P F, A M P, P M Q, and the rest such like are equal,
h) 4, 8. 1. equiangled, and equilateral [h]. And forasmuch as upon every one of the lines cut of the Cube are set two triangles, as the triangles A L M and B L M, there shall be made 12 triangles. And forasmuch as under every one of the eight angles of the Cube are subtended the other eight triangles, as the triangle A M P, &c. of twelve and eight triangles shall be produced twenty triangles, equal and equilateral, con-
i) 25. de. 11. teining the solid of an Icosahedron [i], which shall be inscribed in the given
k) 1. def. 15. Cube A B C [k], the invention of the demonstration of this dependeth of the ground of the former. Wherefore, In a given Cube, &c. Which was required to be done.

COROLLARIE I.

The Diameter of a Sphere which conteineth an Icosahedron, conteineth two sides,

sides, to wit, the side of the Icosahedron, and the side of the Cube which conteineth the Icosahedron.

For if we draw the line A B, it shall make the angles at the point A right angles; for that it is a parallel to the sides of the Cube: Wherefore the line which coupleth the opposite angles of the Icosahedron at the points F and B, conteineth in power the line A B (the side of the Cube) and the line A F (the side of the Icosahedron) [l] which line F B is equal to l) 47. 1.
the diameter of the sphere, which conteineth the Icosahedron [m]. m) 16. 13.

COROLLARIE II.

The six opposite sides of the Icosahedron divided into two equal parts, their sections are coupled by three equal right lines, cutting one another into equal parts, and perpendicularly in the center of the Sphere which conteineth the Icosahedron.

For these three lines are the three which couple the centers of the bases of the Cube, which do in such sort in the center of the Cube cut one another [n], and therefore are equal to the sides of the Cube. But right lines n) Co. 3. 15.
drawn from the center of the Cube to the angles of the Icosahedron, every one of them shall subtend the half side of the Cube, and the half side of the Icosahedron (which half sides contein a right angle:) Wherefore those lines are equal. Whereby it is manifest that the foresaid center is the center of the sphere which conteineth the Icosahedron.

COROLLARIE III.

The side of a Cube divided by an extream and mean proportion, maketh the greater segment the side of an Icosahedron described in it.

For the half side of the Cube maketh the half of the side of the Icosahedron the greater segment: Wherefore also the whole side of the Cube maketh the whole side of the Icosahedron the greater segment [o], o) 15. 5.
for the sections are like [p]. p) 2. 14.

COROLLARIE IV.

The sides and bases of the Icosahedron, which are opposite to one another, are parallels.

Forasmuch as every one of the opposite sides of the Icosahedron, may be in the parallel lines of the Cube, to wit, in those parallels which are opposite in the Cube, and the triangles which are made of parallel lines, are parallels [q]. Therefore the opposite triangles of the Icosahedron, as q) 15. 11.
also the sides, are parallels to one another.

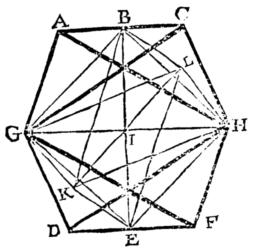
PROP. 15. PROBL. 15.

In an Icosahedron given A C D F, *to inscribe an Octohedron* B G E H K L.

Construction. Let [a] there be taken the three right lines which cut one another into two equal parts perpendicularly, and which cou- a) 2 C. 14. 15
ple the sections into two equal parts of the sides of the Icosahedron; which let be B E, G H, and K L, cutting one another in the point I, and draw the lines B G, G E, E H, and H B.

Demon-

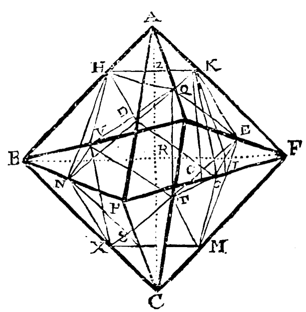
Demonstration FOrasmuch as the angles at the point I are (by Construction) right angles, and are contained under equal lines; the
b) 4. 1. bases G B and H E shall make a square [b]. Likewise unto those bases shall



be equal the lines drawn from the points K and L, to every one of the points B, G, E, and H; and therefore the triangles which make the Pyramis B G E H K, shall be equal and equilateral. And by the same reason shall the rest of the triangles which make the other Pyramis B G E H L upon the same base B G E H, be equal and equilateral. Wherefore B G E H K L
c) 22. d. 11. shall be an Octohedron [c], and shall
d) 1. dc. 15. be inscribed in the Icosahedron [d]: Wherefore, In an Icosahedron, &c. Which was required to be done.

PROP. 16. PROBL. 16.

In an Octohedron given ABCFPL, *to inscribe an Icosahedron.*

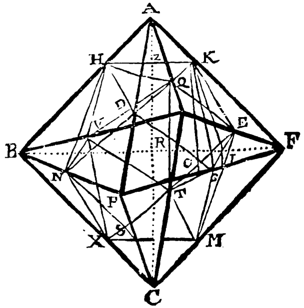


Construction LEt the six angles of the Octohedron be A, B, C, F, P, and L, and draw the lines A C, B F, and P L, cutting one another
a) 2. Cor. of 14. 13. perpendicularly in the point R [a]; and let every one of the twelve sides of the Octohedron be divided by an extream and mean proportion in the points H, X, M, K, D, S, N, G, V, E, Q, and T; and let the greater segments be the lines B H, B X, F M, F K, A D, A Q, C S, C I, P N, P G, L V, and L E, and draw the lines H K, X M, G E, N V, D S, and Q T.

Demonstration FOrasmuch as in the triangle A B F, the sides are cut proportionally, to wit, as the line B H is to the line H A, so
b) 2. 14. is the line F K to the line K A [b]; therefore the line H K shall be a parallel
c) 2. 6. to the line B F [c]. And forasmuch as the line A C cutteth the line H K in the point Z, and the line Z K is a parallel to the line R F, the line R A
d) 2. 6. shall be cut by an extream and mean proportion in the point Z [d]; to wit, shall be cut like unto the line F A; and the greater segment thereof
e) 3. 1. shall be the line Z R. Unto the line Z R put the line R O equal [e], and
f) 33. 1. draw the line K O. Now then the line K O shall be equal to the line Z R [f]. Draw the lines K G, K E, and K I. And forasmuch as the triangles A R F
g) 6. 6. and A Z K, are equiangled [g], the sides A Z and Z K shall be equal to
h) 4. 6. one another [h]; for the sides A R and R F are equal: Wherefore the line Z K

Z K shall be the lesser segment of the line R A. But if the greater segment R Z be divided by an extream and mean proportion, the greater segment thereof shall be the line Z K, which was the lesser segment of the whole line R A [i]. And forasmuch as the two lines F E and F G, are equal to the i) 5. 13.
two lines A H and A K, to wit, each are lesser segments of equal sides of the Octohedron, and the angles H A K and E F G are equal, namely, are right angles [k], the bases H K and G F shall be equal [l]. And by the k) 14. 13.
same reason the other lines X M, N V, D S, and Q T, may be proved l) 4. 1.
equal. And forasmuch as the lines A C, B F, and P L, do cut one another in two equal parts, and perpendicularly by Construction; the lines H K and G E (which subtend angles of triangles like unto the triangles whose angles the lines A C, B F, and P L, subtend,) are cut in two equal parts in the points Z and I [m]; so also are the other lines N V, X M, D S, m) 4. 6.
and Q T, (which are equal to the lines H K and G E) cut in like sort, and they shall cut the lines A C, B F, and P L, like. Wherefore the line K O (which is equal to R Z) shall make the greater segment the line R O, which is equal to the line Z K, (for the greater segment of R Z was the line Z K) and therefore the line O I shall be the lesser segment: When as the whole line R I is equal to the whole line R Z. Wherefore the squares of the whole line K O and of the lesser segment O I, are triple to the square of the greater segment R O [n]: Wherefore the line K I, which conteineth n) 4. 13.
in power the two lines K O and O I, is in power triple to the line R O [o]; o) 47. 1.
for the angle K O I is a right angle. And forasmuch as the lines F E and F G (which are the lesser segments of the sides of the Octohedron) are equal; and the line F K is common to them both, and the angles K F G and K F E (of the triangles of the Octohedron) are equal; the bases K G and K E shall [p] be equal: And therefore the angles K I E and K I G, which p) 4. 1.
they subtend, are equal [q]: Wherefore they are right angles [r]. Wherefore q) 8. 1.
the right line K E (which conteineth in power the two lines K I and I E [s], r) 13. 1.
is in power quadruple to the line R O (or I E); for the line K I is proved s) 47. 1.
to be in power triple to the same line R O: But the line G E is double to the line I E. Wherefore the line G E is also in power quadruple to the line I E [t]. Wherefore the two lines K E and G E are equal. And by the t) 20. 6.
same reason may the rest of the lines, to wit, H K, H N, N V, V X, and X S, and the other lines which couple the sections of the sides of the Octohedron, be proved equal to the same lines K E and G E. Wherefore the triangles described of them, to wit, G E K, G K D, G D S, G S M, and G M E, shall be equal and equilateral [v], making a solid angle at the point G, which is therefore the angle of an Icosahedron [w], v) 8. 1.
and is set in the section G of the side P F. And by the same reason may be w) 16. 13.
proved that the rest of the eleven solid angles of the Icosahedron, are set in the sections of every one of the sides of the Octohedron, to wit, in the points E, N, V, H, K, M, X, D, S, Q, and T. Wherefore there are twelve angles of the Icosahedron. Moreover, forasmuch as every one of the bases of the Octohedron do each contein triangles of the Icosahedron, as in the Pramis A B C F P (which is the half of the Octohedron) the triangle F C P receiveth in the sections of his sides the triangle G M S, and the triangle C P B conteineth the triangle N X S, and the triangle B A P conteineth the triangle H N D: And moreover, the triangle A P F conteineth the triangle K D G, and the same may be proved in the opposite Pyramis A B C F L: Wherefore there shall be eight triangles. And forasmuch

asmuch as besides these triangles, to every one of the solid angles of the Octohedron, are subtended two triangles, as the triangles KEG and MEG, to the angle F, and the triangles HNV and XNV to the angle B, also the triangles NDS and GDS to the angle P; likewise the triangles DHK and ♃HK to the angle A: Moreover the triangles E♃T and V♃T to the angle L, and finally, the triangles SXM and TXM to the angle C, these twelve triangles being added to the eight former triangles, shall produce twenty triangles equal and equilateral coupled together; which shall make an Icosahedron x. And it



x) 35. d. 11. y) 1. dc. 15. shall be inscribed in the Octohedron given ABCFPL y; for the twelve angles thereof are set in twelve like sections of the sides of the Octohedron. Wherefore, In an, &c. Which was required to be done.

COROLLARIE I.

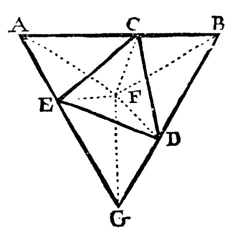
The side of an equilateral triangle being divided by an extream and mean proportion, a right line subtending within the triangle, the angle which is conteined under the greatest segment and and the lesser, is in power duple to the lesser segment of the same side.

For the line KE which subtendeth the angle KFE of the triangle AFL, which angle KFE is conteined under the two segments KF and FE, was proved equal to the line HK, which conteineth in power the two lesser segments HA and AK z; for the angle HAK is a right angle:
z) 47. 1. Wherefore the line KE or HK is in proportion duple to the line AK.

COROLLARIE II.

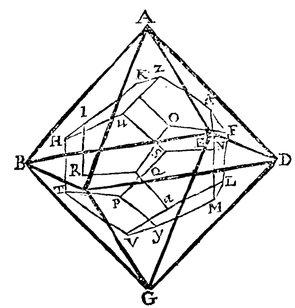
The bases of the Icosahedron are concentrical (that is, have one and the same center) with the bases of the Octohedron which conteineth it.

For suppose that ABG be the base of an Octohedron, conteining ECD the base of an Icosahedron; and let the center of the base ABG be the point F, and draw the lines FA, FB, FC, and FE. Now then the two lines FA and AE shall be equal to the two lines FB and BC; for they are lines drawn from the center, and are also lesser segments, and they contein the halves of equal angles.
a) 4. 1. Wherefore a the bases FC and FE are equal, and by the same reason unto them shall be equal the other line FD. Wherefore making the center the point F, with the distance FE, describe a circle, and it shall be circumscribed about the



the triangle CED, and so shall the point F the center of the base of the Octohedron, be the center of CED, the base of the Icosahedron.

PROP. 17. PROBL. 17.



In an Octohedron given ABGDEC, *to inscribe a Dodecahedron.*

Construction Let the twelve sides of the Octohedron be cut by an extream and mean proportion a. It was manifest that of the right lines which couple these sections are made twenty triangles, of which eight are concentrical with the bases of the Octohedron b. a) 16. 15. b) 3 Cor. of 16. 15.

Demonstration If therefore in every one of the centers of the twenty triangles be inscribed c every one of the twelve angles of the Dodecahedron, we shall find that eight angles of the Dodecahedron are set in the eight centers of the bases of the Octohedron, to wit, those angles I, u, ϑ, O, M, a, P, and X, and of the other twelve solid angles, there are two in the centers of the two triangles which have one side common under every one of the solid angles of the Octohedron, to wit, under the solid angle A, the two solid angles K and Z; under the solid angle B, the two solid angles H and T; under the solid angle G, the two solid angles Y and V; under the solid angle D, the two solid angles F and L; under the solid angle E, the two solid angles S and N; and under the solid angles C, the two solid angles ♃ and R. And forasmuch as in the Octohedron are six solid angles, under them shall be subtended twelve solid angles of the Dodecahedron; and so are made twenty solid angles, composed of twelve equal and equilateral superficial Pentagons d; which therefore contein a Dodecahedron e. And it is inscribed in the Octohedron f, for that every one of the bases of the Octohedron do receive angles thereof. Wherefore, &c. Which was required to be done. c) 5. 15. d) 5. 15. e) 24. d. 11. f) 1. def. 15.

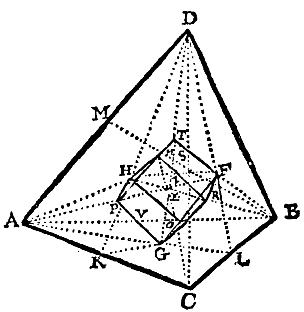
PROP. 18. PROBL. 18.

In a trilateral and equilateral Pyramis, to inscribe a Cube.

Construction Let the base of the given Pyramis be ABC, and his top the point D; and let it be comprehended in a Sphere a; and let the center of that sphere be the point E, and from the solid angles A, B, C, and D, draw right lines passing by the center E, unto the opposite bases of the Pyramis, and they shall fall perpendicularly upon the bases, and shall also fall upon the centers of the circles which contein the bases b. Let the center of the triangle ABC be the point G; and let the center of the triangle ADC be the point H, and of the triangle ADB, let the point N be the center; and lastly, let the point F be the center of the other tri- a) 13. 13. b) C. 13. 13.

triangle D B C. And let the right lines falling upon those centers be D E G, B E H, C E N, and A E F. And by those centers G, H, N, and F, let there be drawn from the angles to the opposite sides, these right lines A G L, D H K, B N M, and D F L, which shall fall perpendicu-
c) C. 12. 13. larly upon the sides B C, C A, A D, and C B c; and therefore they shall
d) 3. 3. cut them into two equal parts in the points K, L, and M d. Again let the lines which were drawn from the solid angles to the opposite bases be divided into two equal parts, to wit, the line D G in the point T, the line C N in the point O, the line A F in the point P, and the line B H in the point R, and draw the lines H T, F T, H O, and F O.

Demonstration FOrasmuch as the lines G K and G L, which are drawn from the center of one and the same triangle A B C, to the sides, are equal, and the lines DK and DL are equal, for they are the perpendiculars of equal and like triangles; and the line D G is common to them.
e) 8. 1. Wherefore e the angles K D G and L D G are equal. And forasmuch as the lines H D and D F are drawn from the center of equal circles, which contein the equal triangles A D C and D B C, therefore they are equal; and the line D T is common to them both, and they contein equal angles, as before hath been proved. Wherefore the bases H T and F T are
f) 4. 1. equal f. And by the same reason if we draw the lines C F and C H, may we prove that the other lines H O and F O are equal to the same lines H T and F T, and also to one another. Wherefore also after the same manner may be proved that the rest of the lines which couple the centers of the triangles, and the sections of the perpendicular into two equal parts, as the lines N P, G R, G P, R N, N T, P H, G O, and R F, are equal. And forasmuch as from every one of the centers of the bases are drawn three right lines to the sections into two equal parts of the perpendiculars, and there are four centers, it followeth that those equal right lines so drawn are twelve in number, of which every three and three make a solid angle in the four centers of the bases, and in the four sections, into two equal parts of the perpendiculars: Wherefore that solid hath eight angles, conteined under twelve equal sides, which make six quadrangled figures, to wit, H O F T, P G R N, P H O G, G O F R, F R N T, T N P H. Now we will prove that those quadrangled figures are rectangled.



Forasmuch as upon D C the common base of the triangles A D C and B D C, falleth the perpendiculars A S and B S, which are drawn by the centers H and F, either of these lines H S and S F shall be the third part of either of these lines A S and S B; for the line A H is double to the line
g) C. 12. 13. H S, and divideth the base D C into two equal parts g: Wherefore in the triangle A B S, the sides A S and B S are cut proportionally in the points H

H and F; and therefore the line H F is a parallel to the side A B h. h) 2. 6.
Wherefore the triangles A S B and H S F are equiangled i: Where- i) 6. 6.
fore the base H F shall be the third part of the base A B k. We k) 4. 6.
may also prove that the line T O is the third part of the line D C; for the lines E C and E D, which are drawn from the center of the Sphere which conteineth the Pyramis are equal; and the line E N (which is drawn from the center to the base) is the third part of the line E C,
so also is the line G E the third part of the line E D l; for it is the sixth l) C. 13. 13.
part of the diameter of the Sphere which conteineth the Pyramis. And the line O N is the half of the whole line N C. Wherefore the residue E O is the third part of the line E C, and so also is the line E T the third part of the line E D. Wherefore the line T O in the triangle D E C is a
parallel to the line D C, and is a third part of the same m; as the line m) 2, 4. 6.
H F was proved the third part of the line A B. But A B and D C being sides of the Pyramis, are equal: Wherefore the lines H F and T O being
the third parts of equal lines, are equal n. Wherefore o the angles H T F n) 15. 5.
and T F O are equal; and by the same reason, the angles opposite to them, o) 8. 1.
to wit, the angles F O H and O H T are equal to one another; and also are equal to the said angles H T F and T F O; but these four angles are
equal to four right angles p: Wherefore the angles of the quadrangle p) C. 32. 1.
H O F T are right angles. And by the same reason may the angles of the other five quadrangled figures be proved right angles. Now resteth to prove that the foresaid quadrangles are each of them in one and the same plain.

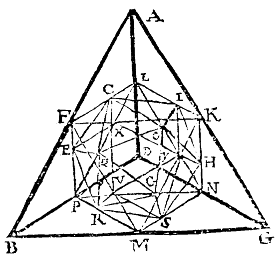
Take the quadrangle H O F T, and forasmuch as in the triangle A S B, the line H F is proved a parallel to the line A B; therefore it cutteth the
lines S V and S B proportionally, in the points I and F q. Now then foras- q) 2. 6.
much as S F was proved the third part of the line S B, the line S I shall also be the third part of the line S V. Moreover, forasmuch as the line V S which coupleth the Sections into two equal parts of the opposite sides of the Pyramis, to wit, of the sides A B and D C, is by the center E di-
vided into two equal parts r (for it is the diameter of the Octohedron in- r) Co. 2. 15.
scribed in the Pyramis) therefore the line S T is two third parts of the half line S E. And by the same reason, Forasmuch as in the triangle D E C, the line T O is proved to be a parallel to the side D C, it shall in the same
triangle cut the lines C E and S E proportionally, in the points O and I s. s) 2. 6.
But the line E O is proved to be a third part of the line E C: Wherefore the line E I is also a third part of the line E S. Wherefore the residue I S shall be two third parts of the whole line E S. Wherefore the point I cutteth either of the lines T O and H F. Wherefore the two lines H I F
and T I O, cutting one another, are in one and the same plain t. And there- t) 2. 11.
fore the points H, T, F, and O, are in one and the same plain. Wherefore the rectangled figure H O F T being quadrilateral and equilateral, and in one and the same plain, is a square, by the definition of a square. And by the same reason may the rest of the bases of the solid be proved to be squares equal and plain, or superficial: Now then the solid is comprehended of six equal squares (which are conteined of twelve equal sides) which squares make eight solid angles, of which four are in the centers of the bases of the Pyramis, and the other four are in the middle sections of the four perpendiculars. Wherefore the solid H O F T P G R N is a
Cube v, and is inscribed in the Pyramis w. Wherefore, In a trilateral, v) 21. d. 11.
&c. Which was required to be done. w) 1. d. 15.

COROLLARIE

The line which cutteth into two equal parts the oppoſite ſides of the Pyramis, is triple to the ſide of the Cube inſcribed in the Pyramis, and paſſeth by the center of the Cube.

For the line S E V, whoſe third part the line S I is, cutteth the oppoſite ſides C D and A B into two equal parts: But the line E I (which is drawn from the center of the Cube to the baſe) is proved to be a third part of the line E S: Wherefore the ſide of the Cube which is double to the line E I, ſhall be a third part of the whole line V S, which is (as hath been proved) double to the line E S.

PROP. 19. PROBL. 19.



In a trilateral equilateral Pyramis given A B G D, *to inſcribe an Icoſahedron.*

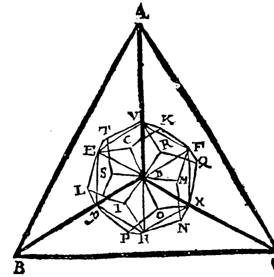
Conſtruction Let each of the ſides of the given Pyramis be divided into two equal parts in the points F, M, K, L, P, and N, and in every one of the baſes of that Pyramis, deſcribe the triangles L F P, P M N, N K L, and
a) 4. 1. F M K, which triangles ſhall be equilateral [a]; for the ſides ſubtend equal angles of the Pyramis, conteined under the halves of the ſides of the ſame Pyramis: Wherefore the ſides of the ſaid triangles are equal: Let
b) 30. 6. thoſe ſides be divided by an extream and mean proportion [b], in the points C, E, Q, R, S, T, H, I, O, V, Y, and X. Now then thoſe ſides are cut in-
c) 2. 4. to the ſame proportion [c]; and therefore they make the like Sections
d) 9. 5. equal [d]. Now I ſay that the foreſaid points do receive the angles of the Icoſahedron inſcribed in the Pyramis A B G D.

In the foreſaid triangles, Let there again be made other triangles, by coupling the ſections, and let thoſe triangles be T R S, I O H, C E Q, and V X Y, which ſhall be equilateral; for every one of their ſides do ſubtend equal angles of equilateral triangles, and thoſe ſaid equal angles are conteined under equal ſides (to wit, under the greater ſegment and the
e) 4. 1. leſſer.) And therefore the ſides which ſubtend thoſe angles are equal [e].

Demonſtration Now let us prove that at each of the foreſaid points, as for example at T, is ſet the ſolid angle of an Icoſahedron. Foraſmuch as the triangles T R S and T Q O are equilateral and equal, the four right lines T R, T S, T Q, and T O, ſhall be equal. And foraſmuch as F P N K is a ſquare, cutting the Pyramis A B G D into two
f) Co. 2. 15. equal parts [f], the line T H ſhall be in power double to the line T N or
g) 47. 1. N H [g]. For the lines T N or N H are equal, for that by Conſtruction they are each of them leſſer ſegments, and the line R T or T S is in power
h) C. 16. 15. double to the ſame line T N or N H [h]; for it ſubtendeth the angle of the triangle conteined under the two ſegments. Wherefore the lines T H, T S,

T S, T R, T Q, and T O, are equal, and ſo alſo are the lines H S, S R, R Q, Q O, and O H, which ſubtend the angles at the point T equal. For the line Q R conteineth in power the two lines P Q and P R, the leſſer ſegments, which two lines the line T H alſo conteined in power. And the reſt of the lines do ſubtend angles (of equilateral triangles) conteined under the greater and the leſſer ſegments. Wherefore the five triangles T R S, T S H, T H O, T O Q, and T Q R, are equilateral and
equal, making the ſolid angle of the Icoſahedron at the point T, [i] in the i) 16. 13.
ſide P N of the triangle P N M. And by the ſame reaſon, in the other ſides of the four triangles P N M, N K L, F M K, and L F P, (which are inſcribed in the baſes of the Pyramis) (which ſides are twelve in number, ſhall be ſet twelve angles of the Icoſahedron, conteined under twenty equal and equilateral triangles, of which four are ſet in the four baſes of the Pyramis, to wit, thoſe four triangles T R S, H O I, C E Q, and V X Y, four triangles are under four angles of the Pyramis; that is, the four triangles C I X, Y S H, E R V, and T Q O, and under every one of the ſix ſides of the Pyramis are ſet two triangles, to wit, under the ſide D G, the triangle T H S and T H O; under the ſide D B, the triangles R Q E and R Q T; under the ſide D A the triangles C O Q and C O I; under the ſide A B the triangles E X C and E X V; under the ſide B G the triangles S V R and S V Y; and under the ſide A G, the triangles I Y H and I Y X. Wherefore the ſolid being conteined under twenty equilateral and equal triangles, ſhall be an Icoſahedron [k], and ſhall be inſcribed k) 23. d. 11.
in the Pyramis A B G D [l]; for all his angles do at one time touch the l) 1. def. 15.
baſes of the Pyramid. Wherefore, In a trilateral, &c. Which was required to be done.

PROP. 20. PROBL. 20.



In a trilateral equilateral Pyramis given A B G D, *to inſcribe a Dodecahedron.*

Conſtruction Let each of the ſides of the given Pyramis be cut into two equal parts, and draw the lines which couple the ſections, which being divided by an extream and mean proportion, and right lines being drawn by the ſections, ſhall receive twenty triangles, making an Icoſahedron [a]. a) 19. 15.

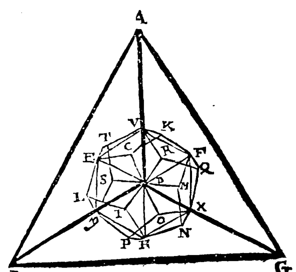
Now then if we take the centers of thoſe triangles, we ſhall there find the twenty angles of the Dodecahedron inſcribed in it [b]. And
foraſmuch as four baſes of the foreſaid Icoſahedron are concentrical b) 5. 15.
with the baſes of the Pyramis [c], there ſhall be placed four angles c) 2 Cor. of 6. 15.
of the Dodecahedron, to wit, the four angles E, F, H, and D, in the four centers of the baſes; and of the other ſixteen angles, under every one of the ſix ſides of the Pyramis are ſubtended two, to wit, under

under the side A D, the angles C and K; under the side B D, the angles L and I; under the side G D, the angles M and N; under the side A B, the angles T and S; under the side B G, the angles P and O; and under the side A G, the angles R and Q; so there rests four angles, whose true places we will now appoint.

Demonstration. FOrasmuch as a Cube conteined in one and the same Sphere with the Dodecahedron, is inscribed in the same Dodecahedron [d]; it followes that a Cube and a Dodecahedron circumscribed about it, are conteined in one and the same bodies, for that their angles meet in the same points. And as was proved [c] that four angles of the Cube inscribed in the Pyramis are set in the middle sections of the perpendiculars which are drawn from the solid angles of the Pyramis to the opposite bases. Wherefore the other four angles of the Dodecahedron are also as the angles of the Cube set in those middle sections of the perpendiculars, to wit, the angle V, is set in the middest of the perpendicular A H; the angle Y in the middest of the perpendicular B F; the angle X in the middest of the perpendicular G E; and lastly, the angle D in the middest of the perpendicular D, which is drawn from the top of the Pyramis to the opposite base. Wherefore those four angles of the Dodecahedron may be said to be directly under the solid angles of the Pyramis, or they may be said to be set at the perpendiculars. Wherefore the Dodecahedron after this manner set, is inscribed in the Pyramis given [f]; for that upon each of the bases of the Pyramis is set an angle of the Dodecahedron inscribed. Wherefore, In a, &c. Which was required to be done.

d) 17. 13. and 8. 15.

c) 18. 15.



f) 1. def. 15.

PROP. 21. PROBL. 21.

In every one of the regular Solids to inscribe a Sphere.

Demonstration. IT was declared [a] that the five regular Solids are so conteined in a sphere that right lines drawn from the center of the sphere, or of the solid inscribed, to every one of the angles of the solid inscribed, are equal, which right lines therefore make Pyramids, whose tops are in the center of the Sphere, or of the Solid, and the bases are every one of the bases of those Solids. And forasmuch as those bases are in every Solid equal and like to one another, and described in equal circles, those circles shall cut the Sphere; for the angles which touch the circumference of the circle, touch also the Superficies of the Sphere: Wherefore perpendiculars drawn from the center of the Sphere to the bases, or to the plain Superficies of the equal circles, are equal: Wherefore making the center the center of the Sphere which conteineth the Solid, and the distance some

a) 13, 14, 15, 16, and 17. of the 13.

some one of the equal perpendiculars, describe a Sphere, and it shall touch each of the bases of the Solid, neither shall the Superficies of the Sphere passe beyond those bases, when as those perpendiculars are the least lines which are drawn from the center to the bases. Wherefore, We have [b], &c. Which was required to be done.

b) C. 18. 13.

COROLLARIE.

The regular figures inscribed in Spheres, and also the Spheres circumscribed about them, or conteining them, have one and the same center.

Namely, their Pyramids, the angles of whose bases touch the Superficies of the Sphere, do from those angles cause equal right lines to be drawn to one and the same point, making the tops of the Pyramids in the same point; and therefore they make the centers of the Spheres in the same tops, when as the right lines drawn from those angles to the crooked Superficies, wherein are set the angles of the bases of the Pyramids are equal.

An Advertisement.

Of these Solids, only the Octohedron receiveth the other Solids inscribed one within another. For the Octohedron conteineth the Icosahedron inscribed in it, and the same Icosahedron conteineth the Dodecahedron inscribed in the same Icosahedron; and the same Dodecahedron conteineth the Cube inscribed in the same Octohedron. And lastly, the same Cube circumscribeth the Pyramis inscribed in the said Octohedron. But this happeneth not in the other Solids.

The End of the Fifteenth Element of EUCLIDE, *after* CAMPANE *and* FLUSSAS.

THE

THE

SIXTEENTH ELEMENT OF EUCLIDE.

THE ARGUMENT.

HAving in the Fifteenth Book shewed how to inscribe the five regular Solids one within another, we shall here in this Sixteenth Book compare those Solids so inscribed one with another, and declare their Passions and Properties; which in this Book (aacording to *Flussas*) is excellently well performed, for the which he deserveth perpetual praise and commendations: In the perusal and practice whereof the Studious Reader shall take much delight, and have occasion offered him to invent and contrive greater variery of Properties and Passions incident to the said Regular Bodies.

PROPOSITIONS, and THEOREMES.

PROPOSITION 1. THEOREM 1.

A Dodecahedron and a Cube inscribed in it, and a Pyramis inscribed in the same Cube, are conteined in one and the same Sphere.

Demon-

Demonstration FOr the angles of the Pyramis are set in the angles of the Cube wherein it is inscribed [a]; and all the angles of the Cube are set in the angles of the Dodecahedron circumscribed about it [b]. And all the angles of the Dodecahedron are set in the Superficies of the Sphere [c]. Wherefore those three Solids inscribed one within another, are conteined in one and the same Sphere [d]: Therefore, A Dodecahedron, &c. Which was to be demonstrated.

a) 1. 15.
b) 8. 15.
c) 17. 13.
d) 1. def. 15.

COROLLARIE.

These three Solids likewise are set in one and the self-same Icosahedron, or Octohedron, or Pyramis.

For they are inscribed in one and the same Icosahedron [e], and they are inscribed in one and the same Octohedron [f]. Lastly, they are inscribed in one and the same Pyramis [g]; for the angles of all those Solids are set in the centers of the bases of the circumscribed Icosahedron, or Octohedron, or Pyramis.

e) 5, 11, & 12. of the 15.
f) 4, 6, & 16. of the 15.
g) 18, & 19. of the 15.

PROP. 2. THEOR. 2.

The proportion of a Dodecahedron circumscribed about a Cube, to a Dodecahedron inscribed in the same Cube, is triple to an extream and mean proportion.

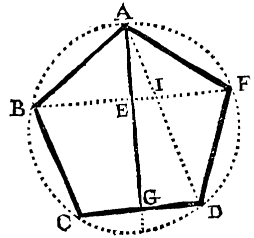
Demonstration FOrasmuch as [a] it was proved that the side of a Dodecahedron inscribed in a Cube, is the lesser segment of the side of that Cube divided by an extream and mean proportion; and the side of the Dodecahedron circumscribed about the same Cube is the greater segment of the side of the same Cube [b], the side of the Dodecahedron circumscribed shall be to the side of the Dodecahedron inscribed; as the greater segment of a right line divided by an extream and mean proportion, is to the lesser segment of the same, which proportion is called an extream and mean proportion [c]. But the proportion of like Solid Polihedrons is triple to the proportion of the sides of like proportion [d]: Wherefore the proportion of the Dodecahedron circumscribed about the Cube, is to the Dodecahedron inscribed in the same Cube in triple proportion of the sides joyned together by an extream and mean proportion: Therefore, The proportion, &c. Which was to be demonstrated.

a) 2 Cor. of 13. 15.
b) 13. 15.
c) 30. 6.
d) C. 17. 12.

PROP. 3. THEOR. 3.

In every equiangled and equilateral Pentagon ABCDF, *a perpendicular* AG *drawn from one of the angles* A, *to the base* CD, *is divided in the point* I *by an extream and mean proportion, by a right line* BF, *subtending the same angle* BAF.

 Demon-



Demonstration FOrasmuch as the angles GAF and GAB are equal [a], and the angles ABF and AFB are equal [b]; therefore the angles remaining at the point E, of the triangles AEB and AEF, are equal; for that they are the residues of two right angles [c]. But the angle EGC is by construction a right angle. Wherefore the lines BF and CD are parallels [d]. Wherefore as the line DI is to the line IA, so is the line GE to the line EA [e]. But the line DA is in the point I divided by an extream and mean Proportion [f]. Wherefore the line GA is in the point E divided by an extream and mean proportion [g]. Wherefore, In every equiangled and equilateral Pentagon, &c. Which was to be demonstrated.

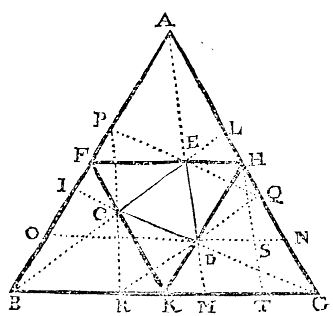
a) 27. 3.
b) 5. 1.
c) C. 32. 1.
d) 28. 1.
e) 2. 6.
f) 8. 13.
g) 2. 14.

COROLLARIE.

The line which subtendeth the angle of a Pentagon, is a parallel to the side opposite to the angle.

As was manifest in the lines BF and CD.

PROP. 4. THEOR. 4.



If from the angles A, B, *and* G, *of the base of a Pyramis* ABG, *be drawn to the opposite sides* AB, BG, *and* GA, *the right lines* GI, AM, *and* BL, *cutting the said sides by an extream and mean proportion, they shall contein the base of the Icosahedron* CDE; *inscribed in the Pyramis, which base shall be inscribed in an equilateral triangle* FKH, *whose angles* C, D, *and* E, *cut the sides of the base of the Pyramis by an extream and mean proportion.*

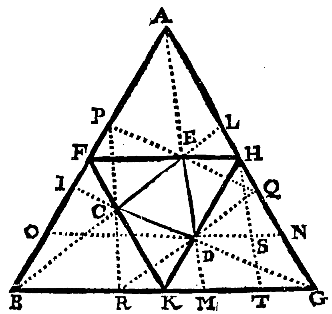
Construction THe equilateral triangle is described by dividing the sides of the base of the Pyramis into two equal parts: And the base of the Icosahedron is inscribed in the Pyramis, by dividing the sides FK, KH,

KH, and HF, by an extream and mean proportion in the points C, D, and E [a]. Again, Let the sides AB, BG, and GA, be divided by an extream and mean proportion in the points I, M, and L [b]; and draw AM, BL, and GI: I say those lines describe the triangle CDE, of the Icosahedron.

a) 19. 5.
b) 30. 6.

Demonstration FOr forasmuch as the lines BG and FH are parallels [c], by the point D, let the line ODN be drawn parallel to either of the lines BG and FH. Wherefore the triangle HDN shall be like to the triangle HKG [d]: Wherefore either of these lines DN and NH, shall be equal to the line DH, the greater segment of the line KH or FH. And forasmuch as the line FO is a parallel to the line HK, and the line OD to the line FH; the line OD shall be equal to the whole line FH, in the parallelogram FODH [e]. Wherefore as the whole line FH is to the greater segment FE, so shall the lines equal to them be, to wit, OD and DN [f]. Wherefore the line ON is divided by an extream and mean proportion in the point D [g]. But the triangles AOD, AFE, and ABM, are like to one another; and so also are the triangles ADN, AEH, and AMG [h]. Wherefore as FE is to EH, so is OD to DN, and BM to MG. Wherefore the line AM cutting the lines FH and ON, like to the line BG, in the points E, D, and M, describeth ED the side of the triangle of the Icosahedron ECD, which is described in the sections E, C, and D, by supposition. And by the same reason, the lines BL and GI, shall describe the other sides EC and CD, of the same triangle. By the point E, let there be drawn to GI, a parallel line PEQ. Now forasmuch as the lines BM and FE are parallels, the line AM is in the point E, cut like to the line AB in the point F [i]: Wherefore the line AE is equal to the line EM, and unto the line EM are also equal either of the lines GD and DI, which are cut like unto the foresaid lines. Again, forasmuch as in the triangle ADI, the lines DI and EP are parallels, as the line DI is to the line EP, so is the line AD to the line AE: But as the line AD is to the line AE, so is the line DG to the line EQ [k]; Wherefore as the line DI is to the line EP, so is the line DG to the line EQ; and alternately, as the line DI is to the line DG, so is the line EP to the line EQ: But the lines DI and IG are equal; wherefore also the lines EP and EQ are equal. And forasmuch as the line AH is equal to the line FH, whose greater segment is the line HN; therefore the whole line AN is divided by an extream and mean proportion in the point H [l]. But as the line AN is to the line AH, so is the line AD to the line AE [m] (for the lines FH and ON are parallels) and again, as the line AD is to the line AE, so [n] is the line AG to the line AQ, and the line AI to the line AP; for the lines PQ and GI are parallels: Wherefore the lines AG and AI are divided by an extream and mean proportion in the points Q and P, and the line AQ shall be the greater segment of the line AG or AB. And forasmuch as the whole line AG is to the greater segment AQ, as the greater segment AI is to the residue AP, the line AP shall be the lesser segment of the whole line AB or AG: Wherefore the line PEQ (which by the point E passeth parallel to the line GI) cutteth the lines AG and BA by an extream and mean proportion in the points Q and P. And by the same reason, the line PR (which by the point C passeth parallely to the line AM) shall fall upon the sections P and R; so also shall the

c) 2. 6.
d) Cor. 2. 6.
e) 34. 1.
f) 7. 5.
g) 2. 14.
h) Cor. 2. 6.
i) 2. 6.
k) 2. 6.
l) 5. 13.
m) 2. 6.
n) 2. 6.



the line R Q (which by the point D, passeth parallelly to the line B L) fall upon the sections R and Q: Wherefore either of the lines P E and E Q shall be equal to the line C D, in the Paralellograms P D and Q C o. And forasmuch as the lines P E and E Q are equal, the lines P C, Q R, R D, and D Q, shall be likewise equal. Wherefore the triangle P R Q is equilateral, and cutteth the sides of the base of the Pyramis in the points P, Q, and R, by an extream and mean proportion. And in it is inscribed the base E C D of the Icosahedron conteined in the foresaid Pyramis: If therefore from the angles, &c. Which was to be demonstrated.

o) 34. 1.

COROLLARIE

The side of an Icosahedron inscribed in an Octohedron, is the greater segment of the line, which being drawn from the angle of the base of the Octohedron, cutteth the opposite side by an extream and mean proportion.

p) 16. 15. For P F K H is the base of the Octohedron, which conteineth the base of the Icosahedron C D E, unto which triangle F K H, the triangle H K G is equal, as hath been proved. By the point H draw to the line M E a parallel H T, cutting the line D N in the point S. Wherefore E S, D T, and E T, are Paralellograms; and therefore the lines E H and M T, are equal; and the lines E M and H T are like cut in the points D
q) 34. 1. and S q. Wherefore the greater segment of the line H T, is the line H S,
r) 2. 6. which is equal to E D, the side of an Icosahedron. But r the line T K is
s) 2. 14. cut like to the line H K, by the parallel D M. And therefore s it is divided by an extream and mean proportion. But the line T M is equal to the line E H. Wherefore also the line T K is equal to the line E F or D H: Wherefore the residues E H and T G are equal. For the whole lines F H and K C are equal. Wherefore K G the side of the triangle H K G, is in the point T divided by an extream and mean proportion, by the right line H T; and the greater segment thereof is the line E D, the side of the Icosahedron inscribed in the Octohedron, whose base is the triangle
t) 16. 15. H K G (or the triangle F K H) which is equal to the triangle H K G t.

PROP. 5. THEOR. 5.

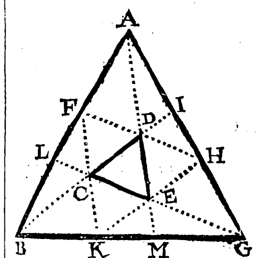
The side A G, *of a Pyramis divided by an extream and mean proportion, maketh the lesser segment* A I *in power double to the side* C E, *of the Icosahedron inscribed in it.*

Construction Let A B G be the base of a Pyramis, and let the base of the Icosahedron inscribed in it be C D E, described of three right

right lines, which being drawn from the angles of the base A B G, cut
the opposite sides by an extream and mean proportion a; to wit, of the three lines A M, B I, and G L. a) 4. 16.

Demonstration Forasmuch as b it was proved that the triangle C D E is inscribed in an equilateral triangle, whose angles cut the sides of A B G, the base of the Pyramis, by an extream and mean proportion; let that triangle be F H K, cutting the line A B in the point F. Wherefore the lesser segment F A is equal to the segment A I c (for the lines A B and A G are cut like:) Moreover the side F H of the triangle F H K, is in the point D cut into two equal parts d; and F C E D is a Paralellogram: Wherefore the lines C E and F D are equal e. And forasmuch as the line F H subtendeth the angle B A G, of an equilateral triangle, which angle is conteined under the greater segment A H, and the lesser segment A F; therefore the line F H is in power double to the line A F, or to the line A I, the lesser segment f. But the same line F H is in power quadruple to the line C E g (for the line F H is double to the line C E:) Wherefore the square of the line A I being the half of the square of the line F H, is in power double to the line C E, to which the line F H was in power quadruple. Wherefore, The side of a Pyramis divided, &c. Which was to be demonstrated.

b) 4. 16.
c) 2. 14.
d) 4. 16.
e) 33. 1.
f) C. 16. 15.
g) 4. 2.



COROLLARIE.

The side of an Icosahedron inscribed in a Pyramis, is a Residual line.

For the diameter of the Sphere which conteineth the five regular bodies, being rational, is in power sesquialtera to the side of the Pyramis h. And therefore the side of the Pyramis is rational, by the definition, which side being divided by an extream and mean proportion, maketh the lesser segment a Residual line i; Wherefore the side of the Icosahedron being commensurable to the same lesser segment (for the square of the Icosahedron is the half of the square of the said lesser segment) is a Residual line.

h) 13. 13.
i) 6. 13.

PROP. 6. THEOR. 6.

The side of a cube conteineth in power half the side of an equilateral triangular Pyramis inscribed in the said cube.

Demonstration Forasmuch as the side of the Pyramis, inscribed in the Cube, subtendeth two sides of the Cube which contein a right angle a, it is manifest b that the side of the Pyramis subtending the said sides, is in power double to the side of the Cube. Wherefore also the square of the side of the Cube is the halfe of the square of the side of the Pyramis: Therefore, &c. Which was to be demonstrated.

a) 1. 15.
b) 47. 1.

PROP.

PROP. 7. THEOR. 7.

The ſide of a Pyramis is double to the ſide of an Octohedron inſcribed in it.

a) 2. 15.
b) Co.39.1.
c) Cor.2.6.
d) 4. 6.

Demonſtration. Foraſmuch as [a] it was proved that the ſide of the Octohedron inſcribed in a Pyramis, coupleth the middle ſections of the ſides of the Pyramis: Wherefore the ſides of the Pyramis and of the Octohedron, are parallels [b]; and therefore [c] they ſubtend like triangles: Wherefore [d] the ſide of the Pyramis is double to the ſide of the Octohedron, to wit, in the proportion of the ſides. Therefore, The ſide, &c. Which was to be demonſtrated.

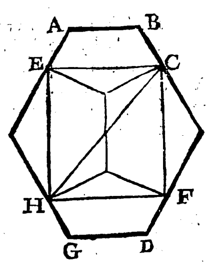
PROP. 8. THEOR. 8.

The ſide of a cube is in power double to the ſide of an Octohedron inſcribed in it.

a) 3. 15.
b) 14. 13.
c) 47. 1.

Demonſtration. It was proved [a] that the diameter of the Octohedron inſcribed in the Cube, coupleth the centers of the oppoſite baſes of the Cube: Wherefore the ſaid diameter is equal to the ſide of the Cube. But the ſame is alſo the diameter of the ſquare made of the ſides of the Octohedron, to wit, is the diameter of the ſphere which conteineth it [b]: Wherefore that diameter being equal to the ſide of the Cube, is in power double to the ſide of that ſquare, or to the ſide of the Octohedron inſcribed in it [c]. The ſide therefore, &c. Which was to be demonſtrated.

PROP. 9. THEOR. 9.



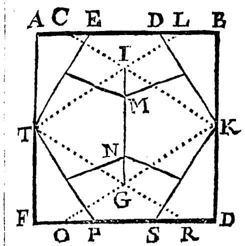
The ſide AB, *of a Dodecahedron* ABGD, *is the greater ſegment of the line which conteineth in power half the ſide* CH, *of the Pyramis inſcribed in the ſaid Dodecahedron.*

b) 1 Cor. of 17. 13.
b) 1. 16.
c) 6. 16.

Demonſtration. For ſeeing that EC, the ſide of the Cube, being divided by an extream and mean proportion, maketh the greater ſegment the line AB, the ſide of the Dodecahedron [a] (for they are conteined in one and the ſame ſphere [b];) and the line EC the ſide of the Cube, conteineth in power the half of the ſide CH [c]. Wherefore AB the ſide of the Dodecahedron, is the greater ſegment of the line EC, which conteineth in power the half of the line CH, which is the ſide of the Dodecahedron inſcribed in the Pyramis. The ſide therefore, &c. Which was required to be proved.

PROP.

PROP. 10. THEOR. 10.



The ſide CL, *of an Icoſahedron, is the mean proportional between the ſide* AB, *of the Cube circumſcribed about the Icoſahedron* CLIGOR, *and the ſide* ED, *of the Dodecahedron* EDMNPS, *inſcribed in the ſame Cube.*

a) 3C.14.15
b) 2C.13.15
c) 3. def. 6.

Demonſtration. Foraſmuch as CL is the greater ſegment of AB [a], and the ſide ED is the leſſer ſegment of the ſame ſide AB [b]: It followeth that AB the ſide of the Cube, being divided by an extream and mean proportion, maketh the greater ſegment CL, the ſide of the Icoſahedron inſcribed in it, and the leſſer ſegment ED the ſide of the Dodecahedron likewiſe inſcribed in it. Wherefore as the whole line AB, the ſide of the Cube, is to the greater ſegment CL, the ſide of the Icoſahedron; ſo is the greater ſegment CL, the ſide of the Icoſahedron, to the leſſer ſegment ED, the ſide of the Dodecahedron [c]. Wherefore, The ſide, &c. Which was required to be proved.

PROP. 11. THEOR. 11.

The ſide of a Pyramis, is in power Octodecuple (that is, as 18 to 1,) to the ſide of the Cube inſcribed in it.

a) 18. 15.
b) 20. 6.
c) 47. 1.

Demonſtration. For by what hath been demonſtrated [a], the ſide of the Pyramis is triple to the diameter of the baſe of the Cube inſcribed in it; and therefore it is in power noncuple to the ſame diameter [b]. But the diameter is in power double to the ſide of the Cube [c]. And the double of noncuple maketh Octodecuple. Wherefore, The ſide, &c. Which was required to be proved.

PROP. 11. THEOR. 11.

The ſide of a Pyramis is in power Octodecuple to that right line, whoſe greater ſegment is the ſide of the Dodecahedron inſcribed in the Pyramis.

a) Co.1.16.
b) 11.16.
c) C.17.13.

Demonſtration. Foraſmuch as the Dodecahedron and the Cube inſcribed in it, are ſet in one and the ſame Pyramis [a], and the ſide of the Pyramis circumſcribed about the Cube, is in power Octodecuple to the ſide of the Cube inſcribed [b]. But the greater ſegment of the ſame ſide of the Cube is the ſide of the Dodecahedron which conteineth the Cube [c]. Wherefore, The ſide, &c. Which was required to be proved.

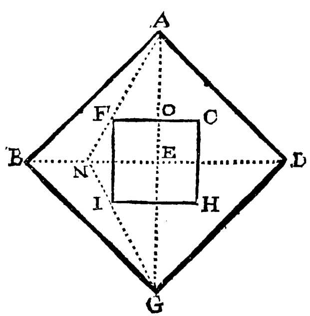
PROP.

PROP. 13. THEOR. 13.

The side of an Icosahedron inscribed in an Octohedron, is in power double to the lesser segment of the side of the same Octohedron.

a) 17. 15.
Demonstration Forasmuch as [a] it was proved, that the side of an Icosahedron inscribed in an Octohedron, coupleth together the two sections (which are produced by an extream and mean proportion) of the side of the Octohedron, which make a right angle, and that right angle is conteined under the lesser segments of the sides of the Octohedron, and is subtended of the side of the Icosahedron inscribed: It followeth therefore that the side of the Icosahedron which subtendeth the right angle, being in power equal to the two lines which contein the
b) 47. 1.
said angle [b], is in power double to each of the lesser segments of the Octohedron which contein a right angle. Wherefore, The side of an Icosahedron, &c. Which was to be demonstrated.

PROP. 14. THEOR. 14.



The sides A B, *of the Octohedron* A B G D E, *and* F I, *of the Cube* F C H I, *inscribed in it, are in power to one another in quadrupla sesquialtera proportion; (that is as* 9 *to* 2.)

Construction Let there be drawn to B E, the base of the triangle A B E, a perpendicular A N; and again, let there be drawn to the same base in the triangle G B E, the perpendicular G N, which A N and G N shall passe by the centers F and I.

a) C. 12. 13.
b) 2. 6.
Demonstration The line A F is double to the line F N [a]: Wherefore the line A O is double to the line O E [b]; for the lines F O and N E are parallels; and therefore the diameter A G is triple to the line F I. Wherefore the power of A G is noncuple (that is, as 18 to 2, or 9 to 1,) to the power of F I. But the line A G is in power double to the side
c) 14. 13.
A B [c]. Wherefore the square of the line A B being the half of the square of the line A G, which is noncuple to the square of the line F I, is quadruple sesquialter to the square of the line F I. The sides therefore, &c. Which was required to be proved.

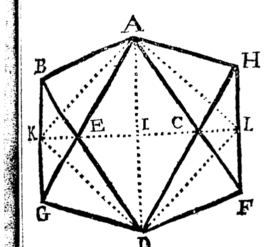
PROP.

PROP. 15. THEOR. 15.

The side of the Octohedron is in power quadruple sesquialter to that right line, whose greater segment is the side of the Dodecahedron inscribed in the same Octohedron.

Demonstration Forasmuch as [a] it was proved, that the side of the Octohedron is in power quadruple sesquialter to the side of the Cube inscribed in it; but the side of the Cube being cut by an extream and mean proportion, maketh the greater segment the side of the Dodecahedron circumscribed about it [b]: Therefore the side of the Octohedron is in power quadruple sesquialter to that right line (to wit, to the side of the Cube) whose greater segment is the side of the Dodecahedron inscribed in the Cube. But the Dodecahedron and the Cube inscribed one within another, are inscribed in one and the same Octohedron [c]. Therefore, The side, &c. Which was required to be proved. a) 14. 16. b) 3 C. 13. 15 c) Co. 1. 16.

PROP. 16. THEOR. 16.



The side B G, *or* E C, *of an Icosahedron* A B G D F H E C, *is the greater segment of that right* B H, *or* K L, *which is in power double to the side* A L, *of the Octohedron* A K D L, *inscribed in the same Icosahedron.*

Demonstration Forasmuch as figures inscribed and circumscribed have one and the same center [a], let the same be the point I. Now right lines drawn by that center to the middle sections of the opposite sides, to wit, the lines A I D and K I L, do in the point I cut one another into two equal parts, and perpendicularly [b]; and forasmuch as they couple the middle sections of the opposite lines B G and H F; therefore they cut them perpendicularly: Wherefore also the lines B G and H F are parallels [c]. Now then draw a line from B to H; and the said line B H shall be equal and parallel to the line K L [d]. But the line B H subtendeth two sides of the Pentagon which is composed of the sides of the Icosahedron, to wit, the sides B A and A H: Wherefore the line B H being cut by an extream and mean proportion, maketh the greater segment the side of the Pentagon [e]; which side is also the side of the Icosahedron, to wit, E C. And unto the line B H, the line K L is equal; and the line K L is in power double to A L, the side of the Octohedron [f]; for in the square A K D L, the angle K A L is a right angle. Wherefore E C the side of the Icosahedron, is the greater segment of the line B H or K L, &c. Which was required to be proved. a) C. 21. 15. b) C. 14. 15. c) 4 C. 14. 15 d) 33. 1. e) 8. 13. f) 47. 1.

 PROP.

PROP. 17. THEOR. 17.

The side of a Cube is to the side of a Dodecahedron inscribed in it, in double proportion of an extream and mean proportion.

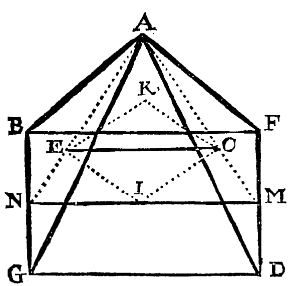
a) 2 C. 13. 15. *Demonstration* FOr it was manifest [a] that the side of a Cube divided by an extream and mean proportion, maketh the lesser segment the side of the Dodecahedron inscribed in it; but the whole is to the lesser segment in double proportion of that in which it is to the greater [b]; b) 10. def. 5. for the whole, the greater segment, and the lesser, are lines in continual c) 3. def. 6. proportion [c]: Wherefore the whole, to wit, the side of the Cube, is to the side of the Dodecahedron inscribed in it, to wit, to his lesser segment, in double proportion of an extream and mean proportion, to wit, d) 2. 14. of that which the whole hath to the greater segment [d].

PROP. 18. THEOR. 18.

The side of a Dodecahedron, is to the side of the Cube inscribed in it, in converse proportion of an extream and mean proportion.

a) 3 C. 13. 15. *Demonstration* IT was proved [a] that the side of a Dodecahedron circumscribed about a Cube, is the greater segment of the side of the same Cube. Wherefore the whole side of the Cube inscribed is to the greater segment, to wit, to the side of the Dodecahedron circumscribed, in an extream and mean proportion: Wherefore by conversion, the greater segment, that is, the side of the Dodecahedron, is to the b) 13. def. 5. whole, to wit, to the side of the Cube inscribed [b], &c. Which was required to be proved.

PROP. 19. THEOR. 19.



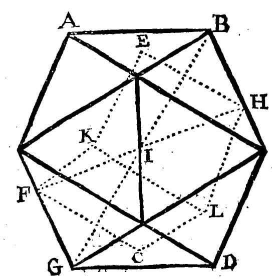
The side GD, *of an Octohedron* ABGDF, *is sesquialter to the side* EC, *of a Pyramis inscribed in it.*

a) C. 14. 13. *Construction* FOr [a] the Octohedron is cut into two quadrilateral Pyramids, one of which let be ABGDF; and let the centers of the circles which contein the four bases of the Octohedron be K, E, I, and C, and draw these right lines KE, EI, IC, CK, and EC. Wherefore KEIC is a square, and one of b) 4. 15. the bases of the Cube inscribed in the Octohedron [b]. And forasmuch as the

the angles of a Cube, and of the Pyramis inscribed in it, are set in the centers of the bases of the Octohedron circumscribed about the Cube [c]; c) 6. 15. and the side of the Pyramis coupleth the opposite angles of the base of the Cube [d]: It is manifest that the line EC is the side of the Pyramis in- d) 1. 15. scribed in the Octohedron ABGDF. I say then that GD the side of the Octohedron, is sesquialter to EC, the side of the Pyramis inscribed in it. From the point A, draw to the bases BG and FD, perpendiculars AN and AM, which [e] shall passe by the centers E and C, and draw the e) C. 12. 13. line NM.

Demonstration FOrasmuch as BGDF is a square [f], the lines NG and f) 14. 13. MD, shall be parallels and equal: For the lines BG and FD, are by the perpendiculars cut into two equal parts, in the points N and M [g]: Wherefore the lines NM and GD shall be parallels and g) 3. 3. equal [h]. And forasmuch as the lines AN and AM, which are the per- h) 33. 1. pendiculars of equal and like triangles, are cut alike in the points E and C, the lines EC and NM shall be parallels [i]. And therefore [k] the triangles i) 2. 6. AEC and ANM shall be like. Wherefore as the line AN is to the line k) Cor. 2. 6. AE, so is the line NM to the line EC [l]. But the line AN is sesquialter l) 4. 6. to the line AE, for the line AE is double to the line EN [m]. Wherefore m) C. 12. 13. the line NM or the line GD, which is equal unto it, is sesquialter to the line EC. Wherefore, &c. Which was required to be proved.

PROP. 20. THEOR. 20.

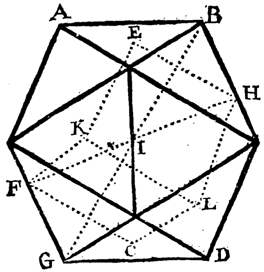


If from the power of the diameter BG, *of an Icosahedron* ABGD, *be taken away the power tripled of the side* EH, *of the Cube inscribed in the Icosahedron, the power remaining shall be sesquitertia to the power of the side* AB, *of the Icosahedron.*

Construction LEt the two bases of the Cube be EHKL, and LKFC joyned; and let the diameter of the Cube be FH.

Demonstration FOrasmuch as the centers of inscribed and circumscribed figures are in one and the same point [a], the diameters BG a) C. 21. 15. and FH, shall in one and the same point cut one another into two equal parts; for by the same Corollary it hath been taught that the tops of equal and like Pyramids, do in that point concurre; let the point be the center I. Now the angles of the Cube which are at the points F and H, are set at the centers of the bases of the Icosahedron [b]: Wherefore the b) 11. 15. line FH shall be perpendicular to both the bases of the Icosahedron. Where-

c) 47. 1. Wherefore the line I B conteineth in power the two lines I H and H B c. But the line H B is drawn from the center of the circle which conteineth the base of the Icosahedron, to wit, the angle B, is placed in the circumference, and the point H is the center. Wherefore the whole line B G



conteineth in power the whole lines F H, and the diameter of the circle (to wit, the double of the line B H) d. But the diameter which is double to the line H B, is in power sesquitertia to the side of the equilateral triangle inscribed in the same circle e; for it is in proportion to the side, as the side is to the perpendicular f. And F H the diameter of the Cube, is in power triple to E H, the side of the same Cube g: If therefore from the power of the diameter B G, be taken away the power tripled of E H, the side of the Cube inscribed; that is, the power of the line F H; the residue (to wit, the power of the diameter of the circle, which is double to the line H B) shall be sesquitertia to the side of the triangle inscribed in that circle, which side is A B the side of the Icosahedron: If therefore, &c. Which was required to be proved.

d) 15. 5.

e) C. 12. 13.

f) Cor. 8. 6.

g) 15. 13.

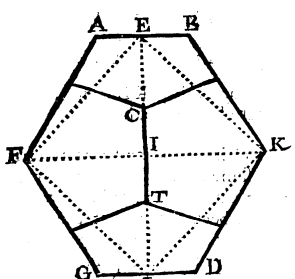
COROLLARIE.

The Diameter of the Icosahedron, conteineth in power two lines, to wit, the Diameter of the Cube inscribed, which coupleth the centers of the opposite bases, and the Diameter of the Circle which conteineth the base of the Icosahedron.

For it was manifest, that B G the diameter, conteineth in power the line F H, which coupleth the centers, and the double of the line B H, that is, the diameter of the circle conteining the base wherein the center H is.

PROP. 21. THEOR. 21.

The side A B, *of a Dodecahedron* A B G D C T, *is the lesser segment of that right line which is in power double to the side* E F, *of the Octohedron* E L F K I, *inscribed in the same Dodecahedron.*



Construction Draw the diameters E L and F K, of the Octohedron. Now they couple the middle sections of the opposite sides of the Dodecahedron A B and G D a; and all those diameters being divided by an extream and mean proportion, do make the lesser segment the side of the Dodecahedron b: Wherefore the side A B is the lesser segment of the line F K. But the line F K conteineth in power the two equal lines E F and E K c; for the angle F E K is a right angle of the square F E K L, of the Octohedron. Wherefore the line F K is in power double to the line E F. Wherefore the line A B (the side of the Dodecahedron) is the lesser segment of the line F K, which is in power double to E F, the side of the Octohedron: Therefore, The side of a Dodecahedron, &c. Which was required to be proved.

a) 9. 15.
C. 3. 17. 13.
b) 4 Cor. of 17. 13.
c) 47. 1.

PROP. 22. THEOR. 22.

The Diameter of an Icosahedron is in power sesquitertia to the side of the same Icosahedron, and also is in power sesquialter to the side of the Pyramis inscribed in the Icosahedron.

Demonstration Forasmuch as it hath been proved a that if from the power of the diameter of the Icosahedron, be taken away the triple of the power of the side of the Cube inscribed in it, there shall be left a square, sesquitertia to the square of the side of the Icosahedron: But the power of the side of the Cube tripled, is the diameter of the same Cube b. And the Cube and the Pyramis inscribed in it, are conteined in one and the same sphere c, and in one and the same Icosahedron d: Wherefore one and the same Diameter of the Cube, or of the Sphere which conteineth the Cube and the Pyramis, is in power sesquialter to the side of the Pyramis e. Wherefore it followeth that if from the Diameter of the Icosahedron be taken away the triple power of the side of the Cube, or the sesquialter power of the side of the Icosahedron: The diameter therefore, &c. Which was required to be proved.

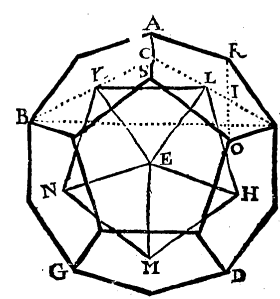
a) 20. 16.

b) 15. 13.
c) 1. 16.
d) Co. 1. 16.

e) 13. 13.

PROP. 23. THEOR. 23.

The side A S, *or* S O, *of a Dodecahedron* A B G D F S O, *is to the side* K L, *of an Icosahedron* K L H M N E, *inscribed in it, as the lesser segment* I F, *of the perpendicular* C F, *of the Pentagon, is to that line* L C, *which is drawn from the center to the side* A S, *of the same Pentagon.*

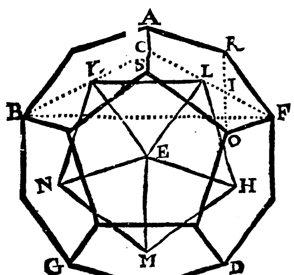


Demon-

Construction FRom the two angles of the Pentagons B A S and F A S, of the Dodecahedron, to wit, from the angles B and F; let there be drawn to the common base A S, perpendicular lines B C and F C, which shall passe by the centers K and L of the said Pentagons [a]: Draw the lines B F and R O. Now forasmuch as the line R O subtendeth the angle O F R, of the Pentagon of the Dodecahedron, it shall cut the line F C by an extream and mean proportion [b]; let it cut it in the point I. And forasmuch as the line K L is the side of the Icosahedron inscribed in the Dodecahedron, it coupleth the centers of the bases of the Dodecahedron; for the angles of the Icosahedron are set in the centers of the bases of the Dodecahedron [c].

a) C. 10. 13.
b) 3. 16.
c) 7. 15.

Demonstration FOrasmuch as in the triangle B C F, the two sides C B and C F, are in the centers L and K, cut like proportionally, the lines B F and K L shall be parallels [d]: Wherefore the triangles B C F and K C L, shall be equiangled [e]. Wherefore as the line C L is to the line K L, so is the line C F to the line B F [f]. But C F maketh the lesser segment the line I F [g]; and the line B F maketh the lesser segment the line S O, to wit, the side of the Dodecahedron [h]; for the line B F, which coupleth the angles B and F, of the bases of the Dodecahedron, is equal to the side of the Cube which conteineth the Dodecahedron [i]: Wherefore as the whole line C F, is to the whole line B F, so is the lesser segment I F, to the lesser segment S O [k]. But as the line C F is to the line B F, so is the line C L to the line K L. Wherefore alternately, [l] as the line I F the lesser segment of the perpendicular of the Pentagon F A S, is to the line L C, which is drawn from the center of the Pentagon, to the base; so is the line S O the side of the Dodecahedron; to the line K L the side of the Icosahedron inscribed in it. Therefore, The side, &c. Which was required to be proved.



d) 2. 6.
e) Cor. 2. 6.
f) 4. 6.
g) 3. 16.
h) 2 C. 13. 15
i) 13. 15.
k) 2. 14.
l) 16. 5.

PROP. 24. THEOR. 24.

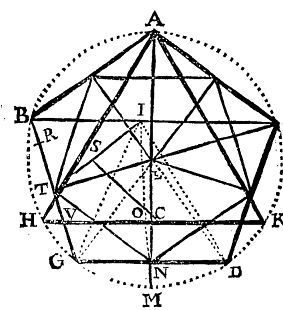
If half of the side of an Icosahedron be divided by an extream and mean proportion, and if the lesser segment thereof be taken from the whole side; and again, from the residue be taken away the third part, that which remaineth shall be equal to the side of the Dodecahedron inscribed in the same Icosahedron.

Construction SUppose A B G D F be a Pentagon, conteining five sides of the Icosahedron [a], and let it be inscribed in a circle, whose center let be E. And upon the sides of the Pentagon let there be raised triangles, making a solid angle of the Icosahedron at the point I [b]. And in the circle A B D, inscribe an equilateral triangle A H K. From the center E, draw to H K the side of the triangle, and G D the side of the Pentagon, a perpendicular line, which let be E C N M; and draw the lines E G, E D, I G, and I D. And divide the line B G into two equal parts in the point T. And draw the lines I N, I T, T N, and E T. And forasmuch as in the perpendiculars I T and I N, are the centers of the circles which contein the equilateral triangles I B G and I G D [c]: Let those centers be the points S and O, and draw the line S O. Divide the line T B the half of B G, the side of the Icosahedron, by an extream and mean proportion in the point R [d]; and let the lesser segment thereof be R B. And forasmuch as the line S O coupleth the centers of the triangles I B G and I G D, it is [e] the side of the Dodecahedron inscribed in the Icosahedron, whose side is the line B G, from the side B G take away B R, the lesser segment of the half side. And from the residue G R, take away the third part G V [f]; then I say that the residue R V is equal to S O, the side of the Dodecahedron inscribed.

a) 16. 13.

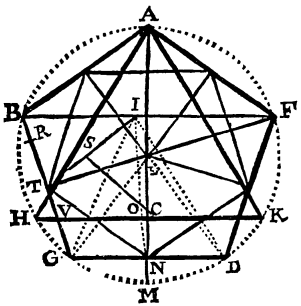
b) 16. 13.
c) Cor. 1. 13.
d) 30. 6.
e) 5. 15.
f) 9. 6.

Demonstration FOrasmuch as the perpendicular E N is in the point C divided by an extream and mean proportion [g], and the greater segment thereof is the line E C, and unto the line E C the line C M is equal [h]. Wherefore the line E C is to the line C N, as the line C N is to the same C N [i]. But as the line E C is to the line C N, so is the whole line E N, to the greater segment E C [k]. Wherefore [l] as the whole line E N is to the greater segment E C, so is the line C M to the line C N. Wherefore the line C M is divided by an extream and mean proportion in the point N, to wit, is divided like unto the line E N [m]. Wherefore the line E M exceedeth the line E N by the lesser segment of his half, to wit, by M N. And forasmuch as E G D is the triangle of an equilateral and equiangled Pentagon A B G D F. Therefore [n] the triangle E T N is like to the triangle E G D. Wherefore as the line E G is to the line E N, so [o] is the line G D to the line N T. Wherefore the line G D (or B G which is equal unto it) exceedeth the line N T by the lesser segment of the halfe of B G; for the line E G did in like sort exceed the line E N. But that lesse segment is the line B R. Wherefore the residue R G is equal to the line T N. And forasmuch as I B G is an equilateral triangle, the perpendicular S T shall be the half of the line S I, which is drawn from the center P. Wherefore the line I T exceedeth the line I S by his third part. And forasmuch as the line S O, which coupleth the sections, is a parallel to the line T N [q]; for the equal perpendiculars I T and I N, are cut like in the points S and O. Therefore the



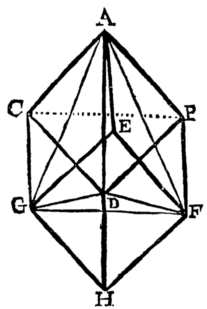
g) Cor. 1. 14.
h) C. 12. 13.
i) 7. 5.
k) 3. def. 6
l) 11. 5.
m) 2. 14.
n) 20. 6.
o) 4. 6.
p) C. 12. 13.
q) 2. 6.

r) Cor. 2.6. s) 4. 6. the triangles ITN and ISO are like [r]. Wherefore as the line IT is to the line IS; ſo [s] is the line TN to the line SO. But the line IT exceedeth the line IS by a third part. Wherefore the line TN exceedeth the line SO by a third part. But the line TN is proved equal to the line RG. Wherefore the line RG exceedeth the line SO by a third part of it ſelfe which is GV. Wherefore the reſidue RV is equal to the line SO, which is the ſide of the Dodecahedron inſcribed in the Icoſahedron, whoſe ſide is the line BG. If therefore half of the ſide of an Icoſahedron be divided by an extream and mean proportion, &c. Which was required to be proved.



PROP. 25. THEOR. 25.

To prove that a given Cube ABCH *is triple to a trilateral equilateral Pyramis* AGDF, *inſcribed in it.*



Demonſtration Foraſmuch as the baſe AFD is common to the Pyramis AFDB and AFDG, the Pyramis AFDB ſhall be ſet without the Pyramis AFDC. Likewiſe the reſt of the baſes of the inſcribed Pyramis are common to the reſt of the Pyramids ſet without, which are the Pyramis AGDC upon the baſe AGD, the Pyramis AGFE upon the baſe AGF, and GDFH upon the baſe GDF,

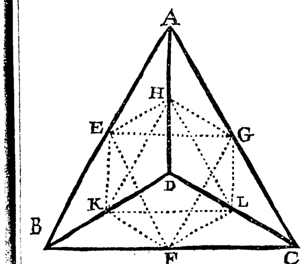
a) 8. de. 11. which Pyramids taken without, are four in number, equal and alike [a]; for every one of them is conteined under three half ſquares of the Cube, and one of the baſes of the Pyramis inſcribed. Wherefore each of them is conteined under the half baſe of the Cube, and the altitude of the Cube. As the Pyramis AEGF hath to his baſe half of the ſquare EH, to wit, the triangle EGF, and hath to his altitude the altitude of the Cube, to wit, the line AE. Wherefore the ſaid Pyramis is the ſixth part of the Cube. For if the Cube be divided into two Priſmes by the Plain CBFG, the priſme ACBGEF ſhall be triple to the Pyramis AEGF, having one and the ſame baſe with it EGF, and one and the

b) 1C.7.12. ſame altitude EA [b]. Wherefore the ſaid outward Pyramis AEGF is the ſixth part of the whole Cube. Wherefore alſo the ſame Pyramis, together with the other three outward Pyramids AFDB, AGDC, and GDFH, ſhall contein two third parts of the Cube. Wherefore the re-

reſidue, to wit, the Pyramis inſcribed AGDF, ſhall contein one third part of the Cube. And therefore converſely, the Cube ſhall be triple to it. Wherefore we have proved that a Cube, &c.

PROP. 26. THEOR. 26.

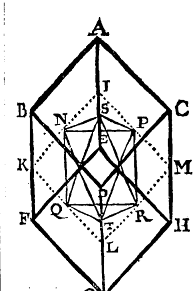
To prove that a trilateral Pyramis ABCD, *is double to an Octohedron inſcribed in it* EGLKHF.



Conſtruction Let the ſides of the Pyramis be cut into two equal parts in the points E, K, F, G, L, and H, inſcribing thereby an Octohedron in the Pyramis [a]. a) 2. 15.
Wherefore the Pyramids AEGH, BEFK, CFGL, and DKHL,
ſall without the Octohedron inſcribed [b]. But the outward Pyramis, to b) 2. 15.
wit, AEGH, and the three other, are like unto the whole Pyramis [c]. c) 7. def. 11.
For the baſes of the whole Pyramis are by parallel lines drawn in them,
cut into like triangles [d], of which the foreſaid Pyramids are made. d) 2. 6.
Wherefore the whole Pyramis is to every one of them in treble proportion of that in which the ſides of like proportion are [e]. But by Con- e) 8. 12.
ſtruction, the proportion of the ſide AB to the ſide AE is double. Wherefore the whole Pyramis ABCD is octuple to the Pyramis AEGH, and ſo is it to each of the Pyramids which are equal to AEGH. For double proportion multiplied into it ſelfe twice, maketh octuple. Wherefore it followeth that AEGH, BEFK, CFGL, and DKHL; taken together, make the halfe of the whole Pyramis ABCD, Wherefore the reſidue, to wit, the Octohedron EGLKHF, is the other half of the Pyramis. Wherefore the Pyramis is double to the Octohedron. Wherefore we have proved that a trilateral equilateral Pyramis, &c.

PROP. 27. THEOR. 27.

To prove that a Cube is ſextuple to an Octohedron inſcribed in it.

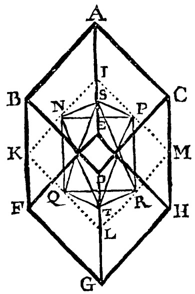


Conſtruction Let the Cube ABCD, EFGH, whoſe four ſtanding lines AE, BF, CH, and DG, be cut into two equal parts in the points I, K, M, and L, and by thoſe points let there be extended a plain KLMI, which ſhall be a ſquare, and parallel to the ſquares BC and FH [a]. Wherefore in it ſhall be the a) 15. 11.
baſe which is common to the two Pyramids of the Octohedron inſcribed in the Cube [b]. Let b) 3. 15.
that baſe be NPRQ, coupling the centers of the baſes of the Cube, & upon that baſe let be ſet the

two

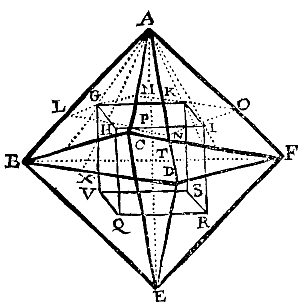
two Pyramids of the Octohedron, which let be N P Q R S and N P Q R T.

Demonstration. FOrasmuch as these two Pyramids taken together have their altitude equal with the altitude of the whole Cube, each of them apart hath to his altitude half the altitude of the Cube, to wit, half the side of the Cube, as the line K B. And forasmuch as the
c) 47. 1. square K L M I is double to the square N R Q P c, the other squares of the Cube shall also be double to the square N R Q P.
And forasmuch as the Cube d is resolved into
d) ult. 15. six Pyramids, whose bases are the bases of the Cube, and the altitudes the lines drawn from the center to the bases, which are equal to half the side of the Cube, it followeth that each of the six Pyramids of the Cube, having his base double to the base of each of the Pyramids of the Octohedron, and the same altitude that the said Pyramids of the Octohedron have, is double to either of the Pyramids of the Octo-
e) 6. 12. hedron e. And forasmuch as each of the Pyramids of the Cube is equal to the two Pyramids of the Octohedron, the six Pyramids of the Cube shall be sextuple to the whole Octohedron. Wherefore we have proved that a Cube is sextuple to an Octohedron inscribed in it.



PROP. 28. THEOR. 28.

To prove that an Octohedron ABCDEF, *is quadruple sesquialter to a Cube* GHIK, VQRS, *inscribed in it.*



Demonstration. FOrasmuch as the lines drawn from the center of the Octohedron, or of the Sphere which conteineth it, unto the centers of the bases of the
a) 21. 15. Octohedron are proved equal a; and the angles of the Cube are set
b) 4. 15. in the centers of these bases b: It followeth that the same right lines are drawn from one and the same center of the Cube, and of the Octohedron;
c) C. 21. 15. for they have each one and the same center c. Let that center be the point T. Wherefore the base B D F C, which cutteth the Octohedron
d) C. 14. 13. into two equal and quadrilateral Pyramids d, shall also cut the
e) C. 39. 11. Cube into two equal parts e. For it passeth by the center T f. And foras-
f) 14. 13. much as the base of the Cube is in the four centers G, H, I, and K, of the bases of the Pyramis A B D F C, a plain L N O M extended by those
g) 4. 15. points, shall be parallel to the plain B D F C g, and shall cut the Pyramis in

the points L, N, O, and M, and the lines L N, B D, and N O, D F, shall be parallels, so also shall the lines O M, F C, and L M, B C, and the square G H I K, of the Cube, shall be inscribed in the square L N O M.
Wherefore the square L N O M is double to the square G H I K h. From h) 47. 1.
the solid angle A, let there be drawn to the plain Superficies B D C F, a perpendicular, which let fall upon it, in the point T; and let the same perpendicular be A T, cutting the plain L N O M in the point P. And it
shall also be a perpendicular to the plain L N O M i. Again, from the an- i) Co. 14. 11.
gle B A D of the triangle A D B, let there be drawn by the center H, of the triangle, to the base, a line A H X. Wherefore the line A X is ses-
quialter to the line A H k. Wherefore the line A H is double to the line k) C. 12. 13.
HX. But the other lines A B, A D, A F, and A C, and the perpendicu-
lar A P T, are cut like unto the line A H X l. Wherefore the line A P is l) 17. 11.
double to the line P T. Wherefore the line A P is the altitude of the Cube; for the line P T is the half thereof. And forasmuch as upon the base G H I K of the Cube, and under the altitude A P, of the same Cube, is set the Pyramis A G H I K, the said Pyramis is the third part
of the Cube m. But unto the Pyramis A G H I K the Pyramis A L N O M m) C. 7. 12.
is double n; for the base of the one is double to the base of the other. n) 6. 12.
Wherefore the Pyramis A L N O M is two third parts of the Cube.
And forasmuch as the Pyramids A L N O M and A B D F C, are like o; o) 7. def. 11.
therefore they are in triple proportion of that in which the sides of like
proportion AH to AX, or A L to A B are p. But the side A B is proved to p) Co. 8. 12.
be sesquialter to the side A L. Wherefore the Pyramis A B C D F is to the Pyramis A L N O M, as 27 is to 8 (which is sesquialter proportion tripled; for the quantity or denomination of sesquialter proportion, to wit, $1\frac{1}{2}$, multiplied into it selfe once, it maketh $2\frac{1}{4}$, which again multiplied by $1\frac{1}{2}$, maketh $3\frac{3}{8}$; that is 27 to 8. But of what parts the Pyramis A L N O M conteineth 8, of the same the Cube conteineth 12, of the same the whole Octohedron (which is double to the Pyramis A B D F C) conteineth 54, which 54 hath to 12 quadruple sesquialter proportion: Wherefore the whole Octohedron is to the Cube inscribed in it in quadruple sesquialter proportion. Wherefore we have proved, &c.

COROLLARIE.

An Octohedron is to a Cube inscribed in it, in that proportion that the squares of their sides are.

For q the side of the Octohedron is in power quadruple sesquialter to q) 14. 16.
the side of the Cube inscribed in it.

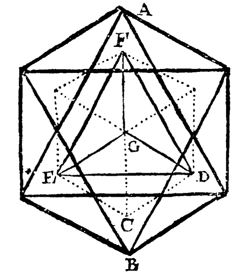
PROP. 29. THEOR. 29.

To prove that a given Octohedron AB, *is tredecuple sesquialter (that is, as* $13\frac{1}{2}$ *to* 1*) to a trilateral equilateral Pyramis* FEGD, *inscribed in it.*

Construction. LEt there be inscribed in the given Octohedron a Cube F C a) 4. 15.
E D a, and in the Cube a Pyramis b. b) 1. 15.

Demonstration. FOrasmuch as the angles of the Pyramis are c set in the c) 1. 15.
angles of the Cube, and the angles of the Cube are set in

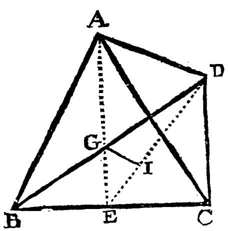
the centers of the baſe of the Octohedron, to wit, in the points F, E, C,
d) 4.15. D, and G d. Wherefore the angles of the Pyramis are ſet in the centers FC and ED of the Octohedron. Wherefore the Pyramis FEDG is
e) 6.15. inſcribed in the Octohedron e. And foraſmuch as the Octohedron AB is to to the Cube FCED inſcribed in it, quadruple ſeſquialter (by the former Prop.) and the Cube CDEF is to the
f) 25.16. Pyramis FEDG inſcribed in it triple f. Wherefore three Magnitudes being given, to wit, the Octohedron, the Cube, and the Pyramis, the proportion of the extreames (to wit, of the Octohedron to the Pyramis) is made of the proportion of the meanes (to wit, of the Octohedron to the Cube, and of the Cube to the Pyramis g.) Now then multi-



g) 11.5. ply the quantities or denominations of the proportions (to wit, of the Octohedron to the Cube, which is $4\frac{1}{2}$, and of the Cube to the Pyramis,
h) def. 6. which is 3 h,) there ſhall be produced $13\frac{1}{2}$, to wit, the proportion of the Octohedron to the Pyramis inſcribed in it. For $4\frac{1}{2}$ multiplyed by 3, produce $13\frac{1}{2}$. Wherefore the Octohedron is to the Pyramis inſcribed in it in tredecuple ſeſquialter proportion. Wherefore we have proved, &c.

PROP. 30. THEOR. 30.

To prove that a trilateral equilateral Pyramis is noncuple to a Cube inſcribed in it.

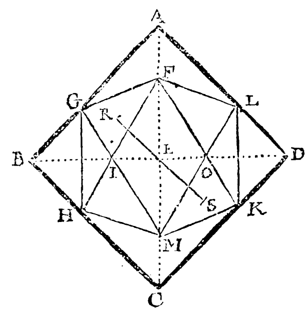


Conſtruction Let the given pyramis be ABCD, whoſe two baſes let be ABC and DBC, and let their centers be G and I, and from the angle A, draw unto the baſe BC, a perpendicular AE, alſo from the angle D, draw unto the
a) 3.3. ſame baſe BC, a perpendicular DE, and they ſhall meet in the ſection E a,
b) Co. 1.3. and in them ſhall be the centers G and I b.

Demonſtration Foraſmuch as the line GI coupleth the centers of the baſes of the pyramis, the ſaid line GI ſhall be the diame-
c) 18.15. ter of the baſe of the Cube inſcribed in the pyramis c. And foraſmuch as
d) C.12.13. the line AG is double to the line GE d, the whole line AE ſhall be triple to the line GE, and ſo is alſo the line DE to the line IE. Where-
e) 2.6. fore the lines AD and GI are parallels e. And therefore the triangles
f) Cor. 2.6. AED and GEI are like f. And foraſmuch as the triangles AED and
g) 4.6. GEI are like, the line AD ſhall be triple to the line GI g. But the line AD is the diameter of the baſe of the Cube circumſcribed about the pyramis ABCD, and the line GI is the diameter of the baſe of the Cube inſcribed in the pyramis ABCD; but the diameters of the baſes are

are equimultiplices to the ſides (to wit, are in power double.) Wherefore the ſide of the Cube circumſcribed about the pyramis ABCD, is triple to the ſide of the Cube inſcribed in the ſame pyramis h. But like h) 15.5.
Cubes are in triple proportion to one another of that in which their ſides are i, and the ſides are in triple proportion to one another. Wherefore triple taken three times, bring forth twenty ſeven euple, which is 27 to 1; i) 33.11.
for the four terms 27, 9, 3, and 1, being ſet in triple proportion, the proportion of the firſt to the fourth (to wit, of 27 to 1,) ſhall be triple to the proportion of the firſt to the ſecond, to wit, of 27 to 9 k. Which k) 10.de.5.
proportion of 27 to 1, is the proportion of the ſides tripled, which proportion alſo is found in like ſolids. Wherefore of what parts the Cube circumſcribed conteineth 27, of the ſame the Cube inſcribed conteines 1. But of what parts the Cube circumſcribed conteineth 27, of the ſame the pyramis inſcribed in it conteineth 9 l. Wherefore of what parts l) 25.6.
the pyramis ABCD conteineth 9, of the ſame the Cube inſcribed in the Pyramis conteineth 1. Wherefore we have proved, &c.

PROP. 31. THEOR. 31.



An Octohedron ABCD, *hath to an Icoſahedron* FGHMKLIO, *inſcribed in it, that proportion which two baſes of the Octohedron have to five baſes of the Icoſahedron.*

Demonſtration Foraſmuch as the ſolid of the Octohedron conſiſteth of eight pyramids, ſet upon the baſes of the Octohedron, and having to their altitude a perpendicular line drawn from the center to the baſe. Let that perpendicular be ER or ES, being drawn from the center E (which center is common to either of the ſolids a) to the centers of the baſes, to a) C.21.15.
wit, to the points R and S. Wherefore ſeeing that three pyramids are equal and like, they ſhall be equal to a priſme ſet upon the ſame baſe, and under the ſame altitude b. But to this priſme is double that priſme which b) Co.7.12.
is ſet upon the ſame baſe, and hath his altitude double, to wit, the whole line RS c; for it is equal to the two equal and like priſmes where- c) C.25.11.
of it is compoſed. Wherefore the priſme ſet upon the baſe of the Octohedron, and having to his altitude the line RS, is equal to ſix piramids, ſet upon ſix baſes of the Octohedron, and having to their altitude the line ER, ſo there remain two pyramids (for in the Octohedron are eight baſes) which ſhall be equal to the priſme which is ſet upon the third part of the baſe of the Octohedron, and under the altitude RS, for priſmes under one and the ſame altitude, are in proportion to one another as are their baſes d. Wherefors the two priſmes which are ſet upon the baſe of the d) Co.7.12.
the

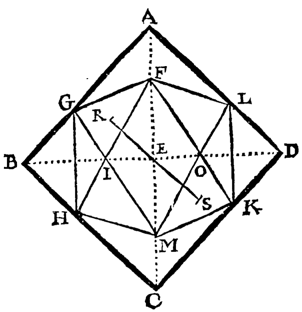
the Octohedron, and upon a third part thereof, and under the altitude R S, are equal to the eight pyramids of the Octohedron, or to the whole Solid of the Octohedron. And forasmuch as the Icosahedron inscribed in the Octohedron, hath his bases set in the bases of the Octohedron [c]; it followeth that the pyramids set upon the bases of the Icosahedron, and having to their tops one and the same center E, are conteined under the same altitude that the pyramids of the Octohedron are conteined under, to wit, under the line E R or E S; and therefore a prisme set upon the base of the Icosahedron, and having his altitude double to the altitude of the Pyramis, to wit, the whole line R S, is equal to the six pyramids set upon the base of the Icosahedron, and under the altitude E R or E S, as we have proved in the Octohedron. Wherefore the twenty pyramids set upon the twenty bases of the Icosahedron, are equal to three prismes set upon the base of the Icosahedron, and under the altitude R S; and morover, to another prisme set upon a third part of the base of the Icosahedron, and under the same altitude R S, which prisme is a third part of the former prisme [f]; for their proportion is as the proportion of the bases. Wherefore two prismes set upon the base of the Octohedron, and a third part thereof, and under the altitude R S, is to four prismes set upon the three bases of the Icosahedron, and a third part thereof, and under the same altitude R S, in the same proportion that the bases are, that is, as four third parts of the base of the Octohedron (which are equal to one base and $\frac{1}{3}$) to 10 third parts of the base of the Icosahedron (which are equal to 3 bases and $\frac{1}{3}$) or as 2 third parts of the base of the Octohedron are to 5 third parts of the base of the Icosahedron. But 2 third parts of the base of the Octohedron, are to 5 third parts of the base of the Icosahedron, as two bases are to 5 bases [g] (for they are parts of equimultiplices.) And two prismes of the Octohedron are to four prismes of the Icosahedron, as the solid of the Octohedron to the solid of the Icosahedron, when as each are equal to each of the solids. Wherefore [h] the solid of the Octohedron is to the solid of the Icosahedron inscribed in it, as two bases of the Octohedron are to five bases of the Icosahedron. Therefore, &c. Which was required to be proved.

c) 17. 15.

f) Co. 7. 12.

g) 15. 5.

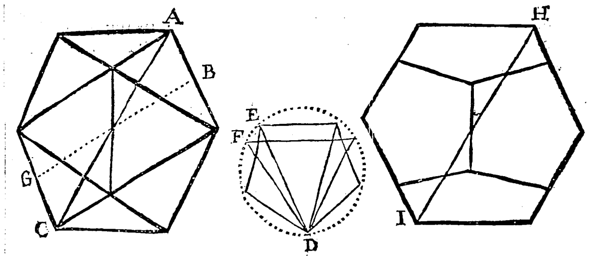
h) 11. 5.



PROP. 32. THEOR. 32.

The proportion of the solid of an Icosahedron A B G C, *to the solid of a Dodecahedron inscribed in it, consisteth of the proportion of the side* D F *of the Icosahedron, to the side* D E *of the Cube, conteined in the same Sphere, and of the proportion*

tion tripled of the Diameter A C, *to the line* B G, *which coupleth the centers of the opposite bases of the Icosahedron.*



Demonstration. Forasmuch as the solid of the Icosahedron A B G C is to the solid of the Dodecahedron H I, being conteined in one and the same sphere, as D F is to D E [a]. But the Dodecahedron whose diameter is H I, is to the Dodecahedron, whose diameter is B G, in triple proportion of that in which the diameter H I is to the diameter B G [b]; and the lines H I and A C are equal by supposition (to wit, the diameters of one and the same sphere.) Wherefore as H I is to B G, so is A C to B G. Wherefore the proportion of the extreames (to wit, of the Icosahedron A B G C, to the Dodecahedron set upon the diameter B G, which coupleth the centers, consisteth [c] of the proportions of the means, to wit, of the proportion of A B G C to the Dodecahedron H I, (which is one and the same with the proportion of D F to D E) and of the proportion of the same H I to the other Dodecahedron set upon the diameter B G, inscribed in the same Icosahedron A B G C [d]; which proportion is triple to the proportion of the line H I (or the line A C) to G B, which coupleth the centers of the opposite bases of the Icosahedron. The proportion therefore, &c. Which was required to be proved.

a) 8. 14.

b) C. 17. 12.

c) 5. def. 6.

d) 5. 15.

PROP. 33. THEOR. 33.

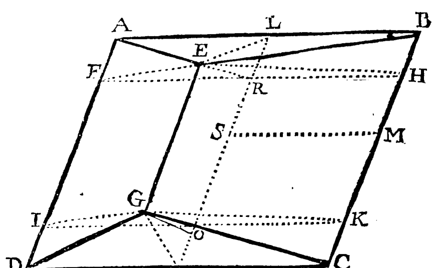
The solid of a Dodecahedron exceedeth the solid of a Cube inscribed in it, by a Parallelepipedon, whose base wanteth of the base of the Cube by a third part of the lesser segment, and whose altitude wanteth of the altitude of the Cube, by the lesser segment of the lesser segment of half the side of the Cube.

Construction. Forasmuch as [a] it was manifest that the base of a Cube inscribed in a Dodecahedron, doth with his sides subtend the angles of four Pentagons, meeting at one and the same side of the Dodecahedron: Let that base of the Cube be A B C D; and let the side whereat four bases of the Dodecahedron circumscribed meet, be E G, which shall contein a solid A E B D G C, set upon the base A B C D. Divide the

a) 17. 13. 18. 15.

the sides A B and D C into two equal parts in the points L and N, and
draw the line L N, which is a parallel to the side E G b. The perpen-
b) 17. 13. diculars also E R and G O, which couple those parallels, are each equal
to halfe of the side E G, and each is the greater segment of half the
side of the Cube. And therefore the whole line E G is the greater seg-
ment of the whole line L N, the side of the Cube c; by the points R and
c) 17. 13. O, draw unto the sides A B and C D, parallel lines F H and I K, and
draw these right lines EF, E H, G I, and G K.

Demonstration Forasmuch as the two lines F H and E R, touching one ano-
ther, are parallels to the two lines I K and G O, touching
also one another, and not being in the same plain with the two first lines.
Therefore the plain Superficies E F H and G I K, passing by those lines,
d) 15. 11. are parallels d; which plains do cut the solid A E B D G C. Wherefore
there are made four quadrangled pyramids set upon the rectangle paralle-
grams L H, L F, N K, and N I, and having their tops the points E and
G. And forasmuch as the triangles G O K and E R H, are equal and
e) 4. 1. like e; to wit, they contein equal angles comprehended under equal sides,
and they are parallels by construction, being set in the plains G I K and E F H, the figures G K H E, O K H R, and G O R E, shall be Parallelograms, by the definition of Parallelograms; and therefore the solid G O K E R H



f) 11. de. 11. is a prisme f. And
by the same reason may the solid G O I E R F be proved to be a prisme. And
forasmuch as upon equal bases N O K C and R L B H, and under equal al-
titudes O G and R E are set pyramids, these pyramids shall be equal to
that pyramis which is set upon the base C K I D (which is double to ei-
ther of the bases N O K C and R L B H) and under the same altitude
g) 6. 12. O G g. And forasmuch as the side G E is the greater segment of the line
h) 33. 1. C B, the line K H which h is equal to the line G E, shall be the greater
i) 2. 14. segment of the same line C B i. Wherefore the residues C K and H B,
shall make the lesser segment of the whole line C B. But as the greater seg-
ment K H is to the two lines C K and H B, the lesser segment, so is the
rectangle Parallelogram O H, to the two rectangle Parallelograms O C
k) 1. 6. H L k. Wherefore the pyramis set upon the base O K M S, conteineth
l) Cor. 7. 12. two third parts of the prisme set upon the same base l. Wherefore the
prisme which is set upon two third parts of the base O K M S, is equal
to the two pyramids N O K C G and R L B H E. For the sections of a
m) Cor. of prisme are to one another, as the sections of the base are m. But the secti-
25. 11. ons of the base are as the sections of the line C B or K M n. Wherefore
n) 1. 6. adde the two pyramids N O K C G and R L B H E, unto the prisme
G O K E F H,

G O K E R H, two third parts of the prisme set upon the base O K M S.
And forasmuch as the line K M is the lesser segment of the whole line B C
(for it is equal to the two lines C K and H B) and the prisme set upon the
base O K H R is cut like unto the line K M, to wit, in each are taken two
thirds, as hath been proved, the prisme equal to the two pyramids, shall
adde unto the prisme G O K E R H, which is set upon the greater seg-
ment K H, two thirds of the lesser segment. Wherefore in the line B C
there shall remain one third part of the lesser segment; and therefore in
the rectangle Parallelogram N B, which is half the base of the Cube,
there shall remain the same third part of the lesser segment. And by the
same reason may we prove that in the other pyramids O N D I G and
R L A F E, and in the prisme G O I E R F is left the same excesse of the
base L A N D, to wit, the third part of the lesser segment. Wherefore
the whole prisme conteined between the triangles I G K and F E H, and
under the length of the greater segment, and two third parts of the lesser
segment of B C the side of the Cube, is equal to the solid composed of
four bases of the Dodecahedron, and set upon the base of the Cube.
Wherefore the base of that prisme wanteth of the whole base of the
Cube, only a third part of the lesser segment, and the altitude of that
prisme was the line G O, which is the greater segment of half the side of
the Cube. And forasmuch as unto the triangle I G K, is double the re-
ctangle Parallelogram set upon the same base I K (the side of the Cube)
and under the altitude G O o. It followeth that three rectangle paralle- o) 41. 1.
logrames set upon the same I K, the side of the Cube, and under the alti-
tude O G, the greater segment of half the side of the Cube, are sextuple
to the triangle I G K. Wherefore these three rectangle parallelogrames do
make one rectangle Parallelogram set upon the base I K, and under the al-
titude of the line G O tripled. But p there are six prismes equal and like p) 7. def. 11.
unto the foresaid prisme, being set upon each of the six bases of the Cube;
which prismes are in proportion to one another as their bases are q. q) 3 C. 7. 12.
Wherefore the solid composed of these six Prismes, shall want of the base
A B C D the third part of the lesser segment, and taking his altitude of
the foresaid rectangle Parallelogram, the said altitude shall be equal to
three greater segments (one of which is G O) of halfe the side of the
Cube.

Now resteth to prove that these three segments want of the side of the
Cube by the lesser segment of the lesser segment of half the side of the
Cube.

Suppose that A B the side of the Cube, be divided into the greater seg-
ment A C, and into the lesser segment C B r; divide into two equal r) 30. 6.
parts the line A C in the point G, and the line C B in the point E. And

unto the line C G, put the line C L equal. Now forasmuch as the lines
A G and G C, are the greater segments of half the line A B, for each
of them is the half of the greater segment of the whole line A B, the lines
E B and E C shall be the lesser segments of half the line A B. Wherefore
the whole line C L is the greater segment, and the line C E is the lesser
 seg-

ſegment. But as the line C L is to the line C E, ſo is the line C E to the reſidue E L. Wherefore the line E L is the greater ſegment of the line C E, or of the line E B, equal thereto. Wherefore the reſidue L B is the leſſer ſegment of the ſame E B (which is the leſſer ſegment of half the ſide of the Cube.) But the lines A G, G C, and C L, are three greater ſegments of the half of the whole line A B, which three greater ſegments

make the altitude of the foreſaid ſolid. Wherefore the altitude of the ſaid ſolid wanteth of A B the ſide of the Cube by the line L B, which is the leſſer ſegment of the line B E, which line B E again is the leſſer ſegment of half the ſide A B of the Cube. Wherefore the foreſaid ſolid conſiſting of ſix ſolids, whereby the Dodecahedron exceedeth the Cube inſcribed in it, is ſet upon a baſe which wanteth of the baſe of the Cube, by a third part of the leſſer ſegment, and is under an altitude wanting of the ſide of the Cube by the leſſer ſegment of the leſſer ſegment of half the ſide of the Cube. Therefore, &c. Which was required to be proved.

COROLLARIE.

A Dodecahedron is double to a Cube inſcribed in it, taking away the third part of the leſſer ſegment of the Cube, and moreover, the leſſer ſegment of the leſſer ſegment of half of that exceſſe.

For if there be given a Cube, from which is cut off a ſolid ſet upon a third part of the leſſer ſegment of the baſe, and under one and the ſame altitude with the Cube, that ſolid taken away, hath to the whole ſolid the proportion of the ſection of the baſe to the baſe [s]. Wherefore from the Cube is taken away a third part of the leſſer ſegment. Again, foraſmuch as the reſidue wanteth of the altitude of the Cube, by the leſſer ſegment of the leſſer ſegment of half the altitude or ſide, and that reſidue is a Parallelepipedon, if it be cut by a plain ſuperficies parallel to the oppoſite plain ſuperficies, cutting the altitude of the Cube by a point, it ſhall take away from that Parallelepipedon a ſolid, having to the whole the proportion of the ſection to the altitude [t]. Wherefore the exceſſe wanteth of the ſame Cube, by the third part of the leſſer ſegment, and moreover, by the leſſer ſegment of the leſſer ſegment of half of that exceſſe.

s) 32. 11.

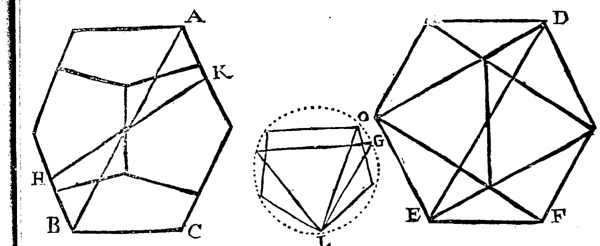
t) 3 Cor. of 25. 11.

PROP. 34. THEOR. 34.

The proportion of the ſolid of a Dodecahedron A H B C K, *to the ſolid of an Icoſahedron* D E F, *inſcribed in it, conſiſteth of the proportion tripled of the diameter* A B, *to that line* K H, *which coupleth the oppoſite baſes of the Dodecahedron, and of the proportion of the ſide* I O *of the Cube, to the ſide* I G, *of the Icoſahedron inſcribed in one and the ſame Sphere.*

Con-

Conſtruction LEt A B be the diameter of the Dodecahedron, and let the line which coupleth the centers of the oppoſite baſes be K H; and let the diameter of the Icoſahedron be D E. Now foraſmuch as one and the ſame circle conteineth the Pentagon of a Dodecahedron, and the triangle of an Icoſahedron deſcribed in one and the ſame Sphere [a]: Let that circle be I G O. Wherefore I O is the ſide of the Cube, and I G the ſide of the Icoſahedron. I ſay then that the proportion of the Dodecahedron A H B C K, to the Icoſahedron D E F inſcribed in it, conſiſteth of the proportion tripled of the line A B to the line K H, and of the proportion of the line I O to the line I G. a) 4. 14.



Demonſtration FOraſmuch as the Icoſahedron D E F is inſcribed in the Dodecahedron A B C, by ſuppoſition, the diameter D E ſhall be equal to the line K H [b]. Wherefore the Dodecahedron ſet upon the diameter K H ſhall be inſcribed in the ſame ſphere, wherein the Icoſahedron D E F is inſcribed: But the Dodecahedron A H B C K is to the Dodecahedron upon the diameter K H, in triple proportion of that in which the diameter A B is to the diameter K H [c], and the ſame Dodecahedron which is ſet upon the diameter K H, hath to the Icoſahedron D E F (which is ſet upon the ſame diameter, or upon a diameter equal to it, to wit, D E.) that proportion which I O the ſide of the Cube, hath to I G, the ſide of the Icoſahedron, inſcribed in one and the ſame ſphere [d]. Wherefore the proportion of the Dodecahedron A H B C K, to the Icoſahedron D E F inſcribed in it, conſiſteth of the proportion tripled of the diameter A B, to the line K H, which coupleth the centers of the oppoſite baſes of the Dodecahedron (which proportion is that which the Dodecahedron A H B C K hath to the Dodecahedron ſet upon the diameter K H) and of the proportion of I O the ſide of the Cube, to I G the ſide of the Icoſahedron (which is the proportion of the Dodecahedron ſet upon the diameter K H, to the Icoſahedron D E F, deſcribed in one and the ſame ſphere [e]:) Therefore the proportion of the ſolid of a Dodecahedron, &c. Which was required to be proved.

b) 7. 15.

c) C. 17. 12.

d) 8. 14.

e) 5. def. 6.

PROP. 35. THEOR. 35.

The ſolid of a Dodecahedron conteineth of a Pyramis circumſcribed about it, two ninth parts, taking away a third part

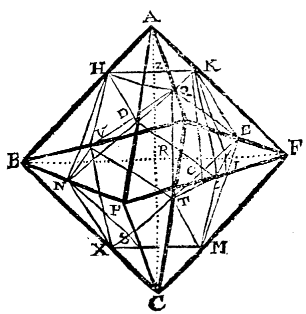
part of one ninth part of the lesser segment (of a line divided by an extream and mean proportion) and moreover the lesser segment of the lesser segment of half the residue.

Demonstration IT hath been proved that the Dodecahedron, together with the Cube inscribed in it, is conteined in one and the same Pyramis a. And it is manifest b that the Dodecahedron is double to the same Cube, taking away the third part of the lesser segment; and moreover, the lesser segment of the lesser segment of half the residue, or of this excesse. But a Pyramis is to the same Cube inscribed in it noncuple c. Wherefore the Dodecahedron inscribed in the Pyramis, and conteining the same Cube twice, taking away the same third of the lesser segment; and moreover, the lesser segment of the lesser segment of half the residue, shall contein two ninth parts of the solid of the Pyramis (of which ninth parts each is equal to the Cube) taking away the same excesse. Therefore, &c. Which was required to be proved.

a) Co. 1. 16
b) C. 33. 16.
c) 30. 16.

PROP. 36. THEOR. 36.

An Octohedron ABCFPL, *exceedeth an Icosahedron* HKEGMXNVDSQT, *by a Parallelepipedon set upon the square of the side of the Icosahedron, and having to his altitude the line which is the greater segment of half the semidiameter of the Octohedron.*



Construction DRaw the diameters AZRC and BROIF, and the perpendicular KO parallel to the line AZR.

Demonstration FOrasmuch as a the triangles KDG and KEQ are described in the bases APF and ALF, of the Octohedron. Therefore about the solid angle there remain upon the base FEG three triangles KEG, KFE, and KFG, which contein a Pyramis KEFG, unto which Pyramis shall be equal and like the opposite Pyramis MEFG, set upon the same base FEG b. And by the same reason shall there of every solid angle of the Octohedron remain two Pyramids, equal and like; to wit, two upon the base AHK, two upon the base BNV, two upon the base DPS, and two upon the base QLT. Now then there shall be made twelve Pyramids, set upon a base conteined of the side of the

a) 16. 15.
b) 8. def. 11.

the Icosahedron, and under two lesser segments of the side of the Octohedron conteining a right angle, as for example, the base GEF. And forasmuch as the side GE subtending a right angle, is c in power double to either of the lines EF and FG, and so the side KH is in power double to either of the sides AH and AK, and either of the lines AH, AK, or EF, FG, is in power double to either of the lines AZ or ZK, which contein a right angle, made in the triangle or base AHK, by the perpendicular AZ. Wherefore it follows that the side GE or HK, is in power quadruple to the triangle EFG or AHK. But the Pyramis KEFG, having his base EFG in the plain FLBP, of the Octohedron, shall have to his altitude the perpendicular KO d, which is the greater segment of the semidiameter of the Octohedron e. Wherefore three Pyramids set under the same altitude, and upon equal bases shall be equal to one Prisme set upon the same base, and under the same altitude f. Wherefore four prismes set upon the base GEF quadrupled (which is equal to the square of the side GE) and under the altitude KO (or RZ the greater segment (which is equal to KO) shall contein a solid equal to the twelve Pyramids, which twelve Pyramids make the excesse of the Octohedron above the Icosahedron inscribed in it. Therefore an Octohedron, &c. Which was required to be proved.

c) 47. 1.
d) 4. def. 6.
e) 16. 15.
f) 1 C. 7. 12.

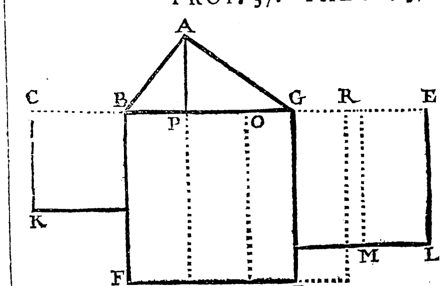
COROLLARIE.

A Pyramis exceedeth the double of an Icosahedron inscribed in it, by a solid set upon the square of the side of the Icosahedron inscribed in it, and having to his altitude that whole line of which the side of the Icosahedron is the greater segment.

For g it is manifest that an Octohedron and an Icosahedron inscribed in it, are inscribed in one and the same Pyramis. It hath been proved likewise h that a Pyramis is double to an Octohedron inscribed in it. Wherefore the two excesses of the two Octohedrons (unto which the Pyramis is equal) above the two Icosahedrons (inscribed in the said two Octohedrons) being brought into a solid, the said solid shall be set upon the same square of the side of the Icosahedron, and shall have to his altitude the perpendicular KO doubled, whose double coupling the opposite sides HK and XM, maketh the greater segment the same side of the Icosahedron i.

g) 19. 15.
h) 26. 16.
i) 1, 2. Cor. 14. 15.

PROP. 37. THEOR. 37.

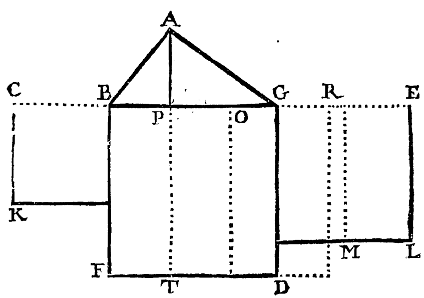


If in a triangle ABG, *having to his base a rational line set* BG, *the sides* AB *and* AG *be commensurable in power to the base, and from the top* A *be drawn to the base a perpendicular*

dicular line A P, *cutting the base, the sections of the base shall be commensurable in length to the whole base, and the perpendicular shall be commensurable in power to the said whole base.*

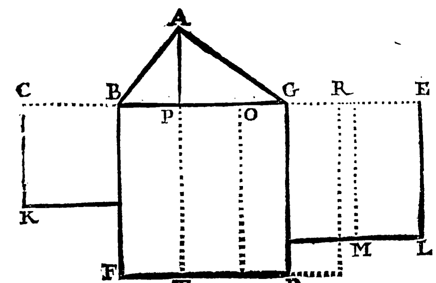
Construction PRoduce on either side the line B G, to the points C and E, and unto the line A G put the line G E equal, and unto the line A B put the line B C equal. And upon the lines C B, B G, and G E, describe the squares B K, B D, and G L, and from the greater of the squares of the lines A B or A G, which let be G L, cut off a Parallelogram
a) 45. 1. E M, equal to the lesser square B K [a], and [b] unto the residue G M, let
b) 45. 1. there be applyed upon the line G D, an equal rectangle Parallelogram O D.

Demonstration FOrasmuch as the angles A P B and A P G are right angles;
c) 47. 1. therefore [c] the line A G conteineth in power the two lines A P and P G, and the line A B the two lines A P and P B. Wherefore how much the line A G conteineth in power more than the line A B, so much also doth the line P G contein in power more than the line B P, to wit, taking away the common square of A P, there is left the excesse of the square P G, above the square of B P. But the square of



A G (which is G L) exceedeth the square of A B (to wit, the square B K) by the rectangle Parallelogram G M or O D, by Construction. Wherefore the square of P G exceedeth the square of B P, by the rectangle Parallelogram O D. And forasmuch as unto the squares of A B and A G, are equal the squares of A P and P B, and of A P and P G, and their excesse is taken away, to wit, the rectangle Parallelogram O D, there shall be left the squares of A P and P O, equal to the squares of A P and P B, and taking away the square of A P, which is common, the residues, to wit, the squares of B P and P O, shall be equal; and therefore their sides (to wit, the lines B P and P O) are equal. And forasmuch as the squares G L and B K are (by supposition) rational, and therefore commen-
d) 15. 10. surable, their excesse O D shall be commensurable unto them [d]; and
e) 9. def. 10. therefore it is rational [e]. Wherefore the rational Parallelogram O D being applyed upon the rational line G D (or B G) maketh the breadth O G rati-

rational and commensurable in length to the whole line B G [f]. But if the f) 20. 10.
whole line B G be commensurable to one of the parts O G, the lines B O,
O G, and B G, shall be commensurable [g]. Wherefore also the line O G g) 15. 10.
shall be commensurable to the half of the line B O; to wit, to the line
P O, or P B [h]. And forasmuch as the two lines P O and O G are commen- h) 12. 10.
surable, the whole line P G shall be commensurable to the line P O, or
to the line P B [i]. Wherefore either of the lines P G and P B shall be com- i) 15. 10.
mensurable unto the whole line G B [k]. Wherefore the lines B G, P B, k) 15. 10.
and P G, have to one another that proportion which numbers have [l]. l) 5. 13.
Wherefore the sections P B and P G, of the base B G, are commensura-
ble in length to the same base [m]. m) 6. 10.



And now that the perpendicular A P is commensurable in power to the base B G, is thus proved. Forasmuch as the square of A B is by supposition commensurable to the square of B G, and unto the rational square
of A B, is commensurable the rational square of P B [n]. Wherefore the n) 12. 11.
residue, to wit, the square of P A, is commensurable to the same
square of B P [o]. Wherefore [p] the square of P A is commensurable to the o) 15. 11.
whole square of B G. Wherefore the perpendicular A P is commensura- p) 12. 10.
ble in power to the base B G [q]. Which was required to be proved. q) 3. de. 10.

In demonstrating of this we made no mention at all of the length of the sides A B and A G, but only of the length of the base B G, for that the line B G is the rational line first set, and the other lines A B and A G, are supposed to be commensurable in power only to the line B G. Wherefore if that be plainly demonstrated, when the sides are commensurable in power only to the base, much more easily will it follow, if the same sides be supposed to be commensurable both in length and in power to the base, that is, if their lengths be expressed by the Roots of square numbers.

COROLLARIE I.

By the former things demonstrated, it is manifest that if from the powers of the base, and of one of the sides be taken away the power of the other side, and if the half of the power remaining be applyed upon the whole base, it shall make the breadth that section of the base which is coupled to the first side.

For from the powers of the base B G, and of one of the sides A G, that

that is, from the ſquares B D and G L, the power of the other ſide A B, to wit, the ſquare B K (or the Parallelogram E M is taken away.) And of the reſidue (to wit, of the ſquare B D, and of the Parallelogram O D or D R, which by ſuppoſition is equal to O D) the half (to wit, of the whole F R, which is P D; for the lines G R and P B are equal to the lines G O and P O) is applyed to the whole line B G or G D, and maketh the breadth the line P G, the ſection of the baſe B G, which ſection is coupled to the firſt ſide A G. And by the ſame reaſon in the other ſide, if from the ſquares B D and B K, be taken away the ſquare G L, there ſhall remain the rectangle Parallelogram F O; for the Parallelogram E M is equal to the ſquare B K, and the Parallelogram G M to the Parallelogram G M to the Parallelogram O D. Wherefore E P the half of the reſidue F O, maketh the breadth B P, which is coupled to the firſt ſide taken A B.

COROLLARIE II.

If a Perpendicular drawn from an angle of a triangle do cut the baſe, the ſections are to the other ſides in power proportional by an Arithmetical proportion.

For it was proved that the exceſſe of the powers of the lines A G and A B, is one and the ſame with the exceſſe of the powers of the lines P G and P B: If therefore the powers do equally exceed one another, they ſhall by an Arithmetical proportion be proportional.

The End of the Sixteenth Element of EUCLIDE, *added by* FLUSSAS.

A

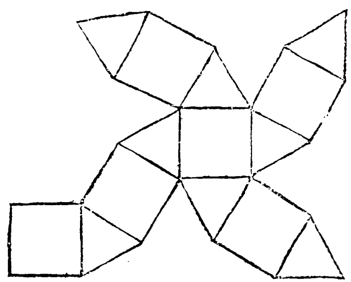
A BRIEFE TREATISE ADDED BY *FLUSSAS*, OF Regular Solids.

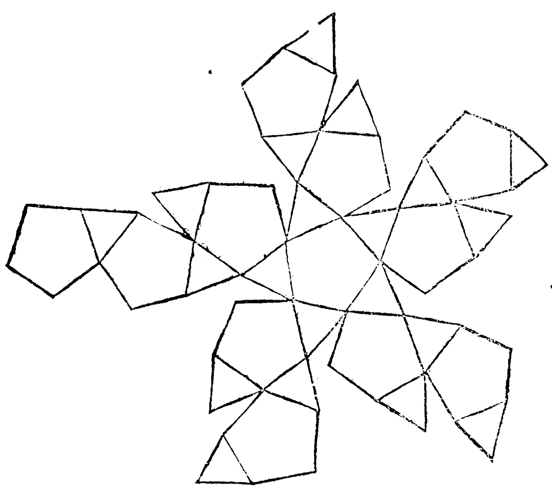
REgular Solids are ſaid to be compoſed and mixt when each of them is transformed into other Solids, keeping ſtill the forme, number, and inclination of the baſes, which they before had to one another; ſome of which yet are transformed into mixt Solids, and other ſome into ſimple. Into mixt, as a Dodecahedron and an Icoſahedron, which are transformed or altered, if you divide their ſides into two equal parts, and take away the ſolid angles ſubtended of plain ſuperficial figures, made by the lines coupling thoſe middle ſections; for the Solid remaining after the taking away of thoſe ſolid angles, is called an Icoſidodecahedron. If you divide the ſides of a Cube and of an Octohedron into two equal parts, and couple the ſections, the ſolid angles ſubtended of the plain ſuperficies made by the coupling lines, being taken away, there ſhall be left a ſolid, which is called an Exoctohedron. So that both of a Dodecahedron and alſo of an Icoſahedron, the Solid which is made ſhall be called an Icoſidodecahedron; and likewiſe the Solid made of a Cube, and alſo of an Octohedron, ſhall be called an Exoctohedron. But the other Solid, to wit, a Pyramis or Tetrahedron, is transformed into a ſimple Solid; for if you divide into two equal parts each of the ſides of the Pyramis, triangles deſcribed of the lines which couple the ſections, and ſubtending and taking away the ſolid angles of the Pyramis, are equal and like unto the equilateral triangles left in each of the baſes, of all which triangles is produced an Octohedron, to wit, a ſimple, and not a compoſed Solid. For the Octohedron hath four baſes,

like in number, form, and mutual inclination with the bases of the Pyramis, and hath the other four bases with like situation opposite and parallel to the former. Wherefore the application of the Pyramis taken twice, maketh a simple Octohedron, as the other Solids make a mixt compound Solid.

DEFINITIONS.

1 *An Exoctohedron is a solid figure conteined of six equal squares, and eight equilateral and equal triangles.*





2 *An*

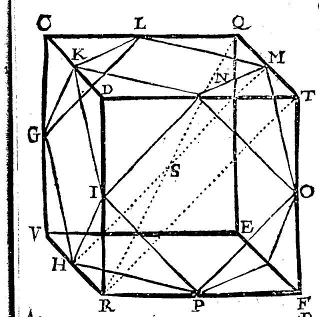
2 *An Icosidodecahedron is a solid figure conteined under twelve equilateral, equal, and equiangled Pentagons, and twenty equal and equilateral triangles.*

For the better understanding of the two former Definitions, and also of the two Propositions following, I have here set two figures, whose figures if you first describe upon pasted paper or such like matter, and then cut them and fold them accordingly, they will represent unto you the perfect formes of an Exoctohedron, and of an Icosidodecahedron.

PROBLEME 1.

To describe an equilateral and equiangled Exoctohedron, and to contein it in a given Sphere, and to prove that the Diameter of the Sphere is double to the side of the said Exoctohedron.

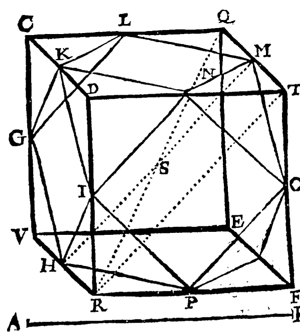
Construction Suppose a Sphere, whose diameter let be AB, and about the diameter AB let there be described a square [a], and a) 6. 4.
upon the square let there be described a Cube [b], which let be CDEF b) 15. 13.
QTVR; and let the diameter thereof be QR, and the center S. Divide the sides of the Cube into two equal parts in the points G, H, I, K, L, M, N, O, P, &c. and couple the middle sections by the right lines IN, NO, OP, PI, and such like, which subtend the angles of the squares or bases of the Cube; and they are equal [c], and contein right angles, as the angle NIP. For the angle NID, which is at the base of the Isosceles triangle NDI, is the half of a right angle, and so likewise is the opposite angle RIP. Wherefore the residue NIP is a right angle, and so the rest. Wherefore NIPO is a square. And by the same reason shall the rest NM LK, KGHI, &c. inscribed in the bases of the Cube, be squares, and they shall be six in number, according to the number of the bases of the Cube. Again, forasmuch as the triangle KIN subtendeth the solid angle D of the Cube, and likewise the triangle KGL the solid angle C, and so the rest which subtend the right solid angles of the Cube, and these triangles are equal and equilateral (to wit) being made of equal sides, and they are the limits or borders of the squares, and the squares the limits or borders of them; as hath been before proved. Wherefore c) 4. 1.



LMNOPHGK is an Exoctohedron by the definition, and is equilateral; for it is conteined of equal subtendant lines, it is also equiangled; for every solid angle thereof is conteined under two superficial angles of two squares, and two superficial angles of two equilateral triangles.

Demonstration FOrasmuch as the opposite sides and diameters of the bases of the Cube are parallels, the plain extended by the right lines Q T and V R, shall be a Parallelogram. And for that also in that plain lyeth Q R, the diameter of the Cube, and in the same plain also is the line M H, which divideth the said plain into two equal parts, and also coupleth the opposite angles of the Exoctohedron; this line M H therefore divideth the diameter into two equal parts [d]; and also divideth it selfe in the same point, which let be S, into two equal parts [e].

d) Co.34.1.
e) 4. 1.



And by the same reason may we prove that the rest of the lines which couple the opposite angles of the Exoctohedron, do in S the center of the Cube, divide one another into two equal parts, for each of the angles of the Exoctohedron are set in each of the bases of the Cube. Wherefore making the center the point S, with the distance S H or S M describe a Sphere, and it shall touch every one of the angles equidistant from the point S.

And forasmuch as A B the diameter of the sphere given, is put equal to the diameter of the base of the Cube, to wit, to the line R T, and the same line R T is equal to the line M H [f], which line M H coupling the opposite angles of the Exoctohedron, is drawn by the center. Wherefore it is the diameter of the Sphere given which conteineth the Exoctohedron.

f) 33. 1.

Lastly, forasmuch as in the triangle R F T, the line P O doth cut the sides into two equal parts, it shall cut them proportionally with the bases, to wit, as F R is to F P, so shall R T be to P O [g]. But F R is double to F P by supposition: Wherefore R T, or the diameter H M, is also double to the line P O, the side of the Exoctohedron. Wherefore we have described, &c. Which was required to be done.

g) 2. 6.

PROBLEME II.

To describe an equilateral and equiangled Icosidodecahedron, and to comprehend it in a sphere given, and to prove that the Diameter being divided by an extream and mean proportion, maketh the greater segment double to the side of the Icosidodecahedron.

Con-

Construction SUppose that the diameter of the sphere given be N L, and [a] divide the line N L, by an extream and mean proportion in the point I, and the greater segment thereof let be N I; and upon the line N I describe a Cube [b]; and about this Cube let there be circumscribed a Dodecahedron [c]; and let the same be A B C D E F H K M O, and divide each of the sides into two equal parts in the points Q, R, S, T, V, X, Y, Z, P, ε, θ, G, &c. and couple the sections with right lines, which shall subtend the angles of the Pentagons, as the lines P G, G V, V Q, Q Y, Y R, R Q, V T, T X, X V, and so the rest.

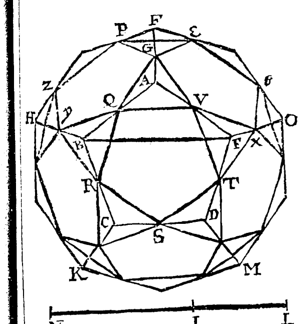
a) 30. 6.
b) 15. 13.
c) 17. 13.

Demonstration FOrasmuch as these lines subtend equal angles of the Pentagons, and those equal angles are conteined of equal sides to wit, of the halves of the sides of the Pentagons; therefore those subtending lines are equal [d]. Wherefore the triangles G Q V, Y Q R, and V X T, and the rest, which take away solid angles of the Dodecahedron, are equilateral.

d) 4. 1.

Again, forasmuch as in every Pentegon are described five equal right lines, coupling the middle sections of the sides, as are the lines Q V, V T, T S, S R, and R Q, they describe a Pentagon in the plain of the Pentagon of the Dodecahedron. And the said Pentagon is conteined in a circle, to wit, whose center is the center of a Pentagon of the Dodecahedron. For the lines drawn from that center to the angles of this Pentagon are equal, for that they are perpendiculars upon the bases cut [e]. Wherefore the Pentagon Q R S T V, is equiangled [f]. And by the same reason may the rest of the Pentagons described in the bases of the Dodecahedron, be proved equal and like.

e) 12. 4.
f) 11. 4.

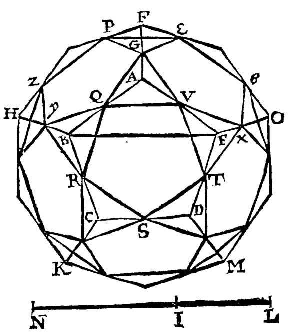


Wherefore those Pentagons are twelve in number: And forasmuch as the equal and like triangles do subtend and take away twenty solid angles of the Dodecahedron; therefore the said triangles shall be twenty in number. Wherefore we have described an Icosidodecahedron by the definition, which Icosidodecahedron is equilateral; for that all the sides of the triangles are equal and common with the Pentagons; and it is also equiangled. For each of the solid angles is made of two superficial angles of an equilateral Pentagon, and of two superficial angles of an equilateral triangle

Now let us prove that it is conteined in the given sphere whose Diameter is N L. Forasmuch as perpendiculars drawn from the centers of the Dodecahedron, to the middle sections of his sides, are the halves of the lines which couple the opposite middle sections of the sides of the Dodecahedron [g]; which lines also [h] do in the center divide one another into two equal parts. Therefore right lines drawn from that point to the angles

g) 3 Cor. of 17. 13.
h) idem.

angles of the Icosidodecahedron (which are set in those middle sections) are equal; which lines are 30 in number, according to the number of the sides of the Dodecahedron; for each of the angles of the Icosidodecahedron are set in the middle sections of each of the sides of the Dodecahedron. Wherefore making the center the center of the Dodecahedron, and the space any one of the lines drawn from the center to the middle sections, describe a sphere, and it shall passe by all the angles of the Icosidodecahedron, and shall contein it.



i) 4 C. 17. 13 And forasmuch as the diameter of this solid, is that right line whose greater segment is the side of the Cube inscribed in the Dodecahedron [i], which side is N I by supposition. Wherefore that solid is conteined in the sphere given, whose diameter is put to be the line N L.

Now let us prove that the greater segment of the diameter is double to Q V the side of the solid. Forasmuch as the sides of the triangle A E B, are in the points Q and V divided into two equal parts, the lines Q V and B E are parallels [k].

k) Co. 39. 1. Wherefore as

l) 2. 6. A E is to A V, so is E B to V Q [l]. But the line A E is double to the line

m) 4. 6. A V. Wherefore the line B E is double to the line Q V [m]. Now the line

n) 2 Cor. of 17. 13. B E is equal to N I, or to the side of the Cube [n]; which line N I is the greater segment of the diameter N L. Wherefore the greater segment of the diameter given is double to the side of the Icosidodecahedron inscribed in the given sphere. Wherefore, We have described, &c. Which was required to be done.

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To the understanding of the nature of this Icosidodecahedron, you must well conceive the passions and proprieties of both these solids, of whose bases it consisteth, to wit, of the Icosahedron and of the Dodecahedron. And although in it the bases are placed oppositely, yet have they to one another one and the same inclination. By reason whereof there lie hidden in it the actions and passions of the other Regular Solids. And I would have thought it not impertinent to the purpose to have set forth the inscriptions and circumscriptions of this Solid, if want of time had not hindred. But to the end the Reader may the better attein to the understanding thereof, I have here following briefly set forth, how it may in or about every one of the five Regular Solids be inscribed or circumscribed; by the help whereof he may, with small travail or rather none at all, having well poised and considered the Demonstrations apperteining to the foresaid five Regular Solids, demonstrate both the inscription of the said Solids in it, and the inscription of it in the said Solids.

Of

Of the inscriptions and circumscriptions of an Icosidodecahedron.

AN Icosidodecahedron may contein the other five regular bodies. For it will receive the angles of a Dodecahedron in the centers of the triangles which subtend the solid angles of the Dodecahedron, which solid angles are twenty in number, and are placed in the same order in which the solid angles of the Dodecahedron taken away, or subtended by them, are. And by that reason it shall receive a Cube and a Pyramis conteined in the Dodecahedron, when as the angles of the one, are set in the angles of the other.

An Icosidodecahedron receiveth an Octohedron, in the angles cutting the six opposite sections of the Dodecahedron, even as if it were a simple Dodecahedron.

And it conteineth an Icosahedron, placing the twelve angles of the Icosahedron in the same centers of the twelve Pentagons.

It may also by the same reason be inscribed in each of the five regular bodies; to wit, In a Pyramis, if you place four triangular bases concentrical with four bases of the Pyramis, after the same manner that you inscribed an Icosahedron in a Pyramis; so likewise may it be inscribed in an Octohedron, if you make eight bases thereof concentrical with the eight bases of the Octohedron. It shall also be inscribed in a Cube, if you place the angles which receive the Octohedron in it, in the centers of the bases of the Cube. Again, you shall inscribe it in an Icosahedron, when the triangles compassed in of the Pentagon bases, are concentrical with the triangles which make a solid angle of the Icosahedron.

Lastly, it shall be inscribed in a Dodecahedron, if you place each of the angles thereof in the middle sections of the sides of the Dodecahedron, according to the order of the Construction thereof.

The opposite plain superficieces also of this solid are parallels. For the opposite solid angles are subtended of parallel plain superficieces, as well in the angles of the Dodecahedron subtended by triangles, as in the angles of the Icosahedron subtended of Pentagons, which thing may easily be demonstrated. Moreover in this solid are infinite properties and passions, springing of the solids whereof it is composed.

Wherefore it is manifest that a Dodecahedron and an Icosahedron, mixed, are transformed into one and the self same solid of an Icosidodecahedron. A Cube also and an Octohedron are mixed and altered into another solid, to wit, into one and the same Exoctohedron. But a Pyramis is transformed into a simple and perfect solid, to wit, into an Octohedron.

If we will frame these two solids joyned together into one solid, this onely must we observe.

In the Pentagon of a Dodecahedron inscribe a like Pentagon, and let its angles be set in the middle sections of the Pentagon circumscribed, and then upon the said Pentagon inscribed let there be set a solid angle of an Icosahedron, and so observe the same order in each of the bases of the Dodecahedron, and the solid angles of the Icosahedron set upon these Pentagons shall produce a solid consisting of the whole Dodecahedron, and

whole

whole Icosahedron. In like sort, if in every base of the Icosahedron, the sides being divided into two equal parts, be inscribed an equilateral triangle, and upon each of those equilateral triangles be set a solid angle of a Dodecahedron, there shall be produced the same solid consisting of the whole Icosahedron, and of the whole Dodecahedron.

And after the same order, if in the bases of a Cube be inscribed squares subtending the solid angles of an Octohedron, or in the bases of an Octohedron be inscribed equilateral triangles subtending the solid angles of a Cube, there shall be produced a solid consisting of either of the whole solids, to wit, of the whole Cube, and of the whole Octohedron.

But equilateral triangles inscribed in the bases of a Pyramis, having their angles set in the middle sections of the sides of the Pyramis, and the solid angles of a Pyramis, set upon the said equilateral triangles, there shall be produced a solid consisting of two equal and like Pyramids.

And now if in these solids thus composed, you take away the solid angles, there shall be restored again the first composed solids, to wit, the solid angles taken away from a Dodecahedron and an Icosahedron composed into one, there shall be left an Icosidodecahedron, the solid angles taken away from a Cube and an Octohedron composed into one solid, there shall be left an Exoctohedron. Moreover, the solid angles taken away from two Pyramids composed into one solid, there shall be left an Octohedron.

Of the nature of a trilateral and equilateral Pyramis.

1 A trilateral equilateral Pyramis is divided into two equal parts, by three equal squares, which in the center of the Pyramis cut one another into two equal parts, and perpendicularly, and whose angles are set in the middle sections of the sides of the Pyramis.

2 From a Pyramis are taken away 4 Pyramids like unto the whole, which utterly take away the sides of the Pyramis, and that which is left is an Octohedron inscribed in the Pyramis in which all the solids inscribed in the Pyramis are conteined.

3 A perpendicular drawn from the angle of the Pyramis to the base, is double to the diameter of the Cube inscribed in it.

4 And a right line coupling the middle sections of the opposite sides of the Pyramis is triple to the side of the same Cube.

5 The side also of a Pyramis is triple to the diameter of the base of the Cube.

6 Wherefore the same side of the Pyramis is in power double to the right line which coupleth the middle sections of the opposite sides.

7 And it is in power sesquialter to the perpendicular which is drawn from the angle to the base.

8 Wherefore the perpendicular is in power sesquitertia to the line which coupleth the middle sections of the opposite sides.

9 A Pyramis and an Octohedron inscribed in it, also an Icosahedron inscribed in the same Octohedron, do contein one and the same sphere.

Of the nature of an Octohedron.

1 Four perpendiculars of an Octohedron, drawn in four bases thereof from two opposite angles of the said Octohedron, and coupled together by those 4 bases, describe a Rhombus, or Diamond figure; one of whose diameters is in power double to the other diameter.

2 For it hath the same proportion that the diameter of the Octohedron hath to the side of the Octohedron.

3 An Octohedron and an Icosahedron inscribed in it, do contein one and the same sphere.

4 The diameter of the solid of the Octohedron is in power sesquialter to the diameter of the circle which conteineth the base, and is in power duple superbipartiens tertias (that is, as 8 to 3,) to the perpendicular or side of the foresaid Rhombus; and moreover is in length triple to the line which coupleth the centers of the next bases.

5 The angle of the inclination of the bases of the Octohedron, doth with the angle of the inclination of the bases of the Pyramis, make angles equal to two right angles.

Of the nature of a Cube.

1 The diameter of a Cube is in power sesquialter to the diameter of his base.

2 And is in power triple to his side.

3 And unto the line which coupleth the centers of the next bases, it is in power sextuple.

4 Again, the side of the Cube, is to the side of the Icosahedron inscribed in it, as the whole is to the greater segment.

5 Unto the side of the Dodecahedron, it is as the whole is to the lesser segment.

6 Unto the side of the Octohedron it is in power duple.

7 Unto the side of the Pyramis it is in power subduple.

8 Again, the Cube is triple to the Pyramis, but to the Cube the Dodecahedron is in a manner double. Wherefore the same Dodecahedron is in a manner sextuple to the said Pyramis.

Of the nature of the Icosahedron.

1 Five triangles of an Icosahedron, do make a solid angle, the bases of which triangles make a Pentagon. If therefore from the opposite bases of the Icosahedron be taken the other Pentagon by them described, these Pentagons shall in such sort cut the diameter of the Icosahedron which coupleth the foresaid opposite angles, that that part which is conteined between the plaines of these two Pentagons shall be the greater segment; and the residue which is drawn from the plain to the angle, shall be the lesser segment.

2 If the opposite angles of two bases joyned together, be coupled by a right line, the greater segment of that right line is the side of the Icosahedron.

3 A line drawn from the center of the Icosahedron to the angles, is in power quintuple to half that line, which is taken between the Pentagons, or of the half of that line, which is drawn from the center of the circle which conteineth the foresaid Pentagon, which two lines are therefore equal.

4 The side of the Icosahedron conteineth in power either of them, and also the lesser segment, to wit, the line which falleth from the solid angle to the Pentagon.

5 The diameter of the Icosahedron conteineth in power the whole line, which

which coupleth the opposite angles of the bases joyned together, and the greater segment thereof, to wit the side of the Icosahedron.

6 The diameter also is in power quintuple to the line which was taken between the Pentagons, or to the line which is drawn from the center to the circumference of the circle which conteineth the Pentagon composed of the sides of the Icosahedron.

7 The dimetient conteineth in power the right line which coupleth the centers of the opposite bases of the Icosahedron, and the diameter of the circle which conteineth the base.

8 Again, the said dimetient conteineth in power the diameter of the circle which conteineth the Pentagon, and also the line which is drawn from the center of the same circle to the circumference: That is, it is quintuple to the line drawn from the center to the circumference.

9 The line which coupleth the centers of the opposite bases, conteineth in power the line which coupleth the centers of the next bases, and also the rest of that line of which the side of the Cube inscribed in the Icosahedron is the greater segment.

10 The line which coupleth the middle sections of the opposite sides, is triple to the side of the Dodecahedron inscribed in it.

11 Wherefore if the side of the Icosahedron, and the greater segment thereof be made one line, the third part of the whole is the side of the Dodecahedron inscribed in the Icosahedron.

Of the Dodecahedron.

1 The diameter of a Dodecahedron conteineth in power the side of the Dodecahedron, and also that right line to which the side of the Dodecahedron is the lesser segment, and the side of the Cube inscribed in it is the greater segment, which line is that which subtendeth the angle of the inclination of the bases, contained under two perpendiculars of the bases of the Dodecahedron.

2 If there be taken two bases of the Dodecahedron, distant from one another by the length of one of the sides, a right line coupling their centers being divided by an extream and mean proportion, maketh the greater segment the right line which coupleth the centers of the next bases.

3 If by the centers of five bases set upon one base, be drawn a plain superficies, and by the centers of the bases which are set upon the opposite base, be drawn also a plain superficies, and then be drawn a right line, coupling the centers of the opposite bases, that right line is so cut, that each of his parts set without the plain superficies, is the greater segment of that part which is conteined between the plaines.

4 The side of the Dodecahedron is the greater segment of the line which subtendeth the angle of the Pentagon.

5 A perpendicular line drawn from the center of the Dodecahedron to one of the bases, is in power quintuple to half the line which is between the plains.

6 And therefore the whole line which coupleth the centers of the opposite bases is in power quintuple to the whole line which is between the said plains.

7 The line which subtendeth the angle of the base of the Dodecahedron, together with the side of the base are in power the quintuple to the line which is drawn from the center of the circle which conteineth the base, to the circumference.

8 A Section of a sphere conteining 3 bases of the Dodecahedron, taketh a third part of the diameter of the said sphere.

9 The side of the Dodecahedron and the line which subtendeth the angle of the Pentagon, are equal to the right line which coupleth the middle sections of the opposite sides of the Dodecahedron.

THE END.

Euclides Data,

A Commentary or Preface written by the Phylosopher MARINUS, *on* EUCLIDES DATA.

IN the first place we ought to set down (as a Foundation) what that is, which we call *DATUM* or GIVEN; then to consider the profit and utility thereof; and in the third place, to what Art or Science this Tract doth appertein.

The Word *DATUM* therefore is diversly defined, for the Ancients have defined it after one manner, and later Writers after another, whence it follows that it seemeth a difficult thing to give a true explication thereof; for some of them have not delivered the Definition of the Word; but have with much labour and trouble sought certain proprieties thereof, and some others collecting and mingling what hath been delivered by others before, have endevoured to define the Word *DATUM*; but not so exquisitely but that they have contradicted themselves; although what hath been said by all of them, seemes to be grounded on one and the same notion and supposition; for they all take the Word *DATUM* to be a thing comprised; and therefore among such as have endevoured to describe it most simply, and with some simple difference, some of them have taken the Word *DATUM* to be the same with *ORDINATUM*, and so *Appollonius* understands it in his Tract of Inclinations, and in his universal Tract; and so others, as *Diodorus*, takes it to be *COGNITUM* KNOWN; for in this signification he takes the right line and the angles to be given, and all that may arrive to our Knowledge, although we may not be able well to expresse it. But others have believed that it hath the same signification as the Word [*Effabile*] that may be declared, and so *Ptolomie* would have it, who calls those things GIVEN, whose measure is known whether precisely, or neer the matter. Others also have thought the Word *DATUM* to be what is

granted us by the proposer in the Hypothesis; being that in the First Elements, a point given, and a right line given, is diversly taken (that is to say, that who so would give and determine the quantity of a right line) all which things signifie some COMPREHENSION; and therefore of all these Definitions, those are most agreeable, which do most openly declare the COMPREHENSION, as we shall make evident by what follows.

Let us now unfold the diverse opinions of those, who writing the nature of *DATUM* GIVEN, have not taken one simple Marke, or only Character for its Definition; and let us reduce it as in a Summary or Epitomy, to the end we may with the more ease know or number all their differences. Some of them then have defined *DATUM* to be *Ordinatum* and *Porimon* together, and others *Ordinatum* and *Cognitum* together, and others *Porimon* and *Cognitum* together. Wherefore all seem to have so defined it as to have have had regard to the *Comprehension*, or *Assuming* and *Inventi*-*on*, of the thing given; and to the end that we may the better conceive their opinions, and that from the saying of many we may be able to draw a true Definition of what is proposed, we will take notice in the first place of the signification of all the simple terms which they make use of, as also of the terms opposed to them, to wit, *Inordinatum*, and *Incognitum*, *Aporon*, and *Irrational*; for those things appertein to this Geometrical businesse, to natural things, and to Mathematical Discipline.

Now we may call that *Ordinatum* (or Regulated) which doth alwayes keep and observe that for which it is said to be ordered, whether you regard its Magnitude or Species, or touching some other such like thing: It is also thus defined, *Ordinatum* is that which cannot be done in divers manners, but in one only manner, and in some determined place: As for Example, A right line drawn by two given points, is said to be ordered, by reason it cannot be otherwise done, nor in divers manners. But an angle passing by two points is said to be *Inordinatum* (or disordinate and irregular) for that it is made in infinite and divers manners by a great or small circle described by two points *ad infinitum*. Contrariwise, an angle constituted by three points, is said to be *Ordinatum*, as also those things which follow, are said to be *Ordinatum*, as to constitute an equilateral triangle on a right line, for it cannot be diversly made, but unchangeably, on both the extremities of the line. Again, To divide a given right line according to a given proportion, for that cannot be done but in one certain point. The things *Inordinatum* are such as are done contrary to those last mentioned, as to constitute a Scalene triangle, and to divide a right line indefinitely. Wherefore the Problem is ordered, is proposed in the determination, considering that a certain thing may be in one manner said to be *Ordinatum*, and *Inordinatum* in another, as an equilateral triangle, considering the equality of the sides, it is *Ordinatum*, but considering its Magnitude it is *Inordinatum*, being in no wise determined.

But we call that *Cognitum* which is notorious, as clear and comprehended of us, and *Incognitum* that which is not known or comprehended of us, as the length of a way is said to be known, when we know how many Miles it conteins; also that the three lines of a Rectiline triangle are equal to two right angles; and in like manner, that the Binomial is Irrational, such things are known, as also that it is only one right line that can touch a spiral line from a given point without it, from both parts; for if there were yet another line, two right lines would enclose a space, which is impossible. Again, Irrational things are not said to be unknown, but such of them only which are neither known nor comprehended of us.

Porimon is that which we may make and constitute, that is to say, bring to our understanding. Again, it is defined thus, *Porimon* is that which may be exhibited by Demonstration, or which is apparent without Demonstration, as to describe a circle from a center and with a space, as also to constitute not only an equilateral triangle, but also a Scalene; or to find a *Binomium*, or to find two right lines rational, commensurable in power only, and other things which are known infinitely, are *Porimon*; as to describe a circle by two points.

Aporon is wholly opposite to *Porimon*, as for Example, the Quadrature of a circle, for that it hath not as yet been found; although it be certainly known that it may be: Neverthelesse the manner of finding it out hath not been to this present comprehended. But we speak here of that which is already known, which is called *Porimon principale*; for what hath not been as yet made, and yet neverthelesse is possible, is called *Poriston*, (or feasible) although the Construction be yet unknown. But *Aporon* as hath been afore said, is opposite to *Porimon*, and is that whose Nature is not as yet decided, nor well determined.

Effabile, that is to say, rational (or speakable and explicable) is that whose Magnitude, Species, and Position, we may be able to declare; but this Definition is a little too general, for properly, and according to it selfe, *Effabile* is that which is known by certain things, and according to a measure given by position, as of a span, or a fingers breadth, &c.

These things then being thus unfolded, we may easily perceive in what all those things that we have afore spoken of do agree together, and wherein they do differ; and first of all how *Ordinatum* and *Cognitum* do agree together, and likewise their opposites the one to the other, for it cannot be said that any one of those things by counterchanging is the other, nor yet that the one hath not more extent than the other, although they agree in many things, as to describe a right line by two points, and to constitute an equilateral triangle by three Circles. But to square a Circle, that is indeed *Ordinatum*, yet neverthelesse, *Incognitum*. Also that at a point of a spiral line there is but one touch line, that is, of the kind we call *Ordinatum*, and cannot be otherwise done, yet neverthelesse the Demonstration and Construction thereof is not yet known. Again, the indefinite section, and the Construction of the Scalenum is *Cognitum*; but is not *Ordinatum*; in so much as it is manifest that amongst those things which are *Ordinatum*, there are some that are *Cognitum*, and others that are *Incognitum*; and contrariwise, that amongst the things that are *Cognitum*, there are some that are *Ordinatum*, and others that are *Inordinatum*; and therefore those things answer one another, as among Living Creatures, that which hath reason, with that which hath Feet, for there is no equality amongst them, neither doth the one extend more than the other.

In like manner, *Ordinatum* and *Inordinatum* agree together, respecting *Porimon* and *Aporon*; seeing that between them there is a very great resemblance, and because that they do differ only in the manner before expressed; for in truth the spiral line is *Ordinatum*; but it was not *Porimon* before *Archimedes*; and by the same reason those things that are inordinate and known by an infinity of wayes and meanes are *Porimon*, if any one shall undertake to invent their Constitution and Construction. Yet neverthelesse

thelesse they are not ordinate, as to constitute a Scalenum triangle, it being no difficult thing to make known the constitution thereof by an equilateral triangle, yea it is most easie, although it be inordinate, and known by an infinity of wayes.

And in the same manner do agree *Ordinatum* and *Inordiatum*, together with *Effabile* and *Irrationale*; for they agree together in many things, differing neverthelesse by the fore-going reason, seeing those things there mentioned bear no equality to each other, neither doth one thing contein the other; for all Binomiums and such as are taken as Irrationals, are indeed ordinate, but yet they are not *Effabile*, or expressible, or to be unfolded, as the diameter of the square is in respect of its side. Now touching *Effabile* there are divers inordinate; because they are diversly known, and indeterminately, for a Scalenum triangle may be measured by a defined and proposed measure, as explicable, although it be inordinate.

Now it is easie to see the agreement that there is between *Cognitum* and *Porimon*, but it is a difficult thing to expresse or unfold their difference; forasmuch as in their natures they come so neer to one another as that there seems to be an equality between them: Neverthelesse, there will some difference appear to him that shall consider it more strictly; for let it be consented to that there can be only one line that can touch a spiral line in a certain point, that is *Cognitum*, yet notwithstanding the Problem is not *Porimon*, it being not as yet comprehended; so as that all that which is *Cognitum* is not therefore *Porimon*. But all that which is *Porimon* is also *Cognitum*, and therefore *Cognitum* appeares to be of a greater extent than *Porimon*.

Now *Cognitum*, *Porimon*, and *Effabile*, do agree in some certain things, and do differ in other things by the same reason before alledged; for those lines which are there called Irrationals are in truth known; and yet neverthelesse are not *Effabile* or explicable. Contrariwise, every number is indeed *Effabile*, and yet every number is not *Cognitum*. But *Effabile* is alwayes of its own nature expressable, although that some lengths may be now *Effabile*, and at another time not, if it be examined with some other according to one and the same measure. But also that same length is sometimes known, and other times not, though they wholly agree with one another. Now it is a difficult thing to find some thing that shall be *Effabile* and *Incognitum*, for *Cognitum* seemes to be of a greater extent than *Effabile*, and by these things it is manifest that *Porimon* and *Aporon* do differ from RATIONAL or *Effabile*, and from IRRATIONAL, for of IRRATIONAL some of them may be *Porimon*; but of RATIONAL none of them can be Irrationals; and therefore it is very easie to perceive in what the before expressed things agree. Notwithstanding they seem to agree together, in such sort as that *Porimon* seemes to be of a greater extent than *Effabile*.

Now by these things we may come to know the difference of those things that have been before spoken of, for in truth *Effabile* and *Irrationale* are so termed in respect of measure, which notwithstanding is not as yet arrived to our understanding, seeing that some thing, that is rational, may be as yet unknown to us, and in like manner may be rational, and yet may never be comprehended so to be. But *Ordinatum* and *Inordinatum* is so termed according to it self, and according to the proper nature of the thing on which we contemplate, although it be not comprehended by us. As *Archimedes* had perceived some things to be ordinate from the nature of the things, the which *Serenus* had before contemplated. But *Cognitum* and *Incognitum* is spoken in respect of us, so as the things before mentioned do differ among themselves; for these have respect to us, the others, some of them to their proper nature, and the rest to measure.

Having then explained the agreements and differences of the things that have been proposed, it remaines now that we consider what is meant by the Word *DATUM*. for of all those that believe the Word *DATUM* to be that which is consented to by the Proposer in the Hypothesis, are wide from what is sought; because that all the Elements of the things GIVEN are not composed of this sort of GIVEN, which is according to the Hypothesis, as may be seen in those Tracts which have been made on this subject GIVEN. Wherefore waving this opinion, let us judg of the Definitions of others.

Then, that which is consented to or granted in the Hypothesis, is some thing which is consequently known by the principles; but such as make use of Definitions of one only Word, do define it and remarke it by some one of the before mentioned, as hath been spoken in the beginning, so as that almost all seem to have had this common notion of GIVEN, to wit, that it is comprehended even as the Word *DATUM*, doth also manifest it to be; and amongst those, these are the chief that do define it by the Hypothesis or Supposition; and others have had regard to what is consented to or granted. But we making use of the said things as of a Rule and Direction to judge rightly, we may be able to find out a perfect Definition of *DATUM*; for it is certain that it ought to equal and be convertible with the thing defined, which is one thing proper to good Definitions. Now such seemes to be the Definition of the thing proposed, which among the most simple and plain Expositors is defined *Porimon*, and amongst the more acute, that which defineth it to be *Porimon* and *Cognitum* together; but all the rest are imperfect; for that which defineth it *Ordinatum* is not sufficient for the comprehension and knowledge of *DATUM*; because that neither wholly ordinate, nor alone ordinate, is not comprised, seeing that there are things inordinate that have the same condition, as hath been shewn. Again, that reason gives not satisfaction neither, which describes it to be *Cognitum*; for all that is known is not comprehended, although that alone *Cognitum* be comprehended. Moreover, that also is not perfect which defineth it to be *Effabile*, for *Effabile* is not alone comprehended; seeing that some of the Irrationals are also comprehended. In like manner, all *Effabile* is not comprehended, as hath been before declared. Now amongst the Definitions which expound it by one only Word, there remains that which defineth it to be *Porimon*, which seemeth greatly to manifest the comprehension; for whole *Porimon* and alone *Porimon* is comprehended. Wherefore *EUCLIDE* himself useth such a Definition in a Description of all the kinds of GIVEN by him conceived and regarded. But amongst such Definitions as are compounded, that is a perfect Definition which defineth *DATUM* to be *Cognitum* and *Porimon* together, having *Cognitum* for analogical kind, and *Porimon* for difference; but that is imperfect which hath *Ordinatum* and *Porimon* together; for those things which are such, are not alone GIVEN, and that which defineth it *Ordinatum* and *Effabile* together, comprehendeth likewise the GIVEN, with the defect or want. But that of *Cognitum* and *Ordinatum* together, is not to be received or admitted, because

cause it doth exceed what is defined, for such is not given alone: Therefore those only which have declared that *DATUM* is *Cognitum* and *Porimon* together, seem to have atteined the notion of GIVEN, for that which is such is all, and alone comprehended, which two things ought to be in those Definitions that are well given. But the former come neer to those which have thus defined it: *DATUM* is that to which we may find an equal, according to those things we have proposed in the first principles and Hypothesis, of which number *EUCLIDE* is one, making use throughout of the Verb πορίσασθαι, which signifies to exhibit or invent, although he leaves *Cognitum* as a consequent of *Porimon*; some one neverthelesse might reprove him, for that in the first place he hath not defined *DATUM* in general; but immediately some of the kinds of GIVEN, although in his Elements of GEOMETRY; he hath defined the line simple before the Species or Kinds.

What is the Utility and Profit that ariseth from this Tract of DATA, *or things GIVEN.*

AFter having explained universally, and according to what seemed necessary for our present use, what this Word *DATUM* signifieth: It followes to shew the Utility of this Tract. Now this Tract is such, as that it is not only ordained and instituted for its own respect, but for some other thing; for it is very necessary to a place which is called Resolved, and we have already declared elsewhere how much strength a resolved place doth obtein in Mathematical Disciplines, as also in Opticks and Cannons, which come very neer to them, as well for that Resolve is an invention of the Demonstration, as for that in such like things it serves us much for the invention of the Demonstration, or for that it is much more excellent to meet with a Resolutive power, then to enjoy divers particular Demonstrations.

To what Art or Science this Tract is referred.

NOw seeing the consideration of GIVEN is useful and profitable in all these kinds of Arts, for that it serveth much to RESOLUTION, it may well be said to be recalled, not only to one only Science, but to the Mathematicks universally which treat of Numbers, Time, Swiftnesse, and such like things, which treateth likewise of Reasons, as also of proportions, and in a word of all *Medietez*: Wherefore for the perfect and demonstrative Knowledge of things GIVEN, being of so great Utility, *EUCLIDE* hath taken pains to frame this Book of things GIVEN, which Author amongst all such as have composed the Elements of Geometry, hath justly deserved the first place and rank, and who having invented the Elements, or rather the Introductions almost of all Mathematical Disciplines, to wit, of all Geometry in 13 Books, of Astronomy, of Musick, and Opticks, he hath left in writing the Elements RESOLUTIVE, in this Treatise of things GIVEN; but as he was a Geometrician, he hath particularly accommodated to Magnitudes, that, that was of the GIVEN; yet neverthelesse common in other things, which method hath also been observed by him, when treating universally of Reasons and Proportions, he appropriates them to the Magnitudes mentioned in his Fifth Book of Plains.

Now

Now it hath been declared in general what is the meaning of *DATUM*, to what Science it apperteineth, and how profitable the Contemplation thereof is. We will add to what hath been said, the Description of this Science which treats of things GIVEN; seeing that it is (as appeares by what hath been said) a comprehension in all manners of things GIVEN; and of their accidents and proprieties. But having respect to the proposed Book, we shall declare it to be an Elementary Doctrine of the whole Knowledge of things GIVEN, whence it follows that it will be very profitable, as also the things therein conteined, seeing they refer to things GIVEN.

Now this Book is divided according to the Species or Kinds of the things GIVEN, and in the first section are conteined those things which are given by reason. Secondly, such as are given by position: and lastly, such as are given by Species or Kind; for that which is given by Magnitude is simple, and particularly conteined in the others, and principally in those things given by Specis or Kind. Now he hath begun with those things given by Reason and Position, forasmuch as those that are given by Species are constituted of them. *EUCLIDE* gives yet another division to this Book, for that he divides it into universal Magnitudes, Lines, and Superficies, and into Circular Theoremes, which order he hath also observed in the Definitions and Suppositions of this Book. Moreover, he useth a certain way of instructing, which proceeds not by Composition, but by Resolution, as *Pappus* hath amply set down in his Commentaries on this Book.

THE END OF THE PREFACE OF MARINUS.

DEFINITIONS.

1 PLains or Spaces, Lines, and Angles, to which we may find others equal, are said to be given by Magnitude.

2 A Reason is said to be given, when we may find one of the same or equal thereto.

3 Rectiline figures, whose angles are given, and also the reason of the sides to one another, are said to be given by Species or Kind.

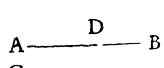
4 Points, Lines, and Angles, which have and keep alwayes one and the same place and situation, are said to be given by Position or Situation.

5 A Circle is said to be given by Magnitude, when the Semidiameter thereof is given by Magnitude.

6 A Circle is said to be given by Position, and by Magnitude when the center thereof is given by Position, and the Semidiameter by Magnitude.

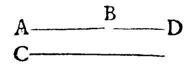
7 Segments of Circles, whose angles and bases are given by Magnitude, are said to be given by Magnitude.

8 Segments of a Circle, whose angles are given by Magnitude, and the bases of the segments by Position and Magnitude, are said to be given by Position and by Magnitude.



9 A Magnitude A B, is greater than another Magnitude C, by a given Magnitude B D, when having taken away the given Magnitude D B, the rest A D, is equal to the other Magnitude C.

10 A Magnitude A B, is lesse than another Magnitude C, by a given magnitude B D, when having added thereto the given magnitude B D, the whole A D, is equal to the other magnitude C.



11 A Magnitude A B, is said to be greater than another magnitude C B, by a given magnitude A D, and in reason, when taking from the same magnitude the given magnitude A D, the rest D B, hath to the other magnitude C B, a given reason.

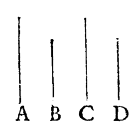
12 A magnitude A B is said to be lesse than another magnitude B C, by a given magnitude A D, and in reason, when the given magnitude A D being added thereto, the whole D B hath to the other magnitude B C, a given reason.

13 A right line is said to be drawn down from a given point, unto a right line given in position, the right line being drawn in a given angle.

14 A right line is said to be drawn up from a given point, to a right line given in position, the right line being drawn in a given angle.

15 A right line is against another right line in position, when it is drawn parallel thereto thorough a given point.

PROPOSITION 1.



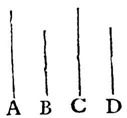
Two magnitudes A and B, being given, the reason they have to one another A to B, is also given.

Demonstration FOr seeing that the magnitude A is given [a], we may find one equal thereto, which let be C. Again, forasmuch as the magnitude B is given, we may also find one equal to that, and let that be D. Therefore seeing that A is equal to C, and B to D, as A is to C [b], so is B to D, and by permutation, [c] as A shall be to B, so C shall be to D. Therefore [d] the reason of A to B is given, for it is the same reason as of C to D, as we have found, and which ought to be demonstrated.

a) 1 def.
b) 7. 5.
c) 16. 5.
d) 2 def.

PROP. 2.

If a given magnitude A, hath to some other magnitude B, a given reason, that other magnitude B, is also given by magnitude.



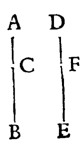
Demonstration FOr seeing that A is given, we may find one equal thereto, which let be C: And forasmuch as the reason of A to B, is also given, we may find [a] one of the same. Let it be found, and (a) 2. def.) let the reason be of C to D. Now seeing that as A is to B, so C is to D; and by permutation, as A is to C, so B is to D: But A is equal to C, Therefore [b] B shall be also equal to D. (b) 14. 5.) Therefore [c] the magnitude B is given, seeing that thereto there hath been (c) 1. def.) found one equal, to wit, D.

PROP. 3.

If given magnitudes AB and BC, are compounded, that magnitude AC, that is compounded of them shall be also given.

Demonstration FOr seeing that AB is given, we may find one equal to it, which let be DE. Again, seeing that BC is given, we may also find one equal to that, which let be EF. Wherefore seeing that DE is equal AB, and EF is equal to BC, the whole AC [a] is equal to the whole DF. Therefore AC (a) 2 ax. 1.) is given, seeing that DF is proposed equal thereto.

PROP. 4.



If from a given magnitude AB, there be taken away a given magnitude AC, the remaining magnitude CB, is also given.

Demonstration FOrasmuch as AB is given, we may find one equal thereto, which let be DE. Again, seeing that AC is given, we may also find one equal to it, which let be DF. Seeing then that the Magnitude AB is equal to the magnitude DE, and the magnitude AC to the magnitude DF; the rest CB [a] shall be equal to the rest FE. Wherefore CB is given, (a) 3 ax. 1.) for to it there hath been found an equal, to wit, FE.

PROP. 5.

If a magnitude AB, hath a given reason to some part thereof AC, it will have also a given reason to the part remaining CB.

Demonstration LEt DE be exposed as a given magnitude, and seeing that the reason of the magnitude AB, to the magnitude AC, is given, [a] we may find one of (a) 2. def.) the same, which let be DE to DF; therefore the reason of the same DE to DF is given. But DE being given, so is [b] also its part DF; and conse- (b) 2. prop) quently, [c] the rest FE: Therefore [d] seeing that DE and FE are given, the (c) 4. prop.) reason of the same DE to FE, is also given. And forasmuch as DE is to (d) 1. prop.) DF, as AB is to AC, and by conversion, as DE to FE, so AB is to CB. But the reason of DE to FE is given, as hath been demonstrated; (19. 5.) therefore the reason of AB to CB is also given.

SCHO-

SCHOLIUM.

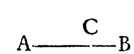
From this it is evident that if a Magnitude hath to some part thereof a given reason, by division, the reason that one part hath to the other, shall be also given. For seeing that as DE is to FE, so is AB to CB; by division, as DF to FE, so AC to CB. But it hath been demonstrated that the parts DF and FE are given, and consequently, their reason is also given: In like manner, therefore the reason of AC to CB is given.

PROP. 6.

If two magnitudes AB and BC, having to one another a given reason, are compounded, the magnitude AC, compounded of them, shall also have a given reason to each of them AB and BC.

Demonstration LEt the given magnitude DE be exposed, and seeing that the reason of AB to BC is given; let there be made one and the same of the said DE to EF; therefore the reason of the same DE to EF is given; and therefore [a] the magnitude DE being given, both the one and the other of them DE and FE, (a) 2. p.) is given. Wherefore [b] the whole DF shall be also given. Therefore [c] the (b) 3. p.) reason of the same DF to each of them DE and EF, shall be given. And (c) 1. p.) forasmuch then as AB is to BC, so is DE to EF; in compounding, [d] as (d) 18. 5.) AC is to BC, so is DF to EF: Therefore by conversion, as AC to AB, so is DF to DE. Therefore as the whole DF is to each of the other magnitudes DE and EF, so the whole AC is to each of the Magnitudes AB and BC: Therefore [e] the reason of the same AC to each of the magni- (e) 2. def.) tudes AB and BC, is given.

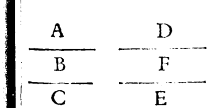
PROP. 7.



If a given magnitude AB, be divided according to a given reason AC to CB, each segment AC and CB is given.

Demonstration FOr seeing the reason of AC to CB is given, the reason of [a] AB to each of them (AC and CB) is also given. But (a) 6. p.) AB is given: Therefore [b] each of the segments AC and CB, is also (b) 2. p.) given.

PROP. 8.



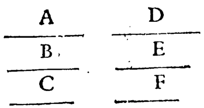
Magnitudes A and C, which have to one and the same a given reason B, shall be to one another in a given reason, A to C.

Demonstration FOr let the given magnitude D be exposed, and seeing that the reason of A to B is given, Let the same be done of the said D to E. Now seeing that D is given, [a] E is also given. Again, seeing (a) 2. p.) that the reason of B to C is given, let the same be done of E to F. But E is given, and therefore F is also given. But seeing that D is given, [b] the (b) 1. p.) reason of the same D to F is given; and seeing that as A to B, so D to E,

and

c) 22. 5. and as B to C, so is E to F; in reason of equality, [c] as A is to C, so is D to F; but the reason of D to F is given. Therefore the reason of A to C is also given.

PROP. 9.



If two or more magnitudes A, B, *and* C, *are to one another in a given reason, and that the same magnitudes* A, B, *and* C, *have to other magnitudes* D, E, *and* F, *given reasons, although they be not the same, those other magnitudes* D, E, *and* F, *shall be also to one another in given reasons.*

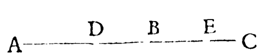
Demonstration. FOrasmuch as the reason of A to B is given, as also that of A to D, the reason of D to B shall be given: But the reason of B to E is also given; therefore the reason of the same D to E shall be in like manner given. Again, seeing that the reason of B to C is given, and also that of B to E, the reason of E to C shall be given. But the reason of C to F is also given. Therefore [a] the reason of E to F shall be given.
a) 8. p. But it hath been demonstrated that the reason of D to E is also given; and therefore [b] the reason of D to F shall be given. Therefore the magnitudes D, E, and F, are to one another in given reasons.
b) 8. p.

PROP. 10.

If a magnitude A B, *be greater than another magnitude* B C, *by a given magnitude, and in reason, the magnitude* A C *compounded of both, shall be also greater then that same magnitude, by a given magnitude, and in reason: But if that compounded magnitude be greater then the same magnitude, by a given magnitude, and in reason; either the remainder shall be also greater then that same by a given magnitude, and in reason; or else the same remainder is given with the following, to which the other magnitude hath a given reason.*

Demonstration. FOr seeing that A B is greater than B C by a given magnitude, and in reason, let the given magnitude A D be taken away.
a) 11 def. Therefore [a] the reason of the remainder D B to B C is given;
b) 6. p. and in compounding, [b] the reason of D C to B C is also given. But the magnitude A D is also given; Therefore A C is greater than the same B C by a given magnitude, and in reason.

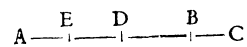
Again, Let the magnitude A C be greater than the magnitude B C, by a given magnitude, and in reason: I say that the rest A B, is either greater then the same B C by a given magnitude and in reason, or that the same A B, with that which followeth, to which B C hath a given reason, is given.



For

Forasmuch as the magnitude A C is greater than the magnitude B C, by a given magnitude, and in reason, cut off from it the given magnitude: Now the same given magnitude is either lesse than the magnitude A B, or greater: Let it in the first place be lesse, and let it be A D. Therefore the reason of the remainder D C to C B is given. Wherefore by division, the reason of D B to B C is given. But the magnitude A D is also given; therefore the magnitude A B is greater [c] then the magnitude c) 11 def.
B C by a given magnitude, and in reason. Now let the given magnitude be greater than the magnitude A B, and let A E be put equal thereto; therefore [d] the reason of the remainder E C to C B is given; and by con- d) 11 def.
version, [e] the reason of the same B C to B E, is also given. But the same e) 5. p.
E B with B A is given, for that the whole A E is given: Therefore there is given A B, with that which followes, B E, to which B C hath a given reason.

PROP. 11.



If a magnitude A B, *be greater than a magnitude* B C, *by a given magnitude, and in reason, the same magnitude* A B, *shall be also greater than the magnitude compounded of them by a given magnitude, and in reason, and if the same magnitude be greater then the two others together by a given magnitude, and in reason, that same magnitude shall be also greater then the rest by a given magnitude, and in reason.*

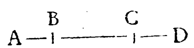
Demonstration. FOr seeing that the magnitude A B is greater then B C by a given magnitude, and in reason; let there be taken from it a given magnitude A D: Therefore [a] the reason of the rest D B to D C, a) 11 def.
is given, and therefore [b] the reason of D C to B D shall be also given: b) 6. p.
Let the same be done of A D to D E, therefore the reason of the same A D to D E is given. But A D is given, therefore [c] D E is also given, c) 2. p.
and consequently, [d] the rest A E, is also given. But seeing that as A D is to d) 4. p.
D E, so is D C to B D; by permutation, [e] as A D is to D C, so is D E to e) 16. 5.
D B: Therefore by compounding, [f] as A C is to C D, so is E B to D B; f) 18. 5.
and by permutation, [g] as A C is to E B, so is D C to D B. But the reason g) 16. 5.
of D C to D B is given: Therefore also is A C to E B, and consequently, that of E B to A C. But it hath been demonstrated that A E is given, Therefore [h] A B is greater then A C by a given magnitude, and in h) 11 def.
reason.

But now let A B be greater then A C by a given magnitude, and in reason: I say that the same A B is also greater then the rest B C, by a given magnitude, and in reason.

For seeing that A B is greater than A C by a given magnitude, and in reason, Let the given magnitude A B be cut off there-from: Therefore [i] the reason of the remainder E B to A C is given, and consequently, also i) 11 def.
shall be given that of A C to E B. Let the same be done of A D to D E, Therefore the reason of A D to D E is given; and by conversion, [k] the k) 5. p.
reason of A D to A E shall be also given, and consequently that of A E to A D. Now A E is given, Therefore the whole A D [l] shall be also gi- l) 2. p.
ven, and seeing that as the whole A C is to whole E B, so the part cut off A D,

m) 19. 5. A D, is to the part cut off E D, so also shall be [m] the remainder D C to the remainder D B. But the reason of A C to E B is given: Therefore also shall n) Schol. 5. p. be given that of D C to D B. Wherefore by division, [n] the reason of B C to D B is given; and consequently also shall be given that of D B to B C. But o) 11 def. it hath been demonstrated that A D is given: Therefore [o] A B is greater then the same B C by a given magnitude, and in reason.

PROP. 12.



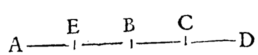
If there be three magnitudes A B, B C, *and* C D, *and that the first* A B, *with the second* B C, *to wit* A C, *be given. But the second* B C, *with the third* C D, (*to wit,* B D, *be also given: Either the first* A B *shall be equal to the third* C D, *or the one shall be greater then the other by a given magnitude.*

Demonstration. FOrasmuch as each of the magnitudes A C and B D are given, the given magnitudes are either equal to one another, or inequal. Let them be first equal: Therefore A C is equal to B D, take a) 3 ax. 1. away the common magnitude B C, and there will remain [a] A B, equal to C D. But suppose them to be inequal, as in this second figure, and let B D be greater then A C: Let then B E be put equal to A C. Now seeing that A C is given, B E is also given. But the whole B D is also given, the rest E D [b] shall be so also; and forasmuch as B E is equal to A C, taking away the common magnitude B C [c], there will remain A B equal to C E. But E D is given: Therefore C D is greater then A B by the given magnitude E D.

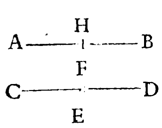
b) 4. p.
c) 3 ax. 1.

SCHOLIUM.

And if the first with the second, to wit, A C, *were greater then the second with the third, to wit,* B D, *as in the other figure,* C E *would be made equal to the same* B D, *and by the same reasons as was above demonstrated, that* A E *is given and equal to* C D; *and therefore* A B *greater then* C D *by a given magnitude.*



PROP. 13.

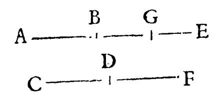


If there be three magnitudes A B, CD, *and* E, *and that the first of them* A B, *hath a given reason to the second* C D; *but the second* C D, *is greater then the third* E, *by a given magnitude, and in reason, also the first* A B, *shall be greater then the third* E, *by a given magnitude, and in reason.*

Demonstration. FOr seeing that C D is greater then E by a given magnitude, and in reason, let the given magnitude C F be taken there-

there-from: Therefore the reason of the rest F D to E is given. And forasmuch as the reason of A B to C D is given, let the same be done of A H to C F. Therefore the reason of the same A H to C F is given. But C F is given: Therefore [a] A H is also given. And seeing that as the whole a) 2. p. A B is to the whole C D, so the part cut off A H is to the part cut off C F, and so [b] also the rest H B is to the rest F D, the reason of the same H B to b) 19. 5. F D is also given. But the reason of F D to E is also given: Therefore [c] the reason of H B to E is given. But it hath been demonstrated that A H c) 8. p. is given: Therefore [d] A B is greater then the said E by a given magnitude, and in reason. d) 11 def.

PROP. 14.

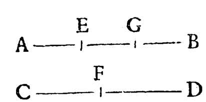


If two magnitudes A B *and* C D, *have to one another a given reason, and that to each of them there be added a given magnitude, to wit,* B E *and* D F; *either the whole* A E, *and* C F, *shall have to one another a given reason, or the one shall be greater then the other by a given magnitude, and in reason.*

Demonstration. FOr seeing that each of those magnitudes B E and D F, is given, [a] the reason of the said B E and D F is also given; a) 1. p. and if that reason be the same with that of A B to C D, that of the whole A E to the whole C F, [b] shall be the same; and therefore the reason of b) 12. 5. the said A E to C F is given.

Now let the reason of B E to D F be not the same, with that of A B to C D, and let it be as A B to C D, so B G to D F. Therefore the reason of the said B G to D F is given. But the magnitude D F is given, therefore [c] B G is also given; and seeing that the whole B E is given, [d] the c) 2. p. rest G E shall be also given. But forasmuch as A B is to C D, as B G is to d) 4. p. D F, [e] so also is the whole A G to the whole C F; and therefore the reason of the said A G to C F is given: But the magnitude G E is given: e) 12. 5. Therefore [f] the magnitude A E is greater then the magnitude C F by a f) 11 def. given magnitude, and in reason.

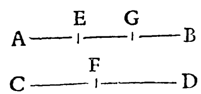
PROP. 15.



If two magnitudes A B *and* C D, *have to one another a given reason, and that from each of them be taken away a given magnitude (to wit, from the magnitude* A B *the magnitude* A E, *and from the magnitude* C D *the magnitude* C F) *the remaining magnitudes* E B *and* F D, *either shall have to one another a given reason, or the one of them shall be greater then the other by a given magnitude, and in reason.*

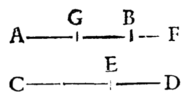
Demonstration. FOr seeing that each magnitude A E and C F is given, the reason of A E to C F is given; and if it be the same with that

that of A B to C D, that of the remainder E B to the remainder F D,
a) 19. 5. [a] shall be also the same; and therefore the reason of the said E B to F D shall be also given. But if it be not the same,



Let it be as A B to C D, so A C to C F. Now the reason of A B to C D is given, therefore also that of A G to C F shall be
b) 2. p. given. But C F is given, therefore [b] A G is
c) 4. p. given. But A E is also given, therefore [c] the rest E G is given; and seeing that as A B is to C D, so the part cut off
d) 4. p. A G is to the part cut off C F, and so also is [d] the rest G B to the rest F D; the reason of the said G B to F D is also given. Therefore seeing
e) 11 def. that E G is given, E B is greater then F D [e] by a given magnitude, and in reason.

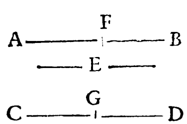
PROP. 16.



If two magnitudes A B *and* C D, *have to one another a given reason, and that from one of them, to wit,* C D, *there be taken away a given magnitude* D E, *and to the other* A B *there be added a given magnitude* B F, *the whole* A F *shall be greater then the rest* C E, *by a given magnitude, and in reason.*

Demonstration FOr seeing that the reason of A B to C D is given, let the
a) 2. def. same be made of B O to D E: Therefore [a] the reason of
b) 2. p. the said B G to D E is given. But D E is given, therefore [b] B G is also
c) 3. p. given. But B F is also given, therefore [c] the whole G F is given. And seeing that as A B is to C D, so the part cut off B G, is to the part cut off
d) 19. 5. D E; and [d] so also is the remainder A G to the remainder C E; the reason of the said A G to C E is given: But G F is given, Therefore the magnitude A F is greater then the magnitude C E by a given magnitude, and in reason.

PROP. 17.

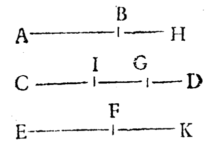


If there be three magnitudes A B, E, *and* C D, *and that the first* A B *be greater then the second* E, *by a given magnitude, and in reason. But the third* C D *be also greater then the same second* E, *by a given magnitude, and in reason; the first* A B *shall have to the third* C D, *either a given reason, or else the one shall be greater then the other by a given magnitude, and in reason.*

Demonstration FOr seeing that A B is greater then E by a given magnitude, and in reason, let the magnitude A F be taken away: Therefore the reason of the remainder F B to E is given. Again, seeing that C D is greater then the said E by a given magnitude, and in reason, let the given magnitude C G be cut off there-from; and the rea-
a) 8. p. son of the remainder G D to E shall be given: Therefore [a] the reason of F B

F B to G D shall be also given. But to the said F B and G D are added the given magnitudes A F and C G: Therefore the whole A B and C D
b) 14. p. [b] shall either have to one another a given reason, or the one shall be greater then the other by a given magnitude, and in reason.

PROP. 18.

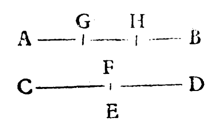


If there be three magnitudes A B, C D, *and* E F, *and that the one of them, to wit,* C D, *be greater then either of the other* A B *or* E F, *by a given magnitude, and in reason; either the two others* A B *and* E F, *shall have to one another a given reason, or the one shall be greater then the other by a given magnitude, and in reason.*

Demonstration FOrasmuch as the magnitude C D is greater then the magnitude A B by a given magnitude, and in reason, let the given magnitude D G be taken there-from: Therefore the reason of the remainder C G to A B is given. Let the same be made of G D to B H, Therefore the reason of the said D G to B H is given. But D G is given,
a) 2. p. Therefore [a] B H is also given. And seeing that as C G is to A B, so is G D
b) 12. 5. to B H, [b] so also is the whole C D to the whole A H, the reason of the said C D to A H shall be also given.

Again, seeing that the same C D is greater then E F by a given magnitude, and in reason; let the magnitude D I be cut off there-from: Therefore the reason of the remainder C I to E F is given: Let the same be made of D I to F K. Therefore the reason of the said D I to F K shall be also given. But D I is given, Therefore F K is also given. And seeing
c) 12. 5. that as C I is to E F, so is I D to F K; so also is the whole [c] C D to the whole E K; the reason of the said C D to E K shall be given. But the rea-
d) 8. p. son of the same C D to A H is also given: Therefore [d] the reason of the said A H to E K shall be given. And seeing that from the said A H and E K, the given magnitudes B H and F K are cut off, the magnitudes A B
e) 15. p. and E F [e] are either in a given reason to one another, or the one is greater then the other by a given magnitude, and in reason.

PROP. 19.

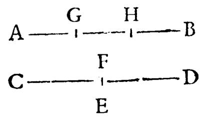


If there be three magnitudes A B, C D, *and* E, *and that the first* A B, *be greater then the second* C D, *by a given magnitude, and in reason; and that the second* C D *be greater then the third* E, *by a given magnitude, and in reason; also the first magnitude* A B *shall be greater then the third* E, *by a given magnitude, and in reason.*

Demonstration FOr seeing that C D is greater then E by a given magnitude, and in reason; let the given magnitude C F be taken there-from: Therefore the reason of the remainder F D to E is given.

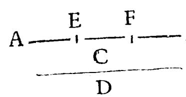
 Again,

Again, ſeeing that A B is greater then the ſame C D by a given magnitude, and in reaſon: Let the magnitude A G be taken there-from: Therefore the reaſon of the remainder G B to C D is given: Let the ſame be made of G H to C F: Therefore the reaſon of the ſaid G H to C F is given. But C F is given: Therefore alſo G H is given, and then A G is alſo given, the whole [a] A H ſhall be alſo given. But as G B is to C D, ſo is G H to C F, and ſo alſo [b] the remainder H B to the remainder F D: Therefore the reaſon of the ſaid H B to F D is given. But the reaſon of the ſame F D to E is alſo given: Therefore the reaſon of H B to E is in like manner given, and ſo is alſo the magnitude A E: Wherefore the magnitude A B [c] is greater then E by a given magnitude, and in reaſon.



a) 3. p.
b) 19. 5.
c) 11 def.

OTHERWISE.

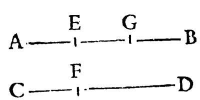


Conſtruction LEt there be three magnitudes A B, C, and D, and let A B be greater then C by a given magnitude, and in reaſon; but let C be alſo greater then D, by a given magnitude, and in reaſon: I ſay that A B is greater then D by a given magnitude, and in reaſon.

Demonſtration FOraſmuch as A B is greater then C by a given magnitude, and in reaſon, let the given magnitude A E be cut off therefrom: Therefore the reaſon of the remainder E B to C is given. But the magnitude C is greater then the magnitude D by a given magnitude, and in reaſon; therefore [d] E B is greater then D by a given magnitude, and in reaſon: Wherefore let the given magnitude E F be cut off therefrom; and the reaſon of the remainder F B to D ſhall be given. But A F [e] is given. Therefore [f] A B is greater then D by a given magnitude, and in reaſon.

d) 13. p.
e) 3. p.
f) 11 def.

PROP. 20.



If there be two given magnitudes, A B *and* C D, *and that from them there be taken magnitudes* A E *and* C F, *having to one another a given reaſon; either the remaining magnitudes* E B *and* F D, *ſhall have to one another given reaſons, or elſe the one ſhall be greater then the other by a given magnitude, and in reaſon.*

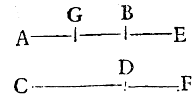
Demonſtration FOr ſeeing that both the magnitudes A B and C D, are given, the reaſon of the ſaid A B to C D is [a] alſo given; and if it be the ſame as of A E to C F, that of the remainder E B to the remainder F D ſhall be [b] alſo the ſame; and therefore the reaſon of the ſaid E B to F D ſhall be alſo given. But if it be not the ſame, let it be ſo as that A E be to C F, as A G to C D. Now the reaſon of the ſaid A E to C F is given: Therefore the reaſon of the ſaid A G to C D is given. But C D is given, Therefore [c] A G is alſo given. But the whole A B is likewiſe given, Therefore [d] the remainder B G is given. And ſeeing that as A E

a) 1. p.
b) 19. 5.
c) 3. p.
d) 4. p.

A E is to C F, ſo is A G to C D, and alſo the remainder E G to the remainder F D, the reaſon of the ſaid E G to F D is given. But G B is alſo given: Therefore the magnitude E B is greater [c] then the magnitude B D by a given magnitude, and in reaſon.

c) 11 def.

PROP. 21.

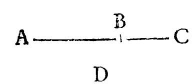


If there be two magnitudes given A B *and* C D, *and to them be added other magnitudes* B E *and* D F, *having to one another a given reaſon: Either the whole* A E *and* C F *ſhall have to one another a given reaſon, or elſe the one ſhall be greater then the other by a given magnitude, and in reaſon.*

Demonſtration FOr ſeeing that both the magnitudes A B and C D are given their reaſon [a] is alſo given; and if it be the ſame reaſon as of B E to D F, the reaſon of the whole A E to the whole C F ſhall be alſo given; for it ſhall be [b] the ſame. But if it be not the ſame, Let it be as B E is to D F, ſo B G to C D: Therefore the reaſon of the ſaid B G to C D is given. But C D is given, Therefore [c] alſo B G ſhall be given. But the whole A B is given, Therefore alſo the [d] remainder A G ſhall be given. And ſeeing that as B E is to D F, ſo is B G to C D, and and alſo [e] the whole G E to the whole C F, the reaſon of the ſaid G E to C F ſhall be likewiſe given. But A G is given, Therefore the magnitude A E is greater then the magnitude C F by a given magnitede, and in reaſon.

a) 1. p.
b) 12, 5.
c) 2. p.
d) 4. p.
e) 12, 5.

PROP. 22.

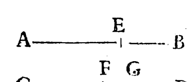


If two magnitudes A B *and* B C, *have to ſome other magnitude* D, *a given reaſon, alſo their compound magnitude* A C, *ſhall have to the ſame magnitude* D, *a given reaſon.*

Demonſtration FOr ſeeing that each magnitude A B and B C, hath a given reaſon to D, the reaſon [a] of A B to B C is given; and by compounding, [b] the reaſon of A C to B C is given. But that of B C to D is alſo given, Therefore [c] the reaſon of the ſaid A C to D ſhall be likewiſe given.

a) 8. p.
b) 6. p.
c) 8. p.

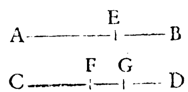
PROP. 23.



If the whole A B *be to the whole* C D *in a given reaſon, and that the parts* A E *and* E B, *be to the parts* C F *and* F D *in given reaſons, although they be not the ſame, the whole (to wit,* A B, A E, *and* B E,) *ſhall be to the whole (to wit,* C D, C F, *and* F D,) *in given reaſons.*

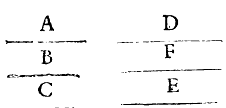
Demon-

Demonstration. FOr seeing that A E is to C F in a given reason, let the same be made of A B to C G; therefore the reason of the said
a) 19. 5. A B to C G is given; and consequently, also that [a] of the rest E B to the rest F G. But the reason of F D to the same E B is also given: Therefore
b) 8. p. the reason of F D to F G [b] is likewise given; and therefore [c] that of F D
c) 5. p. to the remainder G D is also given. But the reason of A B to each of the
d) 8. p. magnitudes C D and C G is given: Therefore [d] also the reason of C D to
e) 5. p. C G is given, and again [e] that of C D to the remainder G D. But the
f) 8. p. reason of F D to D G is given, Therefore also [f] that of the same C D to
g) 5. p. F D, and consequently that of [g] C D to the remainder F C; and therefore also the reason of C F to F D shall be given. But the reason of E B to F D is proposed to be given; therefore the reason of C F to E B shall be given.



Again, for that the reason of A B to C D is given; and also that of C D to each of those F C and F D, the reason of
h) 8. p. the same A B to each of the said F C and F D [h], shall be likewise given. But the reason of the said F D to E B is given: Therefore the reason of A B to B E shall be also given, and consequently A B to the remainder
i) 5. p. [i] A E. Wherefore by division [k] the reason of A E to E B shall be likewise
k) Sch. 5. p. given. But the reason of E B to F D is given. Therefore also that of A E to F D. In like manner, seeing that the reason of C D to A B is given; and that of A B to each of his parts A E and E B; also the reason of the said
l) 8. p. C D to each of the said A E and E B, [l] shall be given: Wherefore each of the magnitudes A B, C D, A E, E B, C F, and F D, is to each of the others in a given reason.

PROP. 24.



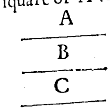
If of three right lines A, B, *and* C, *proportional*, A *to* B, *as* B *to* C, *the first* A *hath to the third* C *a given reason, it will also have to the second* B *a given reason.*

Demonstration. FOr, Let there be exposed another right line D, and seeing that the reason of A to C is given: Let the same be made of D to F; therefore the reason of D to F is given. But D is given, therefore F is also given; betwixt the two right lines D and F, let there
a) 13. 6. be taken [a] a mean proportional E. Therefore the rectangle made
b) 17. 6. under D and F is equal [b] to the square of E. But the same rectangle of D
c) 3 def. and F is [c] given: (for all the angles of that rectangle are given, being right angles, and the reasons that the sides have to one another are also given;) therefore the square of E is given, and consequently the same
d) 14. 2. right line E is also given (for one equal thereto may be found [d] seeing
e) 1. p. that the rectangle of D and F is given.) But D is given, therefore [e] the rea-
f) 1. 6. son of D to E is given, and as A is to C, [f] so D is to F. But as A is to C so the square of A is to the rectangle of A and C, and also as D is to F, so the square of D is to the rectangle of D and F. Therefore as the square of A is to the rectangle of A and C, so the square of D is to the rectangle of D and F. But the rectangle of A and C is equal to the square of B, (seeing that

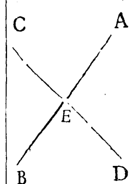
that A, B, and C, are proportional) and that of D and F to the square of E, Therefore as the square of A is to the square of B, so the square of D is to the square of E: Wherefore [g] as A is to B, so D is to E. But the g) 12. 6.
reason of D to E is given, Therefore [h] also the reason of A to B is given. h) 2. def.

OTHERWISE.

Demonstration. FOrasmuch as the reason of A to C is given, and that as A is to C, so the square of A and C, the reason of the said square of A to the rectangle made of A and C, is also given. But to that rectangle made of A and C the square of B is equal (seeing that A, B, and C, are proportional;) therefore the reason of the square of A to the square of B is given; and by consequence, the reason of the line A to the line B is given; for to each of them A and B, we have exhibited an equal to the proper square of each one.



PROP. 25.



If two lines AB *and* CD, *given by position do intersect, the point* E *in which they intersect one another, is given by position.*

Demonstration. FOr if it change its place, the one or the other of the lines A B and C D, would change its position: But so it is that by Supposition it changeth not: Therefore [a] the point E is given by position. a) 4. def.

PROP. 26.

A———B

If the extremities A *and* B, *of a right line* AB, *be given by position, that same right line* A B *is given by position and by magnitude.*

Demonstration. FOr if the point A remaining in its place, the position, or the magnitude of the right line A B shall change, the point B will fall elswhere. But so it is, that by Supposition it doth not fall elswhere. Therefore the right line A B is given by position, and by magnitude.

PROP. 27.

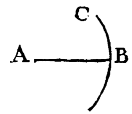
A———B

If one of the extremities A *of a right line* AB, *given by position and magnitude be given, the other extremity* B *shall be also given.*

Demonstration. FOr if the point A remaining in its place, the point B shall change and fall in some other place, either the position of the right line A B, or its magnitude would change: But so it is that according to the Supposition, neither the one nor the other doth change. Therefore the point B is given.

OTHER-

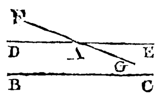
OTHERWISE.



Construction ON the center A, with the distance A B, describe the circumference BC.

a) 6 def. *Demonstration* THerefore [a] that circumference B C is given by position. But the right line A B is also given by position; therefore the point [b] B is given. b) 25. p.

PROP. 28.

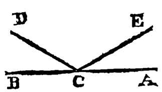


If by a given point A, *there be drawn a right line* D A E, *against another right line* B C, *given by position*, *the right line* D A E *so drawn*, *is given by position.*

Demonstration FOr if it be not given, the point A remaining in its place, the position of the right line D A E may change: Let it then change if it be possible, and fall elswhere, remaining parallel to B C, and let it be the line F A G: Therefore B C is parallel to the said line F A G. But [a] the same B C is also parallel to D A E: Therefore [b] D A E is parallel to the said line F A G, which is absurd; seeing they joyn together and meet in A: Therefore the position of the right line D A E falls not elswhere. Wherefore the said line D A E is given by position.

a) 13 def.
b) 30. 1.

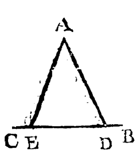
PROP. 29.



If to a right line A B, *given by position*, *and to a point* C *given therein*, *there be drawn a right line* C D, *which shall make a given angle* A C D, *the line drawn* C D, *is given by position.*

Demonstration FOr if it be not given by position, the point C remaining in its place, the position of the line C D observing the magnitude of the angle A C D, will fall elswhere. Let it fall elswhere then if it be possible, and let it be C E. Therefore the angle A C D is equal to the angle A C E, the greater to the lesser, which is absurd. Therefore the position of the right line C D, shall not fall elswhere; and therefore the said line C D is given by position.

PROP. 30.



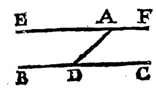
If from a given point A, *be drawn to a right line* B C, *given by position, a right line* A D, *making a given angle* A D B, *that line drawn* A D *is given by position.*

Demonstration FOr if it be not given, the point A remaining in its place, the position of the right line A D keeping the magnitude of the angle A D B will change.

change. Let it change then, and let it be the right line A E: Therefore the angle A D B is equal to the angle A E B, the greater [a] to the lesser, which is absurd. Therefore the position of the right line A D doth not change; and therefore the said line A D is given by position. a) 16. 1.

OTHERWISE.

Construction BY the point A let there be drawn the line E A F, parallel to the right line B C.



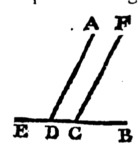
Demonstration THen seeing that by the given point A, and against the right line B C, given by position, there is drawn the right line E F, those lines E F and B C are parallels. But on the same lines doth also fall the right line A D. Therefore [b] the angle F A D is equal to the given angle A D B; and therefore it is also given. Wherefore to the right line E F given by position, and to the given point A therein, there is drawn the right line A D, making the given angle F A D. Therefore [c] the said line A D is given by position.

b) 29. 1.
c) 29. p.

OTHERWISE.

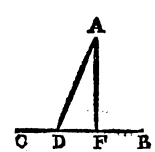
Construction IN the line B C E, let there be taken the given point C, and by the same let there be drawn the line C F, parallel to the said D A.

Demonstration FOrasmuch as A D and F C are parallels, and that on them there doth fall the right line B C E, the angle F C B is equal [d] to the given angle A D B; and therefore it is also given. And seeing that the right line B C is given by position, and that to a given point C therein, there is drawn the right line F C, making the given angle F C B, that same line F C [e] is given by position. But by the given point A, opposite to the line F C given by position, there is drawn the line A D. Therefore the said line [f] A D is given by position.



d) 29. 1.
e) 29. p.
f) 28. p.

OTHERWISE.



Construction IN the right line B C assume some point at F, and draw A F.

Demonstration FOrasmuch as each point A and F is given, the right line A F is given [g] by position. But the line B C is also given by position. Therefore * the angle A F D is given. But by Supposition, the angle A D F is given: Therefore D A F (which is the residue [h] of two right angles) is given; and seeing that to the right line A F given by position, and to the given point therein A there is drawn the right line D A, making the given angle D A F, [i] that same line D A is given by position.

g) 26. p.
h) 32. 1.
i) 29. p.

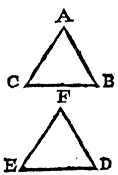
SCHOLIUM.

* *EUCLIDE supposeth here that two right lines being given by position, and inclining to one another do make a given angle, which some do demonstrate after this manner.*

 Demon-

Demonstration FOrasmuch as the two right lines given by position, do incline to one another, the inclination of those lines is given. But the angle is the inclination of the lines: Therefore the angle which makes the right lines given by position, and inclining to one another, is given.

Another thus demonstrateth it.



Construction LEt there be two right lines inclining to one another, as A B and C B, given by position, and in the line A B let there be taken a given point A, and in B C also some point, as C; and let the right line A C be drawn.

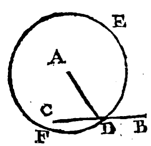
Demonstration SEeing that as well the point B, as each of the points A and C, is given, [k] the three right lines A B, B C, and A C, are given by magnitude. Wherefore of three direct lines equal unto them, a triangle may be constituted: Let there then be made the triangle F D E, having the side F D equal to the side A B, the side F E equal to the side A C, and the base D E equal to the base B C.

k) 26. p.

Seeing then the angles comprized of equal right lines are equal, we have found the angle F D E equal to the angle A B C; and therefore the same [l] angle A B C is given.

l) 1 def.

PROP. 31.



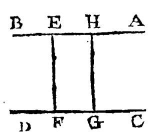
If from a given point A there be drawn to a right line given by position B C, a right line A D, given by magnitude, that line A D shall be also given by position.

Construction FRom the center A, with the distance A D, let the circle D E F be described.

Demonstration FOrasmuch as the center A is given by position, and the semidiameter A D by magnitude, the circle D E F [a] is given by position. But the right line B C is also given by position: Therefore the point of intersection D [b] is given, and seeing that the point A is also given: [c] the right line A D is given by position.

a) 6 def.
b) 25. p.
c) 26. p.

PROP. 32.



If unto parallel right lines A B and C D, given by position, there be drawn a right line E F, making the given angles B E F and E F D, the line drawn E F shall be given by magnitude.

Construction FOr let there be taken in the line C D a given point G, and from that point let be drawn G H parallel to F E.

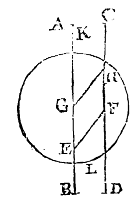
Demonstration FOrasmuch as the lines E F and H G are parallels, and that on them doth fall the line C D; [a] the angle E F D is equal

a) 29. 1.

equal to the angle F G H. But the angle E F D is given, therefore the angle F G H is also given. And forasmuch as to the right line C D given by position, and to the point G given in the same, there is drawn the right line G H, making the given angle F G H, [b] the said line G H is given by position. But A B is also given by position, Therefore [c] the point H is given. But the point G is also given: Therefore [d] the line G H is given by magnitude, and is [e] equal to E F. Wherefore [f] the said line E F is given by magnitude.

b) 29. p.
c) 25. p.
d) 26. p.
e) 34. 1.
f) 1 def.

PROP. 33.



If unto parallel right lines A B and C D, given by position, there be drawn a right line E F, given by magnitude, that line E F shall make the given angles B E F and D F E.

Construction FOr let there be taken in the right line A B the point G, and by that point let there be drawn the line G H, parallel to E F.

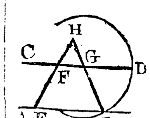
Demonstration THerefore E F is equal to the said [a] G H. But E F is given by magnitude, Therefore G H is also given by magnitude. But the point G is given; and therefore if on that point, with the distance G H, there be described a circle, [b] that circle shall be given by position: Let it be then described, and let it be H K L, the said circle H K L is therefore given by position. But the line C D which doth cut the circumference K H L in H, is also given by position. Therefore the said point of intersection H [c] is given. But the point G is given: Therefore [d] the right line G H is given by position. But the right line C D is also given by position: Therefore [e] the angle G H F is given. But to that angle [f] the angle F E D is equal: Therefore the angle E F D is given; and therefore also the angle B E F; for that it is the residue of the summe of two [g] right angles.

a) 34. 1.
b) 6 def.
c) 25. p.
d) 26. p.
e) Sch. 30. p.
f) 29. 1.
g) 29. 1.

OTHERWISE.

Construction LEt there be taken in the right line C D the point G, and let G D be put equal to E F, then from the center G, with the distance G D, let there be described the circle D B H, and draw G B.

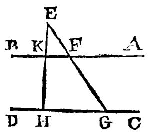
Demonstration FOrasmuch as the center D is given by position, and the semidiameter G D by magnitude, the circle B D H [h] is given by position. But the line A B is also given by position: Therefore [i] the point B is given. But the point G is also given, Therefore [k] the right line G B is given by position. But the right line C D is also given by position: Therefore [l] the angle B G D is given. Wherefore if E F be parallel to B G, the angle E F D [m] shall be given, and consequently also the other angle B E F. But the right lines B G and E F being not parallels, let them meet in the point H. Forasmuch as E B is parallel to F G, and E F is equal to G D, that is to say to B G; also F H [n] shall be equal to G H (for E H and B H being cut proportionally [o] by the parallel F G, as E F is to F H, so is B G to G H; and by permutation, as E F is to B G, so is



h) 6 def.
i) 25. p.
k) 26. p.
l) Sch. 30. p.
m) 29. 1.
n) 14. 5.
o) 2. 6.

p) 5. 1. is F H to G H:) Therefore [p] the angle H F G is equal to the angle H G F,
q) 15. 1. but the said angle H G F is given (for that it is equal [q] to the given angle B G D:) Therefore the angle H F G is also given. But to that angle the angle B E F is equal; and therefore is given, as also the remaining angle E F G.

PROP. 34.

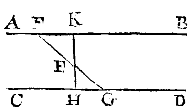


If from a given point E, *there be drawn unto parallel right lines* A B *and* C D, *given by position, a right line* E F G, *that right line* E F G *shall be divided in a given reason (to wit, as* E F *to* F G.

Construction FOr from the point E let there be drawn the line E H, perpendicular to the line C D.

Demonstration FOrasmuch as from the given point E there is drawn to the line C D the right line E H, making the given angle
a) 30. p. E H G [a] the said line E H is given by position, but both the one and the
b) 25. p. other lines A B and C D is also given by position. Therefore [b] the points of intersection K and H, are given. But the point E is also given: There-
c) 26. p. fore [c] each line E K and K H is given. Wherefore [d] the reason of the
d) 1. p. said E K to K H is given. But as E K is to K H, so is E F to F G; (for in the triangle G E H the line K F being parallel to H G, the sides E H and E G are cut proportionally:) Therefore the reason of the said E F to F G is given.

OTHERWISE.



Construction TO the parallel right lines given by position, A B and C D, let there be drawn from the point E the right line F E G: I say that the reason of G E to E F is given.

Demonstration FOr from the point E let there be drawn to C D the perpendicular E H, and produced to the point K; seeing therefore that from the point E to the right line C D, given by position, there
a) 30. p. is drawn the line E H, making the given angle E H G, [a] the said line E H is given by position. But each line A B and C D is also given by po-
b) 25. p. sition: Therefore [b] each point of intersection H and K is given. But the
c) 26. p. point E is also given, Therefore [c] each of the lines E H and E K is given
d) 1. p. by magnitude; and therefore [d] the reason of the said E H to E K is given.
e) 4. 6. But [e] as E H is to E K, so is E G to E F (for the opposite angles at the point E being equal, and the lines A B and C D parallels, the triangles E H G and E K F are equiangled; and therefore as E H is to E G, so is E K to E F; and by permutation as E H to E K, so is E G to E F.) Therefore the reason of the said lines E G to E F is given.

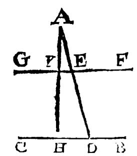
PROP. 35.

If from a given point A, *to a right line* B C, *given by position, there be drawn a right line* A D, *which let be divided in* E, *in a given reason (to wit, as* A E *to* E D, *and that*

that by the point of section E, *there be drawn a right line* F E G, *opposite to the right* B C, *given by position, the line* F G *drawn shall be given by position.*

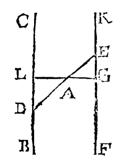
Construction FOr from the point A, let there be drawn the line A H, perpendicular to the line B C.

Demonstration FOr seeing that from the given point A there is drawn to B C given by position, the right line A H, making the
given angle A H D, [a] the said line A H is given by position. But B C a) 30. p.
is also given by position: Therefore [b] the point H is given. But the b) 25. p.



point A is also given: Therefore [c] the line A H is given by magnitude and by position. And seeing that c) 26. p.
[d] as A E is to E D, so is A K to K H, and that d) 2. 6.
the reason of A E to E D is given, also the reason of A K to K H is given; and by compounding, [e] the e) 6. p.
reason of A H to A K is given. But A H is given by magnitude: Therefore [f] also A K is given by magnitude. f) 2. p.
But A K is also given by position, and the point A is given: Therefore [g] the point K is also given; and seeing that by the said g) 27. p.
given point K there is drawn the line F G, opposite to the right line B C given by position; the said line F G [h] is given by position. h) 28. p.

PROP. 36.



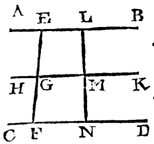
If from a given point A, *there be drawn to a right line* B C *given by position, a right line* A D, *and to it be added a right line* A E, *having to the same* A D *a given reason, and that by the extremity* E *of the added line* A E, *there be drawn a right line* F E K, *opposite to the line* B C, *given by position, that same line* F E K *shall be given by position.*

Construction FOr from the point E let there be drawn to the line B C, the perpendicular A L, and let it be prolonged to the point G.

Demonstration FOrasmuch as from the given point A, there is drawn to the right line B C, given by position, the right G L, which
makes the given angle G L D, [a] that line G L is given by position. But a) 30. p.
B C is also given by position, Therefore [b] the point L is given; and seeing b) 26. p.
that the point A is also given the line [c] A L is given. But forasmuch as the c) 26. p.
reason of A E to A D is given, and that [d] as the said A E is to A D, so d) 4. 6.
is A G to A L; (because the triangles A L D and A G E are equiangled) the reason of A G to A L is also given. But A L is is given by magnitude:
Therefore [e] A G is given by magnitude. But it is also given by position, e) 2. p.
and the point A is given: Therefore [f] the point G is also given. And see- f) 27. p.
ing that by the same given point G, there is drawn the line F K, opposite
to the right line B C, given by position, [g] the said line F K is given by g) 28. p.
position.

PROP.

PROP. 37.



If unto parallel right lines A B *and* C D, *given by position, there be drawn a right right line* E F, *divided in the point* E, *in a given reason (to wit, of* E G *to* G F;) *but if by the point of section* G, *there be drawn opposite to the right lines* A B *or* C D, *given by position, a right line* H G K, *that line drawn shall be given by position.*

Construction FOr let there be taken in the line A B the given point L, and from that point let there be drawn the line L N, perpendicular to C D.

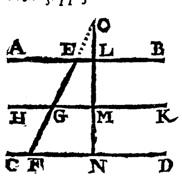
Demonstration SEeing that from the given point L, there is drawn to the right line C D, the line L N, making the given angle
a) 30. p. L N D, the said L N [a] is given by position. But C D is also given by po-
b) 25. p. sition: Therefore the point N [b] is given. But the point L is also given:
c) 26. p. Therefore [c] the line L N is given; and seeing that the reason of F G to G E is given, and that * as F G is to G E, so is N M to M L, the reason
d) 6. p. of the said M N to M L is given; and in compounding, [d] the reason of L N to L M is also given. But L N is given by magnitude, therefore M L
e) 2. p. is [e] given by magnitude. But it is also given by position, and the point L
f) 27. p. is given: Therefore the point M [f] is also given. And considering that by the said point M there is drawn the right line K H, opposite to the right line C D, given by position, the said line K H is also given by position.

SCHOLIUM.

* *EUCLIDE supposeth here, that as* F G *is to* G E, *so* N M *is to* M L; *but by another it is thus demonstrated.*

The lines E F *and* L N *are parallels or not parallels: Let them in the first place be parallels, and forasmuch as by Construction the lines* E L, F N, E F, *and* L N, *are parallels,* E N *shall be a parallelogram; and therefore the side* E F *is equal to the side* L N. *Again, seeing that* M G *is parallel to* F N, *and* G F *to* M N, G N *shall be also a parallelogram; and therefore the side* G F *is equal to the side* M N. *Wherefore the equal sides* E F *and* L N, *shall have to the equal sides* F G *and*
g) 7. 5. M N, [g] *one and the same reason. Therefore as* E F *is to* F G, *so is* L N *to* M N;
h) 17. 5. *and in dividing,* [h] *as* G E *to* G F, *so is* L M *to* M N.

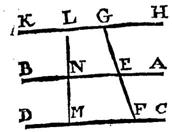
Now suppose that the lines E F *and* L N *be not parallels, but that they meet in the point* O. *Forasmuch as in the triangle* O F N *there is drawn* H K, *parallel to* F N *one of the sides;*
i) 2. 6. [i] *the sides* O F *and* O N *are divided reciprocally; and therefore as* F G *is to* G O, *so is* N M *to* M O. *Again, seeing that in the triangle* O G M *there is drawn* E L, *parallel to the side* G M, *the sides* O G *and* O M *are*
k) 2. 6. *divided proportionally: Wherefore* [k] *as* O E *is to* E G,
l) 18. 5. *so is* O L *to* L M; *and by compounding,* [l] *as* O G *is to* E G, *so is* O M *to* L M; *but it hath been demonstra-*



m) 22. 5. *ted that as* F G *is to* G O; *so is* N M *to* M O; *therefore in reason of equality,* [m] *as* F G *is to* G E, *so is* N M *to* M L.

PROP.

PROP. 38.

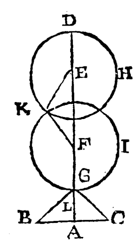


If unto parallel right lines A B *and* C D, *there be drawn a right line* E F, *and that to it there be added some other right line* E G, *which hath a given reason to the same* E F, *but if by the extremity* G, *of the added line* E G, *there be drawn a right line* H K, *against the parallels given by position* A B *and* C D, *the line drawn* H K *shall be also given by position.*

Construction FOr, Let there be taken in the line A B, the given point N, and from thence let there be drawn to C D the perpendicular N M, and let it be prolonged to the point L.

Demonstration FOrasmuch as from the given point N there is drawn to the right line C D, given by position, the right line N M ma-
king a given angle N M F, the said angle N M F [a] is given by position. a) 30. p.
But the line C D is also given by position: Therefore [b] the point M is gi- b) 25. p.
ven. But the point N is also given: Therefore [c] the line N M is given, and c) 26. p.
for that the reason of E G to E F is given, and that [d] as E G is to E F, so d) Sch. 37. p
is L N to N M, the reason of L N to N M is also given: But N M is given,
therefore L N is [e] also given. But the point N is given: Therefore [f] the e) 2. p.
point L is also given. Seeing then that by the given point L there is drawn f) 27. p.
the right line H K, opposite to the line A B given by position, [g] the said g) 28. p.
line A K is also given by position.

PROP. 39.



If all the sides of a triangle A B C *are given by magnitude, the triangle is given by Kind.*

Construction FOr, Let there be exposed the right line D G given by position, ending in the point D; but being infinite towards the other part G, and therein let there be taken D E, equal to A B.

Demonstration NOw seeing the said A B is given by magnitude, D E is so also; but the
same D E is also given by position, and the point D is given: Therefore a) 27. p.
[a] the point E is given.

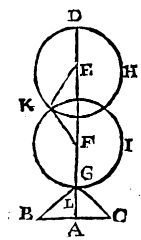
Again, Let E F be put equal to B C; and seeing that B C is given by magnitude, E F shall be so also. But the said E F is in like manner given
by position, and the point E is given: Therefore [b] the point F is given. b) 27. p.

Furthermore, Let F G be taken equal to A C. Now forasmuch as the said A C is given by magnitude, F G is so also. But F G is also given by position, and the point F is given: Therefore the point G is also given. Now from the center E, with the distance E D, let there be described
the circle D H K, [c] and that circle shall be given by position. Again, on c) 6. def.
the center F, and distance F G, let there be described the circle G L K.

There-

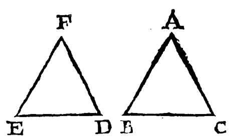
d) 6 def. Therefore [d] the said circle G L K is given by position; and therefore [e] the
e) 25. p. point of intersection K is given. But each of the points E and F is given:
f) 26. p. Therefore each line [f] E K, E F, and F K, is given by position and magnitude. Therefore the triangle E K F is given * by kind; but it is equal and alike to the triangle A B C; and therefore the triangle A B C is also given by kind.

SCHOLIUM.



** EUCLIDE supposeth here that a triangle whose sides are given by magnitude and position, is given by kind; but the antient Interpreters demonstrate it in a manner thus. Forasmuch as the right lines* K E *and* E F *are given,*
g) 1. p. *g the reason which they have to one another is given. Also the right lines* E F *and* F K *being given, their reason is also given; and in like manner, the reason of the said* E K *and* F K *is given. Again, seeing that the same lines* K E
h) Sch. 30. p. *and* E F *are given by position, h the angle* K E F *is given by magnitude: Moreover, the right lines* E F *and* F K *being given by position, the angle* E F K *is given by magnitude, as is also the residue* E K F, *and so in the triangle*
i) 5 def. E K F *are all the angles given, and also the reasons of the sides: Therefore i the said triangle* E K F *is given by kind.*

PROP. 40.



If the angles of a triangle A B C, *are given by magnitude, the triangle is given by kind.*

Construction LEt there be exposed the right line D E, given by position and by magnitude; and let there be constituted at the point D the angle E D F, equal to the angle C B A; but in the point E the angle D E F, equal to the angle B C A; therefore the third angle B A C is equal to the third angle D F E.

Demonstration FOr each of the angles constituted in the points A, B, and C, is given: Therefore each of those which are posited in the points D, F, and E, is also given; and seeing that to the
a) 29. p. right line D E given by position, and to the point D given therein, there is drawn the right line D F, which makes the given angle E D F, [a] the line D F is given by position; and by the same reason, the line E F is gi-
b) 25. p. ven by position: Therefore [b] the point F is given by position. But each
c) 26. p. of the point D and E is given: Therefore [c] each of the lines D F, DE, and E F, is given by magnitude. Wherefore the triangle D F E is given by kind; and is alike to the triangle A B C: Therefore the triangle A B C is given by kind.

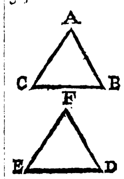
PROP. 41.

If a triangle A B C, *hath one angle* B A C *given, and that the two sides* B A *and* A C, *which do constitute it, have*

to

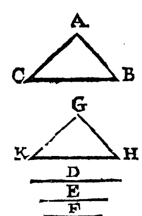
to one another a given reason, the triangle is given by kind.

Construction FOr, Let there be exposed the right line D F given by magnitude and position. But thereon and at the given point F, let there be constituted the angle D F E equal to the angle B A C.



Demonstration NOw the angle B A C is given: Therefore also the angle D F E is given, And seeing that to the right line D F given by position, and from the give point F therein, is drawn a right line
F E, making the given angle D F E, [a] the said line F E is given by position. a) 29. p.
But seeing that the reason of A B to A C is given, let the some be made of D F to F E, then let D E be drawn. Therefore the reason of D F to F E is gi-
ven. But D F is given: Therefore [b] F E is given by magnitude. b) 2. p.
But the same F E is also given by position, and
the point F is given. Therefore [c] the point E is also given. c) 27. p.
But each of the points D and F is given: Therefore [d] d) 26. p.
each of the right lines D F, F E, and D E, is given by position and magnitude. Wherefore [e] the triangle D E F is given by kind. e) 39. p.
And seeing that the two triangles A B C and D E F have an angle equal to an angle, that is to say, the angle B A C to the angle D F E, and the
sides which constitute those equal angles, proportional; [f] the triangle f) 6. 6.
A B C is alike to the triangle D E F. But the triangle D E F is given by kind: Therefore the triangle A B C is given by kind.

PROP. 42.



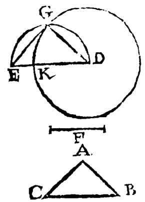
If the sides of a triangle A B C, *be to one another in given reasons, the triangle* A B C *is given by kind.*

Construction FOr, Let there be exposed the right line D, given by magnitude, and seeing that the reason of B C to A C is given, let the same be made of D to E.

Demonstration NOw D is given, therefore [a] E is also given. a) 2. p.
Again, seeing that the reason of A C to A B is given, let the same be made of
E to F. Now E is given, therefore [b] F is also given. Now of three b) 2. p.
right lines, equal to the three given right lines D, E, and F, (and of which three lines, two of them, in what manner soever they be taken, are greater then the other:) Let there be constituted the triangle G H K, in such sort as D may be equal to H K; but E is equal to K G, and G H equal to F; therefore each of the said lines H K, K G, and G H, is given by
magnitude: Wherefore [c] the triangle H G K is given by kind. c) 39. p.
And seeing that as B C is to C A, so is D to E, and that D is equal to H K, and E to K G, as B C is to C A, so H K is to K G. Again, seeing that as C A is to A B, so is E to F, and that E is equal to K G, and F to G H; as C A is to A B, so is K G to G H. But it hath been demonstrated that as B C is to C A, so is H K to K G: Therefore by reason of equality, as B C is to
A B, so is H K to G H. Therefore [d] the triangle A B C is also given by d) 5. 6.
kind.

PROP. 43.

If the sides BC *and* DA, *about one of the acute angles of a rectangled triangle* ABC, *have to one another a given reason, that triangle is given by kind.*



Construction Let there be exposed the right line DE given by magnitude and position, and on it let there be described the semicircle DGE: Therefore [a] the semicircle DGE is given by position.

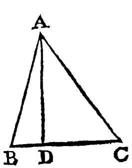
a) 6 def.

Demonstration For the line DE being given, and divided in two equal parts, the center of the said circle is given by position, and the semidiameter by magnitude. And forasmuch as the reason of BC to BA is given, let the same be made of DE to F: Therefore the reason of DE to F is given. But DE is given, therefore F [b] is also given. Now BC is greater then [c] AB: Therefore ED is [d] also greater then F. Let DG be fitted equal to F, and let EG be drawn; then on the center D, with the distance DG, let the circle GK be described. Now that circle [e] is given by position, seeing that the center D is given, and the semidiameter DG also given by magnitude. But the semicircle DGE is also given by position: Therefore [f] the point of intersection G is given. But the points D and E are also given, therefore [g] each of the right lines DE, DG, and EG, is given by position and magnitude. Wherefore [h] the triangle DGE is given by kind. And seeing that the triangles ABC and DGE have an angle equal to an angle, to wit, the right angle BAC to the right angle [i] DGE, and the sides about the angles CBA and EDG proportional. But each of the others ACB and DEG lesse then a right angle: Those triangles ABC and DEG [k] are alike. But the triangle DGE is given by kind: Therefore the triangle ABC is also given by kind.

b) 2. p.
c) 19. 1.
d) 14. 5.
e) 6 def.
f) 25. p.
g) 26. p.
h) 39. p.
i) 31. 3.
k) 7. 6.

PROP. 44.

If a triangle ABC, *hath one angle* B *given, and that the sides* BA *and* AC, *about another angle* BAC, *have to one another a given reason, the triangle* ABC *is given by kind.*



Construction Now the given angle B is either acute or obtuse, (for it was a right angle in the fore-going prop.) Let it be in the first place acute, and from the point A let AD be drawn perpendicular to BC.

Demonstration Therefore the angle ADB is given: But the angle B is also given; and therefore the third angle BAD is given: Wherefore [a] the triangle ABD is given by kind; and therefore [b] the reason of BA to AD is given. But the reason of the same BA to AC is also given: Therefore [c] the reason of AD to AC is given, and the angle ADC is a right angle: Wherefore the triangle [d] ACD is given by kind: Therefore the

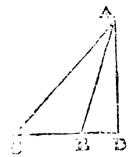
a) 40. p.
b) 3 def.
c) 8. p.
d) 43. p.

[e] the angle C is given. But the angle B is also given; and therefore the other angle BAC is given: Therefore [f] the triangle ABC is given by kind.

e) 3. def.
f) 40. p.

Construction Now let the angle ABC be obtuse, and on the side CB prolonged, let there be drawn the perpendicular AD.

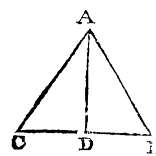
Demonstration Forasmuch as the angle ABC is given, the angle ABD which followes it, shall be given. But the angle ABD is also given: Therefore the third angle DAB is given. Wherefore [g] the angle ABD is given by kind; and therefore [h] the reason of DA to AB is given. But the reason of AB to AC is also given: Therefore [i] the reason of DA to AC is given, and the angle D is a right angle: Therefore the triangle DAC is given by kind, and therefore the angle ACB is given. But the angle ABC is also given: Therefore the third angle BAC is given. Wherefore the triangle ABC is given by kind.



g) 40. p.
h) 3 def.
i) 8. p.

PROP. 45.

If a triangle ABC, *hath one angle* BAC *given, and that the line compounded of the two sides* AB *and* AC, *about the said given angle* BAC, *hath to the other side* BC *a given reason, the triangle* ABC *is given by kind.*



Construction For, Let the angle BAC be divided into two equal parts by the line AD, therefore [a] the angle CAD is given.

a) 7. p.

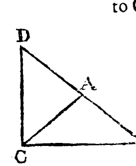
Demonstration Seeing that as AB is to AC, so [b] is BD to CD; by compounding, [c] as the line compounded of CAB is to CA, so is BC to CD, and by permutatiou, as the line compounded of CAB is to CB, so is CA to CD. But the reason of the line compounded of CAB to BC is given; therefore the reason of CA to CD is also given, and the angle CAD is given. Therefore [d] the triangle ACD is given by kind, and therefore the angle C is given. But the angle BAC is also given: Therefore the third angle B is given: Wherefore [e] the triangle ABC is given by kind.

b) 3. 6.
c) 18. 5.
d) 44. p.
e) 40. p.

OTHERWISE.

Construction Let BA be prolonged directly unto the point D, in such sort as that AD may be equal to AC, and let CD be joyned.

Demonstration Forasmuch as the reason of the line compounded of CAB to CB is given, and that AD is equal to AC, the reason of the whole line BD to BC is given. But the angle ADC is also given, for it is the half of the the given angle BAC (for that the said angle BAC [f] is equal to the two internal angles ACD and ADC, which are [g] equal to one another, being the sides AC and AD are equal:) Wherefore the triangle BDC [h] is given by kind, and therefore the angle B is given. But the

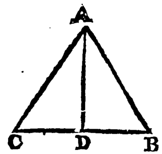


f) 32. 1.
g) 5. 1.
h) 44. p.

i) 40. p.

the angle B A C is alſo given : Therefore the remaining angle A C B is given : Wherefore [i] the triangle A B C is given by kind.

PROP. 46.



If a triangle A B C *hath one angle* B *given, and that the line* C A B *compounded of the two ſides* A C *and* A B, *about another angle* B A C, *hath to the other ſide* B C *a given reaſon, the triangle* A B C *is given by kind.*

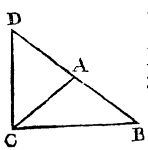
Conſtruction FOr let the angle B A C be divided into two equal parts, by the line A D.

a) 41. p.
b) ·. def.

Demonſtration THerefore (as hath been ſhewn in the foregoing Prop.) the compound line C A B is to C B, as A B is to B D. But the reaſon of the ſaid compound line C A B to C B is given : Therefore alſo the reaſon of A B to B D is given. But the angle B is alſo given : Therefore the triangle A B D [a] is given by kind; and therefore [b] the angle B A D is given. But the angle B A C is double to that of B A D; and therefore it is alſo given. Therefore the third angle C is given. Wherefore the triangle A B C is given by kind.

OTHERWISE.

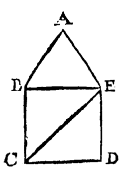
Conſtruction LEt B A be prolonged directly, and let A D be put equal to A C, and let C D be joyned.



c) 41. p.
d) 3 def.

Demonſtration FOraſmuch as the reaſon of the line compounded of C A B to C B is given, and that A D is equal to A C, the reaſon of B D to B C is given; and the angle B is alſo given : Therefore the triangle C D B [c] is given by kind; and therefore [d] the angle D is given : Therefore the angle B A C which is double to B D C, is alſo given : Wherefore the other angle A C B is given; and therefore the triangle A B C is given by kind.

PROP. 47.



Rectiline figures as A B C D E, *given by kind, are divided into triangles given by kind.*

Conſtruction FOr let the right lines E B and E C be drawn.

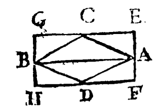
a) 3. def.
b) 41. p.
c) 4. p.

Demonſtration FOraſmuch as the rectline figure A B C D E is given by kind, the angle [a] B A E is given, and the reaſon of the ſide A B to A E is alſo given : Therefore [b] the triangle B A E is given by kind. Wherefore the angle A B E is given. But the whole angle A B C is alſo given : Therefore [c] the remaining angle E B C is given. But the reaſon of the ſide A B to the ſide B E, and alſo that of A B to B C is given : Therefore the

d) 8. p.
e) 41. p.

[d] the reaſon of B C to B E is given, and the angle C B E is alſo given : Therefore [e] the triangle B C E is given by kind. By the ſame diſcourſe it may be demonſtrated that the triangle C D E is given by kind. Therefore rectiline figures given by kind divide themſelves into triangles given by kind.

PROP. 48.



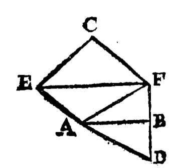
If on one and the ſame right line A B, *are deſcribed triangles as* A C B *and* A B D, *given by poſition, thoſe triangles ſhall have to one another a given reaſon, as* A C B *to* A B D.

Conſtruction FOr from the points A and B, let there be drawn at right angles on the line A B, the lines A E and B G, and prolonged unto the points F and H; but by the points C and D, let there be drawn the lines E C G and F D H, parallel to A B.

a) 3. def.
b) 40. p.
c) 8. p.
d) 3 def.
e) 8. p.
f) 1. 6.
g) 41. 1.
h) 15. 5.

Demonſtration FOraſmuch then as the triangle A B C is given by kind, [a] the reaſon of C A to B A is given, and the angle C A B alſo given; but the angle B A E is given : Therefore the remaining angle C A E is alſo given; but the angle C A E is given; and therefore the other angle A C E is alſo given. Wherefore [b] the triangle A E C is given by kind. Now the reaſon of E A to A B [c] is given; (for [d] the reaſon of E A to A C, and that of A C to A B is given;) and in like manner, the reaſon of F A to A B is given. Therefore [e] the reaſon of E A to A F is given; but as A E is to A F, ſo [f] the parallelogram A H to the parallelogram A G; but A C B is [g] the half of A H, and A D B the half of A G; therefore the reaſon of the triangle A C B to the triangle A D B is given; for it is the ſame reaſon with that of A H to A G [h]; that is to ſay, of E A to A F, which is given.

PROP. 49.



If on one and the ſame right line A B *there be deſcribed any two rectiline figures* A E C F B *and* A D B, *given by kind, they ſhall have to one another a given reaſon (to wit,* A E C F B *to* A D B.)

a) 47. p.

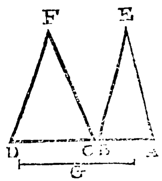
Conſtruction FOr let the lines F A and F E be drawn: Therefore each of the triangles [a] A B F A F E, and E C F is given by kind.

b) 48. p.
c) 6. p.
d) 48. p.
e) 8. p.
f) 6. p.
g) 48. p.
h) 8. p.

Demonſtration SEeing that on one and the ſame right line E F there are deſcribed the triangles E C F and E A F, given by kind; the reaſon of E C F to E A F [b] is given. Therefore, by compounding, [c] the reaſon of A E C F to E A F is given. But the reaſon of the ſaid E A F to F A B is given, [d] being they are triangles given by kind, deſcribed on one and the ſame right line A F: Therefore [e] the reaſon of A E C F to F A B is given. Wherefore by compounding, [f] the reaſon of A E C F B to F A B is given. But the reaſon of the ſame F A B to A B D [g] is given : Therefore [h] the reaſon of A E C F B to A B D is alſo given.

PROP.

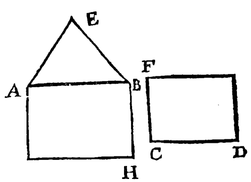
PROP. 50.



If two right lines A B *and* C D, *have to one another a given reason, and that on those lines there be described rectiline figures* A E B *and* C F D, *alike, and alike posited, they will have to one another a given reason.*

Demonstration TO the two lines A B and C D, let there be taken a third proportional: Therefore as A B is to C D, so is C D to G. But the reason of A B to C D is given: Therefore the reason of C D to G is also given:
a) 8. p. Wherefore [a] the reason of A B to G is given. But [b] as A B is to G, so is
b) Cor. 19, 20. 6. A E B to C F D: Therefore the reason of the same A E B to C F D is given.

PROP. 51.

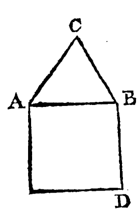


If two right lines A B *and* C D, *have to one another a given reason, and that upon them there be described any rectiline figures* A E B *and* C F D, *given by kind, they will have to one another a given reason, (to wit, that of* A E B *to* C F D.*)*

Construction FOr on A B, let the rectangled figure A H be described alike and alike posited to D F.

Demonstration NOw D F is given by kind: Therefore also A H is given by kind. But A E B is also given by kind, and described on
a) 49. p. the same line A B: Therefore [a] the reason of A E B to A H is given: And seeing that the reason of A B to C D is given, and that on those lines are described the rectiline figures A H and D F alike, and alike posited, the
b) 50. p. reason [b] of the said line A H to D F is given. But the reason of A E B to
c) 8. p. A H is also given: Therefore the reason [c] of A E B to D F is given.

PROP. 52.



If on a right line A B, *given by magnitude, there be described a figure* A C B, *given by kind, that figure* A C D *is given by magnitude.*

Construction FOr on the same line A B, let the square A D be described. Therefore A D is given by kind * and by magnitude.

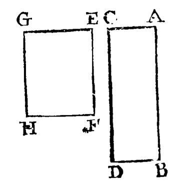
Demonstration SEeing that on the right line A B, are described the two rectiline figures A C B

and A D, given by kind, [a] the reason of A C B to A D is given: a) 29. p.
Therefore [b] A C B is given by magnitude. b) 2. p.

SCHOLIUM.

* *The ancient Interpreter hath noted here that every square is given by kind; for that all the angles thereof are given; being all equal and right angles: But also the reasons of the sides are given; for those sides being all equal, their reasons are also equal. Moreover, whensoever a square is exposed, a square equal thereto may be exhibited; and therefore the square is given by magnitude, as also each side thereof.*

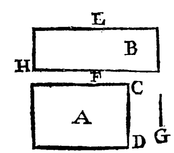
PROP. 53.



If there be two figures A D *and* E H, *given by kind, and that one side* B D, *of the one, hath to a side* F H *of the other, a given reason; the other sides shall have also to the other sides given reasons.*

Demonstration FOr seeing that the reason of B D to F H is given, and also that [a] of a) 3. def.
B D to B A, [b] the reason of the said A B to F H b) 8. p.
is given. But the reason of the same F H to F E
[c] is also given: Therefore [d] the reason of A B to E F is given. In like c) 3 def.
manner also the reasons of the other sides to the other sides are given. d) 8. p.

PROP. 54.



If two figures A *and* B, *given by kind, have to one another a given reason, also their sides shall be to one another in a given reason.*

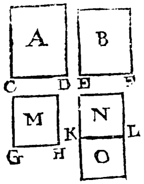
Construction FOr either the figure A is alike and alike posited to B, or is not: Let it in the first place be alike and alike posited; and let there be taken the line G, a third proportional to the lines C D and E F.

Demonstration AS C D is to G, [a] so is A to B. But the reason of A to B is given; therefore also the reason of C D to G is given. b) Cor. 19, 20. 6.
And seeing that C D, E F, and G, are proportional, [b] also the reason of b) 24. p.
C D to E F is given. But A and B are given by kind: Therefore [c] the c) 53. p.
other sides shall have given reasons to the other sides.

Now let the figure A be not alike to the figure B, and let there be described on E F the figure E H, alike and alike posited to A: Therefore the figure E H is given by kind; but the figure B is also given by kind:
Therefore [d] the reason of B to E H is given; and therefore the reason of d) 49. p.
A to the same E H [e] is also given: But A is alike to E H: Therefore (by e) 8. p.
what is abovesaid) the reason of C D to E F is given; and in like manner the reason of the other sides to the other sides is given.

OTHER-

OTHERWISE.



Conſtruction LEt there be expoſed the given line GH: Now either the figure A is alike to the figure B, or not. Let it in the firſt place be alike, and let it be as CD is to EF, ſo is GH to LK; then on GH and LK let the figures M and N be deſcribed alike and alike poſited to the ſaid A and B, which figures M and N ſhall be conſequently given by kind.

Demonſtration THerefore ſeeing that as CD is to EF, ſo is GH to LK, and that on thoſe lines CD, EF, GH, and LK, are deſcribed the figures A, B, M, and N, alike and alike poſited; [f] as A is to B, ſo is M to N. But the reaſon of A to B is given: Therefore the reaſon of M to N is given. But [g] M is given, conſidering that it is a figure given by kind, deſcribed on a right line given by magnitude; therefore N is alſo given.

f) 22.6.
g) 52.p.

Conſtruction 2. NOw, on LK let the ſquare O be deſcribed: Therefore [h] the figure O is given by kind.

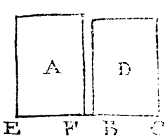
h) Sch.52.p.

Demonſtration 2. WHerefore the reaſon of K to N is given. But N is given: Therefore K is given; and conſequently, [i] alſo KL. But GH is given: Therefore [k] the reaſon of GH to KL is given. But as GH is to LK, ſo is CD to EF. Therefore the reaſon of CD to EF is given; and therefore the figures A and B being given by kind, [l] the other ſides of the ſame figures ſhall alſo have to the other ſides given reaſons. But if the figures be not alike, the later part of the demonſtration here above muſt be obſerved.

i) Sch. 52.p.
k) 3.p.
l) 53.p.

PROP. 55.

If a Space A, *be given by kind, and by magnitude, the ſides thereof ſhall be given by maginnude.*



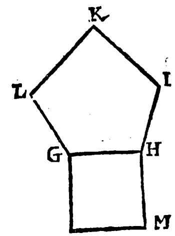
Conſtruction FOr, Let the right line BC given by poſition and by magnitude, be expoſed; and thereon let there be deſcribed the ſpace D, alike and alike poſited to A; therefore the ſaid ſpace D is given by kind.

Demonſtration FOr that it is deſcribed on the line BC, given by magnitude, it is alſo [a] given by magnitude. But the figure A is alſo given: Therefore [b] the reaſon of A to D is given. But thoſe figures A and D are given by kind: Therefore [c] the reaſon of the line EF to the line BC is given. But BC is given: Therefore [d] EF is alſo given. But the reaſon of the ſame EF to FG is given: Therefore [e] FG is given. And by the ſame reaſons it may be demonſtrated that each of the other ſides are given by magnitude.

a) 52.p.
b) 3.p.
c) 54.p.
d) 3.def.
e) 2.p.

OTHER-

OTHERWISE.



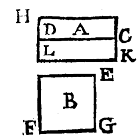
Conſtruction LEt the ſpace GHIKL be given by kind and by magnitude: I ſay that the ſides thereof are given by magnitude. For on the right line GH let there be deſcribed the ſquare GM; therefore [f] GM is given by kind.

f) Sch.52.p.

Demonſtration BUt the ſpace GHIKL is alſo given by kind: Therefore [g] the reaſon of the ſame ſpace GK to GM is given. But GK is given by magnitude: Therefore [h] GM is alſo given by magnitude; and ſeeing that GM is the ſquare of the line GH, [i] that line GH is given by magnitude. Wherefore in like manner, each of the other lines HI, IK, KL, and LG, is given.

g) 49.p.
h) 2.p.
i) Sch.52.p.

PROP. 56.



If two equiangled Parallelograms A *and* B, *have to one another a given reaſon, as one ſide* CD *of the firſt* A, *is to one ſide* FG, *of the ſecond* B; *ſo the other ſide* GE, *of the ſecond* B, *is to that to which* DH *the other ſide of the firſt* A, *hath the given reaſon that the Parallelogram* A *hath to the Parallelogram* B.

Conſtruction FOr let HD be prolonged directly to L, ſo that as CD is to FG, ſo HD may be to DL; and finiſh the Paralellogram DK.

Demonſtration SEeing that as CD is to FG, ſo HD is to DL, and [a] that CD is equal to KL; as LK is to FG, ſo is GE to DL; and thus the ſides about the equal angles DLK and EGF are reciprocally proportional: Wherefore [b] DK is equal to B; and therefore ſeeing the reaſon of A to B is given, and that B is equal to DK, the reaſon of A to DK is given. But as [c] A is to DK (that is to B) ſo is HD to DL; therefore the reaſon of HD to DL is alſo given, and ſeeing that as CD is to FG, ſo GE is to DL, and that the right line HD hath to DL a given reaſon; to wit, that which the ſpace A hath to the ſpace B; as CD is to FG, ſo GE is to that to which HD hath the given reaſon that the ſpace A hath to the ſpace B, that is to ſay, the reaſon of HD to DL.

a) 34.1.
b) 14.6.
c) 1.6.

PROP. 57.

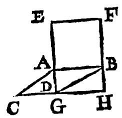
If a given ſpace AD *be applyed to a given right line* AB, *in a given angle* CAB, *the breadth* CA *of the application is given.*

Conſtruction FOr on AB, let there be deſcribed the ſquare AF; therefore [a] the ſame AF is given: Let the lines EA, FB, and CD, be prolonged to the points G and H.

a) Sch.52.p

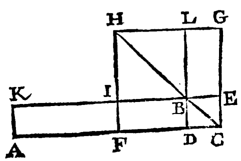
 Demon-

Demonſtration SEeing therefore that each ſpace A D and A F is given, their reaſon is alſo given. But [b] A D is equal to A H:
b) 36. 1.
Therefore the reaſon of A F to A H is given: Wherefore the reaſon of E A to A G is given. (for [c] it is the ſame with that of
c) 1. 6.
A F to A H.) But E A is equal to A B; therefore the reaſon of A B to A G is given. Now ſeeing that the angle C A B is given, and the angle G A B alſo given, the reſidue C A G is given. But the angle C G A is alſo given, being a right angle: Therefore the remaining angle A C G is given. Wherefore the triangle [d] C A G is given by kind. Therefore the reaſon of C A to A G is given. But the reaſon of A B to the ſame A G is alſo given: Therefore the reaſon of C A to A B is given; and the ſaid A B is given: Wherefore C A is alſo given.
d) 40 p.



PROP. 58.

If a given ſpace A B, *be applyed to a given right line* A C, *wanting by a figure* D E, *given by kind, the breadths of the defects are given.*



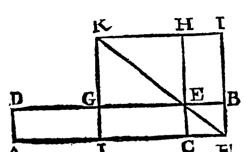
Conſtruction FOr let A C be divided in two equal parts in the point F: Therefore as well A F as F C is given. On the ſaid line F C let there be deſcribed the rectangle figure F G alike and alike poſited to D E. Therefore F G is given by kind.

Demonſtration SEeing the figure F G is deſcribed on the right line F C given by magnitude, the ſaid rectiline F G is [a] alſo given by magnitude. But F G is equal to A B and I L; (for [b] A I and F E being equal, and [c] F B and B G alſo equal, the Gnomon I C L is equal to A B; and therefore their added figure I L, common to both, F G ſhall be equal to A B anp I L:) Therefore the figures A B and I L together are given by magnitude. But A B is given by magnitude: Therefore [d] the remaining figure I L is alſo given by magnitude. But it is alſo given by kind, ſeeing it is [e] alike to D E: Therefore [f] the ſides of the ſame I L are given: Wherefore I B is given; and ſeeing that it is equal [g] to F D, the ſame F D is alſo given. But F C is given, therefore the remainder D C [h] is given; and [i] in a given reaſon to B D, and therefore [k] B D is given.

a) 52. p.
b) 36. 1.
c) 43. 1.
d) 4. p.
e) 24. 6.
f) 55. p.
g) 34. 1.
h) 4. p.
i) 3. def.
k) * p.

PROP. 59.

If a given ſpace A B *be applyed according to a given right line* A C, *exceeding it by a figure* C B *given by kind, the breadths of the exceſſes* C E *and* C F *are given.*

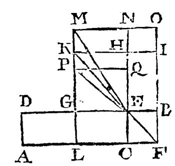


Conſtruction FOr D E being divided into two equal parts in G, let there be deſcribed on G E the rectiline figure G H, alike and alike poſited to C B.

Demonſtration NOw ſeeing that C B is alike to G H, thoſe figures C B and G H * are about one and the ſame diameter, and G H is given by kind, as is C B. But it is deſcribed on the given line G E: Therefore [a] the ſame G H is alſo given by magnitude. But A B is given: Therefore A B and G H are given by magnitude. Now thoſe figures A B and G H, are equal to L I, (for A G, L E, and E I, being equal, the Gnomon G F H is equal to A B; and therefore adding G H common to both, L I ſhall be equal to A B and G H;) therefore L I is given by magnitude; but it is alſo given by kind, being it is [b] alike to C B. Therefore [c] the ſides of the ſaid L I are given, ſeeing it is equal to G E: Therefore [d] the remainder C F is given, and in a given reaſon [e] to C E. Wherefore [f] C E is given.

a) 32. p.
b) 24. 6.
c) 55. p.
d) 4. p.
e) 3 def.
f) 2. p.

SCHOLIUM.



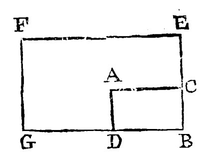
* *EUCLIDE ſuppoſeth here that* C B *and* G H *are about one and the ſame diameter, but we ſhall thus demonſtrate it: Let* C B *and* G H *be two alike Parallelograms diſpoſed as above, that is to ſay, that the equal angles joyn together in* E, *the ſide* C E *meets directly with his homologal ſide* E H, *and the ſide* B E, *his correſpondent ſide* E G; *and let the diameter* F E *be drawn I ſay that the ſaid diameter* F E *prolonged, will paſſe by the point* K; *that is to ſay, the Parallelograms* G H *and* C B, *conſiſt about one and the ſame diameter. For if it be denied, the diameter* E F *being produced, will paſſe above the point* K, *or below it. Let it in the firſt place paſſe above it, and let it cut* G K, *prolonged in the point* M, *and by the point* M, *let there be drawn* M N, *parallel to* K H, *which ſhall meet* E H, *prolonged in the point* N, *and* F B *in* O.

Demonſtration FOraſmuch as the Paralellograms G N and C B are with the Parallelogram L O about one and the ſame diameter, they are [g] alike to one another. Wherefore as F C is to C E, ſo is E G to G M. In like maner, ſeeing the parallelograms C B and G H are alike, as F C is to C E, ſo is E G to G K: Therefore [h] as E G is to G M, ſo is E G to G K. Wherefore [i] G M and G K are equal, a part to the whole, which is abſurd: By the ſame reaſons it may be demonſtrated; that the diameter prolonged will not fall below the point K: Therefore the Parallelograms C B and G E conſiſt about one and the ſame diameter.

g) 24. 6.
h) 11. 5.
i) 9. 5.

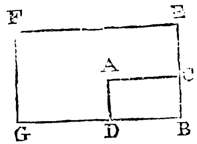
PROP. 60.

If a Paralellogram A B, *given by kind and by magnitude, be augmented or diminiſhed by a Gnomon* C F D, *the breadths of the Gnomon (conſiſting of the lines* C E *and* D G) *are given.*



Demonſtration FOr ſeeing that A B is given, and the Gnomon C F D alſo given, the whole Parallelogram B F is given: But it is alſo given by kind, ſeeing it is alike to B A: Therefore [a] the ſides of the ſame
a) 55. p.

ſame B F are given; and therefore each of the lines B E and B G is given. But each of the lines B C and B D is given; therefore each of the remaining lines C E and D G is alſo given.

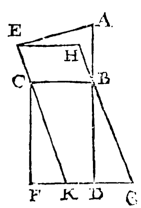


Conſtruction Now let the Paralellogram B F, given by kind and by magnitude, be diminiſhed by the given Gnomon C F D: I ſay that each of the lines C E and D G is given.

Demonſtration 2. For ſeeing that B F is given, and the Gnomon C F D given, the remaining figure A B is alſo given. But it is alſo given by kind,
b) 55. p. ſeeing it is alike to B F: Therefore b the ſides of the ſaid A B are given, and therefore each of the lines C B and B D is given. But each of the lines B E and B G is given: Therefore alſo each of the remaining lines C E and D G is given.

PROP. 61.

If to one ſide of a figure A B C E, *given by kind, there be applyed a ſpace Parallelogram* C D, *in a given angle* B C F, *and that the given figure* A C *hath to the Parallelogram* C D *a given reaſon, the Parallelogram* C D *is given by kind.*



Conſtruction For by the point B, let B H be drawn parallel to C E, and by the point E let E H be drawn parallel to C B, and let E C and H B be prolonged to the points K and G.

Demonſtration Foraſmuch as the angle B C E is given, and the reaſon of
a) 3 def. E C to C B, a the parallelogram C H is given * by kind.
But the figure A B C E is alſo given by kind, and is deſcribed on the ſame
b) 48. p. line B C, as the Parallelogram C H given by kind is: Therefore b the
reaſon of the figure A B C E to the Parallelogram C H is given. But by
Suppoſition, the reaſon of the ſaid figure A B C E to the Parallelogram
c) 36. 1. C D is alſo given; and C D is c equal to C G: Therefore d the reaſon of
d) 8. p. C H to C G is given. Wherefore the reaſon of the line E C to the line
e) 1. 6. C K is given; (for e as C H is to C G, ſo is E C to C K.) But the reaſon
f) 8. p. of E C to C B is alſo given: Therefore f the reaſon of the ſaid C B to
C K is given. And ſeeing that the angle E C B is given, alſo the follow-
g) 13. 1. ing angle B C K g is given. But the angle B C F is propoſed given; and
4. p. therefore the remaining angle F C K is given. Alſo the angle C K F is gi-
h) 29. 1. ven, for that h it is equal to the angle B C K: Therefore the other angle
i) 40. p. C F K is given: Wherefore i the triangle F C K is given by kind; and
therefore the reaſon of F C to C K is given. But the reaſon of C B to the
k) 8. p. ſame C K is alſo given: Therefore k the reaſon of F C to C B is given;
and the angle B C F is alſo given. Wherefore the Parallelogram C D
is given by kind.

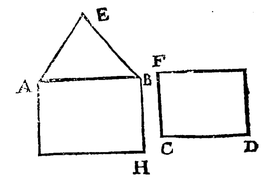
SCHO-

SCHOLIUM.

* *Although it be manifeſt that a Parallelogrrm that hath one angle given, and the reaſon of the ſides about the ſame angle alſo given, is given by kind, as* Euclide *doth here declare, ſo it is notwithſtanding that the ancient Interpreter doth thus demonſtrate it.*

Seeing that in the Parallelogram C H *the angle* E C B *is given, the angle* C E H *is alſo given; for the right line* E C *falling on the Parallels* E H *and* C B, *doth make the two internal angles on the ſame part equal to two right angles. And therefore ſeeing that the angle* E C B *is given, the other angles are given; and ſeeing that the reaſon of* E C *to* C B *is given, and that* B H *is equal to* C E, *and* E H *to* B C, *the reaſon of the ſides to one another is alſo given.*

PROP. 62.

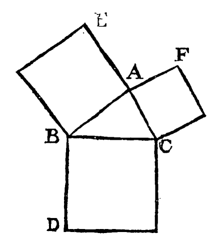


If two right lines A B *and* C D, *have to one another a given reaſon, and that on one of them* A B, *there be deſcribed a figure* A E B, *given by kind; but on the other* C D, *a ſpace Parallelogram* D F, *in a given angle* D C F, *and that the figure* A E B *hath to the Parallelogram* D F *a given reaſon, the Parallelogram* D F *is given by kind.*

Conſtruction For on the line A B let there be deſcribed the Parallelogram A H, alike and alike poſited to D F.

Demonſtration Seeing then that the reaſon of A B to C D is given, and that on thoſe lines are deſcribed the rectiline figures A H
and F D, alike and alike poſited, a the reaſon of A H to F D is given. a) 50. p.
But the reaſon of F D to A E B is alſo given: Therefore b the reaſon of b) 8. p.
A H to A E B is given. But the angle A B H is alſo given, being equal to
the angle F C D, and ſo the figure A E B is given by kind; and to A B
one of the ſides thereof, the Parallelogram A H is applyed in a given an-
gle A B H, and the reaſon of the ſaid figure A E B to the ſaid Parallelo-
gram A H is given: Therefore c the Parallelogram A H is given by kind; c) 61. p.
and therefore E D which is alike thereto, is alſo given by kind.

PROP. 63.

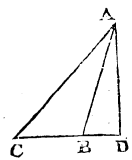


If a triangle A B C *be given by kind, the ſquare* B E, C D, *and* C F, *which is deſcribed on each of the ſides, ſhall have a given reaſon to the triangle* A B C.

Demonſtration For ſeeing that on one and the ſame right line B C, there are deſcribed the two rectiline figures A B C and C D,

a) 49. p. C D given by kind, [a] the reason of the same A B C to C D is given; and
therefore the reason of the squares B E and C F, to the triangle A B C,
is also given.

PROP. 64.

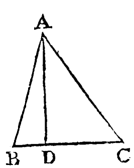


If a triangle A B C, *hath an obtuse angle* A B C *given, that space of which the side* A C *subtending the obtuse angle* A B C, *is more in power then the sides* A B *and* B C, *that comprehend the said angle, shall have a given reason to the triangle* A B C.

Construction Let the line C B be prolonged directly, and from the point
A let the perpendicular A D be drawn: I say that the
space of which the square of the line A C doth exceed the squares of the
a) 12. 2. lines A B and B C, that is to say, [a] the double of the rectangle conteined
under C B and B D, shall have a given reason to the triangle A B C.

Demonstration For seeing that the angle A B C is given, the angle A B D
is also given. But the angle A D B is also given; therefore
b) 40. p. the other angle B A D is given: Wherefore [b] the triangle A B D is given
c) 2. def. by kind; therefore [c] the reason of A D to D B is given. But as A D to
d) 1. 6. D B, so [d] the rectangle of A D and B C is to the rectangle of B C and
B D. But the reason of A D to B D is given: Therefore also is the reason
of the rectangle of A D and B C to the rectangle of B C and B D given:
Wherefore the reason of the double of the said rectangle B C and B D
to the rectangle of A D and B C is also given. But the said rectangle of
A D and B C hath also a given reason to the triangle A B C (to wit, double
c) 41. 1. reason; for the rectangle is [e] double to the triangle) therefore the reason of
the double of the rectangle of B C and B D [f] to the triangle A B C is given.
f) 8. p. But the same double of the rectangle of C B and B D is that space of
which the square of the line A C doth exceed the squares of the lines A B
and B C: Therefore the same space hath a given reason to the triangle
A B C.

PROP. 65.



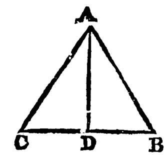
If a triangle A B C, *hath one acute angle* A C B *given, that space, of which the side subtending the said acute angle is lesse in power then the sides comprehending the same acute angle, shall have a given reason to the triangle.*

Construction From the point A let there be drawn the line A D, perpendicular to B C: I say that space of which the square of the
a) 13. 2. line A B is lesse then the squares of the lines A C and C B, that is to say,
[a] the double of the rectangle of B C and C D, hath a given reason to the
triangle A B C.

Demonstration For seeing that the angle C is given, and the angle A D C
also given, the other angle D A C is given: Wherefore
the

the triangle [b] A D C is given by kind; and therefore the reason of A D to b) 40. p.
D C is given, and consequently also [c] that of the rectangle of B C and c) 1. 6.
C D to the rectangle of B C and A D: Therefore the reason of the dou-
ble of the rectangle of B C and C D to the rectangle of B C and A D is
given. But the reason of the same rectangle of B C and A D to the tri-
angle A B C is given: (for [d] the rectangle is double to the triangle) There- d) 41. 1.
fore [e] the reason of the double of the rectangle of B C and C D to the e) 8. p.
triangle A B C is given. And seeing that the same double of the rectangle
of B C and C D is that whereof the square of the line A B is lesse then
the squares of the lines A C and B C, that space of which the square of
the line A B is lesse then the squares of the lines A C and B C, shall have
a given reason to the triangle A B C.

PROP. 66.

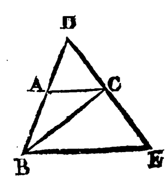


If a triangle A C B, *hath one angle* B *given, the rectangle made of the lines* A B *and* B C, *conteining the same angle, shall have a given reason to the triangle.*

Construction For from the point A let A D be
drawn perpendicular to C B.

Demonstration Therefore seeing that the angle B is given, and also the
angle A C B; the other angle C A D is likewise given.
Wherefore the triangle A C B [a] is given by kind; and consequently the a) 40. p.
reason of A C to A D is given. But as A B is to A D, [b] so the rectangle of b) 1. 6.
A C and C B is to the rectangle of C B and A D: Therefore the reason of
the rectangle of A C and C B to the rectangle of C B and A D is given.
But the reason of the said rectangle of C B and A D to the triangle A C B
is also given; (for that it is double reason, the rectangle being double [c] to c) 41. p.
the triangle:) Therefore [d] the reason of the rectangle of A C and C B to d) 8. p.
the triangle A B D is given.

PROP. 67.



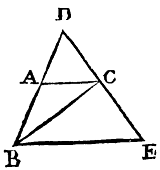
If a triangle A B C, *hath one angle* B A C *given, that Space by which the square of the line compounded of the two sides* B A *and* A C, *that contein the same given angle* B A C *doth exceed the square of the other side, it shall have a given reason to the triangle* A B C.

Construction For let B A be prolonged in such sort as that A D may be equal
to A C, then having drawn D C E infinitely, from the
point B let B E be drawn parallel to A C, meeting the said D E in the
point E.

Demonstration Forasmuch as A D is equal to A C, [a] D B is equal to a) 4. 6. and
B E; (for the two triangles A D C and B D E are alike) 14. 5.
and from the top B is drawn to the base D E, the right line B C: There-
fore

fore * the rectangle of D C and C E, with the ſquare of B C, is equal to the ſquare of B D; but the ſame B D is compounded of B A and A C; therefore the ſquare of the compound of A B and A C is greater then the ſquare of B C, of the rectangle of D C and C E.

Now I ſay that the rectangle of D C and C E hath a given reaſon to the triangle A B C: Foraſmuch as the angle B A C is given, the angle D A C is alſo given. But each of the angles A D C and A C D is given,
b) 40. p. it being the half of the angle B A C which is given. Therefore [b] the triangle A D C is given by kind; and therefore the reaſon of D A to D C
c) 50. p. is given. Therefore [c] the reaſon of the ſquare of the ſaid D A to the ſquare of D C is alſo given. And ſeeing that as
d) 2. 6. B A is to A D, [d] ſo is E C to C D, and alſo as
e) 1. 6. B A is to A D, [e] ſo is the rectangle of B A and A D to the ſquare of A D; and as E C is to C D,
f) 1. 6. [f] ſo alſo is the rectangle of E C and C D to the ſquare of C D; by permutation, as the rectangle of B A and A D is to the rectangle of E C and C D, ſo is the ſquare of A D to the ſquare of D C. But the reaſon of the ſaid ſquare of A D to the ſquare of D C is given: Therefore the reaſon of the rectangle of B A and A D to the rectangle of E C and C D is alſo given. But A D is equal to A C: Therefore the reaſon of the rectangle of B A and A C to the rectangle of E C and C D is given. But the reaſon of

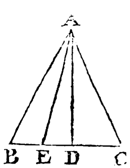


g. 66. p. the rectangle of B A and A C to the triangle A B C [g] is given, becauſe
h) 8. p. the angle B A C is given: Therefore [h] the reaſon of the rectangle E C and C D to the triangle A B C is given. But the rectangle of E C and C D is that whereof the ſquare of the line compounded of B A and A C is greater then the ſquare of B C: Therefore that ſpace to which the ſquare of the line compounded of B A and A C is greater then the ſquare of B C, ſhall have a given reaſon to the triangle A B C.

SCHOLIUM.

** EUCLIDE ſuppoſeth in this place that when in an Iſoſceles triangle a right line is drawn from the top to the baſe, the ſquare of that line, with the rectangle conteined under the ſegments of the baſes, is equal to the ſquare of either of the other legges, which the ancient Interpreter doth thus demonſtrate.*

Conſtruction LEt A B C be an Iſoſceles triangle, whoſe legges are A B and A C; and from the top A let A D be drawn to the baſe B C: I ſay that the ſquare of A D with the rectangle of B D and D C, is equal to the ſquare of either of the legges A B or A C.



Demonſtration NOw the line A D is perpendicular to B D, or not: Let it in the firſt place be perpendicular: Therefore it will cut the baſe B C into two equal parts in the point D; and therefore the rectangle conteined under B D and D C is equal to the ſquare of the ſaid B D, and adding to them the common ſquare of A D, the rectangle of B D and D C with the ſquare of A D, ſhall be equal to the ſquares of D B and A D. But to thoſe ſquares of
i) 47. 1. A D and D B [i] the ſquare of A B is equal: Therefore the

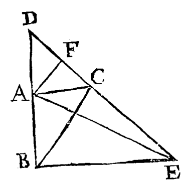
the ſquare of A B is equal to the rectangle of B D and D C, and the ſquare of A D together.

Now ſuppoſe A D not to be perpendicular, but that from the point A there doth fall on B C the perpendicular A E, that being ſo, B C ſhall be cut into two parts equally in the point E, and unequally in D. Where-
fore the rectangle of B D and D C, with the ſquare of D E, [k] is equal to k) 5. 2.
the ſquare of B E; and adding the common ſquare of A E, the rectangle of B D and D C, with the ſquares of D E and A E, ſhall be equal to
the ſquares of B E and A E. But [l] the ſquare of A D is equal to the two l) 47. 1.
ſquares of D E and A E: Therefore the rectangle of B D and D C, with the ſquare of A D is equal to the ſquares of B E and A E. But to theſe ſquares of B E and A E the ſquare of A B is equal: Therefore the ſquare of A D, with the rectangle of B D and D C, is equal to the ſquare of A B.

OTHERWISE.

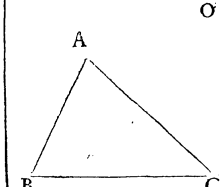
Conſtruction HAving done, as in the foregoing Demonſtration, from the point A, let A F be drawn perpendicular to C D, and let A E be drawn.

Demonſtration FOraſmuch as the angle B A C is given, the half thereof A C F ſhall be alſo given. But the angle A F C is given; and therefore the triangle A F C is given by kind: Therefore the reaſon of A F to F C is given. But the reaſon of C D to the ſame F C is alſo
given, ſeeing that C D is double to F C: Therefore [m] the reaſon of m) 8. p.
C D to A F is given; and therefore alſo the reaſon of the rectangle of C D and E C, to the rectangle of A F and E C, is
given; (for it is the ſame reaſon [n] as that of C D n) 1. 6.
to A F.) But the reaſon of the rectangle of A F and F C to the triangle A C E is given; ſeeing it
is double [o] to the ſame triangle. Therefore the rea- o) 41. 1.
ſon of the rectangle of C D and C E to the triangle A C E is alſo given. But the triangle A C E is
equal to the triangle A B C [p], they being both p) 37. 1.
conſtituted on one and the ſame baſe A C, and
between the ſame parallels A C and B E: There- q) 8. p.



fore [q] the reaſon of the rectangle of C E and C D to the triangle A B C is given. But the ſaid rectangle of C E and C D is the ſpace by which the ſquare of the line compounded of A B and A C, is greater then the ſquare of B C: Therefore that ſpace by which the ſquare of the line compounded of A B and A C is greater then the ſquare of B C, hath a given reaſon to the triangle A B C.

OTHERWISE.

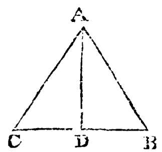


FOr the given angle A is either a right, acute, or obtuſe angle: Let it in the firſt place be ſuppoſed a right angle: Therefore the ſquare of the line compounded of B A C, is greater then the ſquare of B C, by twice the
rectangle of B A and A C; (ſeeing that [r] the r) 47. 1.
ſquare of B C is equal to the ſquares of B A and A C; and the ſquare of the line com-
pounded of B A C [s] is equal to thoſe two s) 5. 2.
ſquares

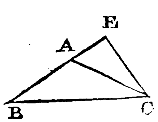
squares of B A and A C, and twice the rectangle of the said B A and A C:) Wherefore the reason of double the rectangle of B A and A C to the triangle A B C is given.

Construction NOw let the angle C be supposed acute, and from the point A let there be drawn on C B the perpendicular A D.

Demonstration FOrasmuch as the triangle C A B is an Oxigonium triangle, and the perpendicular A D being drawn, the square of
t) 13. 5. C A and C B are equal [t] to the square of A B with twice the rectangle of C B and C D; adding therefore the common double rectangle of C A and C B, the squares of C A and C B, with the double rectangle of the said C A and C B, that



u) 4. 2. is to say, [u] the alone square of the line compounded of A C B, are equal to the square of A B, with the double of the rectangle of C D and C B, and over and above the double of the rectangle of A C and C B, that is to say, the double of the rectangle conteined under the compound line of A C D and and C B (for the
w) 1. 2. rectangle of A C D and C B is [w] equal to the rectangles of A C and C B, and of C D and C B:) Therefore the square of the line compounded of A C B is greater then the square of A C, by double the rectangle of A C D and C B. And seeing that the angle A C B is given, and the angle B D A also given, the other angle C A D is given: Therefore [x] the
x) 40. p. triangle C A D is given by kind, and therefore the reason of C D to C A is given; and by consequence the reason of the line compounded of A C D
y) 6. p. to C A [y] is also given. Wherefore the reason of the rectangle of those
z) 1. 6. lines compounded of A C D and C B [z] to the rectangle of A C and C B is also given. But the reason of the said rectangle of A C and C B to the
a) 66. p. triangle C A B [a] is given, seeing the angle C is given; therefore the reason of double the rectangle of the line compounded of A C D and C B to the triangle C A B is given.



Lastly, Let the angle B A C be supposed to be obtuse, and having prolonged B A from the point C, let the perpendicular C E be drawn on the said line B A prolonged; and let A F be proposed to be equal to A E.

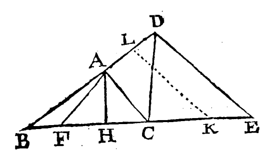
Demonstration FOrasmuch as the angle B A C is obtuse, and the perpendicular C E being drawn, the squares of A B and A C, and the double of the rectangle under B A and A E, or A F, are all alike equal
b) 12. 2. [b] to the square of B C, and adding the common double rectangle of B A and A C, the squares of the said A B and A C, with the double of the rectangle of the same A B and A C, that is to say, [c] the square of the
c) 4. 2. line compounded of B A C, and the double of the rectangle of B A and A F are together equal to the square of B C, with the double of the rectangle of B A and A C. Let the common double of the rectangle of B A and A F be taken away, and there will remain the square of the line compounded of B A C, equal to the square of B C, with the rect-
d) 1. 2. angle of A B and C F; (for the rectangle of A B and A C is equal [d] to the two rectangles of A B and A E, and of A B and C F:) Therefore the square

square of the line compounded of B A C is greater then the square of B C by the double of the rectangle of A B and C F. And forasmuch as the angle B A C is given, the angle C A E [e] is given. But the angle e) 13. 1.
A E C is also given; therefore the other angle A C E is given: Wherefore [f] the triangle A C E is given by kind, and therefore the reason of f) 40. p.
C A to A E, that is to say, to A F is given. Therefore [g] the reason of the g) 5. p.
said C A to F G is also given. But the reason of the same C A to C E is given; therefore [h] the reason of C E to C F is also given. Where- h) 8. p.
fore the reason of the rectangle of E C and A B to the rectangle of F C and A B is given; (for the rectangle is to the rectangle [i] as C E is to i) 2. 6.
C F) and also that of the rectangle of A C and A B, to the rectangle of E C and A B. Therefore [k] the reason of the rectangle of F C and A B to k) 8. p.
the rectangle of A C and A B is given. But the reason of the rectangle of A C and A B to the triangle A B C [l] is given: Therefore also the reason l) 66. p.
of the double of the rectangle of F C and A B, to the triangle A B C is given. But the same double of the rectangle of F C and A B is that, whereof the square of the line compounded of B A C is greater then the square of B C, whereof that space of which the square of the line compounded of B A C is greater then the square of B C, hath a given reason to the triangle A B C.

OTHERWISE.

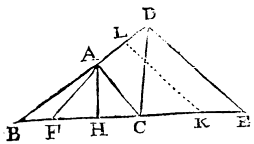
Construction LEt the line B A be prolonged to the point D, in such sort as A D may be equal to A C, and let C D be drawn.

Demonstration FOrasmuch as the angle B A C is given, each of the angles A D C and A C D, which is the half thereof shall be also given; and therefore the other angle D A C is a also given: Therefore [m] the triangle A C D is given by kind. m) 40. p.
Wherefore the reason of A C to C D is given. And forasmuch as the angle A D C is given: Let each of the angles D E C and A F C be made † equal to the said A D C: Therefore seeing that the angle B D C is equal to the angle D E C, and the angle D B E is common to the triangles D B E and D B C, the other angle B D E is equal to the other angle B C D; and therefore the triangle B D E is equiangled to the triangle



B D C. Therefore [n] as E B is to B D, n) 4. 6.
so is B D to C B: Wherefore the rectangle of E B and C B, that is to say, o) 5. 2.
[o] the rectangle of E C and C B, [p] with the square of C B is equal, [q] to p) 5. 2.
the square of B D, that is to say, to the square of the line compounded of q) 17. 6.
B A C; for A D is equal to A C; and therefore the rectangle of E C and C B, with the square of C B, that is to say, the square of the line compounded of B A C is greater then the square of the rectangle of B C and C E: I say therefore that the reason of the said rectangle of B C and C E to the triangle A B C is given. Forasmuch as the angle B D E is equal to the angle B C D, and the angle A D C equal to the angle A C D, the other angle C D E is equal to the other angle A C B: But the angle D E C is also equal to the angle A F C; therefore the remaining angle

angle C A F is equal to the remaining angle D C E. Wherefore the trian-
r) 4. 6. gle A F C is equiangled to the triangle D C E; and therefore [r] as C A is to A F, so is C D to C E; and by permutation, as A C is to C D, so is A F to C E. But the reason of A C to C D is given: Therefore also the reason of A F to C E is given. From the point A let A H be drawn perpendicular to B C: Forasmuch as the angle A F C is given, and the angle A H F also given, the third angle H A F is given: Wherefore [s] the tri-
s) 40. p. angle A H F is given by kind; and by consequence the reason of A F to A H is given. But the reason of A F to C E
t) 8. p. is also given: Therefore [t] the reason of A H to C E is given; and therefore the reason of the rectangle of A H and
u) 1. 6. B C [u] to the rectangle of B C and C E is also given. But the reason of the rectangle of A H and B C to the tri-
w) 41. 1. angle A B C is likewise given; (for the rectangle [w] is double to the triangle) and the rectangle of B C and C E is that whereof the square of the line compounded of B A C is greater then the square of B C. Therefore that space of which the square of the line compounded of B A C is greater then the square of B C by a given reason to the triangle A D C.



SCHOLIUM.

† *The ancient Interpreter pretending to shew the construction of the angle* D E B *equal to the angle* A D C, *saith that on the line* B D *and in the point* D, *the angle* B D E *ought to be made, equal to the angle* B C D, *and that the right lines* B C *and* D E *be drawn until they intersect in* E, *in such sort as he supposeth the angle* B C D, *to be given, but it is not.*

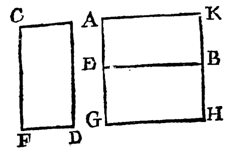
The same Interpreter afterward shews how there may universally from a given point, be drawn a right line given by position to a right line, making an angle equal to a given angle. But we will also reject this way, seeing we have elsewhere shewn another more brief and easie. For example, if we would from the point D *draw to the line* B C *given by position a right line, making an angle equal to a given angle* A D C, *as is here required, we have no more to do but to assume the point* K *in the said line* B C, *and there make the triangle* C K L *equal to the given angle* A D C: *If the line* K L *doth meet with the point* D, *it shall be the line required. But if it meet not with it, from the point* D *let there be drawn the line* D E, *parallel to the said* K L, *cutting* B C *prolonged in* E, *and the angle* D E C *shall be equal to the given angle* A D C, *for on the two parallel lines* L K *and* D E, *there*
x) 29. 1. *doth fall the line* B E; *and therefore the angle* D E C [x] *is equal to the angle* L K C, *which hath been made equal to the given angle* A D C; *and by consequence the same angle* D E C *is also equal to* A D C.

PROP. 68.

If two Parallelograms A B *and* C D, *have to one another a given reason, and that a side hath also a given reason to a side, the other side shall have likewise a given reason to the other side.*

Con-

Construction Let the reason of B E to F D be given: I say the reason of A E to F C is also given. For to the right line E B let there be applyed the Parallelogram F H, equal to the Parallelogram C D, and constituted in such sort as A E and E G may make one right line: † Therefore K B and B H will also make one right line.



Demonstration Forasmuch as the reason of A B to C D is given, and that E H is equal to the said C D; the rea-
a) 1. 6. son of A B to E H is given; and therefore the reason of A E to E G is also given. Seeing therefore that E H is equal and equian-
b) 14. 6. gled to C D, as [b] E B is to F D, so is F C to E G. But the reason of E B to F D is given: Therefore also the reason of F C to E G is given. But the reason of A E to the same E G is also given: Therefore the reason of A E to F C is given.

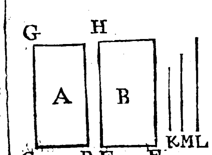
SCHOLIUM.

† *EUCLIDE having posited* A E *and* E G *directly in one right line, presently concludeth that* K B *and* B H *shall also make a right line; but we shall demon-*
b) 13. 1. *strate it thus. Seeing the lines* A E *and* E G *are posited directly, the angles* A E B *and* B E G [b] *are equal to two right angles; and seeing that* A B *is a Parallelogram, the lines* A K *and* E B *are parallels, on which the line* A E *doth fall; and*
c) 29. 1. *therefore the two internal angles* A *and* B E A [c] *are also equal to two right angles, and taking away the common angle* B E A, *there will remain the angle* A, *equal to the angle* B E G; *and consequently their opposite angles* E B K *and* H *are also equal to one another: Again, seeing that* B G *is a Parallelogram, the two lines* B E *and* H G *are Parallels, on which* B H *doth fall; and therefore the two internal angles* H *and* E B H [c] *are equal to two right angles. But it hath been demonstrated that* H *is equal to* E B K: *Therefore the two angles* E B K *and* E B H *are also equal to*
d) 14. 1. *two right angles; and therefore* [d] *the two lines* K B *and* B H *do meet directly according to EUCLIDE.*

OTHERWISE.

Construction Let the given right line K be exposed, and seeing that the reason of A to B is given, let the same be made of K to L; therefore the reason of K to L is also given.

e) 2. p. *Demonstration* But K is given; therefore [e] L is also given. Again, seeing that the reason of C D to E F is given, let the same be made of K to M: Therefore the reason of K to M is given. But K is given,
f) 1. p. therefore [f] M is also given; and therefore the reason of L to M is given. Now seeing that A is
g) 23. 6. equiangled to B, [g] the reason of the said A to B is compounded of that of the sides, that is to say of C D to E F, and of C G to E H. But also the reason of K to L is compounded of K to M, and of M to L; therefore the reason compounded of C D to E F, and of C G to E H, is the same with that which is compounded of K

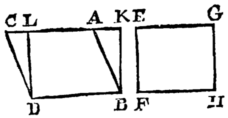


to

to M, and of M to L (the reaſon of K to L being the ſame as of A to B:) But the reaſon of C D to E F is the ſame as of K to M: Therefore the other reaſon of C G to E H is alſo the ſame as of M to L. But the ſaid reaſon of M to L is given: Therefore alſo the reaſon of C G to E H is given.

PROP. 69.

If two Parallelograms C B *and* E H, *having the angles* D *and* F *given, and that a ſide hath alſo a given reaſon to a ſide; in like manner the other ſide ſhall have a given reaſon to the other ſide.*



Conſtruction LEt the reaſon of B D to F H be alſo given: I ſay that the reaſon of A B to E F is given. For if C B be equiangled to H E, it is manifeſt by the precedent Propoſition; but if it be not equiangled thereto, let the right line D B be conſtituted, and in the given point B therein, let the angle D B K be made equal to the angle E F H, and finiſh the Parallelogram D K.

Demonſtration FOraſmuch as each of the angles B K L and B A K is given, † the other angle K B A is given: Wherefore the triangle
a) 40. p. [a] A B K is given by kind; and therefore the reaſon of A B to B K is given. But
b) 35. p. reaſon of C B to E H is ſuppoſed to be given, and [b] C B is equal to D K; therefore the reaſon of D K to E H is given; and ſeeing that D K is equiangled to E H, and the reaſon of the ſaid D K to E H is given, as alſo
c) 68. p. that of D B to F H, [c] the reaſon of B K to F E is given. But the reaſon of
d) 29. 1. the ſaid B K to B A is alſo given: Therefore [d] the reaſon of A B to F E is given.

SCHOLIUM.

† *EUCLIDE ſuppoſeth there that a Parallelogram having one angle given, all the other angles are alſo given, and as well the Ancient Interpreters as others, do give the reaſons why, the angle* F *being given, the other angle* E *ſhall be alſo given, it being the remainder of two right angles, for that on the parallel lines* E G
e) 29. 1. *and* F H *there doth fall the line* E F, *which makes* [e] *the two internal angles (of the*
f) 34. 1 *ſame part)* F *and* G, *equal to two right angles. But to thoſe angles* [f] *the oppoſite angles* G *and* H *are equal, and therefore they are alſo given.*

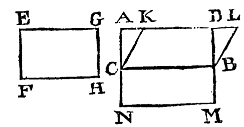
From whence it follows that the angles B D C *and* F *being given by ſuppoſition, all the other angles of the two Parallelograms* C B *and* E H, *are alſo given: Therefore the angle* D B K *having been made equal to the angle* F, *the angle* K *ſhall be equal to the angle* E, *and given as that is: But the angle* B A L *which is oppoſite to the given angle* B D C, *is alſo given; and therefore* B A K *which is the remainder of two right angles, ſhall be alſo given; in ſuch ſort as in the triangle* A B K, *the two angles* B A K *and* B K A *are given, as EUCLIDE doth declare in this place.*

PROP. 70.

If of two Parallelograms A B *and* E H, *the ſides about the equal angles, or about the unequal angles (yet neverthelesse given angles) have to one another a given reaſon, to wit* (A C *to* E F, *and* C B *to* F H) *alſo the ſame Parallelograms* A B *and* E H *ſhall have to one another a given reaſon.*

Conſtruction FOr let A B be prolonged to E H, and on the right line C B let the Parallelogram C M be applyed equal to the Parallelogram E H, in ſuch ſort as A C may be direct to C N; that is to ſay, that A C and C N make one right line; and by conſequence D B ſhall be [a] directly with B M. a) Sch. 68. p.

Demonſtration FOraſmuch then as C M is equiangled and equal to E H, the ſides about the equal angles ſhall be reciprocally [b] proportional: Wherefore as B C is to H F, ſo is F E to N C. But the reaſon of B C to H F is given: Therefore the reaſon of F E to N C is alſo given. But the reaſon of A C to the ſame E F is given: Therefore [c] the reaſon of A C to N C is alſo given. Wherefore the reaſon of A B to C M is given; (for it is the ſame [d] as of A C to C N.) But C M is equal to E H: Therefore the reaſon of A B to E H is given. b) 14. 6. c) 8. p. d) 1. 6.

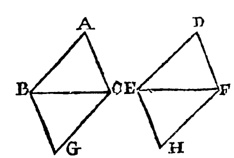


Conſtruction NOw ſuppoſe A B not to be equiangled to E H, and on the right line C B, and in the given point C therein: Let there be conſtituted the angle B C K, equal to the given angle F, and ſo finiſh the Parallelogram C L.

Demonſtration FOraſmuch as the angle A C B is given, and the angle B C K alſo given, the remaining angle A C K is given: Therefore the triangle A C K [e] is given by kind; and therefore the reaſon of A C to C K is given: But the reaſon of A C to E F is alſo given: Therefore the reaſon of C K to E F is given. But the reaſon of B C to H F is alſo given, and the angle B C K is equal to the angle F; therefore (by the firſt part of this Prop.) the reaſon of C L to E H is given. But to the ſaid C L, A B is equal: Therefore the reaſon of A B to E H is given. e) 40. p.

PROP. 71.

If of two triangles A B C *and* D E F, *the ſides about the equal angles* A *and* D, *or elſe about the unequal angles (yet neverthelesse given angles) have to one another a given reaſon (to wit* A B *to* D E *and* A C *to* D F) *the ſame triangles ſhall have alſo to one another a given reaſon* A B C *to* D E F.

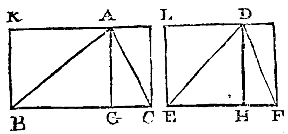


Conſtruction Let the Parallelogram A G and D H be finiſhed.

Demonſtration SEeing that the two Parallelograms A G and D H, have the ſides about the equal angles A and D, or elſe about the unequal angles (neverthelesse given) have a given reaſon to one another, the

a) 70. p. the reason [a] of the Parallelogram A G to the Parallelogram D H is given.
b) 34. p. But the triangle A B C is the half of the Parallelogram A G [b] and the triangle DEF the half of the Parallelogram DH. Therefore the reason of the triangle A B C to the triangle D E F is given.

PROP. 72.

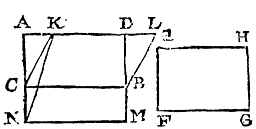


If of two triangles A B C *and* D E F, *the bases* B C *and* E F, *are in a given reason,* B C *to* E F, *and that from the angles* A *and* D, *there be drawn to those bases the right lines* A G *and* D H, *making the equal angles* A G C *and* D H F, *or else unequal (yet neverthelesse given) which shall have to one another given reasons* A G *to* D H, *those triangles* A B C *and* D E F *shall have also a given reason to one another, to wit,* A B C *to* D E F.

Construction For let the Parallelogram K C and L F be finished.

Demonstration FOrasmuch as the angles A G C and D H F are equal, or unequal (yet given) and that the angle A G C [a] is equal to
a) 29. 1. the angle K B C. But the angle D H F equal to the angle L E F, the angles at the points B and E are equal, or else unequal (yet given) and for that the reason of A G to D H is given, and A G is equal to K B. But D H is equal to L E, also the reason of K B to L E is given. But the reason of B C to E F is also given, and the angles at the points B and E are equal,
b) 70. p. or else unequal (yet given:) Therefore [b] the reason of the Parallelogram K C to the Parallelogram L F is given; and therefore the reason of the triangle A B C to the triangle D E F is given, seeing those triangles
c) 41. 1. [c] are the one half of the Parallelograms.

PROP. 73.



If of two Parallelograms A B *and* E G, *the sides about the equal angles* C *and* F, *or else about the unequal angles (but neverthelesse given) are in such sort to one another, that as the side* C B *of the first, is to the side* F G *of the second; so the other side* E F *of the second, is to some other right line* C N. *But that the other side* A C *hath also to the same right line* C N *a given reason, those Parallelograms will have also to one another a given reason* A B *to* E G.

Construction FOr in the first place, Let the Parallelogram A B be equiangled to E G, and having placed C N directly to A C: Let the Parallelogram C M be finished.

Demon-

Demonstration FOrasmuch then, as C B or N M its equal, is to F G, so is E F to C N, and that the angles N and F are equal (for N is equal to the angle A C B, which is put equal to F) the Parallelograms C M and E G [a] are equal: But as A C to C N, so [b] the Parallelogram A B is to the Parallelogram C M or E G: Therefore seeing that the reason of A C to CN is given, the reason of A B to E G is also given. a) 14. 6. b) 1. 6.

Construction 2. NOw suppose the Parallelogram A B not to be equiangled to the Parallelogram E G, and let there be constituted at the given point C in the line C B, the angle B C K, equal to the angle E E G., and so finish the Parallelogram C L.

Demonstration 2. SEeing that each of the angles A C B and K C B is given, the remaining angle A C K is also given. But [c] the angle c) Sch. 69 p.
C A K is given, as also the remaining angle A K C: Therefore [d] the triangle A C K is given by kind; and therefore the reason of A C to C K is d) 40. p.
given. But the reason of the same A C to C N is also given: Therefore [e] the reason of C K to C N is given. And seeing that as C B is to F G; so e) 8. p.
is E F to the right line C N, to which the other side K C hath a given reason, and that the angle B C K is equal to the angle F, the reason of the Parallelogram C L to the Parallelogram E G is given (by the first part of this Prop.) but the Parallelogram C L is equal to the Parallelogram A B: Therefore the reason of the Parallelogram A B to the Parallelogram E G is given.

PROP. 74.

If two Parallelograms (as in the former figure) AB *and* EG, *in equal angles* C *and* F, *or else in unequal angles (yet neverthelesse givenangles) have a given reason to one another, as one side* C B *of the first shall be to one side* F G *of the second, so the other side* E F *of the second, shall be to that to the which the other side* A C *of the first hath a given reason.*

Construction FOr either A B is equiangled or not; suppose it in the first place to be equiangled, and to the right line B C let there be applyed the Parallelogram C M, equal to the Parallelogram E G, and so posited, as that A C and C N may be direct: Therefore [a] D B a) Sch. 68. p.
and B M shall be also direct (that is as one right line.)

Demonstration SEeing that the reason of A B to E G is given, and that C M is equal to E G, the reason of A B to C M is also given; and therefore the reason of A C to C N is given (seeing A B is to C M, [b] as A C is to C N;) and for that C M is equal and equiangled to b) 1. 6.
E G, the sides about the equal angles of the Parallelograms C M and E G, [c] are reciprocally proportional; and therefore as C B is to F G, so c) 14. 6.
is E F to C N. But the reason of A C to C N is given: Therefore as C B is to F G, so is E F to that to which A C hath a given reason.

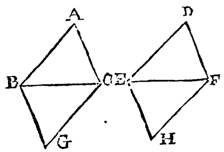
Construction 2. NOw suppose A B not to be equiangled to E G, and in the given point C of the line C B, let there be constituted the

 angle

angle B C K equal to the angle E F G, and finiſh the Parallelogram C L.

Demonſtration 2. Seeing then that the reaſon of A B to E G is given, and
d) 36. 1. [d] that A B is equal to C L, alſo the reaſon of C L to E G is given, and the angle B C K is equal to the angle F, and therefore
e) Sch. 69 p. C L [e] is equiangled to E G: Therefore (by the firſt part of this Prop.) as C B is to F G, ſo is E F to that to the which C K hath a given reaſon. But the reaſon of A C to C K is given; (as appears by what hath been demonſtrated in the latter part of the precedent Prop.) Therefore as C B is to F G, ſo is E F to that to which A C hath a given reaſon.

PROP. 75.



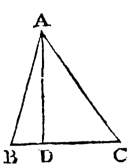
If two triangles A B C *and* D E F, *in equal angles* A *and* D, *or elſe unequal (yet nevertheleſſe given) have to one another a given reaſon, as the ſide* A B *of the firſt, ſhall be to the ſide* D E *of the ſecond, ſo the other ſide* D F *of the ſecond, ſhall be to that right line to the which the other ſide* A C *of the firſt hath a given reaſon.*

Conſtruction For let the Parallelograms A G and D H be finiſhed.

Demonſtration Foraſmuch as the reaſon of the triangle A B C to the triangle D E F is given, alſo the reaſon of the Parallelogram A G to the Parallelogram D H is given.

Seeing therefore that the two Parallelograms A G and D H in equal angles, or unequal angles (nevertheleſſe given) have to one another a given
a) 74. p. reaſon; as [a] A B is to D E, ſo is D F to that to which A C hath a given reaſon.

PROP. 76.



If from the top A *of a triangle* A B C, *given by kind, there be drawn to the baſe* B C, *a perpendicular line* A D, *that line* A D *ſhall have to the baſe* B C *a given reaſon.*

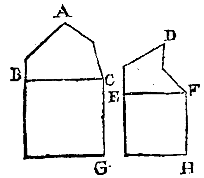
Demonſtration For ſeeing that the triangle A B C is given by kind, the reaſon of A B to B C is given; and the angle B is alſo given. But the angle A D B is given; therefore the other angle B A D is given. Where-
a) 40. p. fore [a] the triangle A B D is given by kind; and therefore the reaſon of A B to A D is given. But the reaſon of A B to B C is given: Therefore
b) 8. p. [b] the reaſon of A D to B C is given.

PROP. 77.

If two figures A B C *and* D E F, *given by kind, have to another a given reaſon, the reaſon alſo ſhall be given of which you*

you

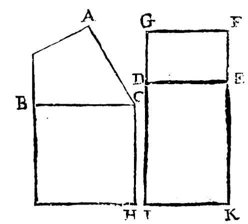
you pleaſe of the ſides of one of the figures, to which you pleaſe of the ſides of the other figure.



Conſtruction For on the right lines B C and E F, let there be deſcribed the ſquares B G and E H.

Demonſtration Foraſmuch as on one and the ſame right line B C, are deſcribed two figures A B C and B G, given by kind,
a) 49. p. [a] the reaſon of the ſaid A B C to B G is given: In like manner, the reaſon of D E F to E H is given; and ſeeing that the reaſon of A B C to D E F is given, and alſo that of the ſame figure A B C to B G; and
b) 8. p. again the reaſon of D E F to E H: [b] the reaſon of B G to E H is given; and therefore the reaſon of B C to E F is alſo given.

PROP. 78.



If a given figure A B C, *hath a given reaſon to ſome rectangled figure* D F, *and that one ſide* B C *hath a given reaſon to one ſide* D E, *the rectangled figure* D F *is given by kind.*

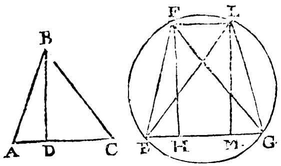
Conſtruction For on the right line B C let the ſquare B H be deſcribed, and to the right line D E, let the Parallelogram D K be applyed equal to B H, in
a) Sch. 68. p. ſuch manner as that G D and D I may be placed directly, [a] and by conſequence F E and E K alſo directly.

Demonſtration Therefore ſeeing that on one and the ſame right line B C are deſcribed the two rectiline figures A B C and B H,
b) 49. p. given by kind; [b] the reaſon of A B C to B H is given. But the reaſon of
c) 8. p. the ſaid A B C to D F is alſo given: Therefore [c] the reaſon of B H to D F is given. But B H is equal to D K: Therefore the reaſon of D K to D F is alſo given. And ſeeing that B H is equal and equiangled to D K,
d) 14. 6. both the one and the other being rectangles, [d] the ſides of thoſe figures are reciprocally proportional; and as B C is to D E, ſo is D I to C H. But by ſuppoſition, the reaſon of B C to D E is given; therefore alſo the reaſon of D I to C H is given; but the reaſon of D I to D G is alſo given:
e) 1. 6. (for D I is to D G [e] as D K to D F:) Therefore [f] the reaſon of D G to
f) 8. p. C H is given. But C H is equal to B C, ſeeing that B H is a ſquare; therefore the reaſon of B C to D G is given. But the reaſon of the ſame B C to D E is alſo given; therefore the reaſon of D E to D G is given,
g) Sch. 61. p. and the angle at D is a right angle: Therefore [g] D F is given by kind.

PROP. 79.

If two triangles A B C *and* E F G, *have an angle* B *equal* to

to an angle F. *But from the equal angles* B *and* F *there be drawn perpendiculars* BD *and* FH, *to the bases* AC *and* EG; *and that as the base* AC *of the first triangle* ABC, *is to the perpendicular* BD, *so also the base* EG *of the other triangle* EFG, *is to the perpendicular* FH, *those triangles* ABC *and* EFG *are equiangled.*



Construction FOr about the triangle EFG let there be described the circle EFLG, then on the right line EG, and in the point E given therein, let there be made the angle GEL, equal to the angle C, and let FL and LG be drawn, and the perpendicular LM.

Demonstration SEeing then that the angle GEL is equal
a) 21. 3. to the angle C, and the angle ELG is equal to the angle EFG, [a] they
being in one and the same segment of the circle; the third angle EGL is
equal to the third angle A. Wherefore the triangle ABC is alike to the
triangle ELG, and the perpendiculars BD and LM are drawn: There-
fore † as AC is to BD, so is EG to LM; but by supposition as AC is to
b) 7. 5. BD; so is EG to FH: Therefore [b] LM is equal to FH. But the said LM
c) 28. 1. is [c] parallel to FH: Therefore [d] FL is also parallel to EG; and there-
d) 33. 1. fore the angle FLE [e] is equal to the angle LEG. But the angle C is also
e) 29. 1. equal to the said angle LEG, and the angle FLE to the angle FGE [f]:
f) 21. 3. Therefore also the angle C is equal to the angle FGE. But by supposition
the angle ABC is equal to the angle EFG: Therefore the third angle
BAC is equal to the third angle FEG: Wherefore the triangle ABC
is equiangled to the triangle EFG.

SCHOLIUM.

† *Now that as* AC *is to* BD, *so* EG *is to* LM, *it is by some thus demonstrated.*
Forasmuch as the angle C *is equal to the angle* GEL, *and the angle* BDC *to the*
angle LME, *each being a right angle, the other angle* CBD *is equal to the other*
g) 4. 6. *angle* ELM: *Therefore* [g] *as* EM *is to* ML, *so is* CD *to* DB. *Again, seeing*
the angle ABC *is equal to the angle* ELG, *and the angle* CBD *to the angle*
ELM, *the remaining angle* ABD *is equal to the remaining angle* MLG; *but*
the angle ADB *is also equal to the angle* LMG; *and therefore the third angle* A
h) 4. 6. *is equal to the third angle* LGM: *Therefore* [h] *as* AD *is to* DB, *so is* GM *to* ML.
But it hath been demonstrated that as CD *is to* DB, *so is* EM *to* ML: *There-*
i) 14. 5. *fore* [i] *as* AC *is to* BD, *so is* EG *to* LM.

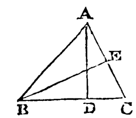
PROP. 80.

If a triangle ABC *hath one angle* A *given, and that the rectangle conteined under the sides* AB *and* AC, *comprising the given angle* A, *hath a given reason to the square of the other side* BC, *the triangle* ABC *is given by kind.*

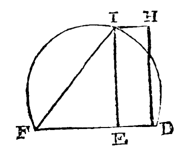
Con-

Construction FOr from the points A and B, let there be drawn the perpendiculars AD and BE.

Demonstration FOrasmuch as the angle BAE is given, and also the angle
AEB, the triangle ABE is given [a] by kind; and there- a) 40. p.
fore the reason of AB to BE is given: Therefore the reason of the rectan-
gle of AB and AC to the rectangle of BE and AC is also given (for it is
the same reason [b] as of AB to BE.) But the re- b) 1. 6.
ctangle of AC and BE is equal to the rectangle of
BC and AD; for that each of those rectangles
is [c] double to the triangle ABC. Therefore the rea- c) 41. 1.
son of the rectangle of AB and AC to the rectangle
of BC and AD is also given. But the reason of the
rectangle of AB and AC to the square of BC is gi-
ven: Therefore [d] also the reason of the rectangle of d) 8. p.



BC and AD to the square of BC is given; and therefore the rea-
son of the right line BC to the right line AD is given. (For that [e] the re- e) 1. 6.
ctangle is to the square as AD to BC.) Now let the right line FG given
by position and magnitude, be exposed; and thereon let there be descri-
bed the segment of a circle FIG, capable of an angle equal to the angle
A. And seeing the said angle A is given, also the angle in the segment
FLG shall be given; and therefore [f] the same segment is given by posi- f) 8. def.
tion. From the point G let there be erected at right angles on the line
FG, the line GH, which [g] is given by position: g) 4. def.
Let it be so made, that as BC is to AD, so FG
may be to GH; and seeing that the reason of BC
to AD is given, also that of FG to GH is given.
But FG is given: Therefore [h] GH is given by mag- h) 2. p.
nitude. But it is also given by position, and the
point G is given: Therefore the point H is [i] also i) 27. p.
given. Now by the point H let there be drawn HI,
parallel to FG, and that line HI shall be given by



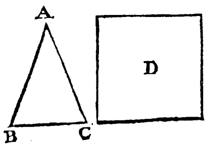
[k] position. But the segment of the circle FIG is also given by position. k) 28. p.
Therefore [l] the point I is given. Let the right lines IF and IG be drawn, l) 25. p.
and the perpendicular IK: Therefore IK is given by position. But the
point I is given, as also each of the points F and G: Therefore [m] each of m) 26. p.
the lines FG, FI, and IG is given by position and magnitude: Where-
fore [n] the triangle FIG is given by kind; and seeing that as BC is to AE, n) 39. p.
so is FG to GH, and [o] that to GH, IK is equal, as BC is to AE, so is o) 34. p.
FG to IK, and the angle A is equal to the angle FIG: Therefore [p] the p) 7[illegible]. p.
triangle ABC is equiangled to the triangle FIG. But FIG is given by
kind: Therefore also the triangle ABC is given by kind.

OTHERWISE.

Construction LEt the triangle ABC, whose angle A is given, and the reason of the rectangle conteined under AB and AC, to the square of BC be given: I say that the triangle ABC is given by kind.

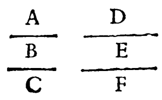
Demonstration FOr seeing the angle A is given, that space of which
the square of the line compounded of BAC is greater
then the square of BC, [q] hath a given reason to the triangle ABC. Now q) 67. p.
let that space be D: Therefore the reason of D to the triangle ABC is
given

given. But the reason of the triangle A B C to the rectangle of A B and
r) 66. p. A C is given; [r] seeing the angle A is given:
s) 8. p. Therefore [s] the reason of the space D to the rectangle of A B and A C is given. But the reason of the rectangle of A B and A C to the square of B C is also given: Therefore [s] the reason of the space D to the square of B C is
t) 6. p. given. Wherefore by compounding, [t] the reason of the space D, with the square of B C to the said square of B C is given:



Therefore the reason of the square of the line compounded of B A C, to the square of B C is given; (for that the space D with the square of B C is equal to the square of the line compounded of B A C;) and therefore
v) Sch. 52. p [v] the reason of the said line compounded of B A C to B C is given. But
w) 46. p. the angle A is also given: Therefore [w] the triangle A B C is given by kind.

PROP. 81.

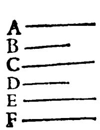


If of three right lines A, B, *and* C, *proportional to three other proportional right lines* D, E, *and* F, *the extreams* A *and* D, C *and* F, *are in a given reason (to wit, as* A *to* D, *and* C *to* F,) *also the meanes* B *and* E *shall be in a given reason, and if one extream hath a given reason to an extream, and the mean to the mean, the other will have also a given reason to the other.*

Demonstration FOrasmuch as the reason of A to D, and of C to F is given, the rectangle of A and D [a] shall have a given reason
a) 70. p.
b) 17. 6. to the rectangle of C and F. But the rectangle of A and D is equal [b] to the square of B; and the rectangle of C and F to the square of E. Therefore the reason of the square of B to the square of E is given; and therefore
c) Sch. 52. p [c] the reason of the line B to the line E is also given.

Again, Let the reason of A to D, and B to E, be given: I say that the reason of C to F is also given. For seeing that the reason of A to D,
d) 50. p. and of B to E is given, also the reason of the square of B [d] to the square of E is given. But the square of B is equal to the rectangle of A and C, and the square of E to the rectangle of D and F: Therefore the reason of the rectangle of A and C to the rectangle of D and F is given. But the
e) 68 p. reason of a side A to a side D is given: Therefore [e] the reason of the other side C to the other side F is also given.

PROP. 82.



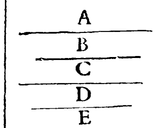
If there be four right lines A, B, C, *and* D, *proportional, as the first* A, *shall be to that line to which the second* B *hath a given reason, so the third* C, *shall be to that to which the fourth* D *hath a given reason.*

Con-

Construction LEt E be the line to which B hath a given reason, and let it be so as that B may be to E, as D is to F.

Demonstration NOw the reason of B to E is given, Therefore also the reason of D to F is given. And seeing that as A is to B, so is C to D. And again, as B is to E, so is D to F, by reason of equality as A is to E, so is C to F. But E is that line to which B hath a given reason, and F that to which D also hath a given reason: Therefore as A is to that to which B hath a given reason, so C is to that to which D hath a given reason.

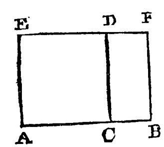
PROP. 83.



If four right lines A, B, C, *and* D, *are in such sort to one another, that of any three of them* A, B, *and* C, *and a fourth* E, *taken proportional, to which that line* D, *which remaines of the four lines, hath a given reason, the four lines* A, B, C, *and* E, *are proportional; as the fourth* D *is to the third* C, *so the second* B *shall be to that to which the first* A *hath a given reason.*

Demonstration FOrasmuch as A is to B as C is to E, the rectangle conteined under A and E [a] is equal to the rectangle conteined a) 16. 6.
under B and C; and seeing that the reason of D to E is given, also shall be given the reason of the rectangle of A and D to the rectangle of A and E
(for [b] it is the same reason as of D to E.) But the rectangle of A and E is b) 1. 6.
equal to the rectangle of B and C: Therefore the reason of the rectangle of A and D to the rectangle of B and C is given. Wherefore [c] as D is to c) 74. p.
C, so is B to that to which A hath a given reason.

PROP. 84.

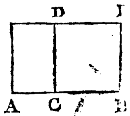


If two right lines A B *and* A E *comprehending a given space* A F *in a given angle* B A E, *and that the one* A B *be greater then the other* A E *by a given line* C B, *also each of the lines* A B *and* A E *is given.*

Demonstration FOr seeing that A B is greater then A E by the given line C B, the remainder A C is equal to A E: Finish the Parallelogram A D. Therefore seeing that A E is equal to A C, the reason of A E to A C is given, and the angle A is also given: Therefore
[a] A D is given by kind. Wherefore the given space A F is applyed to the a) Sch. 61. p.
given right line C B, exceeding it by the given figure A D given by
kind; and therefore [b] the breadth of the excesse is given. Therefore b) 59. p.
A C is given. But C B is also given: Therefore the whole A B is given.
But A E is also given: Therefore each of the right lines A B and A E is given.

PROP.

PROP. 85.



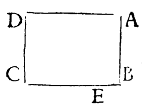
If two right lines A C *and* C D, *do comprehend a given space* A D *in a given angle* A C D, *the line compounded of those lines* A C *and* C D *is given, also each of those lines* A C *and* C D *is given.*

Construction FOr let A C be prolonged to the point B, and let C B be put equal to C D, then by the point B let B F be drawn parallel to C D, and so finish the Parallelogram C F.

Demonstration SEeing then that C B is equal to C D, and the angle D C B is given; for that angle that followes is the given angle; and therefore [a] the Parallelogram D B is given by kind; and again, seeing that the line compounded of A C D is given, and C B is equal to C D, also A B is given. And thus to the right line A B there is applyed the given space A D, deficient by the figure D B given by kind; and therefore [b] the breadths of the defects are also given: Therefore the right lines D C and C B are given. But the compounded line A C D is also given: Therefore [c] each of the lines A C and C D is given.

a) Sch. 61 p. b) 53. p. c) 4. p.

PROP. 86.



If two right lines A B *and* B C, *do comprehend a given space* A C, *in a given angle* A B C, *the square of the one* B C *is greater then the square of the other* A B, *by a given space (yet in a given reason) also each of those lines* A B *and* B C *shall be given.*

Demonstration FOr seeing that the square of B C is greater then the square of A B by a givvn space (yet in a certain reason.) Let the given space be taken away, that is to say, the rectangle conteined under C B and B E: Therefore [a] the reason of the remainder, [b] which is the rectangle conteined under B C and C E to the square of A B is given. And forasmuch as the rectangle † under A B and B C is given, and also that of C B and B E, their [c] reason is given. But as the rectangle under A B and B C is to the rectangle under C B and E B, [d] so A B is to B E; and therefore the reason of A B to B E is given: Wherefore [e] the reason of the square of A B to the square of B E is also given. But the reason of the square of A B to the rectangle under B C and C E is given: Therefore [f] also the reason of the rectangle under B C and C E to the square of B E is given. Wherefore the reason of four times the rectangle under B C and C E to the square of B E is given; and by compounding, [g] the reason of four times the rectangle under B C and C E, with the square of B E to the square of B E is given. But four times the rectangle of B C and C E, with the square of B E, [h] is the square of the compound line B C E: Therefore the reason of the square of the compound line B C E to the square of B E is given: Wherefore [i] the reason of the line compounded of

a) 11 def. b) 2. 2. c) 1. p. d) 1. 6. e) 50. p. f) 8. p. g) 8. p. h) 8. 2. i) 54. p.

of B C and C E to B E is given, and by compounding, [k] the reason of the compound of the lines B C, C E, and B E, that is to say, the double of B C to B E is given; and therefore the reason of the onely line B C to B E is also given. But as B C is to B E, [l] so the rectangle under B C and B E is to the square of B E: Therefore the reason of the rectangle under B C and B E to the square of B E is given. But the rectangle of B C and B E is given: Therefore [m] the square of B E is also given, and consequently the line B E is given. Wherefore B C is also given, seeing that the reason of B E to B C is given. But the space A C is given, and also the angle B: Therefore [n] A B is given. Wherefore each of the lines A B and B C is given.

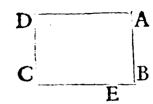
k) 6. p. l) 1. 6. m) 2. p. n) 57. p.

SCHOLIUM.

† *Instead of saying in this place* [what is under, &c.] *we have used this Word Rectangle, it being manifest by what followes that such was the intention of* EUCLIDE, *seeing he makes use in the said Demonstration of the 2d. and 8th. Prop. of the 12th. Element; and also that the space or Parallologram given being not rectangled, it may be reduced thereto, making on* B C, *and in the given point* B, *a right angle* C B A, *so as that there will be two Parallelograms constituted on one and the same base* B C, *and between the same Parallels, as in the 69th. Prop. by meanes whereof this conclusion is drawn.*

¶ Note this serves also for the next Proposition.

PROP. 87.



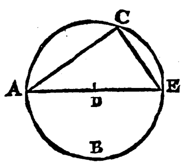
If two right lines A B *and* B C, *do comprehend a given space* A C, *in a given angle* B, *the square of the one* B C, *is greater then the square of the other* A B, *by a given space, also each of those lines* A B *and* B C *shall be given.*

Demonstration FOr seeing that the square of B C is greater then the square of A B by a given space: Let the given space be taken away, and let the rectangle be conteined under B C and B E: Therefore the remainder, [a] which is the rectangle of B C and C E, is equal to the square of A B. And seeing that the rectangle of B C and B E is given, and also the space or rectangle A C; the reason of the said rectangle of B C and B E to A C is given. But as [b] the rectangle of B C and B E is to the rectangle of A B and B C, so is B E to A B: Therefore the reason of B E to A B is given, and therefore [c] the reason of the square of the said D E to the square of A B is also given. But to that square of A B the rectangle of B C and C E is equal: Therefore the reason of the said rectangle of B C and C E to the square of B E is given; and therefore the reason of the quadruple of the said rectangle of B C and C E to the square of B E is also given; and by compounding, [d] the reason of four times the rectangle of B C and C E, with the square of B E, to the said square of B E is given. But four times the rectangle of B C and C E, with the square of B E, [e] is the square of the compound line B C E: Therefore the reason of the square of that compound line B C E to the square of B E is also given; and therefore the reason [f] of the compound line B C E to B E is given. Wherefore by compounding,

a) 2. 2. b) 1. 6. c) 50. p. d) 6. p. e) 8. 2. f) 54. p.

g) 6. p. pounding, g the reaſon of the ſaid compound line BCE and EB, that is to ſay, twice B C to B E is alſo given; therefore the reaſon of the only line B C to B E is given. But the reaſon of the ſame B E to A B is alſo given: Therefore
h) 8. p. h the reaſon of AB to BC is given. And ſeeing that the reaſon of B C to B E
i) 1. 6. is given, and that as the ſaid B C is to B E, ſo the ſquare of B C i to the rectangle of B C and B E, the reaſon of the ſquare of B C to the rectangle of B C and B E is alſo given. But the ſaid rectangle of B C and B E is given, it being that which was taken away, and which was given. Therefore
k) 2. p. the ſquare of B C k is given, and therefore the line B C is given. But the reaſon of the ſame BC to B A is given, therefore A B is alſo given.

PROP. 88.



If in a circle A B C, *given by magnitude, there be drawn a right line* A C, *which ſhall take away a ſegment* A B C, *which doth comprehend a given angle* A E C, *that line* A C *is given by magnitude.*

Conſtruction FOr let D be the center of the circle; and let the diameter thereof A D E be drawn, and let E C be joyned.

a) 31. 3. *Demonſtration* FOraſmuch as the angle A C E is given, for a it is a right angle. But the angle A E C is alſo given, and therefore the
b) 40. p. other angle C A E is given. Wherefore the triangle A C E b is given by kind; and therefore the reaſon of E A to A C is given. But A E is given by magnitude, ſeeing that the circle A B C is given by magnitude. Therefore
c) 2. p. c A C is alſo given by magnitude.

PROP. 89.

If in a circle A B C, *given by magnitude, there be drawn a right line* A C, *given by magnitude, that line* A C *will take away a ſegment* A B C, *comprehending a given angle.*

Conſtruction FOr having taken the point D for the center of the circle, Let the diameter A D E be drawn, as alſo the right line E C.

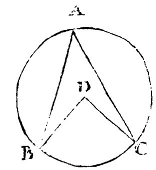
Demonſtration FOraſmuch as each of the right lines A E and AC are given,
a) 1. p. the reaſon of the line A E to A C a is given; and the angle
b) 43. p. A C E is a right angle: Therefore b the triangle A C E is given by kind, and therefore the angle A E C is given.

PROP. 90.

If in the circumference of a circle A B C *given by poſition, and by magnitude, there be taken a given point* B, *and that from that point* B, *to the circumference of the circle* A B C *there doth bend a right line* B A C, *making a given angle* B A C *the other extremity* C *of the bent line ſhall be given.*

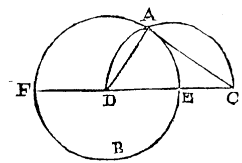
Conſtruction FOr let the center of the circle be D, and let the right lines B D and B C be drawn.

Demon-



Demonſtration FOraſmuch as each point B and D is given, the right line B D, a is given by poſition; and ſeeing that the angle B A C is given, the angle B D C is alſo given. Wherefore to the right line B D given by poſition, and in the point D given therein, there is drawn the right line C D; which makes the given angle B D C; and therefore b the line D C is given by poſition. But the circle A B C is given by poſition and magnitude: Therefore c the right line D C is given by poſition and by magnitude. But the point D is given: Therefore d the point C is alſo given. a) 26. p. b) 29. p. c) 6. def. d) 27. p.

PROP. 91.

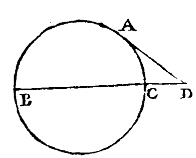


If from a given point C, *there be drawn a right line* C A, *which ſhall touch a circle* A B, *given by poſition; that line* C A *is given by poſition and by magnitude.*

Conſtruction FOr having taken the point D for the center of the circle, let the right lines D A and D C be drawn.

Demonſtration FOraſmuch as each point C and D is given, the right line C D a is given by poſition and by magnitude. But the angle C A D b is a right angle; and therefore the ſemicircle deſcribed on C D ſhall paſſe by the point A: Let it then paſſe by that point, and let the ſemicircle be D A C: Foraſmuch as the ſame D A C c is given by poſition and alſo the circle A B E, d the point A is given. But the point C is alſo given: Therefore e the right line A C is given by poſition and by magnitude. a) 26. p. b) 18. 3. c) 6. def. d) 25. p. e) 26. p.

PROP. 92.

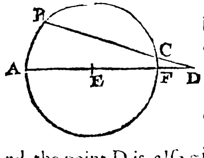


If without a circle A B C, *given by poſition, there be taken ſome point* D, *and from that given point there be drawn a right line* D B, *cutting the circle, the rectangle compriſed under the whole line* B D, *and the part* DC, *between the point* D *and the circumference convex* A C *ſhall be given.*

Conſtruction FOr from the point D let the right line D A be drawn, which ſhall touch the circle in the point A.

Demonſtration THerefore D A a is given by poſition and magnitude; a) 91. p.
and therefore the ſquare of the ſaid D A is b given. But b) 52. p.
the ſaid ſquare of D A is equal c to the rectangle of B D and D C: Therc) 36. 3.
fore the ſaid rectangle of B D and D C is alſo given.

O T H E R W I S E.

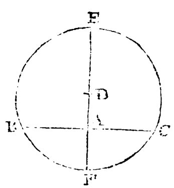


Construction Let E be the center of the circle, and by the same center let there be drawn from the point D the right line D A.

Demonstration Forasmuch as each point D and E is given, the right line D E is [d] given by position and by magnitude. But the circle A B C is given by position and by magnitude: Therefore each point A and F [e] is given, and the point D is also given; and therefore [d] each line A D and F D is given. Wherefore the rectangle of the lines A D and D F is also given. But the said rectangle of A D and D F is equal to the rectangle of D B and D C: Therefore the rectangle of D B and D C is given.

d) 26. p.
e) 25. p.

P R O P. 93.



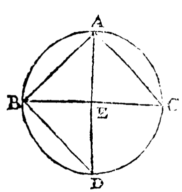
If in a circle given by position there be taken a [illegible] point A, and by that point A [illegible] drawn a right line B C to the [illegible] angle comprised under the [illegible] the same line B C shall be given.

Construction [illegible] let D be taken for the center of the circle, and having drawn the right line A D prolong it to the points E and F.

Demonstration Forasmuch as each point A and D is given, the right line A D [a] is given by position. But the circle B E C is also given by position: Therefore each point E and F is also given by position; and the point A is given. Wherefore each line [b] A E and A F is given: Therefore the rectangle of the same lines A E and A F is given; and is equal to the rectangle [b] of A B and A C: Therefore the said rectangle of A B and A C is given.

a) 26. p.
b) 35. 3.

P R O P. 94.



If in a circle A B C, *given by magnitude, there be drawn a right line* B C, *which doth take away a segment which doth comprehend a given angle* A B C, *and that the said angle being in the segment is cut into two equal parts, the line compounded of the right lines* B A *and* A C, *which comprehend the given angle* B A C *shall have a given reason to the line* A D, *which doth divide that angle into two equal parts; and the rectangle conteined under the line compounded of those lines* B A *and* A C, *comprehending the given angle* B A C, *and that part* E D *of the intersecting line which is below the segment between the base* B C *and the circumference, shall be given.*

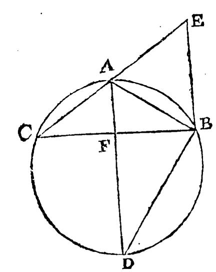
Con-

Construction Let B D be drawn.

Demonstration Forasmuch as in the circle A B C given by magnitude, there is drawn the right line B C, which takes away the segment B A C comprehending the given angle B A C, that line B C [a] is given; and therefore B D is also given: Therefore the reason of B C to B D [b] is given. And seeing that the given angle B A C is cut in two equal parts by the right line A D, as [c] B A is to C A, so is B E to C E; and by compounding, as B A C is to C A, so is B C to C E; and by permutation, as B A C is to B C, so is C A to C E. And seeing that the angle B A E is equal to the angle C A E, and the angle A C E [d] to the angle B D E, the other angle A E C is equal to the other angle A B D; and therefore the triangle A C E is equiangled to the triangle A B D: Therefore [e] as A C is to C E, so is A D to B D. But as A C is to C E, so the line compounded of B A and A C is to B C: Therefore as the compound line B A C is to B C, so is A D to B D; and by permutation, as the compound line B A C is to A D, so is B C to B D. But the reason of B C to B D is given: Therefore the reason of the compound line B A C to A D is also given. Moreover, I say that the rectangle under the compound line B A C and E D is given. For seeing that the triangle A E C is equiangled to the triangle B D E, (for the angle A C E [d] is equal to the angle B D E, and the angle A E C [f] to the angle B E D) as B D is to D E, so is A C to C E. But as A C is to C E, so is also the compound line B A C to B C: Therefore as the compound line B A C is to B C, so is B D to D E. Wherefore the rectangle of the compound line B A C and D E [g] is equal to the rectangle of B C and B D. But the rectangle of B C and B D is given, (for that those lines B C and B D are given:) Therefore the rectangle under the compound line B A C and E D is also given.

a) 88. p.
b) 1. p.
c) 3. 6.
d) 21. 3.
e) 4. 6.
f) 15. 1.
g) 16. 6.

O T H E R W I S E.



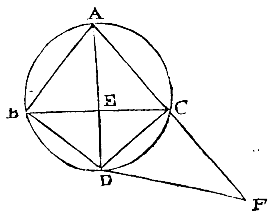
Construction Let C A be prolonged to the point E, and let A E be put equal to B A, and let B E and B D be joyned.

Demonstration Forasmuch as the angle B A C is double to each of the angles C A D and A E B (for the angle B A C is cut into two equal parts by the line A D, and equal [h] to the two angles A B E and A E B, which [i] are equal) the angle A B E is equal to the angle C A D, that is to say, [k] to the angle C B D, adding therefore the common angle A B C, the whole angle A B D shall be equal to the whole angle F B E. But the angle A C B is [k] equal to the angle A D B: Therefore the third angle A E B is equal to the third angle B A D; and therefore the triangle C E B is equiangled to the triangle A B D: Wherefore as C E is to C B, so is A D to B D. But the right line C E is compounded of the two lines C A and A B: Therefore as the compound line B A C is to C B, so is A D to B D; and by permutation, as the compound line B A C is to A D, so is C B to B D. But the reason of C B to B D is given, seeing that each of those lines is given: Therefore the reason of the compound line B A C to A D is also given. And seeing that the triangle C E B is equiangled to the triangle F B D (for the angle A F C is equal [l] to the angle B F D, and the angle E C B [m] to the angle A D B) as E C is to C B, so is B D to D F. But E C is equal to the compound

h) 32. 1.
i) 5. 1.
k) 21. 3.
m) 21. 3.
n) 16. 6.

pound line B A C: Therefore as the compound line B A C is to C B, so is
n) 16.6. B D to D F. Wherefore n the rectangle of the compound line B A C and D F is equal to the rectangle of C B and B D. But the rectangle of C B and B D is given, considering that each of the lines C B and B D is given: Therefore the rectangle of the compound line B A C and D F is given.

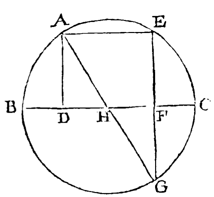
OTHERWISE.



Construction LEt A C be prolonged to F, and let C F be put equal to A B, and let the right lines B D and D F be drawn.

Demonstration FOrasmuch as B A is equal to C F, and o B D
o) 26,29.3. to D C, the two sides A B and B D are equal to the two sides C D and D F, each to his corresponding side, and the angle A B D is equal to the angle D C F,
p) 22.3. 13.1. p seeing that the four sided figure A B D C is within the circle: Therefore the base A D is q equal to the base D F,
q) 4.1. and the angle D A B to the angle D F C. But the angle B A D is given, (being the half of the given angle B A C.) Therefore the angle D F C is so also But D A F is also given: Therefore the triangle A D F is given by kind. Wherefore the reason of F A to A D is given. But A F is the compound of B A and A C, for that C F is equal to A B: Therefore the reason of the compound line B A C to A D is given: The same demonstration will serve to shew that the rectangle conteined under the compound line B A C and E D is given also.

PROP. 95.



If in the diameter B C *of a circle* A B C *given by position, there be taken a given point* D, *and that from that point* D *there be drawn a right line* D A, *to the circumference of the circle. But from the section of the said line there be drawn a right line* A E, *perpendicular thereto, and by the point* E *where that perpendicular doth meet with the circumference, there be drawn a parallel* E F, *to the first line drawn* A D, *that point* F *in which the parallel meets with the diameter, is given; and the rectangle conteined under the parallel lines* A D *and* E F *is also given.*

Construction LEt the right line E F be prolonged to the point G, and let the right line A G be drawn.

Demonstration FOrasmuch as the angle A E G is a right angle, the right line A G is the diameter of the circle. But BC is also the diameter: Therefore the point H is the center of the circle. Now the point D is given; and there-
a) 26.p. fore a the line D H is given by magnitude. But seeing that A D is parallel to EG,
b) 26.p. and A H equal to G H; b D H is equal to F H, and A D to F G; (for the angles
c) 15.1. A H D and F H G c are equal, and D A H and F G H d are also equal.) But the
d) 29.1. line D H is given: Therefore F H is also given. But each of those lines D H and
e) 27.p H F is also given by position, and the point H is given: Therefore e the point F is also given And seeing that in the circle ABC given by position, is taken the given point F, and through the same is drawn the right line E F G; the rectangle under
f) 93.p. E F and F G f is given. But FG is equal to A D. Therefore the rectangle comprehended under A D and E F is given. Which Was to be demonstrated.

THE END OF EUCLIDES DATA.

A BOOK OF THE DIVISIONS OF Superficies:

ASCRIBED TO MACHOMET BAGDEDINE.

Now first put forth, by the pains of JOHN DEE of *LONDON*, and FREDERIC COMMANDINE of *URBIN*.

AS ALSO

A little Book of FREDERIC COMMANDINE, concerning the same matter.

LONDON

Printed by R. & W. LEYBOURN, 1660.

TO THE

Most Illustrious and Most Excellent

Franciscus Maria II:

PRINCE OF

URBIN.

W*Hen* John Dee *of* London, *a man of excellent wit, and singular learning, (Most Illustrious Prince) going away, left with me this little Book of the* Divisions of Superficies, *as a token or testimony of his affection towards me, he added, that nothing could happen more welcome to him from me, than that it might by my means come into the hands of those that are desirous of learning, especially of* the Mathematicks. *Therefore I being moved both with the honest minde and intent of the friendly and most learned man, and also much taken with the marvelous benefit of the Book, for that I knew there was not any such thing extant among us, I have now most willingly endeavoured to satisfie his desire, and as he requested me, have not suffered this Treatise to consist in a Pentagonal division: for what things the Author of the Book hath at large comprehended in many Problems, I have compen-*

 diously

diouſly compriſed and diſpatched in two only; yet ſo, as that from them it may clearly appear, in what ſort thoſe ſections might in other Figures be infinitely produced. In which thing truly, (if I miſtake not) very profitable, I have both obeyed the requeſt of a Friend, and alſo promoted their Studies, who are delighted with this moſt Noble and Excellent kinde of Learning. For it can hardly be conceived, how great a help and Ornament, this attempt will bring to him, that endeavours to be a Geometrician, *ſo as he refuſe not to beſtow good pains therein. Therefore this product of common induſtry (ſuch as it is) I reſolved ſhould come forth adorned with your moſt* Excellent *Name, until I carefully trim up for you ſome more conſiderable teſtimonies of my engagements to you; both for that I have conſecrated all my endeavours to your bounty (to which I am exceedingly obliged) and alſo becauſe I conceived that would be moſt acceptable to* Dr. Dee, *who by the renown of your famous Court was drawn hither, with the pains of a tedious Journey. Farewell, and be pleaſed courteouſly to continue in your defending and cheriſhing of Learning and its Friends.*

Fredericus Commandinus.

TO

TO FREDERIC COMMANDINE OF URBIN.

Iohn Dee of London

Wiſheth Health and all Happineſſe.

HAving now for many years ſet my ſelfe chiefly (my moſt learned *Frederick*) how to preſerve from utter ruine, the moſt famous moniments of our Anceſtors (ſuch as I could) in all the more curious or elegant kinds of Philoſophy; leaſt that either ſo worthy men ſhould be robbed of their due renown, or we longer want the moſt abundant benefit of ſuch Books. I ſay that I ſo beſtowing my pains among other moſt ancient writings of Philoſophers, did at length happen upon this ſmall Book, written, indeed, in a very blinde ilfavoured Character, and alſo by reaſon of its age, hard to be read. But that I might the better ſee, I uſed helps to my ſight: and by often ſtudy and exerciſe therein, I got the knack of reading it. Whereupon being hereby better perſwaded of the excellencie and worth of the Book, I earneſtly wiſhed that the ſame might forthwith be communicated to the Society of Philoſophers. But while I was pondering this in my minde, I found none more worthy than your ſelf (my *Commandine*) in this our age, to enjoy theſe our Labours; who have alſo your own ſelfe revived certain moſt excellent Works of *Archimedes*

and *Ptolemee* almost lost, and have brought them forth into pnblick view in a most magnificent dresse. Therefore this little Book, as a perpetual pledge, even of the affection wherewith I ever embrace you, I commit to you and your trust; and do earnestly beseech you, that you will not suffer this our common labour to go forth into the World destitute of that adornment wherewith you are wont to send abroad others. Yea, I surely hope (if I well know you and your endeavours) that you will some time or other so enrich this subject, as that you will neither permit it to rest in a Pentagonal form; not suffer the *Solids* themselves long to want the like Sections by Plains. Verily, if you would but a little put forward these things, they will by themselves go on to the other kindes of Superficies. But that they may be applyed to *Solids* they will require your sound knowledge, and more than ordinary pains in the Mathematicks. As for the Authors name, I would have you understand, that to the very old Copy from whence I writ it, the name of *MACHOMET BAGDEDINE* was put in Ziphers or Characters, (as they call them) who whether he were that *Albategnus* whom *Copernicus* often cites as a very considerable Author in Astronomie; or that *Machomet* who is said to have been *Alkindus*'s Scholar, and is reported to have written somewhat of the art of Demonstration, I am not yet certain of: or rather that this may be deemed a Book of our *Euclide*, all whose Books were long since turned out of the Greeke into the Syriack and Arabick Tongues. Whereupon, It being found some time or other to want its Title with the *Arabians* or *Syrians*, was easily attributed by the transcribers to that most famous Mathematician among them, *Machomet*: Which I am able to prove by many testimonies, to be often done in many Moniments of the Ancients; and certain friends of mine doe know (that I may bring one of many) that we have by this means restored one small Book, incomparable in occult and mysticall Phylosophie, of that most ancient and excellent Phylosopher *Anaxagoras*, every where row through many ages enobled with the name of *Aristotle* to *Anaxagoras* himselfe, and that by most sure proofs. Yea further, we could not yet perceive so great acutenesse of any *Machomet* in the Mathematicks, from their moniments which we enjoy, as everywhere appears in these Problems. Moreover, that *Euclid* also himselfe wrote one Book περὶ διαιρέσεων, that is to say, *of Divisions*, as may be evidenced from *Proclus*'s Commentaries upon his first of *Elements*: and we know none other extant under this title, nor can we finde any, which for the excellencie of its treatment, may more rightfully or worthily be ascribed to *Euclid*. Finally, I remember that in a certain very ancient piece of Geometry, I have read a place cited out of this little Book in expresse words, even as from amost certain work of *Euclid*. Therefore we have thus briefly declared our opinions for the present, which we desire may carry with them so much weight, as they have truth in them. And if any man shall object to me, that that title of Divisions doth not denote the sections of Magnitudes into their parts, but the divisions of *Genus's* by their differences into *Species*, like as the methodicall divisions of Points, Lines, Angles, Figures, and the like; such as we have in our demonstrated work of Mathematicall *Acribologie* shewed above five huudred: truly I confesse, that this also may in probability be said; but yet how truly, is not more manifest yet to me, than our conjecture is to him. But whatsoever that Book of *Euclid* was concerning Divisions, certainly this is such an one as may be both very profitable for the studies of many, and also bring much honour and renown to every most noble ancient Mathematician; for the most excellent acutenesse of the invention, and the most accurate discussing of all the Cases in each Probleme: and so much for this, I will now direct my discourse to you, who are herein to be greatly intreated by me, that you would with all possible diligence advance your most weighty and usefull labours, which you did yesterday most courteously shew to me in your Study. For so you will make the fairest way to perpetuate your fame, who have in so few years, put forth many Books, so happily, so neatly, and so many of your own: who alone in our time dost adorn the most excellent Princes of Mathematicians, *Archimedes*, *Apollonius*, and *Ptolomeus*, with their due lustre. So you will restore a new and wonderfull livelynesse to Mathematicall Learning much decayed: and so you shall now at last oblige me wholly to your selfe by many engagements. And I intreat you, that so soon as this

this Book is printed, you will be pleased to send a copie or two thereof to the most Noble man, and singular patron of all good Arts, and specially of the Mathematicall, Sir *William Pickering* Knight, my exceeding great friend, living at *London.* For then they will most readily be conveyed to my Library. Now my purpose of Travell calls me away, lest I should be put to undergo a greater trouble of this scorching Season round about us, before I can shelter my selfe in the Roman shades. Farewel, therefore, the honour and renown of Mathematicians; farewel my most courteous *Commandine.* And I very earnestly beseech the great God, that he will vouchsafe to bring your excellent attempts by his special favour, to their desired ends.

URBIN.

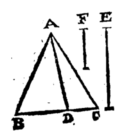
TO THE READER.

I Am here to advertise thee (kinde Reader) that this Author which we present to thee, made use of Euclid *translated into the Arabick Tongue, whom afterwards* Campanus *made to speake Latine. This I thought fit to tell thee, that so in searching or examining the Propositions which are cited by him, thou mightest not sometime or other trouble thy selfe in vain,* FAREWEL.

A BOOK OF THE DIVISION OF SUPERFICIES.

PROP. 1. PROBL. 1.

By a line drawn from an angle of a triangle, to divide that triangle according to a proportion given.



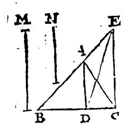
LEt the triangle be A B C, and let it be required to divide the same by a line descending from the angle A in proporiton as E to F.

Divide B C in D in proportion as E to F, and [a] having drawn the line A D, that line A D divides the triangle as is required [b]. a) 12. 6. b) 1. 6.

PROP. 2. PROB. 2.

To divide a triangle given according to a proportion given, by a line drawn from a point assigned in the side of the triangle.

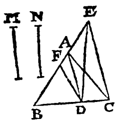
LEt the triangle be A B C, in whose side B C let the point D be assigned, from whence it is required to draw a line dividing the triangle according to the proportion of M to N, draw D A. From that end or extream of B C, towards which I would have the consequent term of the division, which for Examples sake let be the point C. Draw a parallel to D A till it meet in the point E with the line B A extended (that they will meet is manifest [a].) The proportion of M to N will be either equal to the proportion of B A to A E, or greater, or lesser. Let first the proportion be equal, therefore [b] the proportion of the triangle B A D to the triangle A D E, is as the proportion of M to N. But [c] the triangle A D E is equal to the triangle A D C: Therefore [d] the proportion of the triangle A B D to the triangle A D C, is as the proportion of M to N. Which was to be proved. a) 29. 17. 1. b) 1. 6. c) 37. 1. d) 7. 5.



Secondly, Let the proportion of M to N be lesser then the proportion of the line B A to the line A E; therefore divide the line B E according to the proportion of M to N, and let the division fall between B and A [e], it will fall in the point F; and let the line D F be drawn; which line will I e) 8. 5.

say

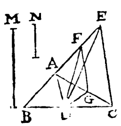
say divide the triangle according to the proportion of M to N.



Demonstr. For having drawn the line DE, the triangle ADE f shall be equal to the triangle ADC. Putting
f) 37. 1. therefore the triangle AFD common to both, the triangle FDE will be equal to the quadrilateral figure AFDC. Seeing that g the triangle BFD is to the triangle
g) 1. 6. FED, as BF is to FE; and by consequence, as M to N; then also the triangle BFD is to the quadrangle AFDC as M to N, and the Proposition is manifest.

Thirdly, Let the proportion of M to N be greater then the proportion of BA to AE. Divide therefore BE in the point F, which will be between A and E, according to the proportion of M to N; and draw FG parallel to CE, until it meet with the line AC in the point G; then joyn GD, I say the line GD will divide the triangle according to the given proportion.

For draw the lines DF and DE, then is the triangle ADE equal to
h) 37. 1. the triangle ADC, and h the triangle ADF is equal to the triangle
i) 37. 1. ADG; therefore i the remaining angles, to wit, the triangle FDE and the triangle GDC are also equal.

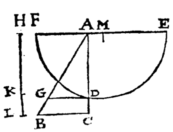


Putting also the triangle ABD common to both the equal triangles AFD and AGD, the triangle BFD will be equal to the quadrilateral figure BAGD. Therefore the triangle FBD is to the triangle FDE, as the quadrilateral figure BAGD is to the triangle GCD. But the triangle FBD is to the triangle FDE, as M is to N, by supposi-
k) 1. 6. tion k. Therefore the proportion of the quadrilateral figure BAGD to the triangle GDC is as the proportion of M to N, as was proposed.

PROP. 3. PROBL. 3.

By a line equidistant to a side assigned of a known triangle, to divide that triangle according to a proportion given.

LEt the given proportion be HK to KL, and the triangle ABC, &c. From the angle A (towards which I would have the antecedent in the proportion sought) protract a line AE perpendicular to CA, and equal to it, and let EA be produced to F, until the proportion of EA to AF be as HL to HK, and making a center in the middest of the line FE, as at M, let the semicircle FDE be described, according to the quantity of the line ME, which semicircle will cut the line AC in the point D; because the line AD is lesse then the line AE, and the line AE is equal to the line AC; drawing then DG parallel to BC: I say that the proportion of the triangle AGD to the superficies GBCD, is according to the proportion of HK to KL.



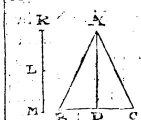
Demonst. For the proportion of the triangle ABC to the triangle
a) 17. 6. AGD, is as AC to AD, a duplicate proportion a. But AC is equal to AE; therefore the proportion of the triangle ABC to the triangle AGD, is as the proportion of AE to AD, duplicate; for the proportion of AE to AD duplicate, is as the proportion of AE to AF b. There-
b) 30. 3. fore

fore the proportion of the triangle ABC to the triangle AGD, is as the proportion of EA to AF. But the proportion of EA to AF is as HL to HK: Therefore the proportion of ABC to AGD is as LH to HK: Therefore severally the proportion of the superficies GBCD to the triangle AGD, is as LK to KH: Therefore contrarily AGD to GBCD, is in proportion as HK to KL. Which was to be proved. 8. 6.

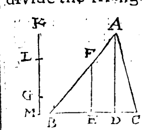
PROP. 4. PROBL. 4.

By a line parallel to a perpendicular line AD, drawn from an angle of a triangle to the base, to divide the triangle according to a proportion given.

LEt the given proportion be KL to LM, and according thereto I would divide the triangle ABC by a line parallel to the perpendicular AD. Divide the line KM according to the proportion of the line BD to DC. As for example, in the first place, Let the division fall in L, the proportion then of KL to LM is as BD to DC; and by consequence, as of the triangle ABD to the triangle ADC a: Therefore a) 1. 6.
the line AD divideth the triangle according to the given proportion.

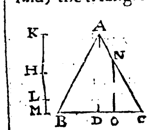


Secondly, Let the proportion of KG to GM, be as the proportion of BD to DC, so as that G be betwixt L and M: I will then divide the triangle ABD accordingly by a line FE, parallel to the side AD; according to the proportion of KL to LG: I say therefore that the proportion of the triangle FBE to the superficies AFEC, is as the proportion KL to LM.



Demonstr. For the proportion of the triangle ADC to the triangle ABD, is as the proportion of MG to GK.
Therefore b joyntly, the proportion of the triangle b) 18. 5.
ABC to the triangle ABD, is as the proportion of MK to KG. But the proportion of the triangle ABD to the triangle FBE, is as the proportion of KG to KL: Therefore according to equal proportionality, c the c) 22. 5.
proportion of the triangle ABC to the triangle FBE, will be as the proportion of MK to KL: Therefore severally the proportion of the superficies AFEC to the triangle FBE, is as the proportion of ML to KL: Therefore on the contrary, the proportion of KL to LM, is as the proportion of the triangle FBE to the superficies AFEC. Which was to be proved.

Thirdly, Let the proportion of KH to HM, be as the proportion of BD to DC, and let H fall betwixt K and L, then divide (by the aforesaid) the triangle ADC according to the proportion of HL to LM, by the line NO parallel to the side AD: I say then that the proportion of the superficies NABO to the triangle NOC, is as the proportion of KL to LM.



Demonst. For the proportion of the triangle ABD to
the triangle ADC, is as KH to HM d. Therefore d) 1. 6. & 11. 5.
joyntly, e the proportion of the triangle ABC to the e) 18. 5.
triangle ADC is as the proportion of KM to HM. But the proportion of the triangle ADC to the triangle NOC, is as the

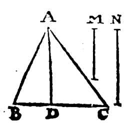
proportion of H M to L M: Therefore according to equal proportionality, the proportion of the triangle A B C to the triangle N O C, is as the proportion of K M to L M: Therefore severally the proportion of the superficies N A B O to the triangle N O C, is as the proportion of K L to L M, which was required.

PROP. 5. PROBL. 5.

To divide a known triangle according to a proportion given by a line parallel to a line drawn from an angle thereof, which is neither parallel to any of the sides, nor any of its perpendiculars.

THis conclusion might be proved as the former: It may also be thus otherwise proved.

First, Let the proportion given be as M to N, and let the triangle be A B C, which I would divide according to the proportion of M to N, by a line parallel to the line A D, which let fall from the angle A, and be neither perpendicular nor parallel to any side of the triangle: Therefore I divide B C according to the proportion of M to N; and let the division fall for example sake in D: I say that A D [a] doth divide the triangle according to the proportion of M to N.

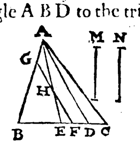


a) 1. 6.

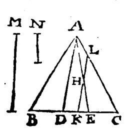
Secondly, Let the division fall twixt B and D, in the point E, so as that the proportion of B E to E C be as the proportion of M to N: Then I put the line B F a mean proportional between B D and B E, and drawing F G parallel to A D: I say that line divideth the triangle according to the Proposition.

Demonstr. I draw the line A E: Therefore the proportion of the triangle A B D to the triangle G B F, is as B D to B F, a double proportion [b]; and therefore is as the proportion of B D to B E. But according to the proportion of B D to B E, is the proportion of the triangle A B D to the triangle A B E: Therefore the proportion is the same of the triangle A B D to the triangle G B F, and to the triangle A B E. Therefore the triangles G B F and A B E are equal: Therefore placing H in the section of the lines A E and G F; it is manifest that the triangles A G H and E F H are equal, to which adding the superficies A H F C, the triangle A E C will be equal to the superficies A G F C; there is therefore the same proportion of the triangle A B E to the triangle A E C, as of the triangle B F G to the superficies A G F C. But the proportion of the triangle A B E to the triangle A E C is as the proportion of M to N given: Therefore the Proposition is manifest.

b) 17. 6.



Let thirdly, the division fall between D and C in the point E, so as that the proportion of B E to E C may be as M to N: I will then put C K a mean proportional between D C and E C: Then drawing K L parallel to A D: I say K L divides the triangle as was required.



For as before, the proportion of the triangle A D C to the triangle L K C, is as the proportion of D C to K C doubled;

doubled; and by consequence, is as the proportion of D C to E C; and according to the same proportion is the triangle A D C to A E C: Therefore the triangles L K C and A E C are equal. Wherefore also the triangles A H L and K H E are equal: Therefore the superfices L A B K is equal to the triangle A B E: Therefore there is the same proportion of the superficies L A B K to the triangle L K C, as of the triangle A B E to the triangle A E C. But that proportion is as M to N: Therefore the proposition is cleer.

NOTE that in this manner the former conclusion may be proved; and this proof is easier then the former.

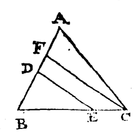
PROP. 6. PROBL. 6.

To divide a known triangle A B C, *according to a proportion given, by a line parallel to any line drawn therein, whether the said line be drawn from an angle or not.*

FIrst, if the line given be parallel to any side of the triangle, then work according to the third Probl. of this Book.

But secondly, if the given line descend from any angle, the Proposition will be had by the aforesaid.

But thirdly, if the assigned line neither descend from any angle of the triangle, nor is parallel to any side, as in the triangle A B C, let the line D E be assigned, which is not parallel to A C; but would meet with it on that side that C is, if it were drawn out at length, then from the angle on that side where that meeting would be, as here from the angle C, let C F be drawn parallel to the line assigned, *viz.* D E: Then (by the aforesaid) let the triangle be divided by a line parallel to C F, according to the proportion given: It is manifest [a] that it is then divided by a line parallel to D E, and so the Proposition is manifest.

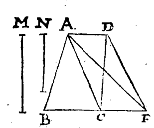


a) 30. 1.

PROP. 7. PROBL. 7.

To divide a known Quadrangle A B C D, *by a line drawn from an angle thereof* A, *according to a proportion given, as* M *to* N.

PRotract the diameter A C, and from the point D protract D F, parallel to A C, till it meet with B C in the point F: Then divide B F as M to N; and let (in the first place) the division fall in C, so that there be the same proportion of B C to C F, as of M to N: I say that A C performes the Proposition.



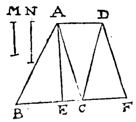
Demonstr. For the triangle A D C is equal to the triangle A F C [a]. But the proportion of the triangle A B C to the triangle A C F is as M to N [b]. Therefore the proportion of the triangle A B C to the triangle A C D, is as the proportion of M to N.

a) 37. 1.
b) 1. 6.

Secondly, Let the division fall in E, between B and C, so as the proportion of B E to E F be as M to N: Then drawing A E, I say the proportion of the triangle A B E to the supeficies A E C D, is as M to N.

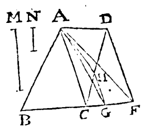
De-

Demonstr. Protracting A F, the triangle A D C will be equal to the trian-
c) 37. 1. gle A F C [c]; and putting the triangle A E C common to both, the superficies A E C D will be equal to the triangle A E F: Therefore there is the same proportion of the triangle A B E to the superficies A E C D,
d) 1. 6. and to the triangle A E F. Since therefore [d] the proportion of the triangle A B E to the triangle A E F, is as M to N; 'tis manifest that the proportion of the triangle A B E to the superficies A E C D, is as M to N. Which was to be proved.



Thirdly, Let the division fall betwixt C and F in G, so as that B G be to G F, as M to N: Draw G H parallel to D F, till it meet with D C in H: Then draw A H, so is the proportion of the superficies A B C H to the triangle A D H, as M to N.

Demonstr. Let A G be produced: Therefore the triangle A H C is equal to the triangle A G C. But also the whole triangle A D C is equal to the whole triangle A F C: Therefore the residue A D H is equal to the residue A F G. Putting therefore the triangle A B C common to both, the triangles A C H and A C G shall be equal; and the superficies A B C H will be equal to the triangle A B G, the proportion therefore of the superficies A B C H to the triangle A D H, is as the proportion of the triangle A B G to the triangle A G F. But the proportion of the triangle A B G to the triangle A G F, is as M to N, as hath been proved.

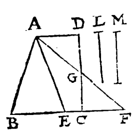


PROP. 8. PROBL. 8.

To divide a known quadrangle A B C D, *having two parallel sides* A D *and* B C, *by a line drawn from a point assigned* E, *in one of the parallel sides, according to a proportion given, viz. as* L *to* M.

Extend B C to F, so that C F be equal to A D, and let A F cut the line
a) 37. 1. D C in the point G; then [a] the triangles A D G and G C F are alike, and the sides A D and C F are equal; and therefore those triangles are equal one to another. Adding therefore A B C G, which is common to both, it is manifest that the Quadrangle A B C D is equal to the triangle A B F.

Then divide B F according to the proportion of L to M, and let the division fall in E; so as that the proportion of B E to B F, may be as L to M: Then draw E A, which performeth the Proposition.

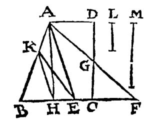


Demonstr. For by reason of the equality of the triangles A D G and C G F, the superficies A E C D is equal to the triangle A E F: Therefore there is the same proportion of the triangle A B E to the superficies A E C D, and to the triangle A E F: But the proportion of A B E to A E F, is as the proportion of L to M; and therefore the proportion of A B E to the residue of the Quadrangle, is as the proportion of L to M. Which was proposed.

Secondly, Let the division fall between B and E in H, so as that B H be to B F, as L to M: Then draw H K parallel to A E, dividing A B in K: Then

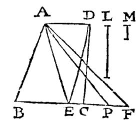
Then draw K E: I say that line K E divides the Quadrangle according to the Proposition.

Demonstr. For I will draw A H, and because A E and K H are parallels, the triangles K A H and K E H shall be equal: Therefore adding K B H to them both, the triangle A B H will be equal to the triangle K B E. But the triangle A K E is equal to the triangle A H E; Therefore adding A E C D common to both, the superficies A K E C D will be equal to the Quadrangle A H C D. But the Quadrangle A H C D is equal to the triangle A H F, as is before shewn: Therefore there is the same proportion of the triangle K B E to the superficies A K E C D, as of the triangle A B H to the triangle A H F, and by consequence as of L to M. Which was to be proved.



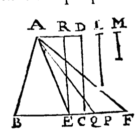
Thirdly, Let the division fall between E and F, and having made the figure, I will cut off from E F a line E P, equal to D A: Then I will divide the line B F in proportion as L to M.

And first of all, let the division fall in P, so as that the proportion of B P be to P F as L to M. Draw E D, which I say doth divide the Quadrangle according to the Proposition.



Demonstr. For draw the line P A, and because E P is equal to A D, and parallel unto it, the triangle A D E will be equal to the triangle A P E. Putting therefore the triangle A B E common, the Quadrangle A B E D will be equal to the triangle A B P; and by consequence, the remaining triangle D E C will be equal to the remaining triangle A P F; because (as hath been before proved) the Quadrangle A B C D is equal to the triangle A B F: Therefore the same proportion is of the Quadrangle A B E D to the trian-
gle D E C, as of the triangle A B P to the triangle A P F [b]. But the pro- b) 19. 5.
portion of A B P to A P F is as L to M: Therefore the proportion of A B E D to D E C, is as L to M. Which was to be proved.

Secondly, Let the division fall between E and P in the point Q, so as that the proportion of B Q to Q F may be as L to M. From A D cut off A K, equal to E Q: Then draw E R, I say that that line divides the Quadrangle, according to the Proposition.

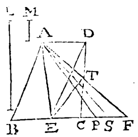


Demonstr. Draw A Q, and because the lines A R and E Q are equal and parallels, the triangles A R E and A Q E are equal, to which adde the triangle A B E common, the Quadrangle A B E R will be equal to the triangle A B Q. But it is proved that the whole Quadrangle A B C D was equal to the triangle A B F: Therefore the remaining Quadrangle R E C D is equal to the remaining triangle A Q F: Therefore there is the same proportion of the Quadrangle A B E R to the Quadrangle R E C D, as of the triangle A B Q to the triangle A Q F; and by consequence as L to M. Which was proposed.

Thirdly, Let the division fall between P and F in S, so as that the proportion of B S to S F, may be as L to M. Divide D C in T, according to the proportion of P S to S F; and draw E T: I say that that line divideth the Quadrangle according to the Proposition.

Demonstr. Draw A S, because therefore the lines A D and E P are equal and

and parallels, the triangles ADE and APE are equal, and by consequence, adding the common triangle ABE, the Quadrangle ABED is equal to the triangle ABP. But the whole Quadrangle ABCD is equal to the whole triangle ABF: Therefore the triangle DEC is equal to the triangle PAF. But the proportion of the triangle DET to the triangle TEC, is as PAS to SAF: Therefore the triangle DET is equal to the triangle PAS, and the triangle TEC equal to the triangle SAF. It was proved that the Quadrangle ABED is equal to the triangle ABP.



Adding the triangle DET to the first, and the triangle PAS equal to it, to the second, the Pentagon ABETD will be equal to the triangle ABS, But it was proved that the triangles TEC and SAF are equal: Therefore there is the same proportion of the Pentagon ABETD to the triangle TEC, as of the triangle ABS to the triangle ASF; and by consequence, as L to M. Which was proposed.

PROP. 9. PROB. 9.

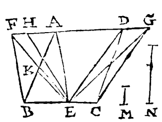
To divide any known Quadrangle ABCD, *according to any proportion given* M *to* N, *by a line drawn from a point* E, *in one of the sides assigned* BC, *not parallel.*

Draw AE and ED, and extend DA at length on both sides, then draw BF parallel to EA, to cut DA extended in F. Likewise draw DG parallel to ED; cutting DA extended in G, then divide FG in protion as M to N.

And first let the division fall between F and A in the point H, so as that the proportion of FA to HG, may be as M to N.

Divide also BA in proportion as FH to HA, and let the division be in the point K, so as that the proportion of BK to KA, may be as FH to to HA: Then drawing KE, you have divided the Quadrangle as was required.

Demonst. For having drawn EF and EG, the triangle AFE will be equal to the triangle ABE,

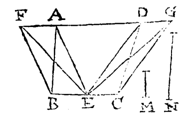


a) 37. 1. and the triangle DGE equal to the triangle DCE. Then adding to both, the triangle AED; the triangle FEG will be equal to the Quadrangle proposed ABCD. And because the triangle AFE is equal to the triangle ABE; and that there is the same

b) 1. 6. proportion of FH to HA, as of BK to KA: Therefore [b] the triangle EHF is equal to the triangle EKB: Therefore also the residue is equal to the residue: Therefore the remaining triangle HEG is equal to the Pentagon AKECD. There is therefore the same proportion of the triangle EKB to the Pentagon AKECD, as of the triangle EHF to the triangle EGH; and therefore also as the line FH to the line HG; and by consequence as M to N. Which was to be proved.

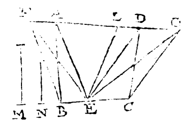
Secondly, Let the division fall in the point A, so as that the proportion of FA to AG may be as M to N, then drawing EA: I say that that line divideth the Quadrangle according to the Proposition.

Demonstr. For the triangle AFE is equal to the triangle ABE: Therefore the triangle AEG the residue, is equal to the Quadrangle AE CD



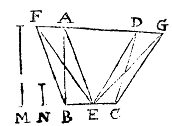
CD the residue: Therefore there is the same proportion of the triangle ABE to the Quadrangle AECD, as of the triangle AFE to the triaegle AEG. Therefore as the line FA to the line AG; and by consequence, as M to N: As hath been provd.

Thirdly, Let the division fall bewteen A and D in the point L, so as that the proportion of FL to LG, may be as M to N: Then I say that the line EL divides the Quadrangle, as is required.

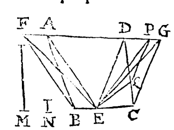


Demonstr. For seeing that the triangles AFE and ABE are equal; and that if to both be added the triangle LEA: Then the triangle LFE will be equal to the Quadrangle ABEL: Therefore the triangle LEG the residue, is equal to the Quadrangle LECD, also the residue: Therefore the Quadrangle ABEL is in proportion to the Quadrangle LECD, as the triangle LFE to the triangle LEG; and by consequence as M to N, as hath been proved.

Fourthly, Let the division fall in the point D: Then because the triangle DGF is equal to the triangle DCE, the triangle remaining DFE, will be equal to the remaining Quadrangle DABE: Therefore the proportion of the Quadrangle ABED, to the triangle DEC, is as the triangle DFE to the triangle DFG; and therefore as the line FD to the line DG; and by consequence as M to N: Therefore DE divides the Quadrangle, as is required.



Fifthly, Let the division fall between D and G, in the point P; so as that the proportion of FP to PG, may be as M to N: From P draw a line PQ, parallel to CG, until it meet with CD in Q; then having drawn EQ, the Quadrangle is divided as is required.



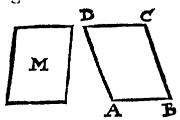
Demonstr. For draw PE, then the triangle DEP will be equal to the triangle DEQ [c]: Putting therefore the triangle AED common, the triangle will be equal to the Quadrangle AEQD; c) 37. 1. also the triangle AFE will be equal to the triangle ABE: Therefore the triangle FEP is equal to the Pentagon ABEGD: Therefore the triangle PEG residue, is equal to the triangle QEC, also residue: Therefore the proportion of the Pentagon ABEQD, to the triangle QEC, is as the triangle FEP to the triangle FEG: Therefore as FP to PG; and by consequence as M to N, as was proposed.

PROP. 10. PROBL. 10.

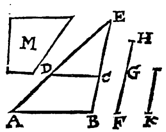
On a known line AB, *and two lines* AD *and* BC, *drawn at any angle at pleasure with it, and on the same side: To make a superficies* ABDC *equal to a superficies given* M, *so that the said superficies shall be included between the lines* AD *and* BC, *and between* AB *and a line* DC, *parallel to* AB.

The

THe two angles D A B and C B A are either equal to two right angles, or greater or lesser: Let them in the first place be equal to two right
a) 44. 1. angles: Therefore A D will be parallel to B C. Then [a] upon A B make a superficies of parallel sides, whose angle let be equal to the angles D A B and C B A; and let the superficies be equal to the superficies M; and the Proposition is manifest.

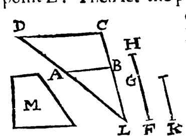


Secondly, Let the two angles D A B and C B A be lesse then two right angles: Therefore A D and B C will meet on that side that C D is, to wit, in the point E. Unlesse therefore the triangle E A B shall be found greater then the superficies M: It is not possible on that side that D C is to constitute the superficies required. Let therefore the triangle E A B be greater then the superficies M; and let the proportion of the triangle E A B to the superficies M, be as F H to F G; and let K be a mean proportional between F H and G H. Then cut off from E B the line E C, which let be in proportion to E B, as K is to F H: Then drawing C D parallel to B A: I say that the superficies A B C D is equal to the superficies M.



b) 17. 6. *Demonstr.* For [b] the triangle B A E is in proportion to the triangle C D E, as B E to C E doubled; and therefore the proportion of F H to K doubled; and by consequence, the proportion of the triangle B A E to the triangle C D E, is as F H to G H: Therefore the everse proportion of the triangle B A E is to the Quadrangle B A D C, as F H is to F G. But F H is in proportion to F G, as the triangle B A E to the superficies M. Therefore there is the same proportion of the triangle B A E to the superficies M, and also to the Quadrangle B A D C: Wherefore the superficies M and the Quadrangle B A C D, are equal. Which is the thing desired.

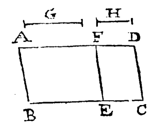
Thirdly, Let the angles D A B and C B A be greater then two right angles, they will then meet on that side that A B is; and let it be in the point E: Then let the proportion of G H to G F be put as the proportion of the triangle A B E to the superficies M; and let K be a mean proportional between F H and G H; and let the proportion of E C to E B, be as F H to K: Then drawing C D parallel to A B: I say the superficies M is equal to the quadrangle A B C D.



Demonstr. For the proportion of the triangle C D E to the triangle B A E is (as is before shewn) as the proportion of F H to G H: Therefore the everse proportion of the triangle C D E is to the quadrangle C D A B as F H to F G: Therefore the disjunct proportion of the triangle A B E is in proportion to the quadrangle A B C D, as G H to G F; and by consequence, as the same triangle A B E to the superficies M: Therefore the quadrangle A B C D, and the superficies M are equal. Which was to be demonstrated.

PROP. 11. PROBL. 11.

To divide a Quadrangle A B C D, *of parallel sides, by a line* F E, *parallel to one of its sides* A B, *according to the proportion of* G *to* H.



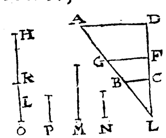
DIvide the line B C in the point E, according to the proportion of G to H; and draw E F parallel to A B; and the Proposition is done.

Demonstr. For [a] there is the same proportion of the quadrangle A B E F to the quadrangle F E C D, as a) 1. 6.
of the line B E to the line E C; and by consequence, as of G to H. As was required.

PROP. 12. PROBL. 12.

To divide a Quadrangle A B C D, *having only two sides parallel,* A D *and* B C, *by a line* G F, *parallel to* A D *and* B C, *according to a given proportion,* M *to* N.

FIrst, The sides A B and D C, must necessarily meet in the point E, and let the proportion of H O to L O, be as the triangle D A E to the triangle C B E: Therefore by conversion and division, the proportion of the triangle C B E will be to the quadrangle D A B C, as L O to L H. Then divide K L in K, so as that the proportion of H K to K L may be as M to N; and let P be a mean proportional between K O and O L; and let the proportion of F E to C E, be as K O to P: Then draw F G parallel to D A. I say that line doth divide the quadrangle, as was required.



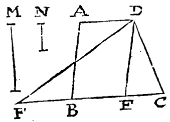
Demonstr. For the proportion of the triangle F G E to the triangle C B E, is as F E to C E double proportion: Therefore also as K O to P double proportion. And by consequence, the proportion of the triangle F G E to the triangle C B E, is as K O to L O: Therefore the everse proportion of the quadrangle F G B C is in proportion to the triangle C B E, as K L to L O. But the triangle C B E is in proportion to the quadrangle A B C D (as is afore shewn) as L O to L H: Therefore by equal proportionality, the quadrangle F G B C is in proportion to the quadrangle A B C D, as K L to L H: Therefore the disjunct proportion of the quadrangle F G B C is in proportion to the quadrangle A G F D, as K L is to K H: Therefore contrarily, the quadrangle A G F D is to the quadrangle G B C F, as K H to K L; and by consequence, as M to N. Which was proposed.

PROP. 13. PROBL. 13.

To divide a Quadrangle A B C D, *having only two sides parallel,* A D *and* B C, *by a line parallel to one side, not parallel* A B *according to any proportion, as* M *to* N.

FRom one of the angles C or D, draw a line D E within the quadrangle, parallel to A B: Then extend E B at length to F; until E F be equal to B E, and divide B F in proportion as M to N.

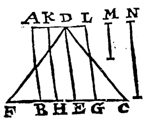
And in the first place, Let the division fall in the point E, so as that the proportion of F E to E C may be as M to N: I say then that the line D E divides the quadrangle, as was required. Mmmm De-



Demonstr. For draw D F, then the proportion of the triangle F D E to the triangle E D C, is as F E to E C; and therefore as M to N. But [a] the quadrangle A B E D is equal to the triangle F D E: Therefore the quadrangle A B E D is in proportion to the triangle D E C, as M is to N. Which was proposed.

a) 1.6. 41. 1.

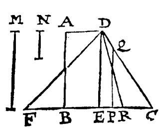
Secondly, Let the division fall betwixt F and E, so as that the proportion of F E to E C be greater then the proportion of M to N: Therefore E C being equally divided in G, the proportion of B E to E G will be greater then that of M to N; by reason that B E is the halfe of F E, and E G the half of E C. Therefore dividing B G in proportion as M to N: Let the division fall betwixt B and E in the point H, so as that the proportion of B H to H G, may be as M to N. Then drawing the line H K parallel to B A: I say the said line H K doth divide the quadrangle, as is required.



Demonstr. For let A D be produced to L, till it meet with G L, parallel to D E. Because therefore E C is double to E G, the quadrangle D E G L will be equal to the triangle D E C. Adding therefore to both of them the quadrangle K H E D; then the quadrangle K H G L will be equal to the quadrangle K H C D. Therefore the quadrangle A B H K is in the same proportion to the quadrangle K H G L, and also to the quadrangle K H C D. But the proportion of the quadrangle A B H K, is to the quadrangle K H G L, as B H to H G; and by consequence, as M to N. Therefore the proportion of the quadrangle A B H K is to the quadrangle K H C D as M to N. Which was proposed.

Thirdly, Let the division fall between F and C in the point R, so as that the proportion of F R to F C, may be as M to N: Draw D R, and [b] divide the triangle D E C, by the line P Q, according to the proportion of the triangle D E R to the triangle D E C; and let P Q be parallel to D E; and let the quadrangle D E P Q be equal to the triangle D E R; and also the triangle Q P C equal to the triangle D R C: Then I say that the line P Q divideth the quadrangle A B C D, as was required.

b) 3. of this.



Demonst. For the triangle F D R is in proportion to the triangle R D C, as M is to N. But the quadrangle A B E D is equal to the triangle F D E, and the quadrangle D E P Q equal to the triangle D E R: Therefore the Pentagon A B P Q D is equal to the triangle F D R. But also the triangle D R C is equal to the triangle Q P C: Therefore the Pentagon A B Q P D is in proportion to the triangle Q P C, as the triangle F D R is to the triangle D R C; and by consequence, as M to N. Which was proposed.

In like manner must you work by a line parallel to the side D C, and so the whole which was proposed, will be manifest.

PROP. 14. PROBL. 14.

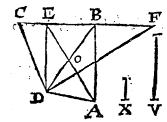
To divide a Quadrangle A B C D, *having one of his sides parallel, according to a proportion given*, V *to* X, *by a line* D E, *parallel to one of its sides* A B.

Draw

DRaw from one of his angles C or D, a line D E, within the quadrangle, parallel to A B: Then draw the lines E A and B D, cutting each other in the point O. Extend C B at length to F, until the proportion of F B to B E be as A O to O E; and draw F D, then divide F C as V to X.

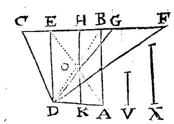
And first of all, Let the division fall in E, so as that the proportion of F E to E C, be as V to X: I say D E divideth the quadrangle, as is required.

Demonstr. For the triangle A D O is in proportion to the triangle O D E, as A O to O E; also the triangle A B O is in proportion to the triangle O B E as A O to O E: Therefore as the aggregand proportion of the triangle B A D is to the triangle B E D, so is A O to O E; and by consequence, as F B to B E; and according to the same proportion, the triangle F D B to the triangle B E D. Therefore the triangle B A D is equal to the triangle F B D: Therefore adding the triangle B D E common to both, the triangle F D E will be equal to the quadrangle A B E D. But the triangle F D E is to the triangle E D C as F E to E C; and by consequence, as V to X: Therefore the quadrangle A B E D is to the triangle E D C, as V is to X. Which was proposed.



Secondly, Let the division fall between F and E (whether within or without the quadrangle, it matters not) as in the point G. Let F G be to G C as V to X, and draw G D: Therefore the triangle F G D to the triangle G D C, will be as V to X. Joyn therefore [a] to the line A B a superficies, equal to the triangle F D G, which may be conteined by the two angles A B C and B A D. Then separating them by the line H K, parallel to A B: I say that the line H K divideth the quadrangle, as was required.

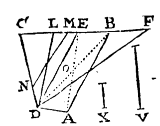
a) 10. prop.



Demonstr. For it will passe within the quadrangle A B E D, because the triangle F D E is equal to the quadrangle A B E D; and the triangle F D G is lesse then the triangle F D E. Since therefore the triangle F D E is equal to the quadrangle A B E D, and the triangle F D G equal to the quadrangle A B H K, the triangle G D E must also be equal to the quadrangle K H E D. Putting therefore the triangle E D C common to both, the triangle G D C will be equal to the quadrangle K H C D: Therefore the quadrangle A B H K is to the quadrangle H K C D, as the triangle F G D is to the triangle G D C; and by consequence, as V to X. Which was proposed.

Thirdly, Let the division fall between E and C, in the point L, so as that F L may be to L C, as V to X: Therefore the triangle F D L is to the triangle L D C as V to X: Then [b] cut off from the triangle D E C a triangle like to it, but equal to the triangle L D C, by the line M N, parallel to E D: I say that line, to wit, M N, divideth the quadrangle, as was required.

b) 3. prop.



Demonstr. For the triangle F D E is equal to the quadrangle A B E D, and the triangle E D L is equal to the quadrangle D E M N; because the triangle M N C is equal to the triangle L D C: Therefore the Pentagon A B M N D is equal to the tri-

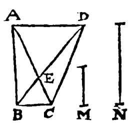
angle

angle F D L: Therefore the Pentagon A B M N D is to the triangle M N C, as the triangle F D L is to the triangle L D C; and by conſequence as V to X. Which was propoſed.

¶ As the quadrangle is divided according to a proportion given by a line parallel to its ſide A B, ſo may it alſo divided by a line parallel to any other of its ſides, which ſoever, and the Propoſition is manifeſt.

PROP. 15. PROBL. 15.

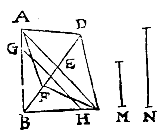
To divide any Quadrangle A B C D, *by a line parallel to one of his diameters* A C, *according to any proportion given,* M *to* N.



DRaw B D, cutting C A in the point E, and divide B D in proportion as M to N. And firſt, Let the diviſion fall in the point E, ſo as that E B may be to E D, as M to N: I ſay then that the diameter A C doth divide the quadrangle A B C D in the proportion required.

Demonſt. For A B E is to A E D as B E is to E D; likewiſe B E C to E D C is as B E to E D, and by conſequence as M to N.

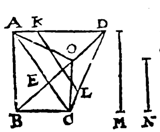
Secondly, Let the diviſion fall between B and E, in the point F; ſo as that the proportion of B F may be to F D as M to N. Then draw the lines F A and F C, and there will be the ſame proportion of the two triangles A B F and C B F joyntly, to the quadrangle A F C D, as of B F to F D. Then from the triangle A B C take the triangle G B H, a) like unto it, and equal to the two triangles A B F and C B F joyntly, by the line G H parallel to A C, which line G H doth divide the quadrangle, as is required.



a) 3. prop.

Demonſtr. For the triangle G B H is equal to the ſuperficies A B C F, the triangle A F C will be equal to the quadrangle A G H E: Therefore adding A D C common to both, the quadrangle A F C D will be equal to the Pentagon A G H C D, the proportion therefore of the triangle G B H to the Pentagon A G H C D, is as the ſuperficies A B C F to the quadrangle A F C D; and by conſequence as M to N. Which was propoſed.

Thirdly, Let the diviſion fall between E and D, in the point O, ſo as that B O may be to O D as M to N. Draw O A and A C: There will be the ſame proportion of the quadrangle A B C O to the ſuperficies A O C D, as of B O to O D; and by conſequence, as M to N: Therefore b) from the triangle A C D cut off the triangle K L D, like to it, but equal to the ſuperficies A O C D, by K L, parallel to A C: I ſay the line K L divideth the quadrangle, as was required.



b) 3. prop. of this Book.

Demonſtr. For the triangle A O C is equal to the quadrangle A C L K: Therefore the quadrangle A B C O is equal to the Pentagon A B C L K, and the triangle K L D equal to the ſuperficies A O C D: Therefore the proportion of the Pentagon A B C L K to the triangle K L D, is as the quadrangle A B C O to the ſuperficies A O C D; and by conſequence as M to N. Which was propoſed.

In

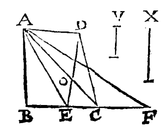
In like manner may the quadrangle A B C D be divided according to a proportion given, by a line parallel to the diameter B D.

PROP. 16. PROBL. 16.

To divide any quadrangle A B C D, *by a line parallel to a line aſſigned* A E, *within the quadrangle, which is neither parallel to any of its ſides, nor either of his diameters according to any proportion given, as* V *to* X.

DRaw the diameters A C and E D, cutting each other in O, produce B C at length to F, until there be the ſame proportion of E C to C F, as of E O to O D; and draw A F: then divide B F in proportion as V to X.

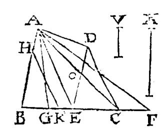
And in the firſt place, Let the diviſion fall in the point E, ſo as that B E may be to E F as V to X: I ſay that line divideth the quadrangle, as is required.



Demonſtr. For the proportion of the triangle A E C to the triangle A C D, is as E O to O D: Therefore as E C to C F; and by conſequence, as the triangle A E C to the triangle A C F: Therefore the triangles A C F and A C D are equal: Therefore the whole quadrangle A E C D is equal to the whole triangle A E F: There is therefore the ſame proportion of the triangle A B E to the quadrangle A E C D, as to the triangle A E F. But the proportion of the triangle A B E to the triangle A E F, is as V to X: Therefore the proportion of the triangle A B E to the quadrangle A E C D, is as V to X. Which was propoſed.

Secondly, Let the diviſion fall between B and E, in the point G, ſo as that B G may be to G F as V to X. Then draw A G, and a) cut off from the triangle A B E the triangle H B K, like unto it, and equal to the triangle A B G, by the line H K, parallel to A E, then will the ſaid line H K divide the quadrangle, as is required.

a) 3. prop. of this.

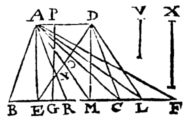


Demonſtr. For the quadrangle A H K E, the reſidue of the triangle A B E, will be equal to the triangle A G E, the reſidue of the ſame triangle A B E. But the quadrangle A E C D is equal to the triangle A E F: Therefore the Pentagon A H K C D is equal to the triangle A G F: Therefore the ſame proportion is of the triangle H B K to the Pentagon A H K C D; as of the triangle A B G to the triangle A G F: Therefore as of B G to G F, and by conſequence as of V to X. Which was required.

Thirdly, Let the diviſion fall between E and F: Therefore becauſe A E is not parallel to C D, draw from one of the two angles D or C, the line D M within the quadrangle, and parallel to A E, then draw A M cutting E D in N; then making L M to M E as D N to N E, [this may preſently be done by drawing D L parallel to A M:] Let therefore L fall on this ſide F, ſo as that D F if it ſhould be drawn, would be parallel to A C. If that be drawn (as here it is) then draw A L. Then the triangle A E L will be equal to the quadrangle A E M D: Therefore divide B F in proportion

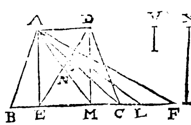
portion as V to X; and now let the diviſion fall between E and L in the point R, ſo as that B R may be to R F, as V to X; then [b] draw P Q parallel to A E, ſo as that the ſuperficies A E Q P may be equal to the triangle A E R. And becauſe the triangle A E L is greater then the triangle A E R, and the triangle A E L is equal to the quadrangle A E M D: Therefore will the quadrangle A E Q P be leſſe then the quadrangle A E M D: I ſay therefore that the line P Q doth divide the quadrangle A B C D. As was required.

b) 10. prop. of this.



Demonſtr. For the quadrangle A E C D is equal to the triangle A E F, and the quadrangle A E Q P is equal to the triangle A E R: Therefore the quadrangle P Q C D the reſidue, is equal to the triangle A R F the reſidue: In like manner, becauſe the quadrangle A E Q P is equal to the triangle A E R, putting the triangle A B E common to both, the quadrangle A B Q P will be equal to the triangle A B R: Therefore the quadrangle A B Q P is to the quadrangle P Q C D, as the triangle A B R to the triangle A R F: Therefore as B R to R; and by conſequence as V to X. Which was propoſed.

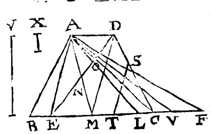
Fourthly, Let the diviſion fall in L, ſo as that B L may be to L F, as V to X: I ſay D M doth divide the quadrangle, as is required.



Demonſtr. For the triangle A E F is equal to the quadrangle A E C D, and the triangle A E L is equal to the quadrangle AEMD: Therefore the triangle A L F reſidue, is equal to the triangle D M C reſidue. In like manner, becauſe the quadrangle A E M D is equal to the triangle A E L; and putting the triangle A B E common, the quadrangle A B M D will be equal to the triangle A B L: Therefore the quadrangle A B M D is to the triangle D M C, as the triangle A B L is to the triangle A L F; and by conſequence as V to X. Which was propoſed.

Fifthly, Let the diviſion fall between L and F, in the point V, ſo as that B V may be to V F as V to X. Draw A V: Therefore becauſe the triangle D M C is equal to the triangle A L F, and A L F is greater then the triangle A V F: The triangle D M C will be greater then the triangle A V F: Therefore from the triangle D M C [c] ſeperate the triangle S T C like unto it, and equal to the triangle A V F, by the line S T, parallel to D M. I ſay therefore that that line S T doth divide the quadrangle, as is required.

c) 30. of this Book.



Demonſtr. For the triangle D M C is equal to the triangle A L F; alſo the triangle S T C is equal to the triangle A V F, the quadrangle D M T S being the reſidue, will be equal to the triangle A L V, alſo the reſidue: Therefore ſince the quadrangle A B M D is equal to the triangle A B L, the Pentagon A B T S D will be equal to the triangle A B V: Therefore the Pentagon A B T S D is to the triangle S T C, as the triangle A B V to the triangle A V F; and by conſequence as V to X. Which was to be demonſtrated.

¶ Note

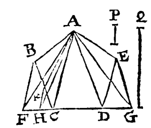
¶ Note that as the quadrangle is divided by a line parallel to a line drawn from an angle thereof, and neither parallel to its ſides nor diameters; ſo may it alſo be divided by a line parallel to a line not drawn from an angle aſſigned, as by drawing a line from any angle of the quadrangle falling within the quadrangle and parallel to a line aſſigned, then the operation muſt be as we have now ſhewn.

PROP. 17. PROB. 17.

To divide a known Pentagon A B C D E *by a line* A K, *drawn from any of its angles as* A, *according to a proportion given* P *to* Q.

DRaw the lines A D and A C, and from the angle B draw B F, parallel to A C, and drawn out till it meet with D C (extended) in F. Likewiſe from the angle E draw E G parallel to A D, till it cut C D (extended) in G. Then drawing A F and A G, the triangle A F G will be [a] equal to the Pentagon A B C D E; by reaſon that the triangle A B C is equal to the triangle A F C, and the triangle A E D is equal to the triangle A G D; then adding the triangle A C D common to both, what we ſaid is clear.

a) 31. 1.



Demonſtr. Divide (firſt of all) the line F G in proportion as P to Q; and let the diviſion fall between F and C in the point H; ſo as that F H may be to H G as P to Q: Then draw H K parallel to B F, till it touch B C in the point K, there is then [b] the ſame proportion of B K to K C, as of F H to H C: Then drawing A K: I ſay that that line doth divide the Pentagon, as was required.

b) 2. 6.

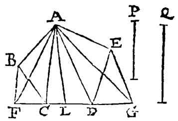
Draw A H, becauſe therefore the triangle A E D is equal to the triangle A G D, adding the triangle A C D common to both, the quadrangle A C D E will be equal to the triangle A C G: Likewiſe becauſe the triangle A K C is equal to the triangle B H C, by reaſon of the Paralleliſme of K H and A C; the Pentagon A K C D E will be equal to the triangle A H G. Alſo becauſe B C is to B K, as F C to F H, there will be the ſame proportion of the triangle A B C to the triangle A B K, as of the triangle A F C to the triangle A F H: Therefore by permutation, the triangle A B C is to the triangle A F C, as the triangle A B K is to the triangle A F H. Since therefore the triangle A B C is equal to the triangle A F C, the triangle A B K will be equal to the triangle A F H: Therefore the triangle A B K will be to the Pentagon A K C D E, as the triangle A F H to the triangle A H G; and therefore as F H to H G; and by conſequence as P to Q. Which was propoſed.

Secondly, Let the diviſion fall in C, ſo as that F C may be to F G, as P to Q; then I ſay, the line A C doth divide the Pentagon, as was required.

Demonſtr. For (as is before ſhewn) the quadrangle A C D E is equal to the triangle A C G; and the triangle A B C is equal to the triangle A F C: Therefore the triangle A B C is in proportion to the quadrangle A C D E, as the triangle A F C to the triangle A C G: Therefore as F C to C G, and by conſequence, as P to Q. Which was propoſed.

Thirdly,

Thirdly, Let the division fall between C and D in the point L; so as that the proportion of F L to L G may be as P to Q. Draw A L: I say the line A L doth divide the Pentagon, as was required.

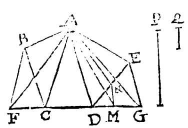


Demonstr. For because the triangle A B C is equal to the triangle A F C, putting the triangle A C L common, the quadrangle A B C L will be equal to the triangle A F L. Likewise putting the triangle A L D to both triangles, to wit, the triangle A E D and A G D, the quadrangle A L D E will be equal to the triangle A L G: Therefore the quadrangle A B C L is to the quadrangle A L D E, as the triangle A F L is to the triangle A L G: Therefore as F L to L G; and consequently as P to Q. Which was proposed.

Fourthly, If the division fall in the point D: Then I say that the line A D divideth the Pentagon according to the Proposition.

The Demonstration is manifest, as it was made appear, when the division fell in the point C, as in the second Case.

Fifthly, Let the division fall between D and G in the point M, so as that F M may be to M G as P to Q. Draw M N parallel to G E, till it touch D E in the point N; then draw A N, which line A N I say doth divide the Pentagon according to the Proposition.

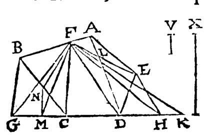


Demonstr. For having drawn A M, it is proved as in the first Case, that the triangle A E N is equal to the triangle A G M, and that the Pentagon A B C D N is equal to the triangle A F M: Therefore the Pentagon A B C D N is to the triangle A N E, as the triangle A F M is to the triangle A M G: Therefore as F M to M G; and by consequence as P to Q. Which was required.

PROP. 18. PROBL. 18.

By a line drawn from a point F, *in the side assigned of a known Pentagon* A B C D E, *to divide the said Pentagon according to a proportion given, as* V *to* X.

DRaw F C; F D, and F E, then drawn B G parallel to F C, and E H parallel to F D, till they meet with C D prolonged on both sides to G and H, then draw A D, cutting F E in the point L: Then extend D H to K, until the proportion of D H to H K, be as D L to L A (this may be done by imagining A K to be produced parallel to L H:) Then draw F G, F H, and F K. Divide then G K according to the proportion of V to X, and let the division fall (in the first place) between G and C, in the point M, so as that the proportion of G M to M K, may be as V to X: Then divide B C in the point N, by a line parallel to B G, and there will be the same proportion of B N to N C, as of G M to M C: Then draw F N. which line F N, I say, doth divide the Pentagon according to the Proposition.

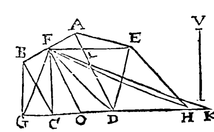


Demonstr. For the triangle F D E is in proportion to the triangle F A E, as D L to L A: Therefore the proportion of D H to H K is as D F H to H F K: Therefore the proportion of the triangle F D E, to the triangle F A E, is as D F H to H F K: Therefore (changing) the proportion of the triangle D F E to the triangle D F H, is as F A E to E H K; but D E H is equal to D F E (by reason of the Parallelisme of F D and F H;) therefore F A E is equal to F H K: Therefore the quadrangle F D E A is equal to the triangle F D K: Adding therefore F D C common to both, the Pentagon F C D E A will be equal to the triangle F C K. On the other side draw F M, and because therefore the triangle F B C is equal to the triangle F G C; and there is the same proportion of B N to N C, as of G M to M C: Therefore the triangle F B N is equal to the triangle F G M, and the triangle F N C is equal to the triangle F M C: Therefore by gathering all together, it is manifest that the Hexagon F N C D E A is equal to the triangle M F K, and the triangle F B N is equal to the triangle F G M: There is therefore the same proportion of the triangle F B N to the Hexagon F N C D E A, as of the triangle F M G to the triangle F M K; and therefore as G M to M K; and by consequence as V to X. Which was proposed.

Secondly, Let the division fall in the point C, so as that the proportion of G C to C K may be as V to X: I say then that the line F C doth divide the Pentagon according to the Proposition.

Demonst. For (as hath been shewn) the Pentagon F C D E A is equal to the triangle F C K, and F B C is equal to F G C; therefore as F B C is to F C D E A, so is F G C to F C K: Therefore as G C is to G K, and consequently, as V to X. Which was proposed.

Thirdly, Let the division fall between C and D in the point O, so as that the proportion of G O to O K may be as V to X: I say then that the line F O divideth the Pentagon, as was required.



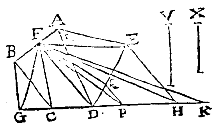
Demonstr. For adding the common triangle F O D to the quadrangle F D E A, and to the triangle F D K (equal to it) the Pentagon F O D E A will be equal to the triangle F O K. Likewise adde the triangle F C G common to the two triangles F B C and and F G C; the quadrangle F B C O will be equal to the triangle F G O: Therefore the same proportion is of the quadrangle F B C O to the Pentagon F O D E A, as of the triangle F G O to the triangle F O K; and therefore as G O to O K; and by consequence as V to X. Which was proposed.

Fourthly, Let the division fall in the point D, so as that the proportion of G D to D K, may be as V to X: I say then that the line F D doth divide the Pentagon according to the Proposition.

Demonstr. For the triangle F C D common to both, being added to the equal triangles F B C and F G C, the Proposition is manifest.

Fifthly, Let the division fall between D and H in the point P, so as that the proportion of G P to P K, may be as V to X. Divide the line D E in the point Q, by the line P Q parallel to E H: There will be therefore the same proportion of D Q to Q E, as of D P to P H: Therefore the line F Q being drawn, I say that line F Q doth divide the Pentagon according to the Proposition.

Demonst. For the whole quadrangle F D E A is equal to the whole triangle F D K. But also the triangle F D Q is equal to the triangle F D P:

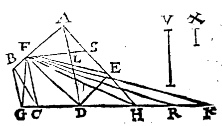


Therefore the quadrangle F Q E A remaining, is equal to the triangle F P K also remaining. Again, the quadrangle F B C D is equal to the triangle F G D. Adding therfore the triangle F D Q to the quadrangle F B C D, and the triangle F D P equal to the triangle F D Q being added to the triangle F G D; it is apparent that the Pentagon F B C D Q is equal to the triangle F G P; there is therefore the same proportion of the Pentagon F B C D Q to the quadrangle F Q E A, as of the triangle F G P to the triangle F P K; and by consequence as V to X. Which was proposed.

Sixthly, Let the division fall in the point H: I say that the line F E doth divide the Pentagon, as was required.

Demonstr. For because the quadrangle F B C D is equal to the triangle F G D, and (as is afore said) the triangle A F E is equal to the triangle F H K, and the triangle F D E is equal to the triangle E D H: Therefore the Pentagon F B C D E is equal to the triangle F G H: Therefore there is the same proportion of the Pentagon F B C D E to the triangle F A E, as of the triangle F G H to the triangle F H K; and therefore as G H to G K; and by consequence as V to X, as was proposed.

Seventhly, Let the division fall between H and K, in the point R, so as that the proportion of G R to R K, may be as V to X: Then divide E A in the point S, so as that the proportion of E S to S A, may be as H R to R K: I say therefore that the line F S doth divide the Pentagon according to the Proposition.



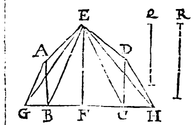
Demonstr. Forasmuch as the triangle A F E is equal to the triangle F H K, and the proportion of E S to S A is as the proportion of H R to R K, the triangle F E S will be equal to the triangle F H R, and the triangle F S A will be equal to the triangle F R K. But the Pentagon F B C D E is equal to the triangle F G H: Therefore the Hexagon F B C D E S is equal to the triangle F G R; there is therefore the same proportion of the Hexagon F B C D E S to the triangle F S A, as of the triangle F G R to the triangle F K R; and therefore as of the line G R to R K; and by consequence as V to X. Which was proposed.

PROP. 19. PROBL. 19.

To divide a Pentagon A B C D E, *having two sides parallel, by a line parallel to those parallel sides, according to a proportion given, as* Q *to* R.

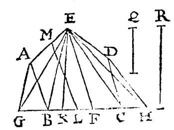
First, Let it be required to performe this by a line parallel to A B, which side let be parallel either to C D or D E: Let it be here parallel to C D: Then draw E F parallel to A B: Then draw E B and and E C: Then draw A G parallel to E B, and D H parallel to E C, till they meet with B C (extended both wayes) in G and H. Then divide G H in proportion as Q to R, and first let the division fall in the point F: Then I say that the line E F doth divide the Pentagon, as was required.

De-



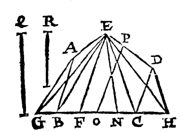
Demonst. For because A G is parallel to E B, having drawn E G, the triangle E A B will be equal to the triangle E G B, and adding the triangle E B F common to both, the triangle E G F will be equal to the quadrangle E A B F; also because D H is parallel to E C, having drawn E H, the triangle E D C will be equal to the triangle E H C, then adding the triangle E F C common to both, the triangle E F H will be equal to the quadrangle E F C D; and before the triangle E G F was proved to be equal to the quadrangle A B F E: Therefore the quadrangle A B F E is to the quadrangle E F C D, as the triangle E G F is to the triangle E F H: Therefore as G F to F H, and consequently, as Q to R. Which was proposed.

Secondly, Let the division fall between G and F in the point K, so as that the proportion of G K to K H, may be as Q to R: Then draw E K. Because therefore the triangle E G K is lesse then the triangle E G F, and the triangle E G F is equal to the quadrangle A B F E; the triangle E G K will be lesse than the quadrangle A B F E: Therefore a) I joyn to A B the superficies A B L M, equal to the triangle E G K, by the line L M, parallel to A B: I say then that the line L M doth divide the Pentagon, as was required. a) 10. of this Book.



Demonstr. For the triangle E G K is equal to the quadrangle A B L M, and the whole triangle E G H is equal to the whole Pentagon A B C D E: Therefore the remaining triangle E K H is equal to the remaining Pentagon M L C D E: Therefore the quadrangle A B L M is in proportion to the Pentagon M L C D E, as the triangle E G K is to the triangle E H K; and by consequence, as Q to R. Which was proposed.

Thirdly, Let the division fall between F and H in the point N; and let E N be drawn, then the triangle E H N will be lesse then the quadrangle E F C D; because it is in proportion lesse then the triangle E H F, which is equal to the said quadrangle: Therefore b) adjoyn to D C the superficies P O C D, equal to the triangle E H N, by the line O P, parallel to C D: I say then that the line O P doth divide the Pentagon, as was required. b) 10.prop. of this.



Demonstr. For because the quadrangle P O C D, is equal to the triangle E N H, and the whole triangle E G H is equal to the whole Pentagon A B C D E, the residual Pentagon A B O P E is equal to the residual triangle E G N: Therefore the proportion of the Pentagon A B O P E is to the quadrangle P O C D, as the triangle E G N is to the triangle E N H; and by consequence as Q to R. Which was proposed.

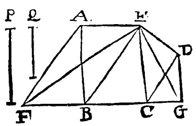
In like manner, also as the Pentagon A B C D E, having two sides parallel, is divided by ratiocination upon B C, opposite to the angle E, intercepted within the parallel sides, so making two sides A B and D E parallel, the Pentagon will be divided by a line parallel to A B, made by ratiocination upon the side E A, opposite to the angle C, intercepted between its two parallel sides A B and D E. And the Proposition is manifest both wayes.

PROP.

PROP. 20. PROBL. 20.

To divide a Pentagon A B C D, *one of whoſe ſides* A B, *is parallel to one of its diameters* C E, *by a line parallel to that ſide, and alſo to that diameter, according to a proportion given,* P *to* Q.

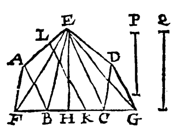
DRaw E B, and make A F parallel to E B, and D G parallel to E C; produce them to cut B C (produced on either ſide) in F and G: Then drawing E F and E G, the triangle E F G is equal to the Pentagon A B C D E, propoſed; as is manifeſt. Then divide F G in proportion as P to Q, and let the diviſion fall either in the point C, or before or after C; and firſt let it fall in the point C: I ſay then that the line E C divideth the Pentagon, as was required.



Demonſtr. For the quadrangle A B C E is equal to the triangle E F C, becauſe the triangle E C D remaining, is equal to the triangle E C G remaining; and the whole pentagon is equal to the whole triangle: Therefore the quadrangle A B C F is in proportion to E C D, as the triangle E F C to the triangle E C G: Therefore as F C to C G, and by conſequence, as P to Q. Which was propoſed.

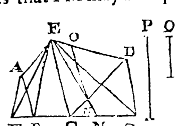
Secondly, Let the diviſion fall between F and C in the point H, ſo as that the proportion of F H to H G, may be as P to Q. Becauſe therefore the quadrangle A B C E is equal to the triangle E F C; and the triangle E F H leſſe then the triangle E F C, the triangle E F H will be leſſe then the quadrangle A B C E: Adjoyn to the line A B [a] the quadrangle A B L K, equal to the triangle E F H, by the line K L, parallel to A B: I ſay the line K L doth divide the Pentagon as is required.

a) 10. of this Book.



Demonſtr. For becauſe the whole pentagon is equal to the whole triangle F E G, and the quadrangle A B K L is equal to the triangle E F H, the remaining pentagon L K C D E is equal to the remaining triangle E H G: Therefore the quadrangle A B K L is in proportion to the pentagon L K C D E, as the triangle E F H is to the triangle E H G; and therefore as F H to H G; and conſequently as P to Q. Which was propoſed.

Thirdly, Let the diviſion fall between C and G, in the point M, ſo as that F M may be in proportion to M G, as P to Q. Becauſe the triangle E D C is equal to the triangle E G C, and the triangle E M C is leſſe then the triangle E G C: Therefore the triangle E M C is leſſe then the triangle E D C. Adjoyne therefore to the line E C the quadrangle E C N O equal to the triangle E M C, by the line N O, parallel to E C [b]. Separate the triangle D O N from the triangle D E C like unto it, and equal to the triangle E G M: I ſay that the line N O doth divide the pentagon according to the Propoſition.



b) 3, 10. of this.

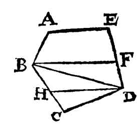
De-

Demonſtr. For becauſe the whole Pentagon A B C D E is equal to the whole triangle F E G, and the triangle O N D equal to the triangle E M G, the Hexagon A B C N O E the reſidue, is equal to the triangle E F M, the reſidue: Therefore the Hexagon A B C N O E is in proportion to the triangle O N D, as the triangle E F M is to the triangle E M G. Therefore as F M to M G, and by conſequence as P to Q. Which was propoſed.

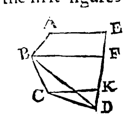
PROP. 21. THEOR. 1.

To any ſide A E, *of a Pentagon* A B C D E, *aſſigned, which is neither parallel to any of his ſides, nor any of his diameters. It is poſſible to draw two right lines within the Pentagon, from (any) two of the three angles not joyned to that ſide, which two lines ſhall be parallel to the ſide before limited.*

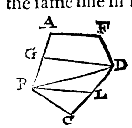
FOr Example, Let it be that in the Pentagon A B C D E, the ſide A E be neither parallel to any ſide thereof, nor to the diameter B D: Then I ſay that from any two of the three angles B, C, or D, two lines may be drawn within the Pentagon, both which ſhall be parallel to the ſide A E; for ſince A E and B D are not parallel, they being extended, would meet either on the part next A B, or elſe on that of E D, if they meet on the part of A B, then the line B F drawn from the point B, parallel to A E, would neceſſarily fall on the ſide E D, as in both the uppermoſt figures: But if they (*viz.* A E and B D) meet on the part of E D: Then the line D G, drawn from D, parallel to A E, muſt needs fall on the ſide A B, as in both the lower-moſt figures of this Propoſition.



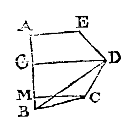
Alſo if A E and B D ſhould meet on the part next A B, as in both the firſt figures of this Propoſition; then the line B F being not parallel to C D, ſhould either meet with it on the part of F D, or on the part of B C: If on the part F D, as in the firſt of the former figures; then from D may be drawn D H, parallel to A E; falling in the ſide B C. But if B F and C D ſhould meet on the part of B C, as in the ſecond figure; then from C may be drawn a line parallel to A E, falling in the ſide E D. We have therefore B F and D H parallel to A E in the firſt figure, and we have B F and C K parallel to the ſame line in the ſecond figure of this Propoſition.



But if A F and B D ſhould meet on the part of B D, as in the two later figures of this Propoſition. Then the line D G (it being not parallel to B C) would meet with it either on the part of G B, or on the part of D C: If on the part of G B, as in the third figure, then may a line be drawn from B, *viz.* B L parallel to A E, and will fall in the ſide C D.



But if G D and B C meet on the part of C D, as in the third figure; then from C may be drawn C M, parallel to the line A E; and will fall in the ſide A B: We have therefore D G and B L in the third figure, and D G and C M in the fourth figure, parallels to the line



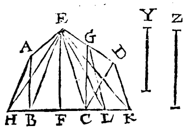
A E,

A E, and falling within the Pentagon: Therefore the whole intended to be declared, is manifeſt.

PROP. 22. PROBL. 21.

To divide a Pentagon A B C D E, *by a line parallel to one of the ſides aſſigned* A B, *which ſide is neither parallel to any of the other ſides* (E D *or* C D) *nor yet to the diameter* E C, *according to a proportion given*, Y *to* Z.

FRom two of the three angles C, D, and E, I draw two lines within the Pentagon, parallel to the ſide A B: Theſe two lines deſcending ſo from the angles, will fall upon the ſame ſide, or on the oppoſite ſides. Let them in the firſt place fall on the oppoſite ſides, and let them be E F and C G, ſo as that F may be in the ſide B C; and let the point G be in the ſide E D. I will reaſon alſo upon the ſide whereupon the neereſt parallel to A B falleth, to wit, upon B C. Draw E B and E C, then draw A H parallel to E B, and D K parallel to E C, till they meet with B C extended on both ſides, in the points H and K; and draw E H and E K; becauſe therefore the triangle E A B is equal to the triangle E H B, and the triangle E D C is equal to triangle E K C, adding the triangle E B C common to both, the Pentagon A B C D E will be equal to the triangle E H K: I draw therefore G L parallel to E C, then protract E L, then divide H K in proportion as Y to Z. The diviſion will fall either in F or in L, or between H and F, or between F and L: Let it fall firſt in F, ſo as that the proportion of F H to H K, may be as Y to Z: I ſay that the line E F doth divide the Pentagon, as is required.



Demonſtr. For the quadrangle E A B F is equal to the triangle E F H, and the quadrangle E D C F is equal to the triangle E K F: Therefore the quadrangle E A B F is in proportion to the quadrangle E D C F, as the triangle E H F is to the triangle E K F; and therefore as H F to F K: and conſequently as Y to Z. Which was propoſed.

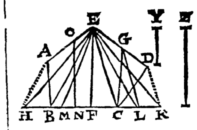
Secondly, If the diviſion fall in L, I ſay then that the line C G doth divide the Pentagon, as is required.

Demonſtr. For becauſe E C and G L are parallels, the triangle E G C is equal to the triangle E L C. But the whole triangle E D C is equal to the whole triangle E K C: Therefore the triangle G C D is equal to the triangle E L K, alſo the quadrangle A B C B is equal to the triangle E H C: Therefore the Pentagon A B C G E is equal to the triangle E H L: Therefore the Pentagon A B C G E hath the ſame proportion to the triangle G C D, as the triangle E H L hath to the triangle E L K: Therefore as H L to L K, and conſequently as Y to Z. Which was propoſed.

Thirdly, Let the diviſion fall between H and F in the point M; and let E M be drawn. Becauſe therefore the triangle E H F is equal to the quadrangle E A B F, and the triangle E H M is leſſe then the triangle E H F: Therefore will the triangle E H M be leſſe then the quadrangle E A B F. Joyn therefore [a] to the line A B a ſuperficies A B N O, equal to the triangle E H M, by the line N O, parallel to A B: I ſay then that the line N O doth divide the Pentagon, as is required.

a) 10. of this.

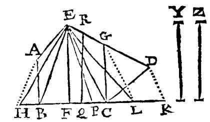
Demonſtr.



Demonſtr. For the Pentagon A B C D E is equal to the triangle E K, and the quadrangle A B N O is equal to the triangle E H M: Therefore the Pentagon O N C D E the remainder, is equal to the triangle E M K remainder: Therefore the quadrangle A B N O is in proportion to the Pentagon O N C D E, as the triangle E H M is to the triangle E M K: Therefore as H M to M K, and by conſequence as Y to Z. Which was propoſed.

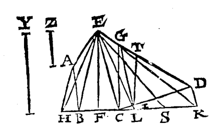
Fourthly, Let the diviſion fall between F and L, in the point P, and let E P be drawn. Becauſe therefore the triangle E F L is equal to the quadrangle E F C G, and the triangle E F P leſſe then the triangle E F L; the triangle E F P will be leſſe then the quadrangle E F C G. Adjoyne therefore [b] to the line E F, the quadrangle E F Q R equal to the triangle E F P, by the line Q R, parallel to E F: I ſay then that the line Q R doth divide the Pentagon, as was propoſed.

b) 10. of this.



Demonſtr. For the triangle E H P is equal to the Pentagon A B Q R E, and the whole Pentagon A B C D E is equal to the whole triangle E H K: Therefore the quadrangle R Q C D remaining, is equal to the triangle E P K: Therefore the Pentagon A B Q R E is in proportion to the quadrangle R Q C D, as the triangle E H P is to the triangle E P K: Therefore as H P to K P, and by conſequence as Y to Z. Which was propoſed.

Fifthly, Let the diviſion fall between L and K, in the point S. Becauſe the paralleliſme of the lines E C and G L, doth conſtitute the triangle E G C, equal to the triangle E L C; alſo the whole triangle E D C to the whole triangle E K C: Therefore the remaining triangle G D C will be equal to the remaining triangle E K L. But having drawn the line E S, the triangle E K S is leſſe then the triangle E K L: Therefore the triangle E K S is alſo leſſe then the triangle G D C: Therefore [c] cut off from the triangle G D C, the triangle T D V like unto it, and equal to the triangle E K S, by the line T V, parallel to G C: I ſay the line T V doth divide the Pentagon as was propoſed.

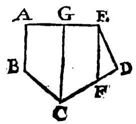


c) 3. of this.

Demonſtr. For the whole Pentagon A B C D E is equal to the whole triangle E H K, and the triangle T D V is equal to the triangle E K S: Therefore the Hexagon A B C V T E remaining, is to the triangle T D V, as the triangle E H S is to the triangle E K S: Therefore as H S to S K, and conſequently as Y to Z. Which was propoſed.

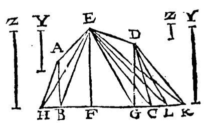
But if the two lines E F and C G, which are parallel to A B, ſhould fall ſo as that the line E F fall on the ſide C D, and the line C G on the ſide A E: Then make the angle C, and you muſt reaſon upon the line A E, as was done before upon B C, and you may atteine your deſire. But if the two lines which were protracted parallel to A B, happen to fall on one and the ſame ſide, then muſt you reaſon upon that ſide: As for Example, Let it be that in the Pentagon A B C D E, the two lines E F and D G protracted parallel to the line A B, fall upon the ſide B C: Then draw A H parallel to E B, and D K parallel to E C: Draw alſo E G,

E G, and a line D L parallel thereto: It is manifest by what hath been premised, that the triangle E H K is equal to the Pentagon A B C D E, and the triangle E H L is equal to the same Pentagon A B C D E, and so is left the triangle D G C equal to the triangle E L K. Divide then H K in proportion as Y to Z; and let the division fall either in F or in L, or betwixt them and the extreames: Therefore in the first place, let the division fall in F, so as that H F may be to F K as Y to Z: I say the line E F doth divide the Pentagon, as was proposed.

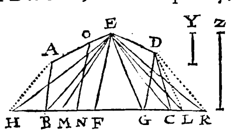


Demonstr. For the quadrangle A B F E is equal to the triangle E H F, and the quadrangle E F C D is equal to the triangle E F K: Therefore the quadrangle A B F E is to the quadrangle E F C D, as the triangle E H F is to the triangle E F K, and by consequence as Y to Z. Which was proposed.

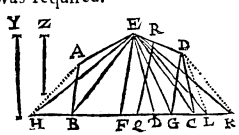
Secondly, Let the division fall in L: I say that the line D G doth divide the Pentagon, as was proposed.



Demonstr. For because the triangle E G D is equal to the triangle E G L; and the quadrangle A B G E is equal to the triangle E H G; the Pentagon A B G D L will be equal to the triangle E H L. But the triangle D G C is equal to the triangle E L K: Therefore the Pentagon A B G D L is to the triangle D G C, as the triangle E H L is to the triangle F L K: Therefore as H L to L K, and consequently, as Y to Z. Which was proposed.

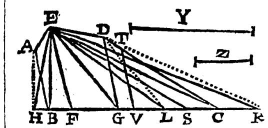


Thirdly, Let the division fall in M, between H and F; having drawn the line
d) 10. of this Book. E M, Let [d] the quadrangle A B N O be made equal to the triangle E H M, by the line N O, parallel to A B: It is manifest therefore as before that the quadrangle A B N O is to the Pentagon O N C D E, as the triangle E H M is to the triangle E M K; and by consequence as Y to Z: Therefore the line O N doth divide the Pentagon, as was required.



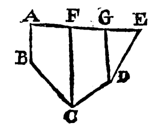
Fourthly, Let the division fall between F and L, in the point P: Then having drawn the line E P, Let there be made
e) 10. of this. [e] the quadrangle E F Q R equal to the triangle E F P: Therefore the Pentagon A B Q R E is equal to the triangle E H P: Therefore the Pentagon A B Q R E is to the quadrangle R Q C D, as the triangle E H P is to the triangle E P K: Therefore as A P to P K, and by consequence as Y to Z. Which was proposed.

Fifthly, Let the division fall in S, between L and K, so as that H S may be to S K as Y to Z. Because therefore (as aforesaid) the triangle D G C is equal to the triangle E L K, the triangle E S K will be lesse
f) 2. of this. then the triangle D G C cut off: Therefore [f] from the triangle D G C another triangle, to wit, T V C like unto it, and equal to the triangle E S K, by the line T V, parallel to D G: I say then that the line T V doth



doth divide the Pentagon, as was required.

Demonstr. For because the triangle T V C is equal to the triangle E S K, and the whole Pentagon A B C D E is equal to the whole triangle E H K: Therefore the Hexagon A B V T D E is equal to the whole triangle E H S: Therefore the Hexagon A B V T D E, is to the triangle T V C, as the triangle E H S is to the triangle E S K; and by consequence as Y to Z. Which was proposed.



But if the two lines which shall be drawn parallel to A B, fall upon the side A E, according to which the lines C F and D G doe fall, constitute the angle C, and reason on the line A E, as we have done on the line B C, and (as before) we shall arrive to our desire.

The End of MACHOMETUS BAGDEDINUS,

OF THE DIVISION OF SUPERFICIES.

A LITTLE BOOK CONCERNING THE DIVISION OF SUPERFICIAL FIGURES.

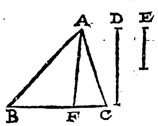
Written in Latine by FREDERICUS COMMANDINUS of URBIN.

PROBLEME I.

To divide a right lined figure according to a proportion given, from a point given in any part of the Ambitus or Circuit thereof, whether the said point be taken in any angle or side of the figure.

A Right lined figure is here taken to be such as is conteined by (an equal number) a like number of sides and angles.

A triangle divided from a point in an angle.

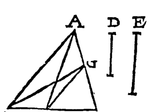


LEt the triangle A B C be divided in proportion as D to E, from a point A in an angle: Divide
a) 10. 6. B C [a] in F, so as that B F may be to B C as D to E; and draw A F, which doth divide the triangle as is required.

Demonstr. For the triangle A B F is to the trian-
b) 1. 6. gle A F C, [b] as B F is to F C, that is, as D to E.

A triangle divided from a point in a side.

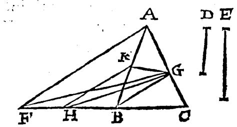
LEt G be a point given in the side A C, from whence the triangle is to be divided as D to E, by a right line. Draw G B, and from A draw a line parallel to G B, as the line A F: Then draw G F, and divide F C in the point H, so as that F H may be to H C, as D to E; the point H will then fall either in B or between F and B, or between B and C. If it fall in B (as here it doth) the line G B performeth the Probleme.



Demonstr. For the triangle G F B is to the triangle G B C, as F B is to B C; that is, as D to E. But the triangle A B G is equal to the triangle G F B (being on the same
c) 37. 1. base) and having the same altitude [c]: Therefore the triangle A B G is to

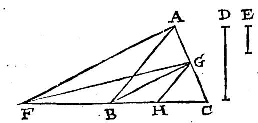
to the triangle G B C, as the triangle G F B is to the same triangle G B C, that is, as D is to E.

But if H fall between F and B: Draw H K parallel to G B, which will cut A B in K: Then draw G H and G K. I say then the line G K doth divide the triangle as was required.



Demonstr. For again, the triangle A B G is equal to the triangle G F B; and adding the triangle G B C common to both, the triangle A B C is equal to the triangle G F C. But the triangle G K B is equal to the triangle G H B: Wherefore the remainder is equal to the remainder, to wit, the triangle A K G to the triangle G F H; and therefore the quadrilateral figure G K B C is equal to the triangle G H C: Therefore the triangle A K G, is to the quadrilateral figure G K B C, as the triangle G F H is to the triangle G H C, that is, as D to E.

But if H fall between B and C, draw G H, which again performeth the Probleme.

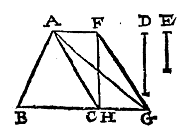


Demonstr. For seeing that the triangle G F B is equal to the triangle A B G. Adding then the triangle G B H common to both, the triangle G F H is equal to the quadrilateral figure A B H G: Therefore the triangle G F H is to the triangle G H C, to wit, D to E, as the quadrilateral figure A B H G, is to the triangle G H C.

If the same point be taken in another, either side or angle the same reasoning is to be used.

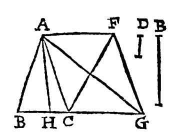
To divide a quadrilateral figure from an angle.

LEt the quadrilateral figure (or quadrangle) A B C F, be divided by a right line from the angle A, in proportion as D to E: Joyne A C, and from F draw a parallel to it, as F G, which will meet with B C produced in G. Joyn A G, then the triangle A C G is equal to the triangle A C F, and adding the triangle A B C common to both, the triangle A B G will be equal to the quadrangle A B C F. Let B C be divided as D to E, now in H, so as that B H may be to H G as D to E. And for that the point H falls in C, the Problem is already performed.



Demonstr. For the triangle A B C is to the triangle A C F, as to the triangle A C G, that is, as D to E.

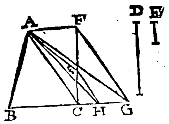
If the point of division H, fall between B and C, the only joyning of A H performeth the Probleme.



Demonst. For the quadrangle A H C F is equal to the triangle A H G: Wherefore the triangle A B H is to the quadrangle A H C F, as the triangle A B H, is to the triangle A H G, to wit, as D to E.

But

But if H fall between C and G. Again to F C, having drawn H K parallel to A C, and joyned A H and A K, the line A K will divide the Quadrangle as was required.

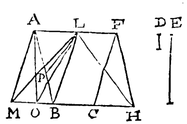


Demonstr. For the triangle A C K is equal to the triangle A C H; therefore the remaining triangle A K F will be equal to the remaining triangle A H G, and the Quadrangle A B C K equal to the triangle A B H: Therefore the Quadrangle A B C K, is to the triangle A K F, as the triangle A B H is to the triangle A H G, that is, as D to E.

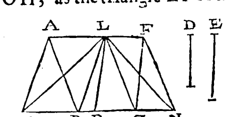
A Quadrangle divided from a point in a side.

Let any point be taken in the side A F, as L, from whence it is required to divide the Quadrangle in proportion as D to E.

Draw the lines L B and L C, and let B C be produced on both sides, and to it from A, let there be drawn the line A M parallel to L B; and from F let F N be drawn parallel to L C; and joyn L M and L H: Then by what hath been shewn, the triangle L M C will be equal to the Quadrangle A B C L; also the triangle L H C will be equal to the triangle L C F, and the whole triangle L M H will be equal to the whole Quadrangle A B C F. Let M H be divided in O, so as that M O may be to O H, as D to E; and joyn L O: The point O then will fall in M C, or in C H, if in M C (by the doctrine before taught) divide the Quadrangle A B C L by a right line drawn from the angle L, which let be L P, so that the parts (*viz.* the triangle L P A) may be to the Quadrangle L P B C, as M O to O C: I say that the line L P doth divide the Quadrangle, as is required.



Demonstr. For the point P will be either in A B or B C: Let it be first in A B: And because the triangle L P A is to the Quadrangle L P B C, as M O is to O C, to wit, the triangle L M O to the triangle L O C; in compounding, the Quadrangle A B C L is to the Quadrangle L P B C, as the triangle L M C is to the triangle L O C, and by permutation. But the triangle L M C, is equal to the Quadrangle A B C L: Therefore also the triangle L O C will be equal to the Quadrangle L B P C, and the triangle L M O will be equal to the triangle L P A; and therefore the remaining triangle L O H will be equal to the Pentagon L P B C F: Therefore the triangle L M O, is to the triangle L O H, to wit, M O to O H, as the triangle L P A is to the Pentagon L P B C F.

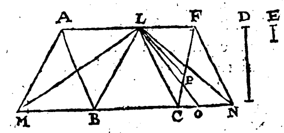


Then let P be in B C as (in the other figure) we will shew in the same manner that M O is to O N, as the Quadrangle A B P L is to the Quadrangle L P C F required.

But if the point of division O, fall in C N: Let the triangle L C F be divided by the line L P, so that the triangle L C P may be to the triangle L P E, as C O to O N, and that which was required is done.

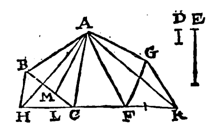
Demonstr. For because the triangle L C P, is to the triangle L P F, as C O

C O to O N; that is, as the triangle L O C to the triangle L O N; in compounding, the triangle L C F is to the triangle L P F, as the triangle L C N is to the triangle L O N; and by permutation, the triangle L C N is equal to the triangle L C F: Therefore also the triangle L O N is equal to the triangle L P F; and the remaining triangle L M O is equal to the pentagon A B C P L. Wherefore as the triangle L M O is to the triangle L O N; that is, as M O to O N, that is, as D to E; so is the pentagon A B C P L to the triangle L P E: Therefore the Quadrangle A B C F, &c. Which was to be done.

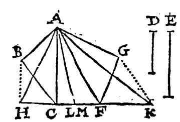


But if the point be given in any other either, angle or side of the Quadrangle, the Proposition is wrought after the same manner.

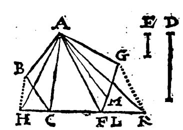
A Pentagon divided from an angle.



Let the pentagon A B C F G be to be divided by a right line drawn from the angle A, in proportion as D to E: Let there be drawn the lines A C and A F, and from B and G, upon C F, extended on both ends; let there be drawn B H parallel to A C, and G K parallel to A F; and having drawn A H and A K, the triangle A H F will be equal to the Quadrangle A B C F, and the triangle A F K will be equal to the triangle A F G, and the whole triangle A H K is equal to the whole pentagon A B C F G: Let H K be cut in L, so as that H L may be to L K, as D to E; the point L then will either be in H F or F K; and if in H F, let the Quadrangle A B C F (by the Rules aforesaid) be divided by A M, issuing from the angle A, so as that the parts thereof may be in proportion as H L to L F: I say the line A M doth divide the Pentagon, as is required.



Demonstr. For (by the reasons aforesaid) the triangle A B M is to the pentagon A M C F G, or as in the other Scheme, the Quadrangle A B C M is to the Quadrangle A M F G, as H L is to L K, required.

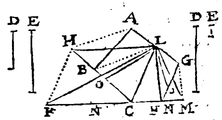


But if L doth fall in F K, in like manner, A M drawn from the angle A, doth divide the triangle A F G in proportion as F L to L K, and sheweth that the pentagon A B C F M is to the triangle A M G, as H L is to L K, that is, as D to E.

To divide a Pentagon from a side.

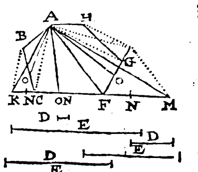
Let the point L be taken in the side A G, from which point a line drawn, doth divide the pentagon in the proportion given, to wit, of D to E. Draw L C and L F, and C B being produced, on the side B, let a triangle (by the Rules before) be made, to wit, the triangle L H C, equal

to the Quadrangle LHCF. Then producing CF on the part C, make the triangle LKF equal to the Quadrangle LHCF, that is, to the pentagon LABCF. And again, it being produced on the part F, make the triangle LFM equal to the triangle LFG; the whole triangle LKM will be equal to the pentagon ABCFG: Therefore let KM be divided in N, ſo as that KN may be to NM, as D to E. And if the point N fall in KF, divide the pentagon LABCF by LO, ſo as that the Quadrangle LABO may be to the Quadrangle OCFL, as KN to NF: The Quadrangle LABO will be to the pentagon OCFGL, as KN to NM: In the ſame manner, it may be demonſtrated that the Hexagon LABCFO, is to the triangle LOG, as KN to NM; that is, as D to E. Which ought to be done.



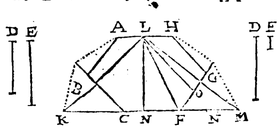
To divide a Hexagon from an angle.

Let the Hexagon be ABCFGH, and the angle A from whence it ought to be divided in proportion as D to E, draw AF, being extended both wayes, make the angle AKF equal to the Quadrangle ABCF, and the triangle AFM equal to the Quadrangle AFGH: Then the whole triangle AKM will be equal to the Hexagon ABCFGH: Then divide KM in the point N, ſo as that KN may be to NM, as D to E. And if the point N fall in KF, divide the Quadrangle ABCF by a line drawn from the angle A, ſo that its parts be in proportion as KN to NF. But if the point N fall in FM, divide the Quadrangle AFGH, as FN to NM, and ſo the Hexagon ABCFGH will be divided as KN to NM; that is, as D to E, required.



To divide a Hexagon from a ſide.

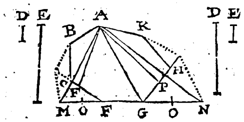
Let there be taken in the ſide AH the point L, from whence a line drawn, may divide the Hexagon given, according to the proportion of D to E. Draw LF, and having produced CF, make the triangle LKF equal to the pentagon LABCF, and the triangle LFM equal to the quadrangle LFGH, ſo the whole triangle LKM will be equal to the whole Hexagon ABCFGH. Then divide KM in N, in proportion as D to E, and if the point N fall in KF. Let the pentagon LABCF be divided from the angle L in proportion as KN to NF. And if it fall in FM, let the Quadrangle LFGH be divided according to the proportion of FN to NM, ſo will the whole Hexagon be divided by a line drawn from L, in proportion as KN to NM, that is, as D to E, required.



To

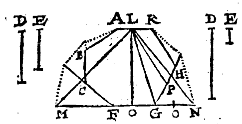
To divide an Heptagon from an angle.

Let the Heptagon ABCFGHK, be divided from the angle A, in proportion as D to E: Let AG be drawn, and make the triangle AMG equal to the pentagon ABCFG, and the triangle AGN equal to the quadrangle AGHK; and the whole triangle AMN equal to the whole Heptagon ABCFGHK: Let MN be cut in the point O, in proportion as D to E. And if O fall in MG: Let the pentagon ABCFG be divided as MO is to OG, with the line AP; and if it fall in GN, divide the quadrangle AGHK as GO to ON, and ſo the Heptagon will be divided in proportion as MO to ON.



To divide an Heptagon from a point in a ſide.

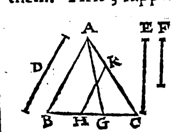
In the laſt place, Let the point L be in the ſide AK, and from L a line be drawn to divide the Heptagon according to the proportion of D to E. Let there be drawn the line LG, and conſtitute the triangle LMG, equal to the Hexagon LABCFG; and let the triangle LGN be made equal to the quadrangle LGHK, and ſo as that the whole triangle LMN will be equal to the Heptagon AECFGHK. Again, Let MN be cut according to the given proportion in O, and if O fall in MG, divide the Hexagon in proportion as MO to MG. But if it fall in GN (as on the left hand) divide the Quadrangle according to the proportion of GO to ON. And the whole Heptagon will be divided as MO is to ON, that is, as D to E given. In like manner one may proceed to other Figures, of how many angles or ſides ſoever. Which was to be done.



PROBLEME II.

To divide a right lined figure GABC, *according to a proportion given* E *to* F, *by a right line parallel to another line given* D.

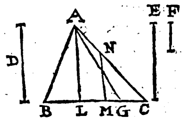
DIvide BC in G, ſo as that BG may be to GC, as E to F; then either the line D is parallel to one ſide of the triangle, or to none of them. Firſt, ſuppoſe it parallel to the ſide AB, and find a mean proportional CH, and by H let HK be drawn parallel to BA: I ſay HK divides the triangle, as is required.



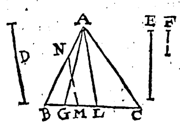
Demonſtr. For having drawn AG, the triangle ABG will be to the triangle AGC, as BG to GC, that is, as E to F; and in compounding, the triangle ABC is to the triangle AGC, as BC to CG. But BC is to CG, as the triangle ABC is to the triangle KHC; a) 19. 6.
[a] for the triangles ABC and KHC are like, and BC to CG hath dupli-

duplicate proportion of BC to EH: Wherefore the triangle KHC is equal to the triangle AGC, and the remaining Quadrangle ABHK is equal to the triangle ABG: Therefore the Quadrangle ABHK is to the triangle KHC, as the triangle ABG is to the triangle AGC; to wit, as E to F.

In like manner, it will be demonstrated, if the given line be parallel to the side BC or CA required.



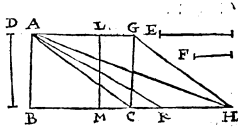
If the given line D be parallel to no side of the triangle, Let AL be drawn parallel to D: Then the point G will fall either betwixt L and C, or betwixt B and L; if betwixt L and C: Let there be taken between LC and CG, a mean proportional; and draw MN parallel to AL: Then the triangle NMC (by what is before demonstrated) will be equal to the triangle AGC; and the Quadrangle ALMN will be equal to the triangle ALG. Wherefore adding the triangle common to both, to wit, ABL, the Quadrangle ABMN will be equal to the triangle ABG: Therefore the Quadrangle ABMN is to the triangle NMC, as E to F.



But if the point G fall betwixt B and L, then also find a mean proportional, as BM, between LB and BG; and let MN be drawn parallel to AL. By the same reason, the triangle NBM will be equal to the triangle ABG, and the Quadrangle ANML is equal to the triangle AGL: Therefore adding the triangle ALC common to both, the Quadrangle ANMC will be equal to the triangle AGC: Therefore the triangle ABC is divided in the proportion given, by a parallel to D, as was required.

A Quadrangle ABCG, *divided by a line parallel to a line given* D, *in proportion as* E *to* F.

Where D is parallel to a side of a Quadrangle, as here to AB: Joyn AC, and from the point G draw GH parallel to AC, which will meet with BC extended in H; then let AH be drawn: Therefore (by what is before said) the triangle ACH is equal to the triangle ACG; and adding to both the common triangle ABC, the triangle ABH will be equal to the Quadrangle ABCG: Let BH be divided in K, so as that BK may be to KH, as E is to F; and let AK be joyned, the side then of the Quadrangle is parallel to BA, or not parallel, and if it be parallel, howsoever the point K doth fall, let b (to the line AB) there be applyed the superficies ABML, equal to the triangle ABK, so as that LM be parallel to the said line AB: I say then that LM performes the Proposition.

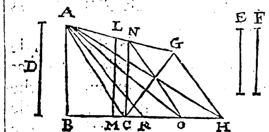


b) 10. of the former book.

Demonstr. For because the triangle ABH is equal to the Quadrangle ABCG, and the triangle ABK is equal to the Quadrangle ABML: The triangle AKH remaining, will be equal to the remaining Quadrangle LMCG: Therefore as the Quadrangle ABML is to the Quadrangle

drangle LMCG; so is the triangle ABK to the triangle AKH. But the triangle ABK is to the triangle AKH, as BK is to KH; that is, as E is to F: Therefore the Quadrangle ABML is to the Quadrangle LMCG, as E to F.

But if CG be not parallel to the side BA: Let there be drawn from one of the points, as CG, within the Quadrangle, a right line, parallel to BA; let that be now CN; and from N, draw NO parallel to AC; and joyn AO: Then the triangle ABO will be equal to the Quadrangle ABCN: Therefore if K fall in O, the line CN performes the Prposition.

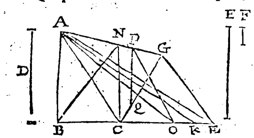


Demonstr. For the Quadrangle ABCN is to the triangle CNG, as the triangle ABO is to the triangle AOH; that is, as BK is to KH, and as E is to F.

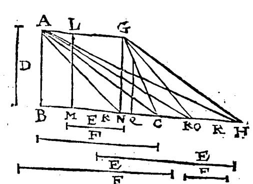
But if K doth fall between BO, apply c to AB a Superficies, equal to the triangle ABK, which let be ABML; so as that ML may be parallel to AB; which may in like manner be demonstrated, to divide the Quadrangle ABCG, as was proposed.

c) 10. of the former book.

Lastly, if K fall between OH, divide the triangle NCG by the line PQ, parallel to NC, in proportion as OK to KH; that is, as the triangle AOK to the triangle AKH; and since the triangle NCG is equal to the triangle AOH, the Superficies NCZP will be equal to the triangle AOK, and the triangle PQG equal to the triangle AKH: Therefore the pentagon ABCQP is equal to the triangle ABK, and the same pentagon ABCQP, is to the triangle PQG, as BK is to KH, that is, E to F.

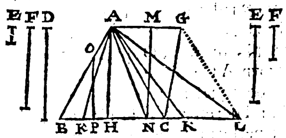


In like manner proceed, if from G there be drawn within the Quadrangle the line GN, parallel to AB, (as in the other figure it doth appear. For having joyned AN and AC, and from G drawn GO, which is parallel to the said AN; and having drawn GH which is parallel to AC. Lastly, Let there be joyned AO and AH: Then the triangle ABO will be equal to the Quadrangle ABNG; and the triangle ABH equal to the Quadrangle ABCG. And if the point K fall in O; then NG performes the Proposition. But if it fall between BO, do as in the former rules. And if it fall between OH, then from the triangle GNC cut off the Superficies GNQP, equal to the triangle AOK; draw PQ parallel to GN, and it is done as was required. But if D be not parallel to any of the sides of the Quadrangle ABCG; draw from one of the points AB, within the Quadrangle, a right line, parallel to D. Let it first be AH, and joyning AC, let GL be drawn parallel (from the point L) to AC, which produced, may meet with BC in L; and joyning AL, the triangle ABL will be equal to the Quadrangle ABCG. Let BL be divided in the point K, so as that BK may be to KL, as E



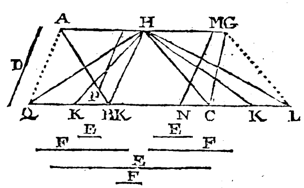
to F. Then the point K will either fall in H, or between H L, or between B H, if it fall in H, the right line A H performeth the Proposition.

But if it fall between H L (by the former Rules divide the *Quadrangle* A H C G in proportion as H K is to K L, by the right line M N, parallel to A H, that is to D, which divideth the *Quadrangle* A B C G, as was proposed.



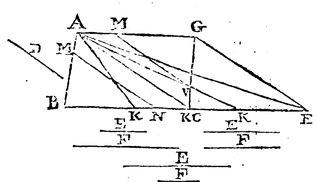
Demonstr. Because the triangle A B H is to the triangle A H K, as B H is to H K; in compounding, the triangle A B K is to the triangle A H K, as B K to K H. But the triangle A H K is in proportion to the triangle A K L, as K H to K L: Therefore from equality, the triangle A B K is to the triangle A K L as H K to K L. But the Quadrangle M N C G is equal to the triangle A K L: Therefore the *Quadrangle* A B N M is to the Quadrangle M N C G, as B K to K L; that is, as E to F.

Lastly, if K fall between B H, having drawn A K, cut from the triangle A B H the Superficies A O P H, equal to the triangle A H K, by O P, parallel to A H; the remaining triangle O B P will be equal to the remaining triangle A B K: Therefore the triangle O B P is to the pentagon A O P C G; as the triangle A B K is to the triangle A K L; that is, as B K to K L, to wit, as E to F.



But if B H be parallel to D, make the triangle H Q B equal to the triangle A B H, and the triangle H B L equal to the Quadrangle H B C G; and dividing Q L in proportion as E to F, in the point K: If K fall in B, the line B H will performe the Probleme. If it fall between B L or *Q* B, then proceed as before newly is shewn.

But if the joyned line A C be parallel to D, make to the triangle A C G an equal triangle, as A C L; and dividing B L in K, according to the proportion given E to F: If K fall in C, the line A C performes the Proposition.



If it fall between C L, cut off from the triangle A C G, the Superficies A C N M, equal to the triangle A C K; drawing also M N parallel to the same A C.

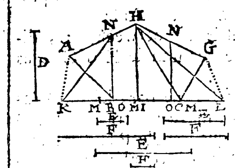
And if it fall between B C, cut from the triangle A B C a Superficies A C N M equal to the triangle A K C, by a right line M N, parallel to A C; and in like manner, it may be demonstrated that the *Quadrangle* A B C G is divided according to the proportion of E to F, as was required.

No otherwise do we proceed if the joyned line B G were parallel to D.

A

A Pentagon divided in proportion as E *to* F, *by a line parallel to* D.

Let there be drawn from some point either in the angle or side, a right line to the base; parallel to D, so as that it cut off a quadrangle on either side, or on the one side a quadrangle, and on the other side a triangle: We make any side of the pentagon lying most opposite to D the base. As in the first figure, Let there be drawn from H a right line H I, parallel to D; and having joyned H B and H C; let there be drawn from A the line A K, parallel to H B; which being produced, may meet with C B in K; and from G, let G L be drawn, parallel to H C, and meeting with B C produced in L; and let H K and H L be joyned: Then the triangle H K I will be equal to the quadrangle A B I H, and the triangle H I L will be equal to the quadrangle H I C G, and the whole triangle H K L will be equal to the whole pentagon.

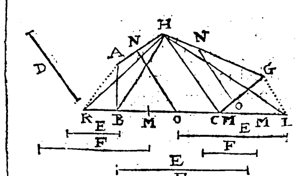


Divide K L in the point M, in proportion as E to F: Therefore M will fall either in I, or between K I: If M fall in I, the right line H I performeth the Proposition.

Demonstr. For the quadrangle A B I H is to the quadrangle H I C G, as the triangle H K I is to the triangle H I L; that is, as K I to I L, to wit; as E to F.

If the point fall between K I, divide (as before is shewn) the quadrangle A B I H in proportion as K M to M I, by a right line N O, parallel to H I. And if it fall between I L, in like manner divide the quadrangle H I C G in proportion as I M to M L, by N O, drawn parallel to H I, and N O will divide the pentagon A B C G H in the proportion given, which will be demonstrated in the same manner as before.

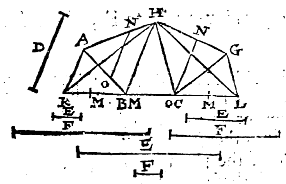
Again, in the following figure, in which H C is parallel to D, by joyning H B, the triangle H K B may be made equal to the triangle H A B, and the triangle H C L equal to the triangle H C G, then the triangle H K C will be equal to the quadrangle A B C H, and the whole triangle H K L equal to the whole pentagon A B C G H: Therefore having divided K L in proportion as E to F, in the point M, if M fall in C, the line H C will performe the Proposition.



If it fall between K C, divide the quadrangle A B C H in proportion as K M is to M C.

But if it fall between C L, divide the triangle H C G in proportion as C M to M L, and so the pentagon will be divided as is required.

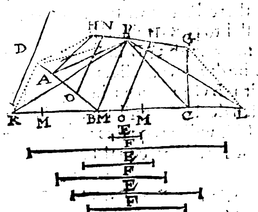
No otherwise shall it be done, if H B be parallel to D, for then will be



be made the triangle HKB, equal to the triangle HAB; and the triangle HBL equal to the quadrangle HBCG: Wherefore if M fall in the point B, the line HB performeth the proposition.

If it fall between KB, Let the triangle HAB be divided in proportion as KM to MB.

But if it fall between BL, divide the quadrangle HBCG as BM to ML, and what was required is done.



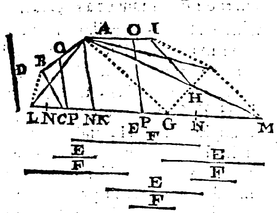
Lastly, if BP be parallel to D (as in this figure) make the triangle PKB equal to the quadrangle PHAB; and the triangle PBL equal to the quadrangle PBCG, and if M fall in B, the same line BP will performe the Proposition.

If it fall between KB, divide the quadrangle PHAB in proportion as KM to MB.

But if it fall between BL, divide the quadrangle PBCG in proportion as BM to BI, and do the like in all other pentagons, and that which was required will be done.

A Hexagon ABCGHI, *divided in proportion as* E *to* F, *by a line drawn parallel to* D.

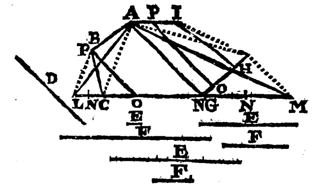
Let there be drawn from any point to the base, a right line parallel to D, so as that it cut off either a quadrangle or pentagon on both sides, or on the one side a triangle or quadrangle, and on the other side a pentagon, (as in the figure.) And let there be drawn from A, a right line AK, parallel to D, and let the triangle ALK be made equal to the quadrangle ABCK, and the triangle AKM equal to the pentagon KGHIA. Then let LM be divided in proportion as E to F, in the point N, which point N will fall either in K, or between LK, or between KM. If it fall in K, the line AK performeth the Proposition.



If it fall between LK, divide the quadrangle ABCK according to the proportion of LN to NK, by a line OP, parallel to AK.

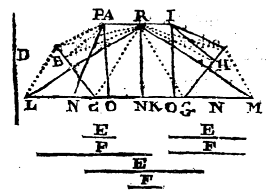
If it fall between KM (by what hath been demonstrated) divide the pentagon AKGHI, in proportion as KN to NM, by the right line OP, parallel to AK.

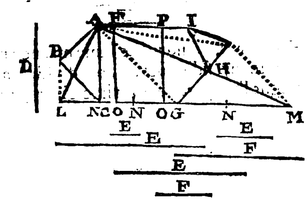
But



But if the joyned line AG be parallel to D, again make the triangle ALG equal to the quadrangle ABCG, and the triangle AGM equal to the quadrangle AGHI, as by other operations also hath been often before shewn.

But if RK be parallel to D, make the triangle RLK equal to the Pentagon RABCK, and the triangle RKM equal to the pentagon KGHIR.

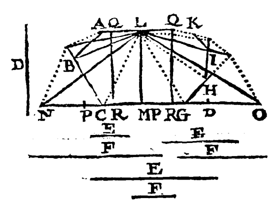




Lastly, if the joyned line AC be parallel to D, make the triangle ALC equal to the triangle ABC, and the triangle ACM equal to the pentagon ACGHI, do the rest as hath been shewn by the former rules, and the Hexagon will be divided as was required.

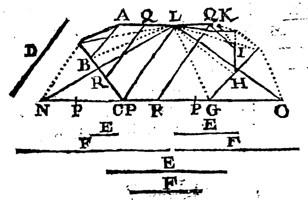
A Heptagon ABCGHLK, *divided in the proportion of* E *to* F, *by a line parallel to a line given, as* D.

Let there be drawn from any point to the base, a right line parallel to D, which shall either cut off a pentagon on both sides, or on the one side a triangle, or quadrangle, or pentagon, but on the other side a Hexagon, or on the one side a quadrangle, and on the other a pentagon, as in the figure, where LM is parallel to D. Make the triangle LNM equal to the pentagon LABCM, and the triangle LNMq ual to the Hexagon LMGHIK, and the triangle LMO equal to the Hexagon LMGHIK, and having cut NO according to the proportion of E to F in the point P. If P fall in M, the right line LM performeth the Probleme.



If it fall between NM, divide the pentagon LABCM according to the proportion of NP to PM, by the line QR, parallel to LM.

If it fall between MO, divide (by what is before taught) the Hexagon LMGHIK according to the proportion of MP to PO, by the right line LM, parallel to D.



But if the joyned line LC be parallel to D, make the triangle LNC equal to the quadrangle LABC, and the triangle LCO equal to the Hexagon LCGHIK, and do the reſt, as is before ſhewn. And the Heptagon will be divided, as was required.

And the like do in all other Heptagons. And in the ſame manner may all other right lined figures, of how many ſides ſoever, be divided according to a proportion given, by a right line parallel to a line given, which was propoſed.

FINIS.

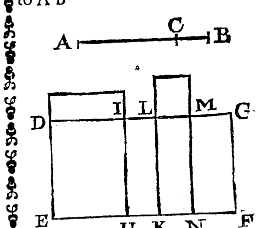
ERRATA.

Pag	marg line	Page line	Read
11		20	*dele* not
19	7		23 d. 1.
20		39	lines
22		33	*dele* alſo
22	8		3 Com. ſent.
28		40	ACF
29		4	will not cut
30	6		15 prop.
30	7		16 prop.
30	9		13 prop
35		16	ECD
35		26	*del* equal
36		26	*dele* equal
36		27	*dele* equal
37		28	GE
37		31	A and D
40	2		13 prop.
40		9	DHG
40		29	BGH
46	6		31. 1.
47	3		34. 1.
58		24	28
59		9	EGH
59		25	CB
60	5		31. 1.
61		37	KG
64		1	RHT
64		35	be a right angle
64		40	EFG
65	4		47. 1.
70	2		47. 1.
71		24	EB
71		30	DE
72			E . . . . 24 F
72			F . . . . G 13
76		13	DB & the ſquare (of CB
84		18	*dele* let
84		21	as
89	3		5. 1
90		35	within at B
90		40	*dele* it
91		25	*dele* but GP & FC are greater than the other ſide GC.
95		40	CE
96		9	FCEG
96		18	FC and GE
98		9	BE
101		4	DAB
102	1		20. 3.
106	3		27. 3.
108	1		18. 3.
108	2		22. 3.

Page	marg line	Page line	Read.
110	3		5. 2.
113		10	AG
118	3		16. 3.
120		1	prop. of the third
120		32	*de.* the equal lines
124	4		26. 3.
124	5		29. 3.
124	6		27. 3.
124		40	$\frac{3}{5}$ of 2 right angls
126		17	CBF
126	4		Cor. 16. 3.
126		40	AF and BF
127		32	of 2 right angles
128		27	equall to BH
134		41	divers
135		41	greater inequa- (lity
138		46	$\frac{5}{22}$
139		25	one of the which
143		7	of the other
143		20	as of D to E
143		24	*dele* and
151		17	and EH be added
153	1		3. 5.
159	2		6. 5.
164	7		15. 5.
165	6		10. 5.
166	4		11. 5.
197	3		19. 5.
177		14	*dele* of
178		24	DEF
178		25	EH
179	3		38. 1.
180	7	33	39. 1.
181		36	as BD
185			*de.* ADB are right (angles
188	2	27	31. 3.
196		24	BC to CG
198			GBC
200	2		43. 1.
202	2		25. 6.
204	7		19. or 20. 6.
206	2		28. 3.
206	3		27. 3.
182		*ult*	in
186		10	there
191	9		12. c. ſ.
193		26	to A and to D
202		5	by 2 and 2
205	6		26. 7.
206		30	any Prime num.
207		6	to E the fourth

Page	marg line	Page line	Read
208		18	A B and C
212	2		39. 7.
216		29	I of B in E
219		36	of D in E
220		11	E and H
220		32	*add* [to the ſide D
226		31	A and C
231		16	D, E and G
234		27	proportional
236		27	as C meaſures G
241		5	A, B and D
242	3		12. c. ſ.
248		8	A, BC, D, & EF
248		17	D is to BC
259		3	*dele* the
259		10	and therefore
261		8	C to a number D
271		34	incomenſurable
276		43	*de* Being equal to the medial AB. AC & H Irrational.
282		9	ſquare rational
287	10		Cor. 24. 10.
289		3	the rectangle
295		17	to the Rational
297		33	no ſquare greater
303	3		22. 10.
310		10	DL and LG
313		33	as the rectangle
324		28	*dele* compound to the ſaid
334		*pe.*	which with
336	8		10. 10.
337	3		23, 10.

344 line 2 & 3, *dele* [and to GH the rectangle HE equal to AB



This Scheme belongs to the 61, 62, and 63, Propoſitions of the Tenth Book.

Errors in the Diagrams.

First Book, Prop. 16. BFC for BEC. Third Book prop. 36 E omitted in the ſecond ſcheme. Prop. 27. C ſtands in place of I. Fourth Book Def. 1. a line omitted from K to L. Prop. 5, a line omitted from A to F. Prop 12, F is in place of L, and F muſt be in center, Fifth Book, Prop. 3, line C is not to be divided. Sixth Book, prop. 1, the line AL omitted. prop, 16, E is put for F in the rectangle. Seventh Book, prop. 14, for F...2, F...3. Eighth Book, prop, 5, for 19, 16. prop 6, for G 9, G 6. prop. 12, for E 6, E 9. prop. 15, for I 22, I 72. prop. 18: for A 13, A 12. Ninth Book, prop. 23 E omited. Tenth Book, prop. 45, V in the firſt, and M in the ſecond ſcheme omited. The Scheme in prop. 49 relates to prop. 50. prop. 51. place 6 between the third and fourth points.

THE GROUND-PLAT
Of the MATHEMATICAL PREFACE of Mr. JOHN DEE.

Sciences and Arts Mathematical are either

- Principal, which are two only
  - Arithmetick
    - Simple, Which dealeth with Numbers onely, and demonstrateth all their properties, and appurtenances, where an Unite is Indivisible.
    - Mixt, Which what ayde of Geometry principal, demonstrateth some Arithmetical Conclusion or purpose.
  - Geometry
    - Simple, Which dealeth with Magnitudes only, and demonstrateth all their properties, passions and appurtenances. Whose point is Indivisible.
    - Mixt, Which what aide of Arithmetick principal, demonstrateth some Geometrical purpose as EUCLIDES ELEMENTS.
  - The use whereof is either
    - In things supernatural, eternal, and divine, by application ascending.
    - In things Mathematical without further application.
    - In things natural both substantial and accidental, visible and invisible, &c. by application descending.
  - The like uses and applications are (though in a degree lower) in the arts Mathematical derivative.
- Derivative from the Principals of which some have
  - The names of the Principals, as
    - Arithmetick Vulgar, which considereth
      - Arithmetick of most usual Numbers, and of Fractions to them apperteining.
      - Arithmetick of Propotions.
      - Arithmetick Circular.
      - Arithmetick of Radical numbers, Simple, Compound, Mixt, and of their Fractions.
      - Arithmetick of Cossick Numbers, with their Fractions. And the great Art of Algiebar.
    - Geometry Vulgar, which teacheth measuring
      - By distance from the thing measured, as
        - How far, from the Measurer any thing is: of him seen, on Land or Water, called Apomecometry.
        - How high or deep from the Level of the Measurers standing, any thing seen of him on Land or Water, called Hypsometry.
        - How broad a thing is, which is in the Measurers view, so it be situated on Land or Water, called Platometry.
        - Of which are grown the feats and Arts of
          - Geodesie, more cunningly to Measure and Survey Lands, Woods, Waters, &c.
          - Geography.
          - Chorography.
          - Hydrography.
          - Stratarithmetry.
      - At Hand
        - All Lengths — Mecometry.
        - All Plaines, as Land, Board, Glasse, &c. — Embadometry.
        - All Solids, as Timber, Stone, Vessels, &c. — Stereometry.
  - Proper names as
    - Perspective, — Which demonstrateth the manners and properties of all Radiations, Direct, Broken, and Reflected.
    - Astronomy, — Which demonstrateth, the Distances, Magnitudes, and all Natural motions apparances and passions, proper to the Planets, and fixed Stars, for any time past, present and to come: in respect of a certain Horizon, or without respect thereof.
    - Musick, — Which demonstrateth by reason, and teacheth by sense, perfectly to judge and order the diversities of Sound high or low.
    - Cosmography, Which wholy and perfectly maketh description of the Heavenly and also Elemental part of the World: and of these parts maketh homologal application, and mutual collation necessary.
    - Astrology, — Which reasonably demonstrateth the operations and effects of the natural beams of light, and secret influence of the Planets, and fixed Stars in every Element and Elemental Body, at all times in any Horizon assigned.
    - Statike, — Which demonstrateth the causes of heavinesse and lightnesse of all things, and of the motions and properties to heavinesse and lightnesse belonging.
    - Anthropographe, Which describeth the Number, Measure, Weight, figure, Situation, and colour of every divers thing, contained in the perfect body of man, and giveth certain knowledge of the figure, Symmetry, weight, characterization, and due local motion of any parcel of the said body assigned, and of numbers to the said parcel apperteining.
    - Trochelike, — Which demonstrateth the properties of all circular motions: Simple and Compound.
    - Helicosophy, — Which demonstrateth the designing of all spiral lines, in Plain, on Cylinder, Cone, Sphere, Conoid, and Spheroid, and their properties.
    - Pneumatithmie, Which demonstrateth by close hollow Geometrical figures (Regular and Irregular) the strange properties (in motion or stay) of the Water, Air, Smoak and Fire, in their Continuity, and as they are joyned to the Elements next them.
    - Menandry, — Which demonstrateth how above Natures vertue and power simple, vertue and force may be multiplyed, and so to direct, to lift, to pull too, and to put or cast from, any multiplyed or simple determined vertue, Weight or force, naturally not so directable, or moveable.
    - Hypogeiody, Which demonstrateth how under the Spherical superficies of the Earth, at any depth to any perpendicular line assigned (whose distance from the perpendicular of the entrance, and the Azimuth likewise, in respect of the said entrance, is known certain way, may be prescribed and gone, &c.
    - Hydragogy, — Which demonstrateth the possible leading of water by Natures law, and by artificial help from any head (being spring, standing, or running water) to any other place assigned.
    - Horometry, — Which demonstrateth, how at all times appointed, the precise usual denomination of time may be known, for any place assigned.
    - Zography, — Which demonstrateth and teacheth, how the intersection of all visual Pyramids made by any plain assigned (the Center, distance and lights being determined) may be by lines and proper colours represented.
    - Architecture, Which is a Science garnished with many doctrines, and divers instructions: by whose judgment, all works by others finished, are judged.
    - Navigation, — Which demonstrateth, how by the shortest good way, by the aptest direction and in the shortest time: a sufficient ship, between any two places (in passage navigable) assigned, may be conducted, and in all storms and natural disturbances changing, how to use the best possible means, to recover the place first assigned.
    - Thaumaturgike, Which giveth certain order to make strange works of the sence to be perceived, and of men greatly to be wondred at.
    - Archimastry, Which teacheth to bring to actual experience sensible, all worthy Conclusions, by all the Arts Mathematical purposed: and by true natural Philosophy concluded; and both addeth to them a farther scope in the terms of the same arts: and also by his proper Method, and in peculiar terms proceedeth, with help of the foresaid arts, to the performance of compleat Experiences, which of no particular Art, are able (formally) to be challenged.

Place this after DEE's Mathematical Preface.